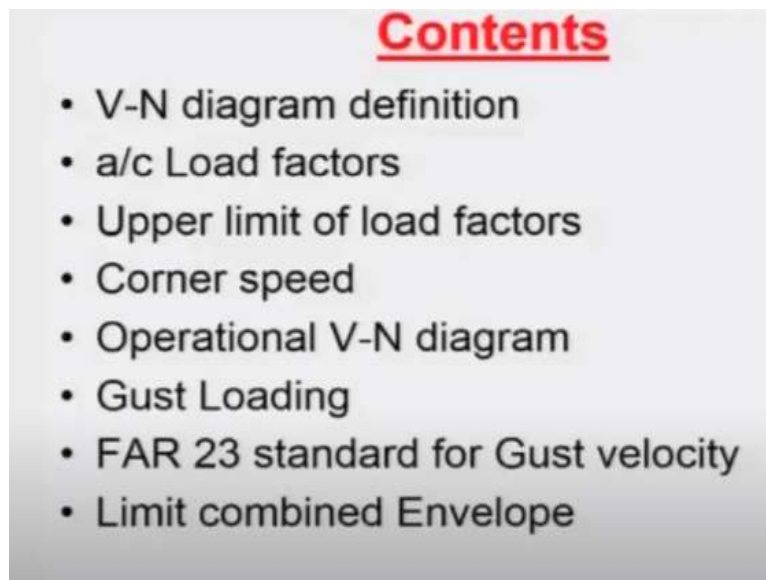


Introduction to Aircraft Design
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Lecture - 79
Limit Maneuver Envelope

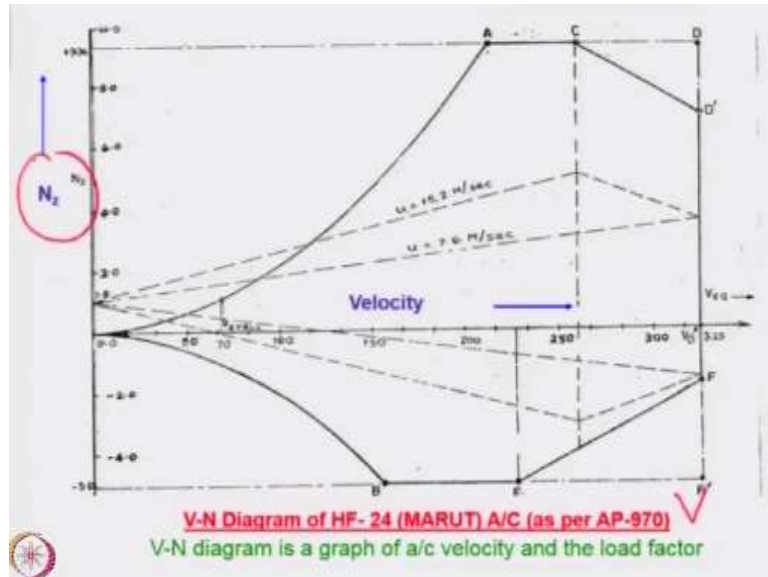
Hello, let us have a look at one of the important diagrams that is drawn by a designer called as the V-N diagram.

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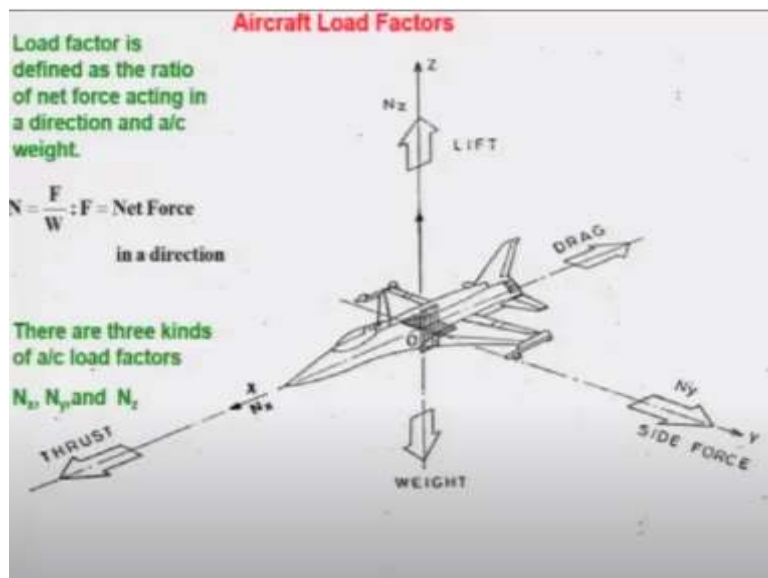
The contents of this talk are as follows. We will first look at the definition of a V-N diagram. We have a look at the load factors acting on an aircraft. We identify what are the upper limits on these load factors. We look at the concept of corner speed and then the operational V-N diagram. Then we look at the effect of gust loading. You see how the regulatory bodies like FAR 23 define a V-N gust loading for V-N diagram. And finally, what is the limit combined envelope.

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Here is a V-N diagram as per the old specifications called as the AP-970. This is for the MARUT aircraft designed in India by Hindustan Aeronautics Limited. It was a transonic bomber aircraft. So the V-N diagram is essentially a graph of aircraft velocity V along the x axis and load factor N_z along the y axis.

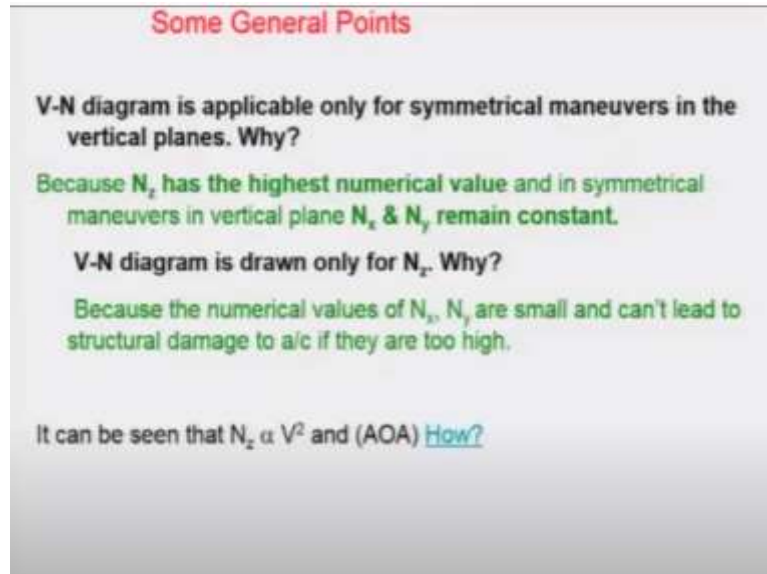
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Let us see what is the meaning of load factor. A typical aircraft has three axes in which we define all the forces and moments. You have the vertical or the z direction in which the lift normally acts. We have the longitudinal or the x direction in which the thrust normally acts. And we have the lateral or the y direction in which the side force N_y normally acts. And the weight of the aircraft acts vertically below along the z axis.

So load factor is nothing but the ratio of the net force acting in a particular direction and the aircraft weight. Force upon W where F can be the force in the vertical, longitudinal or lateral direction and therefore, we have three load factors N_x , N_y and N_z typically defined.

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Now the V-N diagram is applicable only for the symmetrical maneuvers in the vertical planes. What is the reason? The reason is that the highest value, the highest numerical value of the load factor amongst N_x , N_y and N_z normally happens to be for N_z . And also when the aircraft does symmetrical maneuvers in the vertical plane, then the values of N_x and N_y remain constant.

Hence, since it is the largest load acting and also because in symmetrical maneuvers, the other two values remain constant we draw the V-N diagram for the vertical plane. The numerical values of N_x and N_y are small, and they cannot lead to structural damage to the aircraft as compared to N_z . And therefore, we draw the diagram mostly for only N_z versus the velocity.

Now it can be shown that the vertical load factor N_z will be proportional to the square of the velocity and the angle of attack. Let us see how this can be shown.

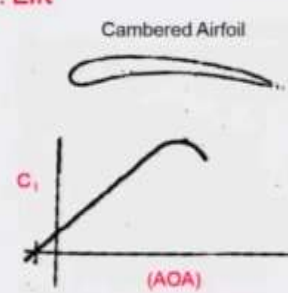
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$N = \frac{L}{W}$

$L = \frac{1}{2} \rho v^2 S C_L$: Lift
 $L = \frac{1}{2} \rho v^2 S (AOA) a_0$

where
 $\rho =$ density of air $C_L =$ Lift Coefficient
 $v =$ a/c speed $S =$ wing area
 $a_0 =$ Lift curve slope

Thus $N_z \propto \rho V^2$
and $N_z \propto (AOA)$



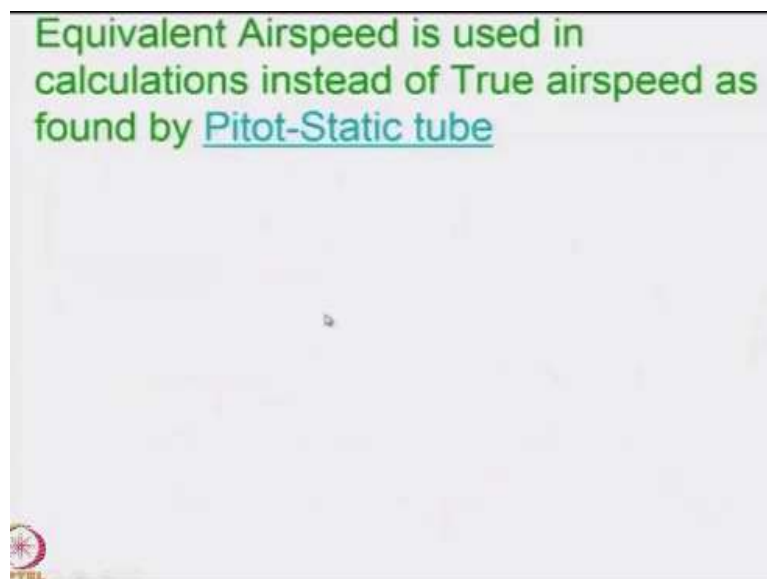
But this would imply that we need to draw a different V-N diagram for every possible altitude.
So how do we eliminate this problem?

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So the ratio N_z would be nothing but ratio of lift over weight. Now the lift is proportional to $\frac{1}{2} \rho V^2 S C_L$. And if we replace C_L by the angle of attack and the lift curve slope, you get S into AOA into a_0 . So now if you take a ratio, so you will find that it will be proportional to V^2 and also proportional to angle of attack α because for a given altitude ρ infinity S and a_0 remain constant.

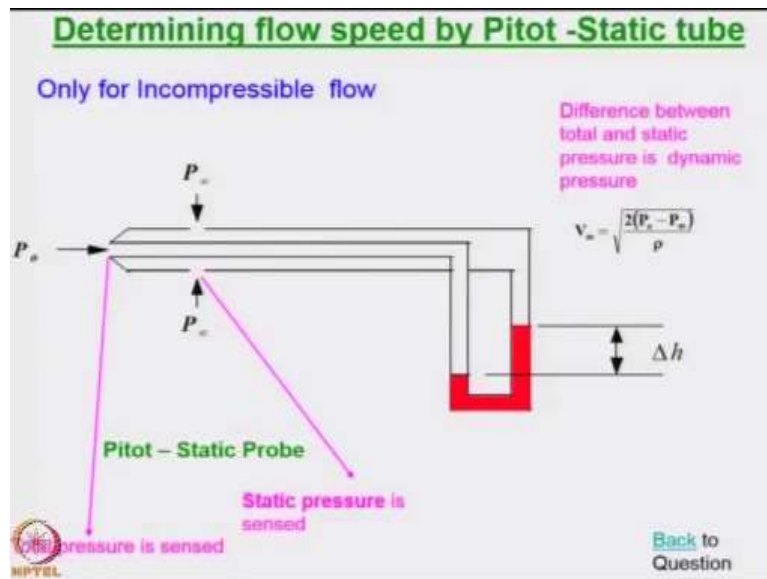
Now what it means is if the ρ is changing, that means if the aircraft is operating at many altitudes, then we need to draw a different V-N diagram for every possible altitude because N_z is proportional to ρ and V^2 . So now, how do we eliminate this problem and how do we manage to draw the V-N diagram only for one particular value?
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Equivalent Airspeed is used in calculations instead of True airspeed as found by Pitot-Static tube



We do this by using equivalent airspeed in the calculations as against the true airspeed which the pitot-static tube normally determines.

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So this figure shows the working of the pitot-static tube. It shows that the tube has got a hole in the front which measures the total pressure and there are holes on the side which give you the static pressure. So when you calculate the difference between the total and the static pressure that is proportional to the square of the velocity. Hence

$$V_{\infty} = \sqrt{\frac{2(P_0 - P_{\infty})}{\rho}}$$

So this is how we can measure the velocity in a pitot-static tube.

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Equivalent Airspeed is used in calculations instead of True airspeed as found by Pitot-Static tube

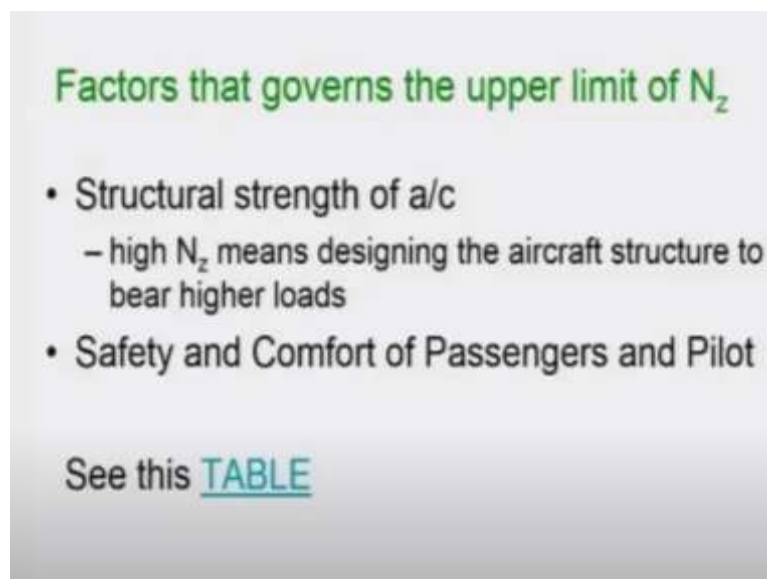
- The speed on Airspeed Indicator **Indicated Airspeed, IAS** is proportional to dynamic pressure
- Taking into account the errors in calibrated instruments we get the **Calibrated Airspeed, CAS**
- And after taking into considerations the compressibility effects we get **Equivalent Airspeed EAS** (the speed at which the a/c would be flying at sea level under same conditions of pressure and temperature)
- By using this Equivalent Airspeed the variable 'ρ' can be eliminated
- So $N_z \propto AOA$
 $\propto V_{eq}^2$ **ONLY**

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But the question is that the speed on the airspeed indicator is the indicated airspeed. Now that is proportional to the dynamic pressure which the air is facing. Is not the true airspeed, but if you take into account the errors in the calibrated instruments, then we get what is called as the calibrated airspeed and also the position. Now if you take care of the compressibility effects, then we get what is called as the equivalent airspeed.

That is the speed at which the aircraft would be flying at sea level under the same conditions of pressure and temperature which the aircraft is facing during flight. So by using equivalent airspeed, the variable ρ can be eliminated. So therefore N_z is proportional to angle of attack and N_z proportional to only V equivalent square. So this is the way in which we can eliminate this problem.

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Now let us have a look at the factors that govern the upper limit of load factor N_z . The upper limit is decided based on the structural strength of the aircraft. High N_z means you have to design the aircraft to bear higher loads. But we have to also keep in mind that an aircraft will typically be carrying passengers and crew members.

So their safety and comfort also must be kept in mind because you might be able to take care of the structural loading that the aircraft is expected to bear but human beings have a limit.

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Typical Limit Load Factors		
Aircraft Type	N (Positive)	N (Negative)
General Aviation-normal	2.5 to 3.8	-1 to -1.5
General Aviation-utility	4.4	-1.8
General Aviation-aerobatics	6	-3
Homebuilt	5	-2
Transport	3 to 4	-1 to -2
Strategic Bomber	3	-1
Tactical bomber	4	-2
Fighter	6.5 to 9	-3 to -6

Observe:- N (negative) is almost half of N (Positive) !!

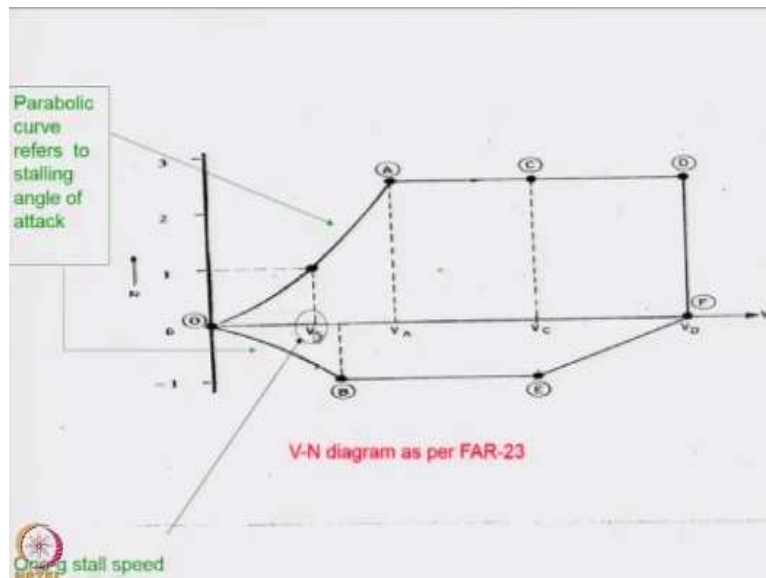
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So there is a table which lists the typical limits on the load factors depending on the aircraft type. So if you have a general aviation aircraft, under categories of normal, utility and aerobatic, you have an increasing value of the positive load factor. In the case of a normal aircraft, the load factor varies from 2.5 to 3.8 using a standard formula. If it is a homebuilt aircraft, the maximum value is normally kept at 5.

For transport aircraft where we have passengers is generally kept between 3 and 4. For bombers because bombers are not expected to do very heavy maneuvers it is 3. But a tactical bomber may need to do some activity and some maneuvers therefore it is 4. But for a fighter aircraft depending on the level of the fighter, and depending on the kind of maneuvers which it is expected to do, the maximum load factor varies from 6.5 to even up to 9.

Beyond 9 it is not specified because it would lead to a severe problem with the pilot on board. Notice the upper limits on the negative load factors are also provided. And these limits generally are approximately half of the limit on the positive load factors.

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So here is a V-N diagram as per FAR part 23 regulations. We notice that there is a curve O to A, O being the origin with zero velocity and zero load factor. From O to A we have a parabolic curve and this parabolic curve corresponds to the stalling angle of aircraft because at this along this curve the aircraft has got the highest lift coefficient or the lowest speed. V_S corresponds to 1g stall speed or level flight stall speed.

So the load factor is 1 and the speed is equal to V_S the stalling speed. The point V_A corresponds to a speed called as the corner speed. Notice that point A corresponds to the highest vertical load factor and also the highest possible speed V_A .

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Corner Speed

- Point A in the graph is important because it corresponds to highest N_z permissible, and also the max. lift coefficient of a/c.

Implications:-

1. It leads to smallest turn radius (tightest turn)
2. And Fastest turn rate

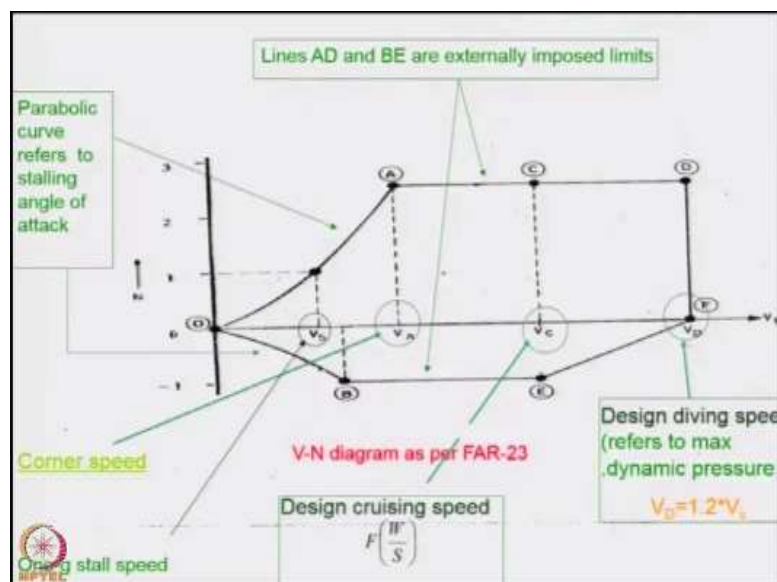
The speed corresponding to this a/c is called the **Design Manoeuvre speed** or **Corner speed**

So the corner speed is a very interesting value. Point A on the graph is important because it corresponds to the highest permissible load factor as well as the high

maximum lift coefficient. So what are the implications of this? The implications is that since you have the highest load factor, you will have and also the maximum lift coefficient you will have the smallest turn radius and the fastest turn rate.

So this particular speed becomes very important when you are comparing two competitive aircraft. The one which has a lower value of corner velocity is better because it is able to maneuver or pull the same amount of g at a lower speed and hence it is more maneuverable. So the speed corresponding to this aircraft is called as the design maneuver speed or the corner speed.

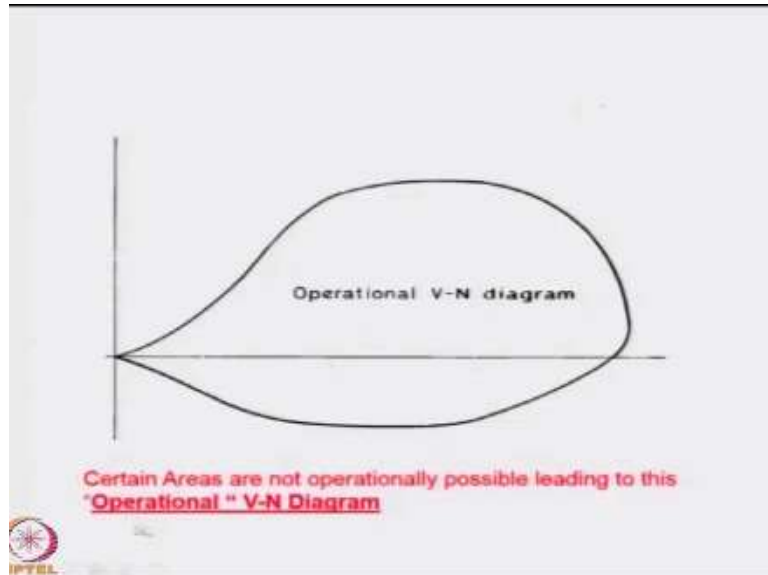
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That is this value V_A . And then you have lines AD and BE which are externally imposed limits on the vertical positive and the vertical negative value of the load factor as prescribed in the table which we just now saw. Now this V_C corresponds to the cruising speed, the design cruising speed, which is a function of the wing loading of the aircraft W/S . And there is also another speed called as the design diving speed.

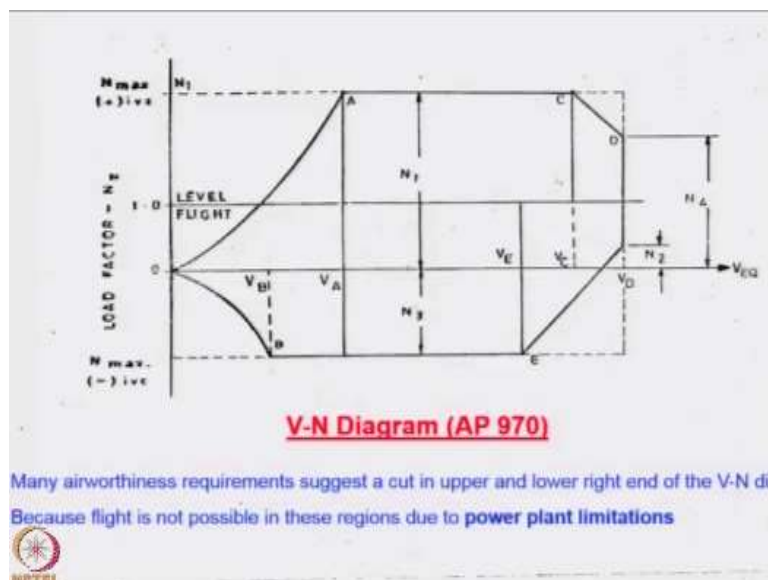
This refers to the maximum dynamic pressure that the aircraft can withstand before it can expect it to undergo structural failure. And that particular speed generally is 20% or so higher than the maximum or design cruise speed, okay.

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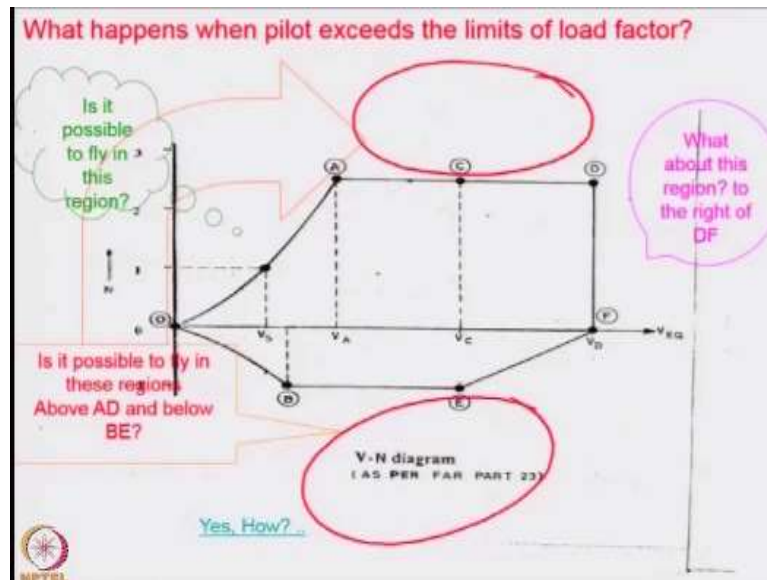
So now certain areas of the V-N diagram are not operationally possible leading to the operational V-N diagram.

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This is the V-N diagram as per aviation publication 970. So notice that in some airworthiness regulations, you are provided cuts or concessional zones in the right top and the right bottom area of the V-N diagram, because flight in these areas are not normally permitted because of the power plant limitations as we will see very shortly.

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Now let us see what happens when the pilot knowingly or unknowingly exceeds the limits of the load factor. Now first question is, is it possible for you to fly in this region on the left of the line OA? The answer is no you cannot because the moment you go into that region, since you are going to be at a speed lower than V_{stall} , then you will actually not be able to fly level. So you will have to come back.

So sustained flight is not possible in this area. So the aircraft cannot be flown in the area on the left of curve OA and curve OB okay. What about to the right of the region of the line DF, what about in this region? No, in this region, we cannot go because you have already exceeded the speed which the power plant can provide you. You will not be able to go in that area generally because of the power plant limitations, okay.

So can you fly in the regions above AD? That is can you fly in this region? Can you fly in this region or in this region? Yes, flight is possible, flight is possible in this region. Let us see how it is possible.

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- Pilot can make the a/c fly in this region if enough engine control power is available
 - But it could lead to structural damage as well as health problems to pilots and passengers
 - But during the **Dive-Pull out Manoeuvre** it is possible that pilot may exceed the N_{\max} prescribed at the lowest point of the dive
- That's why this manoeuvre is called "**checked manoeuvre**"

So in this region, if you have enough engine power and enough control power, you might be able to fly. But the problem is that you will exceed the permitted load factor. So there is a chance that you will either deform the structure or you may even lead to catastrophic failure plus the passengers will have a serious problem, okay.

So during a dive-pull out maneuver when the aircraft is put into a dive and then brought up symmetrically, it is quite possible for the aircraft to exceed this particular value of load factor. And there have been instances when there have been failures of the wing during a dive-pull out. So therefore, this is a dangerous maneuver and it is called as a checked maneuver.

Pilot has to be cautious that the limits specified on the structure are not exceeded during this particular maneuver. Thanks for your attention. We will now move to the next section.