Introduction to Aircraft Design
Prof. Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology, Bombay

Lecture - 67
Refined Sizing

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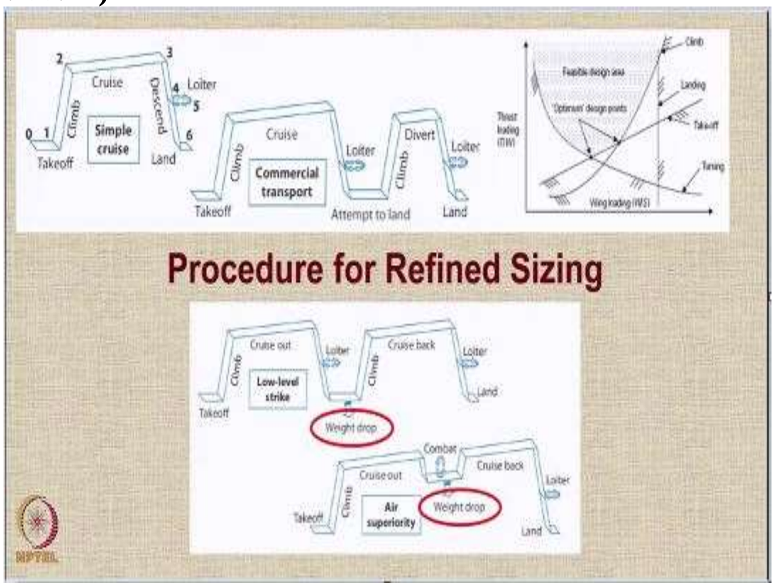

Hello, let us have a look at the procedure for refined sizing. All of you by now know how to carry out initial sizing for an aircraft whose mission profile is given and whose basic aircraft data is known to you. We also know how to do constrained analysis for an aircraft in which we determine the values of W/S and T/W. But there are certain missions for which the method that we have used is not applicable. And for those missions, we need to use the procedure for refined sizing. That is what we will look at today.

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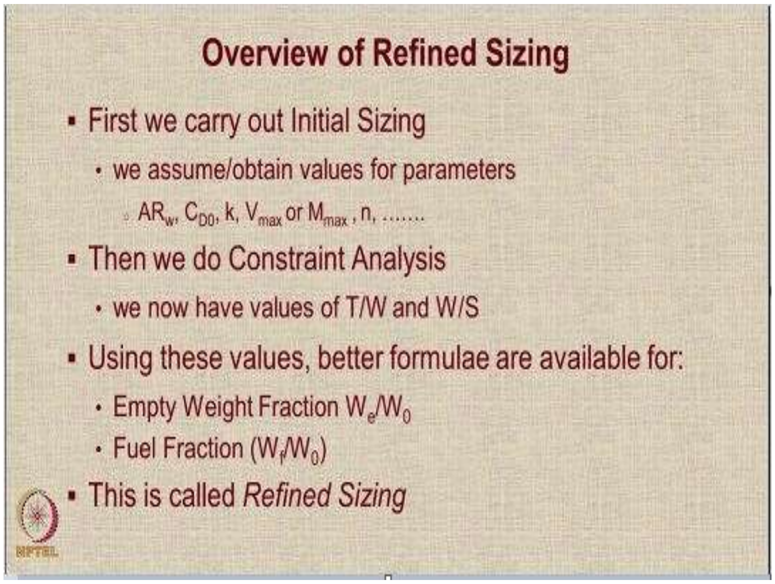

Let us first have an overview of refined sizing. So, first we carry out initial sizing we assume or obtain the values of certain parameters such as the wing aspect ratio, the aircraft zero lift drag coefficient C_{D_0} the induced drag coefficient k, V_{max} or M_{max} load factor n etc. Once we do initial sizing, we have an estimate of W_0 the design gross weight. And since we know the values of various constraints imposed.

We can do constraint analysis and through that we can get the limiting value of thrust to weight ratio or power to weight ratio if it is a turboprop or piston prop powered aircraft. And W/S the wing loading. Now, after this, what we should do is improve our estimates because now we can apply better formula for the estimation of parameters such as the empty weight fraction, the fuel fraction this particular procedure where we used refined formulae for sizing of the aircraft because now we know the value of the T/W and W/S is called as a refined sizing. So, let us see how it is done.

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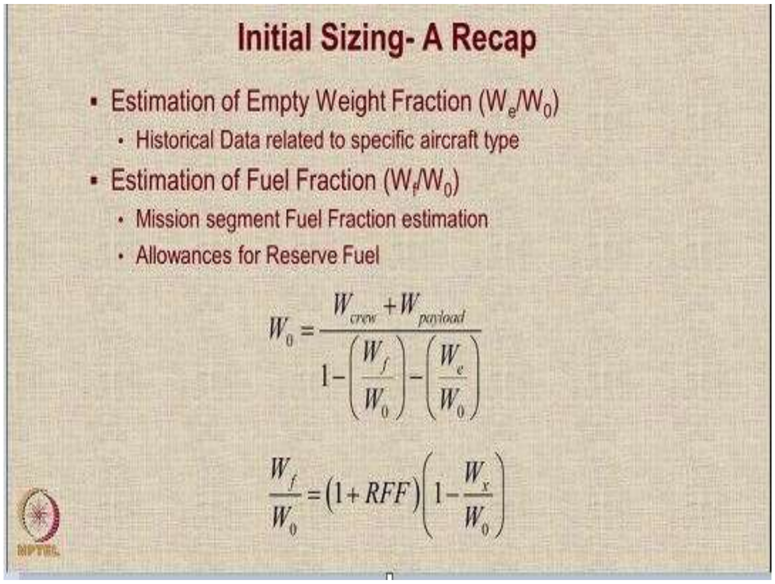

So, let us do a quick recap about initial sizing. If you have not watched the video lectures about initial sizing, I would recommend that you should go and do that before you proceed further. This is only a recap. So initial sizing, the first thing that we do is we estimate the empty weight fraction. For this if you remember, we use historical data related to specific aircraft type, we use the equation $A * W_0^c$.

Where A and c are constants whose values come from those lines, which were drawn based on past experience, there was a separate line for each type and these lines are genuinely parallel to each other roughly parallel to each other. After that, looking at the various segments of the mission, we estimate the fuel fraction for each of those segments. And then since we assume that the only reason why there is a loss of weight of the aircraft is because of fuel consumption.

We just multiply the mission fuel segments and then subtract the product of mission fuel segment from one to get the fuel fraction. So after doing the mission segment fuel fraction estimation, then we put in an allowance for reserve fuel, which was multiplying the mission fuel fraction by 1 - RFF. This was the base formula for estimating W_0 in which the numerator contains 2 quantities W_{crew} and $W_{payload}$.

Which are known to us from regulatory requirements, conventional items and also from the data regarding the aircraft such as $W_{payload}$. The 2 unknowns are $\frac{W_f}{W_0}$ and $\frac{W_e}{W_0}$. And as I mentioned a few minutes ago, the $\frac{W_f}{W_0} = (1 + RFF)\left(1 - \frac{W_x}{W_0}\right)$

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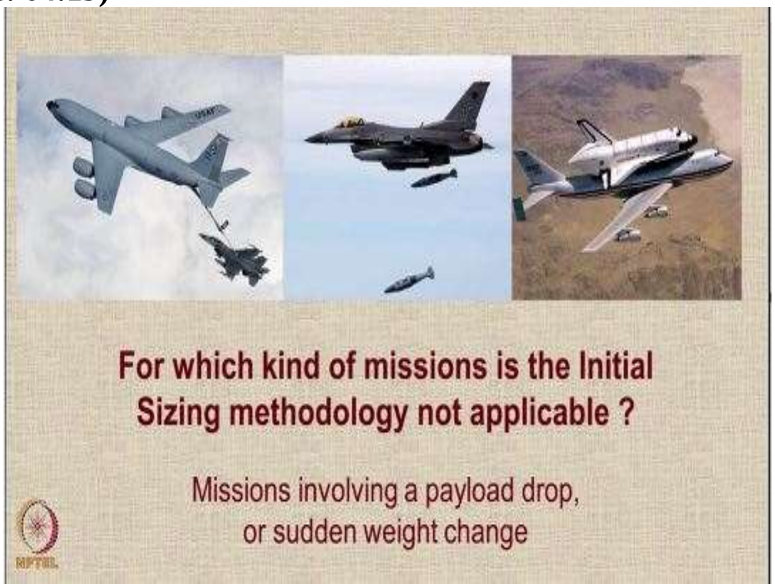

Now, this question I asked even there and I repeat this question, the initial sizing procedure is applicable for certain classes of aircraft, but there are certain missions for which it is not applicable. These are missions which involve a payload drop or when there is a sudden significant change in the weight of the aircraft other than because of fuel consumption due to flying. Actually can think of 3 examples.

For example, there could be a situation when there is air to air refueling in the aircraft. So in this particular phase, the aircraft acquires additional weight in the form of the fuel transferred from the tanker to the onboard tanks, there could be a drop of payload like bombs or armament during combat, which will also involve sudden change in the weight. And there could be a situation where an aircraft piggybacks on some other aircraft such as what you are seeing in this particular slide.

And if that is the case, so, here we see the space shuttle being carried by Boeing 747 or modified aircraft which, so, when you when this aircraft releases the payload, which is the space shuttle, there is a sudden change in the weight. So, the sizing of such aircraft cannot be done using the procedures where which we have discussed earlier and for them we need to look at refined sizing.

So, what we do is, we carry out the initial sizing assume that although the aircraft undergoes change in the payload.

We assume that it has not happened during the mission and keeping the whole aircraft rate fixed except the consumption of fuel we carry out the initial sizing but now, we will look at how to do the initial sizing if there is a sudden change in the payload.

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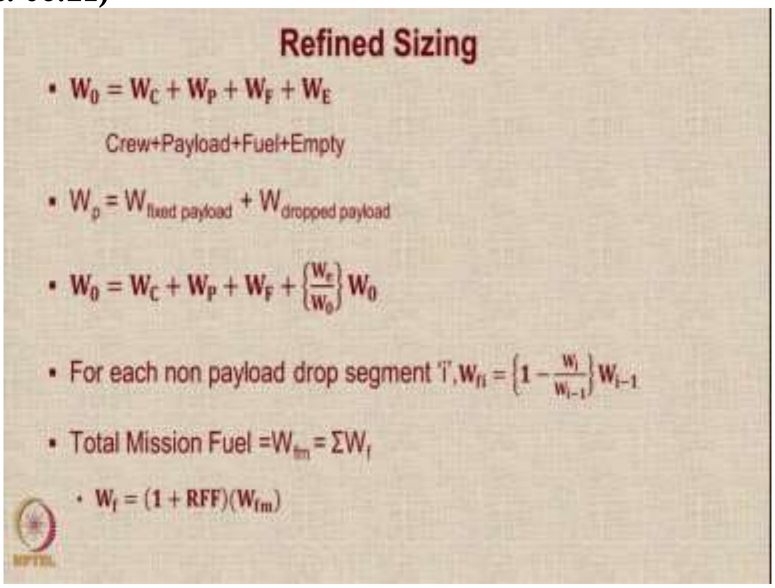

So, in refined sizing, we use the same master formula that the aircraft gross weight is a combination of 4 weights, the crew weight, the payload weight, the fuel weight and the empty weight. And we say that the payload is going to be a combination of fixed payload, which is not released or consumed and the dropped payload. So, we will replace W_e by the $\frac{W_e}{W_0}$. So, that is a straight forward substitution.

Now, what we do is for each segment in which there is no drop in the payload, we use the same procedure. But now we say that the fuel consumed in that segment earlier we were working in ratios, correct, we were looking at the ratio of the weight of the aircraft at the beginning and the end of the particular segment and we multiplied those ratios. Now, we do not work in the form of ratios, we work in fuel consumed in each mission segment.

So, fuel consumed in the mission segment i which does not involve sudden payload change is W_{f_i}, which is equal to

$$W_{f_i} = \left\{1 - \frac{W_i}{W_{i-1}}\right\} W_{i-1}$$

which is what has happened in so far into the weight just at the beginning of this particular segment. So, once you do this for each mission segment, and then if you sum all those W_i's, you W_{f_i}'s you will get W_f after that you can put the reserve fuel fraction like before and get the value of W_f.

So, earlier we were iterating between the LHS and the RHS because for W_e. Now, also we will iterate but we will not be using $\bar{W}_{fuel}$ as we used last time, but we will use W_{fuel} directly.

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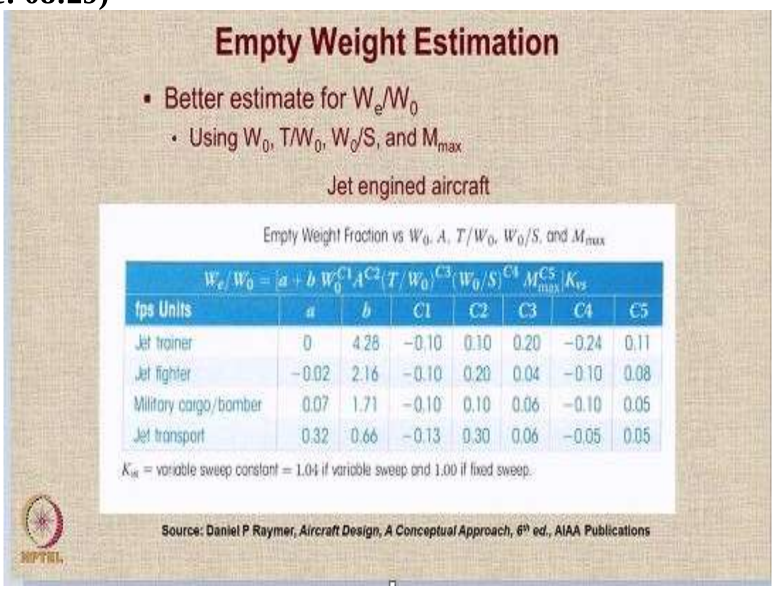

Now, the empty weight estimation earlier it was done using a simple formula $A * W_0^c$ but now we use a better formula, because now we are aware about T/W and W/S values and we also know the maximum Mach number. So for that if you look at the textbook by Daniel Raymer, the sixth edition I have taken this figure from there this chart from there in fps units for jet engine aircraft, you can get a formula for various types of aircraft using the coefficients a b C_1 C_2 C_3 C_4 C_5.

And here are the parameters W_0 ,a, T/W, W/S and M_{max} are used K_{VS} is the same variable sweep constant it is 1.0 if there is no variable sweep fixed sweep aircraft and if it is 1.04 if their curve has variable sweep, so, this is just a more I would say a little bit more accurate formula for Empty weight fraction. Earlier we only had you know $A * W_0^c$ something like that, but now we have more terms in this expression.

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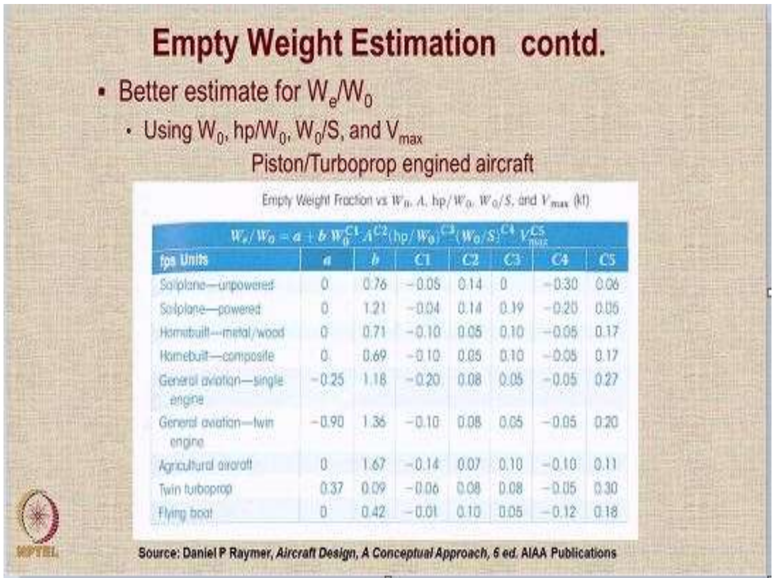

Empty Weight Fraction vs W_0, A, hp/W_0, W_0/S, and V_{max} (kt)

$W_e/W_0 = a + b\, W_0^{C1} A^{C2} (hp/W_0)^{C3} (W_0/S)^{C4} V_{max}^{C5}$

fps Units	a	b	C1	C2	C3	C4	C5
Sailplane—unpowered	0	0.76	−0.05	0.14	0	−0.30	0.06
Sailplane—powered	0	1.21	−0.04	0.14	0.19	−0.20	0.05
Homebuilt—metal/wood	0	0.71	−0.10	0.05	0.10	−0.05	0.17
Homebuilt—composite	0	0.69	−0.10	0.05	0.10	−0.05	0.17
General aviation—single engine	−0.25	1.18	−0.20	0.08	0.05	−0.05	0.27
General aviation—twin engine	−0.90	1.36	−0.10	0.08	0.05	−0.05	0.20
Agricultural aircraft	0	1.67	−0.14	0.07	0.10	−0.10	0.11
Twin turboprop	0.37	0.09	−0.06	0.08	0.08	−0.05	0.30
Flying boat	0	0.42	−0.01	0.10	0.05	−0.12	0.18

The similar formula is also available for aircraft which are powered by piston prop and turboprop engines. So there are there is a list there is a whole table available for the values. Once again I have taken this chart in fps units from the book by Daniel Raymer.

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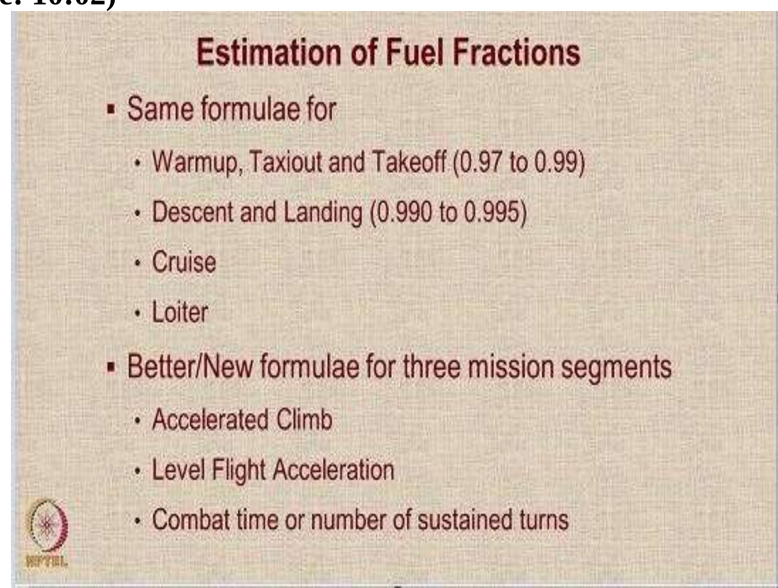

Now, let us move ahead to look at how we estimate the fuel fractions. So the formula are the same for warmup, taxi-out takeoff for descent and landing for cruise and for loiter, but there are certain changes in the way we apply this formula as I will show you soon for 3 missions segments, we have better formula or new Formula one of them is the accelerated climb, the other is the level flight acceleration. And the third is the combat time or the number of sustained turns that are to be done during combat at a particular zone. So, for these 3 we have better and more accurate formula.

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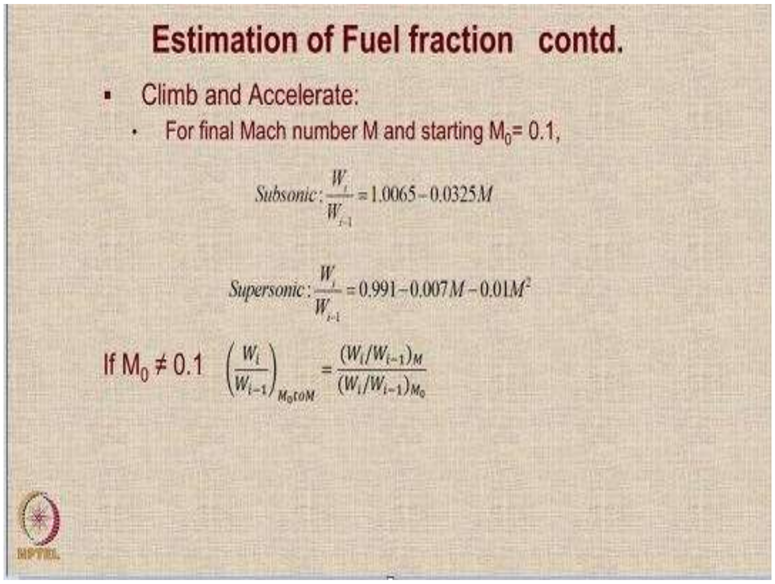

Let us look at first how we can get better estimate of the fuel fractions in the climb and accelerate segments. So, assume that you have a climb or acceleration that starts with M 0 equal to 0.1 which is typically the Mach number at the end of the takeoff segment and then there is a final Mach number M. So, in subsonic aircraft, we can assume that the ratio W_i that is end of climb upon W_{i-1} would be as shown in this formula in terms of Mach number. And if it is supersonic, there is a slightly different formula which involves an n squared term also.

So, what we do is using this we can directly get the value but suppose you are not starting from Mach number 0.1 you are starting from let us say Mach number 0.8 . Let us say if you are accelerating the aircraft from Mach number 0.8 to Mach number 2. Now you do not, you are not starting from Mach number 0.1. So what you do is you first calculate how does the weight fraction going from Mach number 0.1 to 0.2.

And then you calculate the value when you go for a Mach number 0.1 to 0.8. And then if you divide the 2 ratios, you will get the value of the weight fraction when you go from Mach number M_0 to M. So from 0.1 to M_0 and from 0.1 to M, you get these 2 values and then if you divide you will get the value for the weight ratio to go from M_0 to M.

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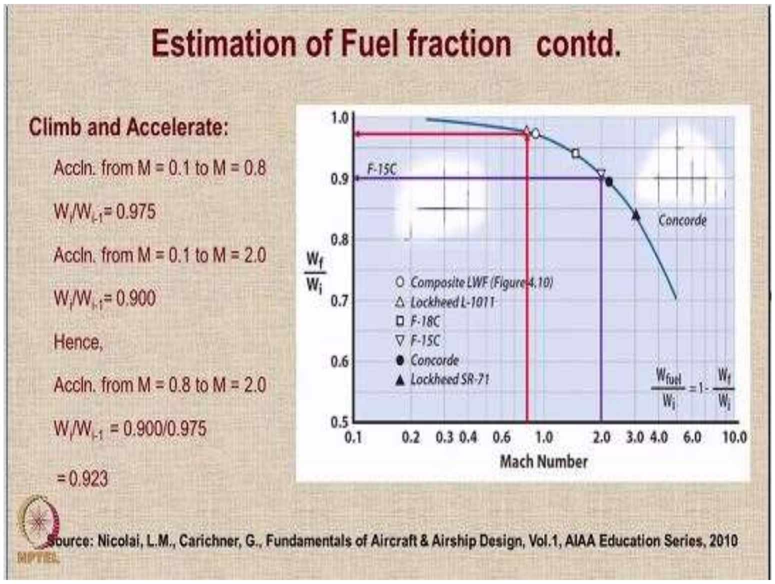

Just to illustrate with an example. Let us so we use this chart either we use the formula given behind or we can also use this chart let us just use this chart as an example. Let us say you would like to accelerate from Mach number 0.1 to 0.8. Now, let us say you want to accelerate from Mach number 0.8 to 2.0 you want to know the fuel fraction in an accelerated climb from my Mach number 0.8 to 2.0.

So, what you do is first you find out what is the fuel fraction in acceleration from Mach number 0.1 to 0.8 for that we can use this chart and you can read out from the graph that the weight ratio is 0.975 then you find out what is the ratio if you accelerate from Mach number 0.1 to 2.0 that is shown by these purple lines it goes to Mach number it goes to 0.9. So, therefore, if you want to know find out the weight ratio to accelerate from Mach number 0.8 to 2.0 you just divide these 2 values when you accelerate from Mach number 0.8 to 2.0.

The weight ratio will be more than 0.9 but less than 0.975. So, it will be the ratio 0.9 divided by 0.975 or 0.923. So, this way, you can get the weight ratios for acceleration from any Mach number to any Mach number.

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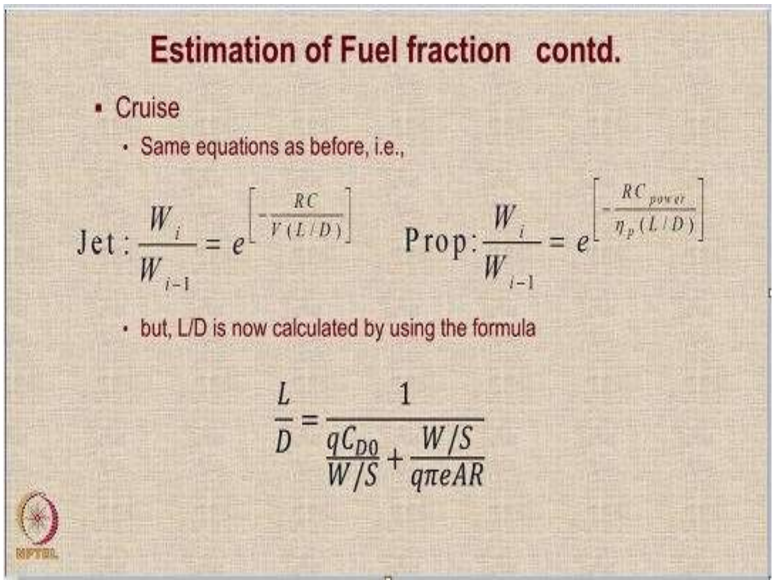

Let us look at cruise. So, earlier also we use this particular formula, the Breguet range equation for jet engine aircraft and this one for prop engine aircraft we use the same formula as before. The only difference is that in refined sizing, the value of L/D is not the one that we assume last time as either L/D max or 0.866 time L / D max actually you can calculate the value of L/D because see you know the value of $C_{D_0}, \frac{W}{S}, e, AR$. So, using this information you can get the actual L/D available to the aircraft use that value and get the weight ratios.

So, it is slightly more accurate rather than assuming L/D as L/D max or L/D is 0.866 L/D max. We actually calculate the L/D which is prevalent during the cruise and use that expression in the appropriate formula either the jet engine aircraft or the propeller engine aircraft.

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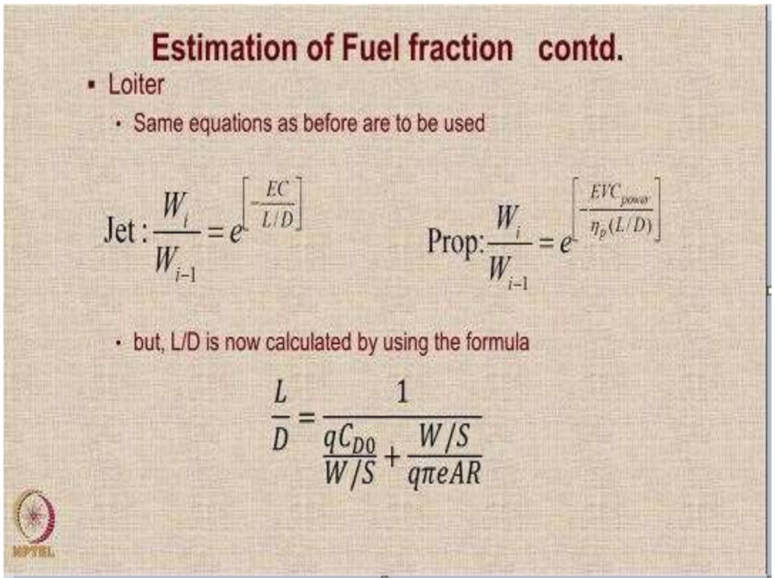

Similarly, if you go to loiter, again we use the same equation for the jet the equation is

$$\frac{W_i}{W_{i-1}} = e^{\left[-\frac{EC}{L/D}\right]}$$

For prop it is

$$\frac{W_i}{W_{i-1}} = e^{\left[-\frac{EVC}{\eta_p(\frac{L}{D})}\right]}$$

the formulae are the same, the difference is that now you can use a better estimate of L/D just as I discussed in the cruise segment.

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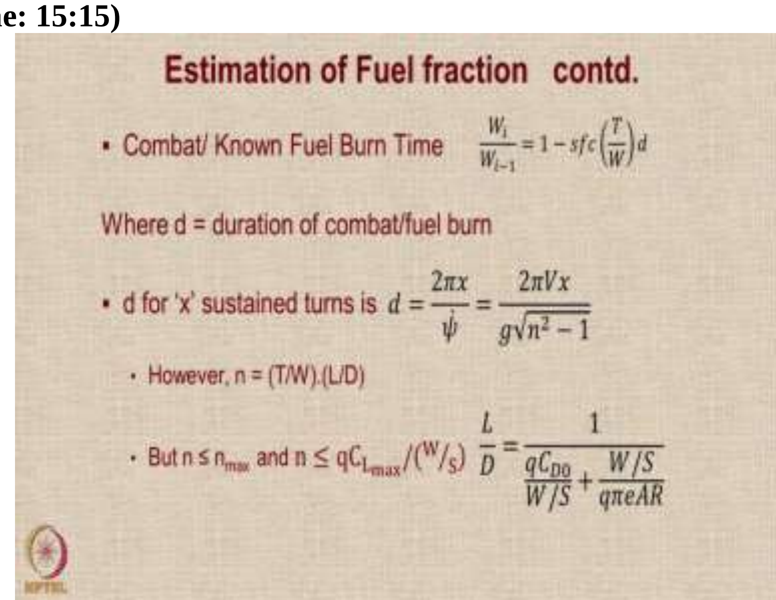

Moving ahead, suppose you are given that the aircraft has to go for a mission in which there is some combat for a particular fuel for a particular time for example, we normally say that the aircraft will go to some distance go down to the zone of combat and then there will be a combat of 20 minutes, so, we know that time, where there is these the duration of the combat or the fuel burn. Now, we know the Sfc of the aircraft in that condition whether afterburner is on or off etc.

And we know it is T / W. So, therefore, you can get the more accurate value of the weight ratio in that mission segment as $\frac{W_i}{W_{i-1}} = 1 - sfc * \frac{T}{W} * d$, where d is the duration of the combat or the fuel burn. So, for that particular segment, where you know that I am going to do a combat for so many minutes under this operating condition, you can get a more accurate value of the fuel fraction.

Now, suppose you are told that you have to go and do say 10 sustained turns at a particular turn rate or a particular radius or a particular g at a particular speed, then what you can do is you can use the duration d as the time taken for doing those turns.

So, you know the L/D value plugin the correct L/D value and ensure that n will be less than or equal to n max and hence n will be less than or equal to $\frac{qC_{L_{max}}}{W/s}$. So, that W/S is also known q is also known. So, using this you should be able to get more accurate value. So, you can get the value of n because T/W is known you can get value of n because n is known you can get the value of $\frac{qC_{L_{max}}}{W/s}$.

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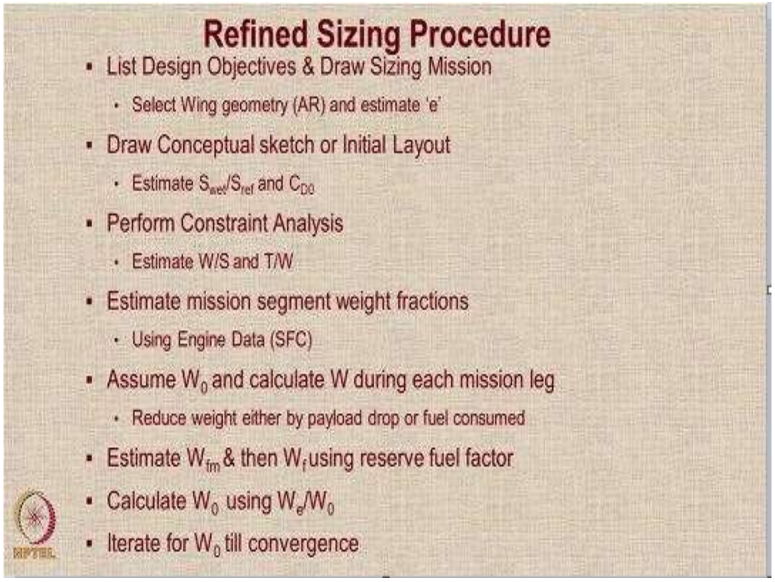

So, just to sum the refined sizing procedure, so, what you do is you list the design objectives and draw the sizing mission that is the mission profile you have to select wing geometry that means aspect ratio mainly and then you have to estimate the value of Oswald efficiency. Remember we had one very large big formula in terms of sweep of the maximum thickness line and the taper ratio etc. And aspect ratio mainly so, you can get the value of e.

So, what you do is draw estimate S_{wet}/S_{ref} and C_{D_0} that is what you do generally when you do initial sizing carry out the constraint analysis where you can get W/S and T/W then you estimate the mission segment fuel fractions using the engine data or the SFC assumed W_0 and calculate W during each mission leg. So, you start with some W_0 and then you keep on reducing the weight by payload drop or by the fuel consume. Finally, you estimate W_{fm} and then W_f using reserve fuel factor. So, then since you know W_e/W_0 as an estimate you calculate W_0 and then you keep on iterating till you achieve some kind of a convergence.

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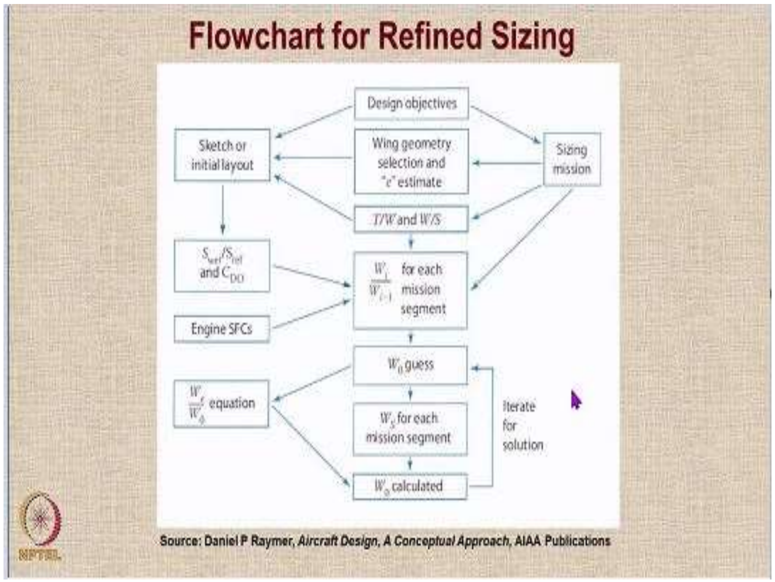

So, here is a flowchart for explaining the same procedure. So, you can see that there is a sizing mission from the mission we get the wing geometry and therefore, you can estimate the value of e from the sizing mission and sustain analysis we also know the value of T/W and W/S. So, from the sketch of the layout, you can get the wetted area ratio, you remember the graph given in Raymer's textbook where you eyeball the aircraft.

So, you can get a $\frac{S_{wet}}{S_{ref}}$ C_{D_0} can be calculated by the detailed procedure for based on the geometry and operating conditions. So, using that you can get for our guess value and then you can iterate for it. So, you will learn this only if you do this calculation.

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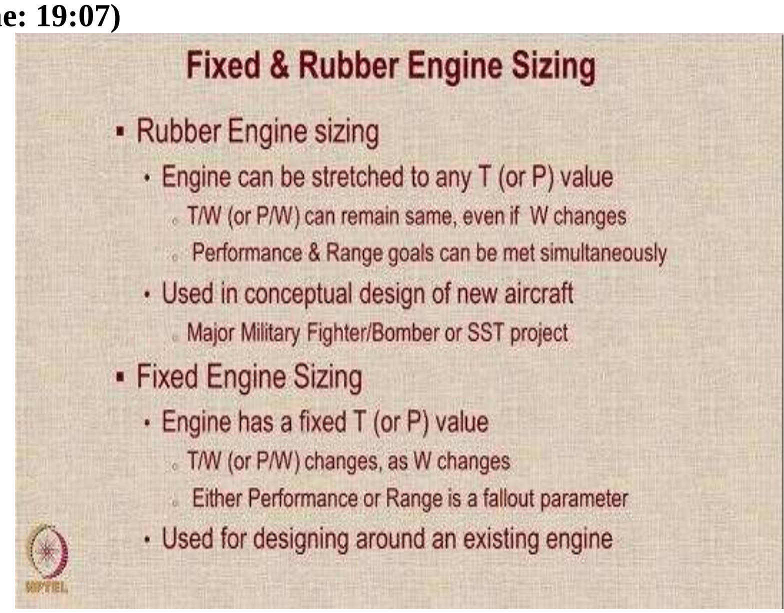

Now, I want to just talk about 2 approaches which are followed in sizing they are called as fixed engine sizing and rubber engine sizing. So, let us see what is meant by rubber engine sizing in rubber engine sizing we assume that the engine of the aircraft is almost like a chewing gum you can stretch it or shrink it to whatever requirement you want. So, we assume that you have an engine which can be stretched to any T or P value and accordingly certain parameters will scale up or down. So, the value of T/W or P/W can remain the same even if W changes.

So, if W increases by 1.5 times you assume that you can get some engine which will have its T also 1.5 times therefore, the T/W remains the same. So, what happens is that in this case whatever are the performance and range goals, you can meet both of them simultaneously, because you assume that there is a theoretical engine available, which is exactly meeting the requirements of T/W for all the segments.

So, therefore, you have an engine available, which can be used or stressed or shrunk to whatever requirement you have this kind of a method is more useful when you do conceptual design of a new aircraft, where you do not want to assume that you are going to design around a specific engine you want to be free to get you can assume here that you know once we complete the design exercise, either somebody will make an engine required for our needs or somebody will scale up their engine in some way to meet our requirements.

So, let us say you have a major military air fighter aircraft or a bomber aircraft or an SST project and you do not want to assume an existing or a specific engine, you can do this rubber engine sizing, but in most practical examples in real life, usually, an aircraft is designed around a given engine. So, in many cases, the practical reality is that nobody is going to make an engine specifically to meet your requirement.

Because the process of designing an engine takes a lot of time and a manufacturer will not create a special program to come up with an engine meeting your requirement. Unless it is a huge or a massive project, what most engine designers do for engine companies do is they tweak their existing engine to meet the requirements that may crop up. But in reality, what we do is we say design an aircraft that can do this, this this this, but use this particular engine, so, the engine has a fixed T or P value it cannot be changed.

So, you cannot have any T/W there will be a fixed T/W depending on the fixed value of T and the W that you obtain. So, in this case, the T/W or P value will change as the W changes unlike in rubber engine sizing where you can have any value of T/W or P/W. So, what happens is either you can say that let us keep range fixed. So, the performance will be whatever is available or you can say I keep my performance fixed that means the maximum speed etc.

Then the range is going to be a fallout parameter you cannot have both performance and range ideally met because your engine is fixed. This one is used when you design an aircraft around an existing engine and this fringe is most of the time this is the reality in aircraft design that you do a fixed engine sizing, but in the classroom or in a theoretical exercise we can always do rubber engine sizing.

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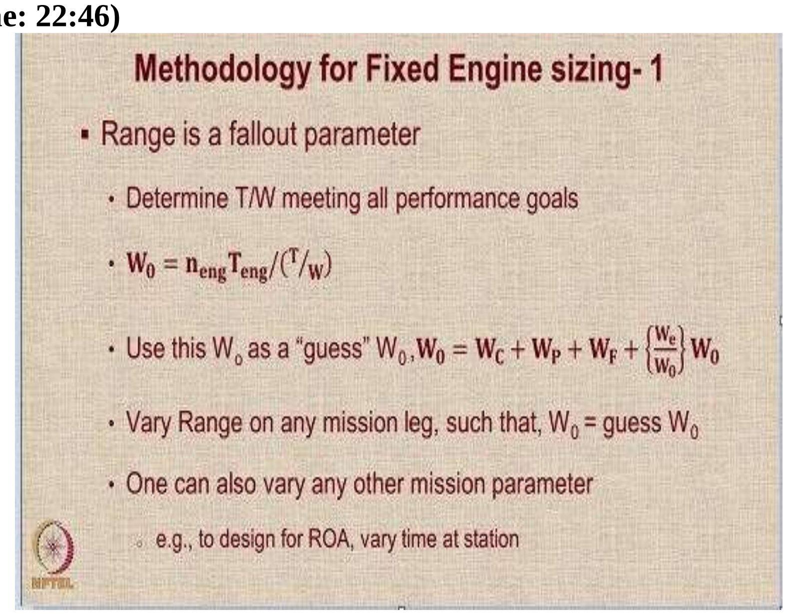

So, let us see what is the procedure to be followed for sizing when you do fixed engine sizing and when you do rubber engine sizing let us look at the fixed engine sizing first in the fixed engine sizing suppose you say that I am not particular about the range, range will be something that I get give me the performance then what you do is you first find out the T/W that meets all your performance requirements forget about the range.

So, W_0 will then be equal to the number of engines into the T engine that is the thrust of the engine available into the T/W that you calculate meeting all the performance goals. So, use this as a guess W_0 and then start doing the refined sizing as I discussed a few slides ago. So what you do then is that once you get the value, you have to now vary the range on any mission leg such that W_0 will be equal to the guess value of W_0.

So, since your T/W is fixed from performance requirements, hence W_0 gets fixed. So when W_0 gets fixed, that means you now also know what is the empty weight, what is you know, what is the weight of the payload and the weight of the crew members. So now you know this much fuel is available for the mission. And with that mission, whatever range you get, you have to live with that.

You can also vary some other parameters like the speed if you want but normally, the range itself is a fallout parameter in most cases, but you might say let us say I want to design for a particular radius of action. Or I want to design an aircraft for a particular time at a station. Let us say I am designing a HALE UAV where I say I wanted to loiter for 30 hours or a particular station. So then that becomes the main mission requirement, but you might say if I vary it can I match the fuel available with the fuel required? This is fixed engine sizing.

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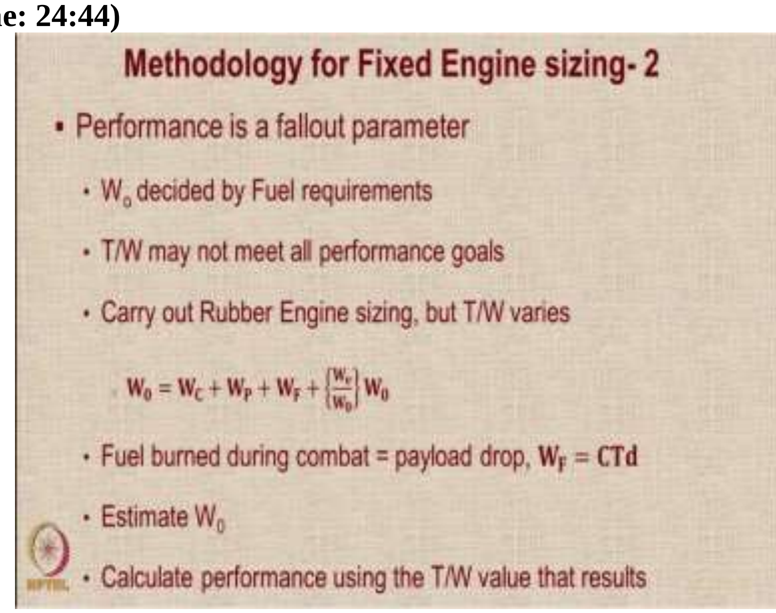

Let us look at the approach and fixed engine sizing followed when performance is the fallout parameter but range is a fixed known parameter. I do not want to compromise on the range. I can compromise on the performance. In this case, W_0 decided by the fuel requirements for the mission and the T/W may not meet all the performance goals. So, what you do you carry out rubber engine sizing, but you vary T/W.

So, T/W is not fixed it varies. So, fuel burn during combat or the payload phase will become simply it will simply be you know C*T*d, d is the duration T is the thrust and c is the sfc. So, with this you just estimate W_0 by assuming by calculating the fuel needs segment and then whatever you get you will get some W_0. Earlier we fixed W_0 by the T/W that we want for performance and known value of number of engines and the engine thrust.

Now, we are saying no, we are going to allow performance to vary but we want to do the mission exactly as required. So, what you do is you estimate W_0 and then you calculate the performance using the T/W value that results.

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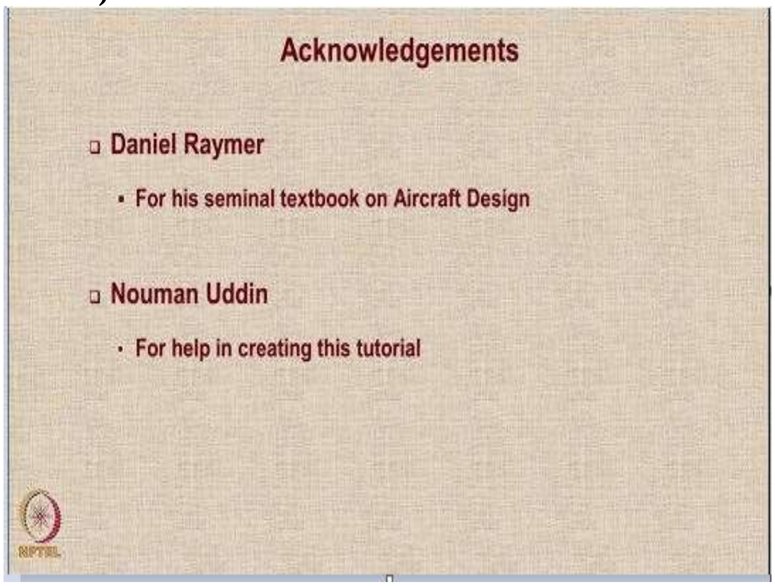

Before I close I would like to thank Daniel Raymer first for his seminal textbook on aircraft design, which I have followed for this particular lecture and also Nouman Uddin my teaching assistant for this course for help in creating this tutorial. Thanks for your attention.