Introduction to Aircraft Design
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Lecture - 66
Tutorial on Constraint Analysis of Military Aircraft: Part-2

Let us have a look at how we incorporate the constraints due to instantaneous turn, there are 2 types of turns a sustained turn and instantaneous turn. In an instantaneous turn, we allow the aircraft to lose the altitude or reduce the speed if needed to achieve a higher turn rate. The requirements specified are as mentioned; we need a turn rate of minimum 18 degrees per second, while operating at 6 kilometres or 6000 meters above mean sea level

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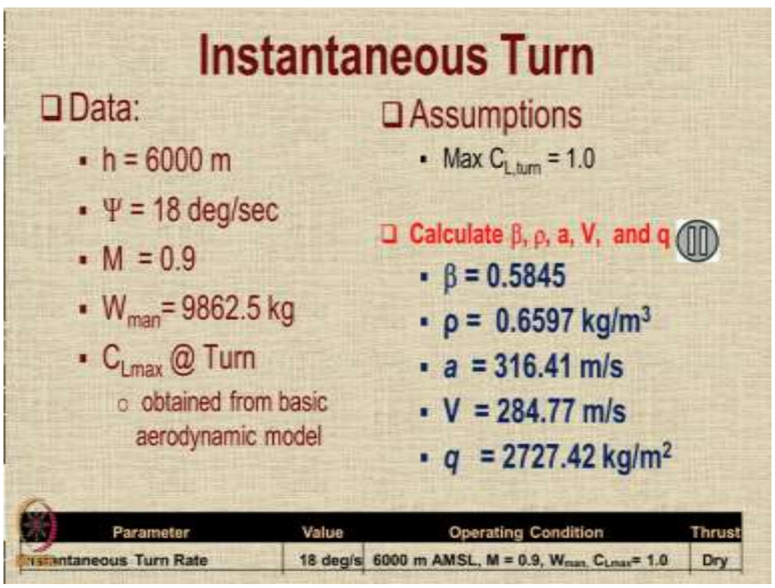

At a Mach number of 0.9, $C_{L_{max}}$ is given as 1.0 and the aircraft weight is equal to the maneuver weight. So, the data is as mentioned in the screen and the manoeuvre weight was already calculated as the weight of the aircraft with the half the fuel and the armaments gone, but the missile and the gun available, the $C_{L_{max}}$ at turn is obtained from the basic aerodynamic model which as we known is 1.0.

So, the assumptions are that the $C_{L_{max}}$ at turn is 1.0. And now, we need to calculate $\beta, \rho, a, V\ and\ q$ to calculate β we need to know what is going to be the aircraft weight at this condition we can assume it to be 0.8, the density will come from the ISA table at 6000 meters either you can calculate it using the standard methods or you can look up the ISA table once you know the altitude you know the ambient temperature and hence you know the sonic speed.

One sonic speed is known because Mach number is known you can calculate the value of V and once density and v are known, you can calculate dynamic pressure q. So, at this stage take a pause and calculate these values the beta comes straightaway from the ratio of W manoeuvre upon W take off and the other parameters come from the standard values from the atmospheric table.

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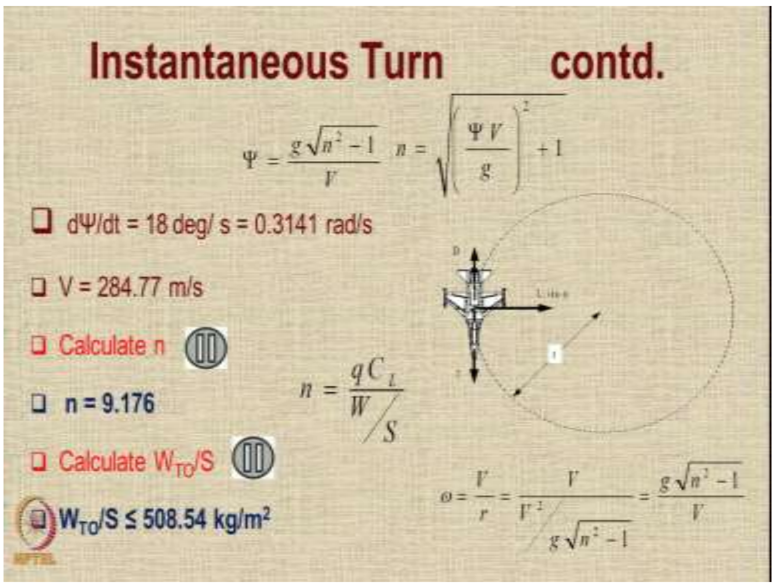

Moving on if this is the top view of an aircraft undergoing an instantaneous turn of radius r, then we see that the omega or the angular velocity

$$\omega = \frac{V}{r} = \frac{g\sqrt{n^2 - 1}}{V}$$

So, here we must note that if you want to have large omega, you should either have a lower velocity V or a higher value of load factor n_z that is n.

So, rearranging this expression in terms of the ψ which is actually omega and again rearranging because we need to know the value of n. So, the $\frac{d\omega}{dt}$ or the turn rate $\dot{y}$ actually is given as 18 degrees per second which is 0.3141 radians per second and the V was also specified which turns out to be 284.77 meter per second by our calculations. So, at this point, I think you can just look at the formula on the top right of the screen and calculate the value of n because $\dot{y}$ is also given and V is also given g is a constant.

So, by putting in the numbers we get the value of n as 9.176. In other words, this aircraft is pulling a g of 9.176 when it undergoes an 18 degree per second turn under these conditions. Now, we also know that the load factor n in turn would be

$$n = \frac{qC_L}{W/S}$$

Hence, we can see that the load factor is directly connected to the wing loading. So, putting in the value of n and q and C_L you can rearrange and calculate the value of W/S.

Remember, you will have to put beta in this expression in the denominator because you need to be sure that you get the value at W_{TO}/S putting in those values, we get the required W_{TO}/S to be less than equal to 508.54 kg per meter square.

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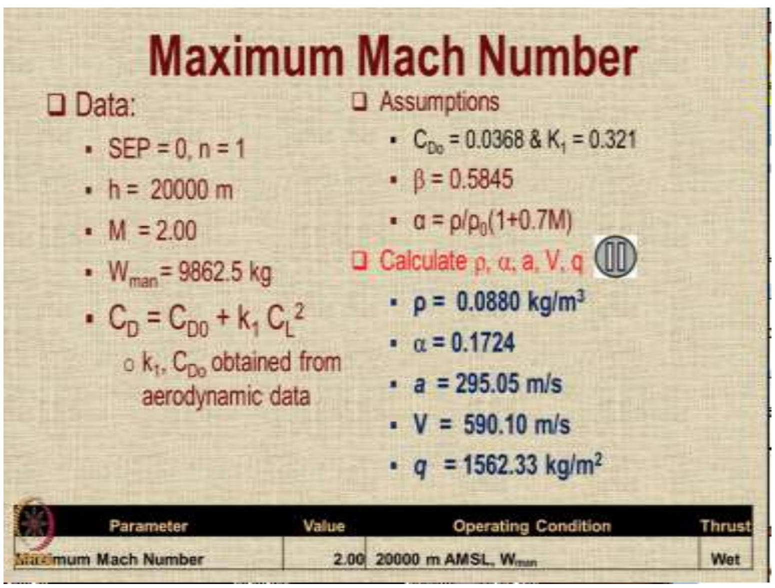

Let us move ahead to the next requirement which is the maximum Mach number. This is also specified in the constraints as a maximum value of 2 while operating at very high altitude of 20,000 meters or 20 kilometres above mean sea level with the weight equal to the manoeuvring weight and the afterburner permitted. So, that is why it is thrust is wet. Moving ahead if you want to look at maximum Mach number it is actually obtainable if you put the specific excess power equal to one with load factor n = 1 in level flight .

The other parameters are already given in the requirements. So, we have to get the values of C_{D_0} and k_1 as mentioned from the aerodynamic data and we know that under these conditions the corresponding values are specified. Beta we have last calculated in the last example, at the maneuver weights the beta is 0.5845. The value of alpha which is the thrust ratio thrust lapse ratio.

Here it would be actually more than one because we are going to go for an afterburner. So, we have this term 1 + 0.7M. So, with this information I think it is enough for you to calculate the values of density rho at 20 kilometres alpha using the expression $\frac{\rho}{\rho_0}$ because you can get the value of ρ at 20 kilometres from the atmospheric tables. Since you know the value of t you can get the value of a and once you know a and you know the Mach number.

You can get the value of V once you know V and ρ you can get q. So, take a pause at this place pause the video and do these calculations. If you really want to practice and learn you need to do these calculations yourself. So, putting in the values as mentioned, these are the numbers we get for the various parameters.

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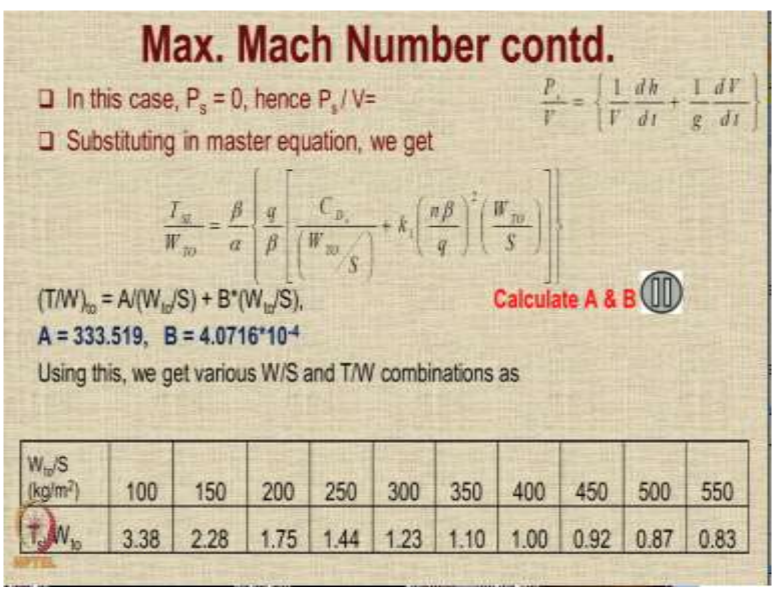

Moving ahead the maximum Mach number calculations. So, we know that the specific excess power is 0 in case of level unaccelerated flight. So, therefore, $\frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$ term in the master equation will vanish. So, therefore, the master equation will simply become as shown in the screen and it can be considered as a standard form of a constant A divided by the wing loading W/S and another constant B into W/S.

So, at this point I think we can calculate the values of A and B using the values of $\beta, q, C_{D_0}, \alpha$ and k_1 and β and q to calculate the value of B. So, pause the video at this stage and do calculate these values what we get is that A is 333.519 and B is $4.0716*10^{-4}$ putting this in the expression we can get an equation that relates T/W versus W/S.

And that will give you the corresponding values. So, what we see here is that if you want to fly aircraft up to Mach number 2, then to have a reasonable value of thrust to weight ratio, you need to have very high wing loading. So, the wing loading the thrust to weight ratio would be 1 if you have a wing loading of 400. And, you know correspondingly you can calculate the values for other conditions.

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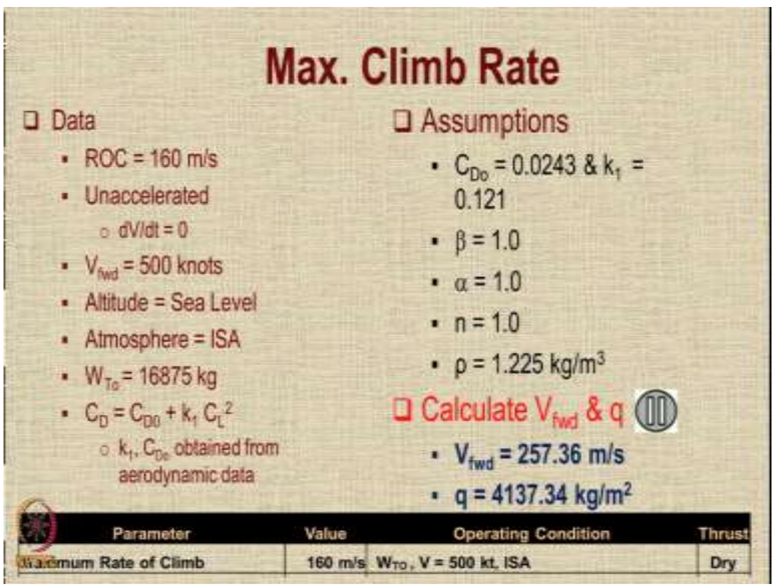

Moving ahead, let us look at how the maximum climb rate requirement would affect in the constraint diagram. It is specified in the requirements that the maximum climb rate is 160 meters per second at the max takeoff weight while flying the aircraft at 500 knots under ISA conditions. And in this case, we assume that the aircraft is not able to use the afterburner, so it is dry thrust condition.

So, putting all these values, we can get a list of the data as mentioned in the screen, once again, the values of K_1 and C_{D_0} have to be obtained from the aerodynamic data. And as per our previous information, the values are as shown their C_{D_0} is 0.0243 and K_1 = 0.1 to 1. Now, since we are just at takeoff weight therefore, beta = 1 since we are not allowed any afterburner alpha = 1 at sea level.

And since it is a unaccelerated therefore n also equal to one and the rho is the density at sea level which is 1.225 kg per meter cube with this information, we can calculate the forward speed V and the value of dynamic pressure q, the forward speed we can be obtained easily by converting 500 knots into meters per second.

So, you should pause the video at this stage and do these 2 calculations, let us move ahead. So, the values obtained would be 257.36 meter per second for the speed and 4137.34 kg per meter square for the dynamic pressure.

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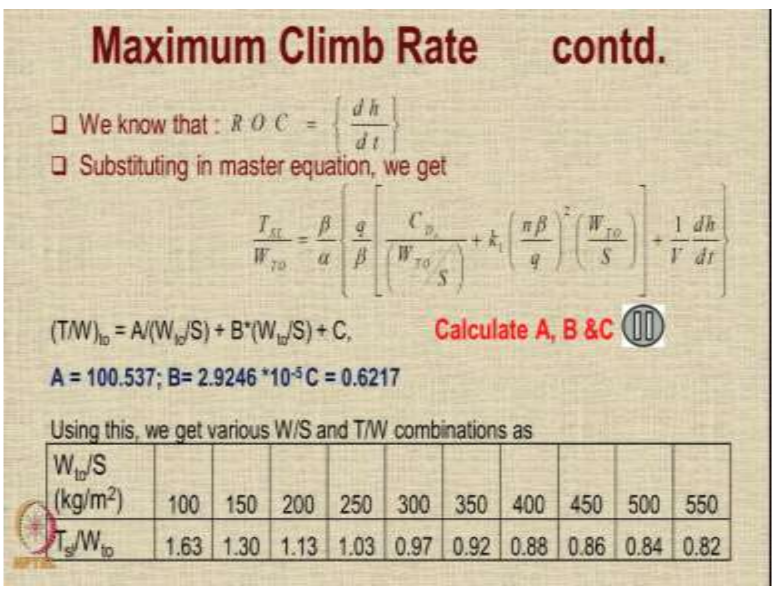

So, continuing further, we know that the ROC = dh/dt and you know if it is unaccelerated then dV/dt is going to be 0. So, the master equation will have as 0 value for the last term which is (1/g)(dv/dt). Now, once again you have an expression between T/W and W/S except that there is now an additional term. So, there will be a constant term which will be (1/V)(dh/dt).

So, that constant term would be C and the value of A and B has to be calculated as earlier but remember that the conditions are different therefore, the values of q β, C_{D_0} etc have changed. So, please pause the video at this stage and do calculate these values. So, once we calculate the values we can see that they are obtained as shown in the screen. Now, putting these numbers in the expression for T/W as a function of W/S, you can get many combinations of W/S and T/W that just meet the constraint.

So, what we see here again is that if you want to have a reasonably good climb rate of let us say 160 meters per second as in this case, then to have reasonable values of thrust to weight ratio, you need to have higher values of wing loading. So, as the values of wing loading are increasing, the thrust to weight ratio required is decreasing.

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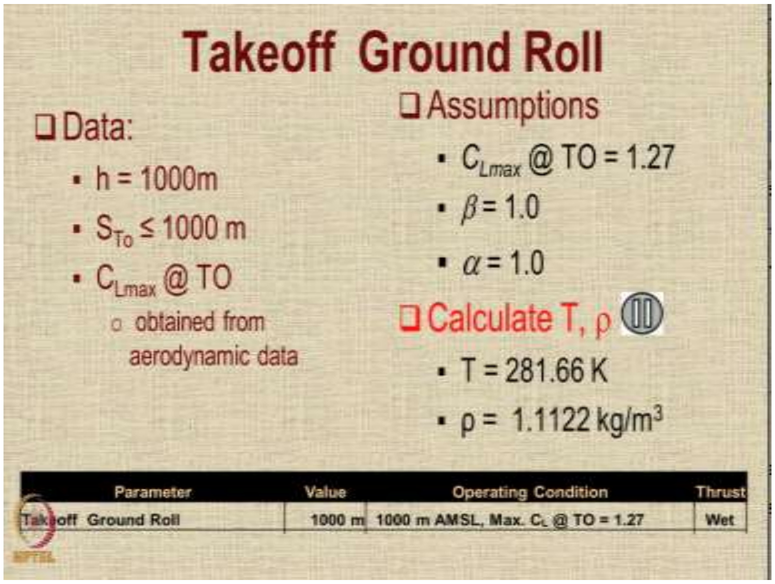

Now, so, far we have used the master equation for all the constraint calculations, but there are a couple of constraints that can be that need to be calculated without the master equation these are the takeoff and the landing roll. So, for takeoff ground roll the requirements specified are as mentioned below we want it to be within 1000 meters while operating from an airport which is at one kilometre above mean sea level.

And from aerodynamic information we know that the maximum C_L at takeoff is 1.27 and you are allowed to use afterburner. So, this information is inserted in our database and obtaining the $C_{L_{max}}$ at takeoff as 1.27 from the aerodynamic information.

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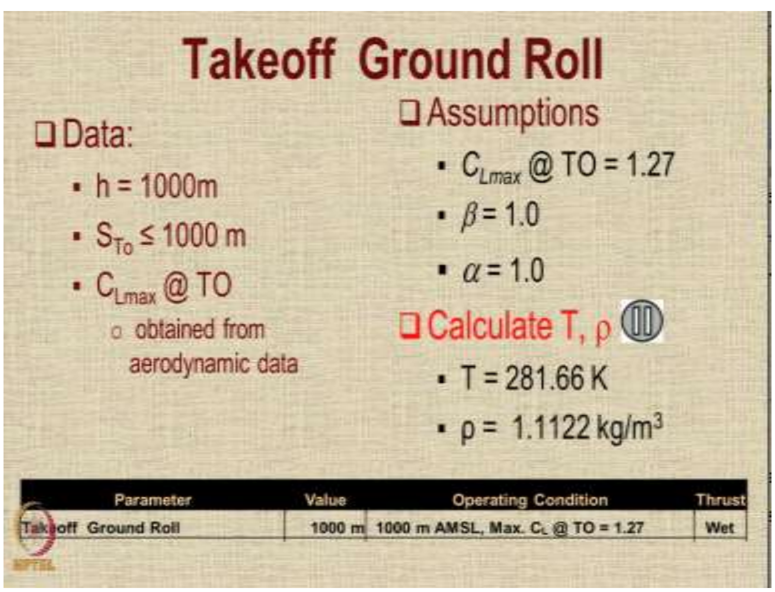

And since we are using the sea level conditions and just add the takeoff condition therefore, beta = 1 and alpha = 1 with this you can calculate the value of T and rho. So, the value of rho but this is this is at 1 kilometre above mean sea level. So, you need to invoke the ISA table and get the values of T and from there the value of density.

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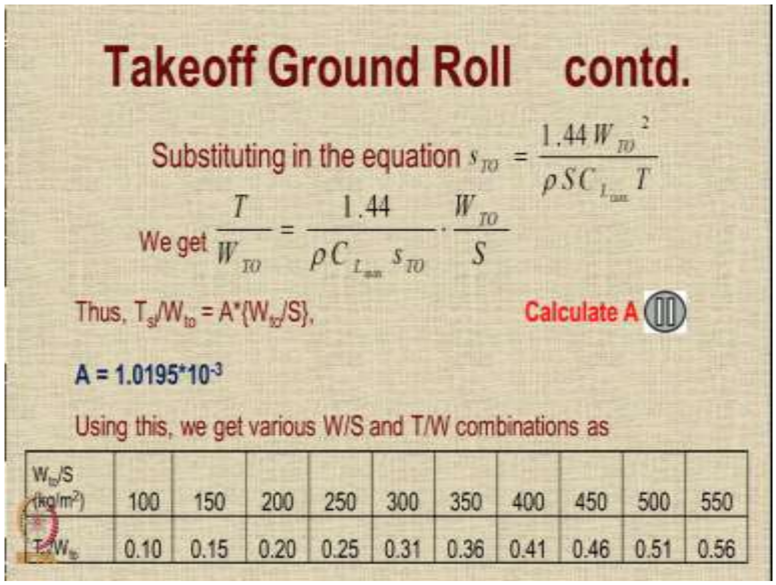

W_{to}/S (kg/m²)	100	150	200	250	300	350	400	450	500	550
T/W_{to}	0.10	0.15	0.20	0.25	0.31	0.36	0.41	0.46	0.51	0.56

Moving ahead we have a simple equation for the take of ground roll which comes straightaway from the fact that the takeoff velocity is 1.2 times V_{stall}. So, by a simple rearrangement we can get T/W is equal to a constant expression times W/S.

So, with this, if you insert the values of S_{TO} which is specified $C_{L_{max}}$ which is also known ρ which is calculated at 1.4 for being a constant, you get a simple relationship between T/W and W/S, notice that this relationship is linear. It is linear. So, you can easily calculate the value of the constant A, take some time to pause the video and calculate this number.

It turns out to be as shown on the screen with this you can get a linear equation combination between W/S and T/W. So, what we notice here is that, if you want to have a low value of T/W, then the wing loading also has to be low. So, we notice here that there is a direct combination if you increase the wing loading, then you also have to increase the W by value to achieve the corresponding requirement.

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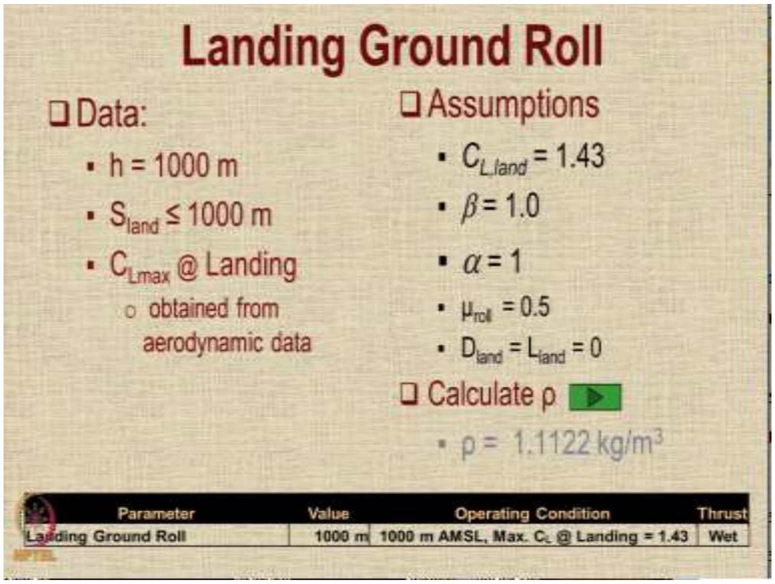

Similarly, we have another last requirement to be considered which is the landing ground roll. The landing ground roll again is specified here the conditions are identical as the previous case except that the C_L at landing because of larger angle of flap deflection is 1.43. Otherwise, the requirement is same 1000 meters at an airport which is at 1000 meters above mean sea level under the ISA conditions.

So, we reproduce the data from the requirements which have been specified and for the $C_{L_{max}}$ we use that information which is available from the aerodynamic calculations alpha = 1 and beta = 1. We are assuming the value of μ_{roll} as 0.5 which is a typical value for the runways which the military operates. And for ease in calculation, let us assume that the drag and the lift acting during landing is 0 we assume that as soon as the aircraft touches the ground, we lose the entire lift.

And immediately we have spoilers which kill the entire which actually kill the entire lift. So, with this all you need to do is to calculate rho but hold on why do we need to calculate because in the previous calculation for the take-off ground roll, we have obtained the value of ρ for the same conditions. So, therefore, we do not need to calculate we just have to look back you can copy and take it from there.

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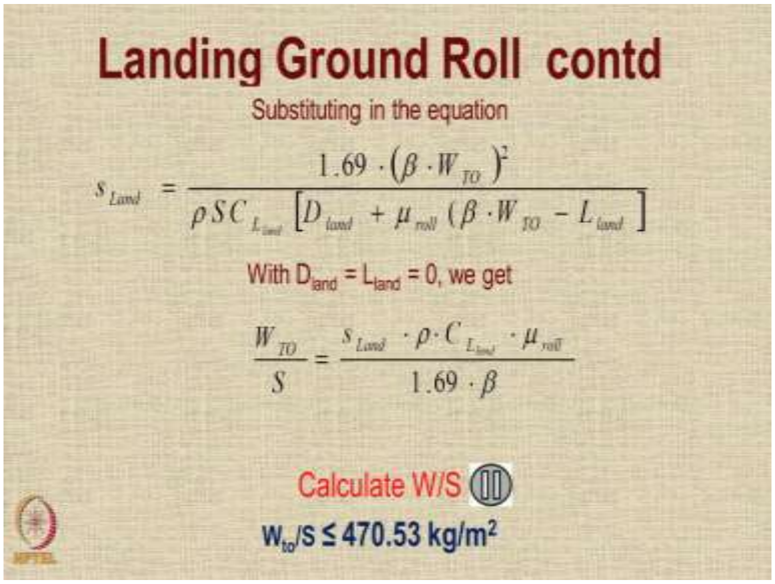

So, moving ahead, we have this long expression for the landing distance calculation this particular expression has been already derived in the lecture on constraint analysis if you have not seen the lecture, I would request you to go back and have a look at that the number 1.69 comes because of the fact that it is 1.3 times the V_{stall} from safety point of view. So, putting D_{land} = 0 putting L_{land} = 0.

You can get a very simple expression in which you have on the left hand side W/S and on the right hand side, we do not have anything to do with the thrust because during landing ground roll, we do not expect the reverse thrust to be considered although we can put reverse thrust and reduce it, but from safety point of view, it is assumed that we should calculate this value without the reverse thrust available to us.

So, therefore, landing ground roll is a constraint that depends only on W/S like the ceiling and like the stalling that we have seen earlier in the in the lecture. So, therefore, S_{land} ρ C_L land μ_{roll} and beta are all available to us. So, with this you can easily calculate the W/S that will exactly give you the required distance. So, hence if a S_{land} cannot increase the value of 1000 W/S cannot increase a particular value, that value is 470.53.

So, we conclude that the landing ground roll puts a constraint that it should the wing loading should be less than or equal to 470.53 kg per meter square. So, now, what we will do is, we are going to plot the constraint diagram by superimposing all these constraint lines one by one.

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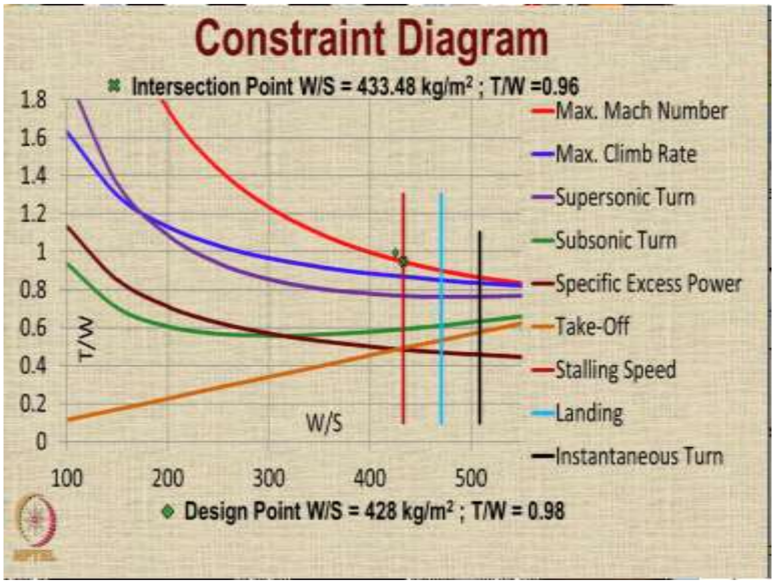

So, first we have on the x axis W/S in kg per meter square and on the y axis we have T/W which is dimensionless and we have plotted this line for the maximum Mach number and the area below this line happens to be infeasible and the area above this line is feasible on this we superimpose the line corresponding to the maximum climb rate. Once again we see that the area above the blue line is feasible and below that is infeasible.

But what we observe is that this line is completely below the red line, which means that in the current problem, the constraint on the climb rate is not really very serious. In fact, if you need the constraint and on the maximum Mach number, automatically you would have met the constraint for the climb rate. So, therefore, you could say that for the current problem, but not for every problem in the current problem it so turns out that the maximum climb rate constraint is actually ineffective.

Similarly, the next constraint you have is the constraint for supersonic turn. So, we see that there is an intersection with the blue line and the area below this line is infeasible. So, while this line also is completely below the red line it also means that the supersonic turn is not really very demanding as far as this aircraft is concerned and other constraint is on subsonic turn that is far less demanding.

So, it is it is actually below all these lines. So, subsonic turn requirement actually can be better than what is specified in the minimum then you have specific excess power once again we find that it is not really a demanding constraint, because that line is also immersed completely below the first 3 lines. And finally, we have the line for takeoff which also is not at all consider a demanding constraint in this case.

See, we do not know this thing a priori. So, we need to do these calculations and only at the end you come to know which constraint is more important or not just by tightening a particular constraint, I can make that constraint as more binding. Now, these were the constraints that depended on the values of both thrust to weight ratio and wing loading. So, since we have lines between these 2 which are either curved or straight.

We can clearly understand that all these constraints as you see on the screen, they depend on W/S and T/W together, but there are some constraints which depend only on T/W and only on W/S. Now, for our military aircraft, the constraints which depend on T/W alone are not normally active. So, we will not we have not even got those values in our constraint table. But, this vertical line corresponds to the constraint on the instantaneous turn due to which the area on the right of this line is infeasible.

So, this particular vertical line is the first constraint that depends only on the blue bias, the instantaneous turn specified, and then we have a landing constraint. So, it turns out that the constraint on landing in this case is more critical than constraint on the instantaneous turn as far as the vertical line is concerned. So, if you meet the constraint for the landing, you have automatically met the constraint for the instantaneous turn, but there is another constraint which is on the stalling speed.

And it turns out that that is more important in this case compared to the other 2. So, if you see now, we have all the 9 constraints now on the diagram. So, now, the feasible region would be the area on the left of the vertical red line corresponding to stalling speed and the area above the red line corresponding to the constraint on the maximum Mach number. So, therefore, in the constraint diagram this is the complete constraint diagram for all the constraints put together.

And we have this intersection point which is the point that corresponds to just meeting all the constraints, the value of Wing loading and thrust to weight ratios are as mentioned, but we would not choose this point as our design point we would choose the design point to be the one that has got some margin, margin for future growth, margin for increasing the weight of the aircraft margin for increased performance requirement.

So, what is prudent is to choose a point that is slightly above and slightly on the left of this point, such as something like this. So, just to round off the numbers, I have taken the designed to be wing loading of approximately 4 and half kg per meter squared around 10% Wing loading better and I have taken it 0.2 as the margin and thrust to weight ratio. So, I have taken the design point to be 428 kg per meter square. And thrust to weight ratio of point nine eight okay. So, this is how we obtain the values of the thrust to weight ratio and the wing loading that meet all our constraints.

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Now, let us see, we have done these calculations for the F 16 C aircraft and we need to really have an idea that all these classroom calculations how far are we from the reality.

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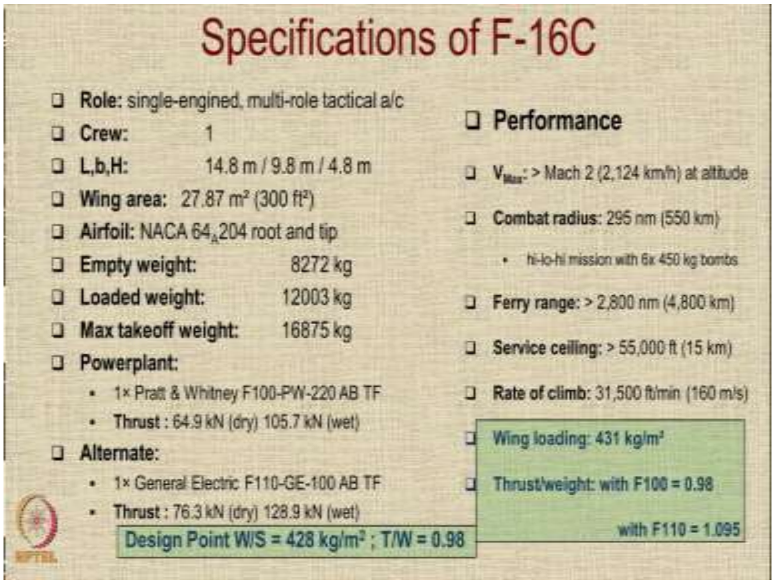

So, let us have a look this is the actual specifications of F 16 C aircraft, in this particular table the L, b and H stands for the length the wingspan and the height respectively. And we also have some information regarding the performance of the aircraft we have incorporated the constraint on the rate of climb and also on the maximum Mach number but combat radius ferry range service ceiling we have not considered because it was not given to us in the in our constraint table.

The actual wing loading of the of this aircraft and the actual cost loading is as shown 431 and 0.98 with F 100 and little bit higher with F 110 engine and the value that we got by our basic calculations was very near to that. So, what it shows is that the calculations that we have done are not very far away from reality. In actual aircraft design far more serious and critical constraints might be considered in the constraint diagram but this is just an example.

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Let us also look a little bit about the armament of F 16 aircraft. This aircraft has a large amount of option for carrying armament as you can see in this figure, but the standard ammunition that is fitted on the wingtips of this aircraft is the AIM 120 advanced medium range air to air missile or AMRAAM.

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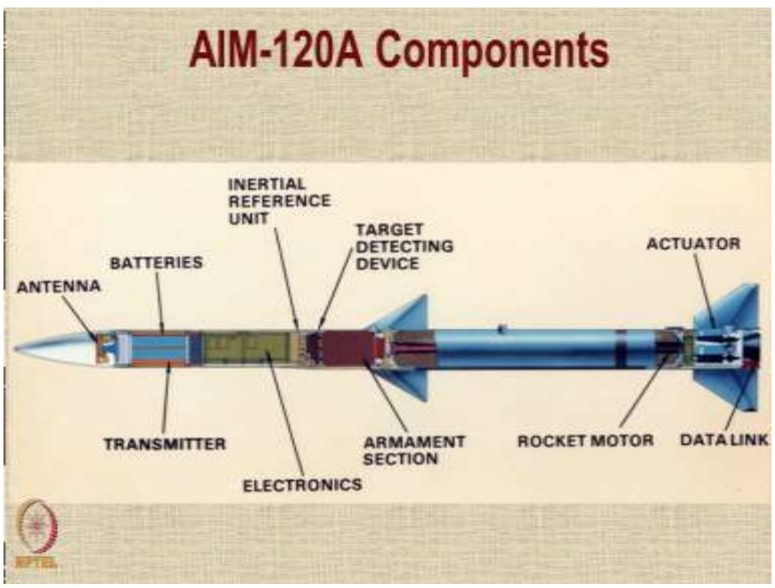

AIM 120 is a missile which as I said is a standard fit on this aircraft. As you can see, it has an antenna in the front and the nose cone which is transferred into the antenna so that you can you know travel towards your target it has batteries and transmitters then there is a inertial reference unit which allows you to you know, measure at what rates you are turning, there is a target detection device. And there is armaments section which is the charge it is powered by a rocket motor. And there are actuators available on the rear to manoeuvre the missile so that it can you know hit the target.

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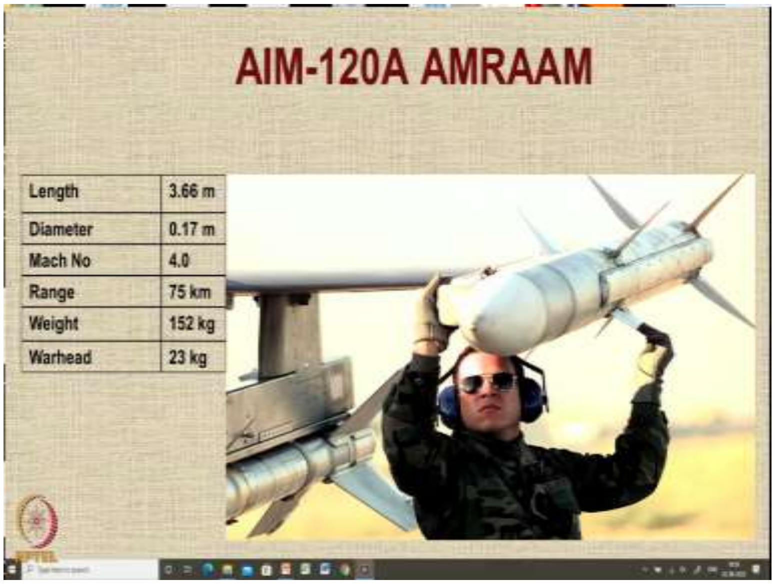

So here is an example of how this design is mounted and fitted at the tip of the; you can see actually 2 of them. But the one at the extreme tip is the one that I am talking about the AMRAAM AIM 120 missile. Here is the data related to this particular missile. You can see that the warhead in this missile is approximately 23 kilograms. And it weighs around 152 kilograms, it is quite heavy, it can travel at Mach number 4 and it has a range of 75 kilometers.

So if you want to out manoeuvre this particular missile, you have to ensure that you are away by at least 75 kilometers otherwise it is going to come at you with Mach 4 and it is very difficult to survive.

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So let us have a look at a small video clip showing how this particular missile works. So you can see the missile on the wingtips mounted here and there it is released proceeding at Mach number 4. And there is another video that will show the firing of the missile and here is a capture of the missile hitting the target.

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Another very interesting weapon that is mounted in this aircraft as a standard fit is the Vulcan.

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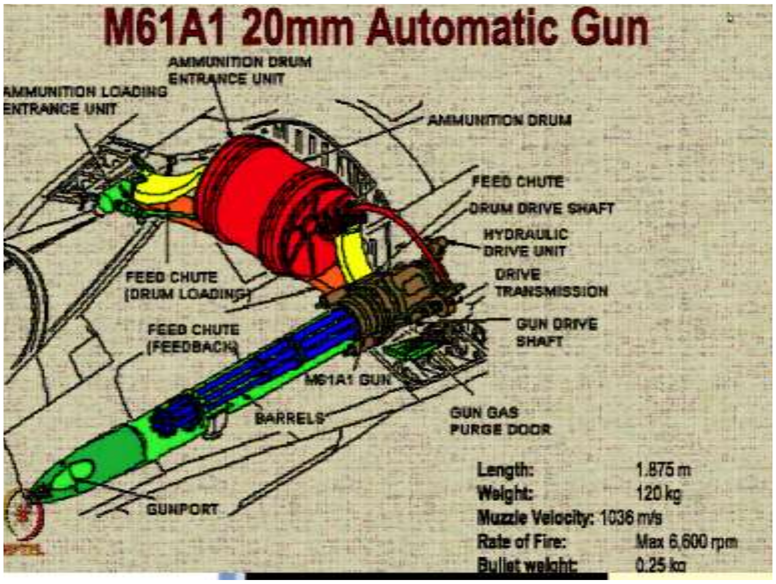

M61 Vulcan air to air cannon air to ground cannon. You can see that this particular cannon is actually quite heavy it is around 120 kilograms and it fires 20 mm Dia bullets, each of the bullet is around 250 grams and it fires at the rate of it can fire up the rate of 6600 RPM. Normally it is fire around 4000 rpm. So this is another very interesting weapon that is installed as a standard fit on the F 16 C aircraft. Thanks for your attention.