**Introduction to Aircraft Design**
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**Lecture - 65**
**Tutorial on Constraint Analysis of Military Aircraft: Part - 1**

Let us have a look at how we can do constraint analysis for a military aircraft. And as an example, we have chosen this very popular and very famous aircraft, which I am sure all of you can recognize by the photograph.

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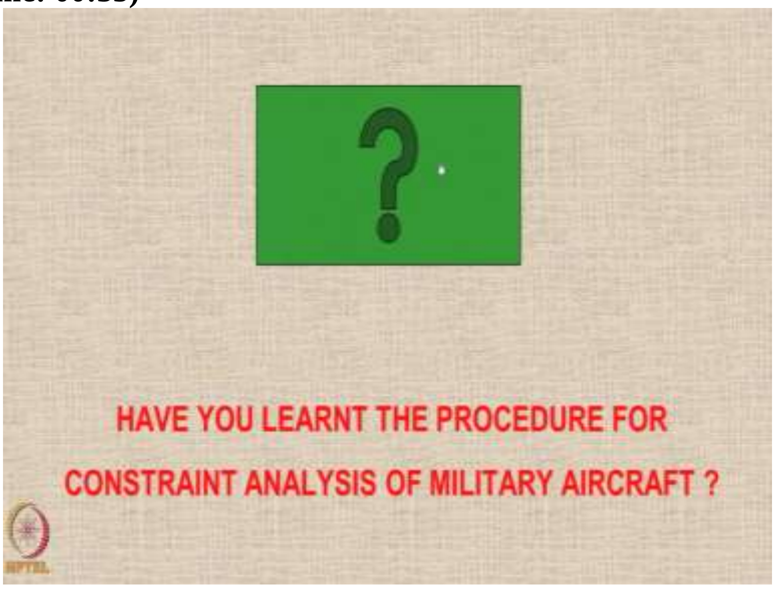


It is a single seat jet engine fighter aircraft. And before we go ahead with the tutorial, I would like to start with a question. You see, this tutorial cannot be appreciated or understood unless you have gone through the procedure for constraint analysis for military aircraft. If you have learnt the procedure, then of course, you can go ahead otherwise, I would request you to go ahead and get yourself familiar and then come again to this tutorial.

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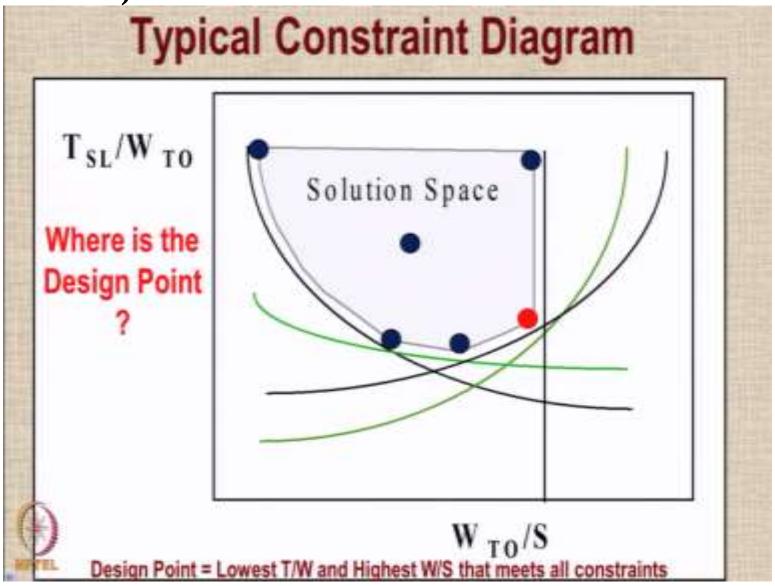


Let us look at a typical constrained diagram which is as shown in the figure where on the x axis we have wing loading $W_{TO}/S_{REF}$. And on the y axis, we have the sea level static thrust divided by the W takeoff of the aircraft the area on the right of the vertical lines and below the curved lines is infeasible. So, the possible solution space is as marked in the figure, please note that the top horizontal line is just a theoretical line in reality you can go up to any value of T/W in the vertical direction.

So, the question is where is the design point? Of course, the design point has to be any point within the solution space. So, it could be this point or it could be this one or the top left junction or the top right junction or any other junction or it could be any point within the solution space. Any of these points could actually be the design point. The question is, which is the design point you will choose?

So, the design point would correspond to the lowest value of the thrust to weight ratio and the highest value of the wing loading that meets all the constraints. And hence, we would choose a point at the rightmost junction of the lines as we have seen in the red colored point in the figure.

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## Mission Requirements (one by one)

| Parameter | Value | Operating Condition | Thrust |
|---|---|---|---|
| Stalling Speed | 300 kmph | 1-g @ Sea Level @ MTOW, $C_{Lmax}$ = 1.0 | Dry |
| Sustained Turn Rate (subsonic) | 9 g | 1500 m AMSL, M = 0.9 | Wet |
| Sustained Turn Rate (supersonic) | 4 g | 9000 m AMSL, M = 1.2 | Wet |
| Instantaneous Turn Rate | 18 deg/s | 6000 m AMSL, M = 0.9, $W_{man}$, $C_{Lmax}$ = 1.0 | Dry |
| Maximum Mach Number | 2.00 | 20000 m AMSL, $W_{man}$ | Wet |
| Specific Excess Power | 150 m/s | 1500 m AMSL, M = 0.9 | Wet |
| Maximum Rate of Climb | 160 m/s | $W_{TO}$, V = 500 kt, ISA | Dry |
| Takeoff Ground Roll | 1000 m | 1000 m AMSL, Max. $C_L$ @ TO = 1.27 | Wet |
| Landing Ground Roll | 1000 m | 1000 m AMSL, Max. $C_L$ @ Landing = 1.43 | Wet |

**Estimation of Maneuver Weight**

$W_{TO}$ = 16875 kg

$W_{man}$ = Weight with 50% fuel + Missiles + Gun = 9862.5 kg

So let us see the mission requirements of this aircraft and let us familiarize ourselves with the requirements one by one. The first requirement is the stalling speed. So it is specified that with engine in the dry condition that means with no afterburner or no reheat this aircraft should have a stalling speed of no more than 300 kilometers per hour. This is the 1 g stalling speed at sea level at max takeoff weight.

In other words level flight at sea level with maximum takeoff weight and the $C_{L_{max}}$ in this condition is assumed to be 1 this is determined by the aerodynamic analysis for this aircraft, which I presume has been already covered in the past. So you can assume either we can assume that the aerodynamic data of the aircraft is made available to us or we assume that we have estimated it ourselves earlier. The next requirement is sustained turn rate in the subsonic condition.

This aircraft is required at 1 and 1/2 kilometers above mean sea level at a Mach number of 0.9 to pull 9 g with an afterburner on so therefore, it will be the wet thrust. Then in supersonic condition, which is at 9000 meters above mean sea level at Mach 1.2. Again with afterburner working this aircraft is expected to pull 4 g's the instantaneous turn rate is no less than 18 degrees per second at 6 kilometers above mean sea level at Mach number equal to 0.9 $C_{L_{max}}$ is again 1, but the weight in this case is what is called as the maneuver weight.

Now the maneuver weight is essentially the weight of the aircraft with half the fuel gone and the 2 missiles and the onboard gun still remaining with the ammunition in the gun. In this case, the aircraft carries 2 AIM 120 AMRAAM missiles on the wingtips and there is a gun which has got some amount of ammunition. So the data that is available is that the takeoff weight the maximum takeoff weight is 16875 kg, but the maneuver weight is only 9862.5 kg.

This is the typical weight of the aircraft when it is in a maneuvering condition at that condition we want to have an instantaneous turn rate of at least 18 degrees per second. The maximum Mach number of the aircraft at the same maneuver weight but at 20 kilometers altitude above mean sea level with afterburner working is expected to be 2, the specific excess power at subsonic conditions of 0.9 or I would say transonic conditions of 0.9 Mach number.

And at the height of 1500 meters above mean sea level is expected to be 150 meters per second. And once again the engine can be with afterburner on the maximum rate of climb of the aircraft is expected to be 160 meters per second. Normally, the rate of climb is measured in meters per minute. So, this would go to around 315,000 I think, meters per minute at maximum takeoff weight with a forward speed of 500 knots under ISA conditions, but during this condition, you are not allowed to use the afterburner.

So, the engine is dry and then we have the takeoff ground roll requirement which is less than a kilometer at an altitude of 1 kilometer above mean sea level at which the maximum $C_L$ during takeoff is expected to be 1.27 and here you can use the afterburner. And finally, the 9th requirement is that the landing ground roll should be no more than 1000 meters again at the same condition of 1000 meters above mean sea level, the airport location and $C_{L_{max}}$ at landing would be 1.43 with the flaps deployed and of course, the engine has no role in this case. But, so, what we can do is these are the requirements.

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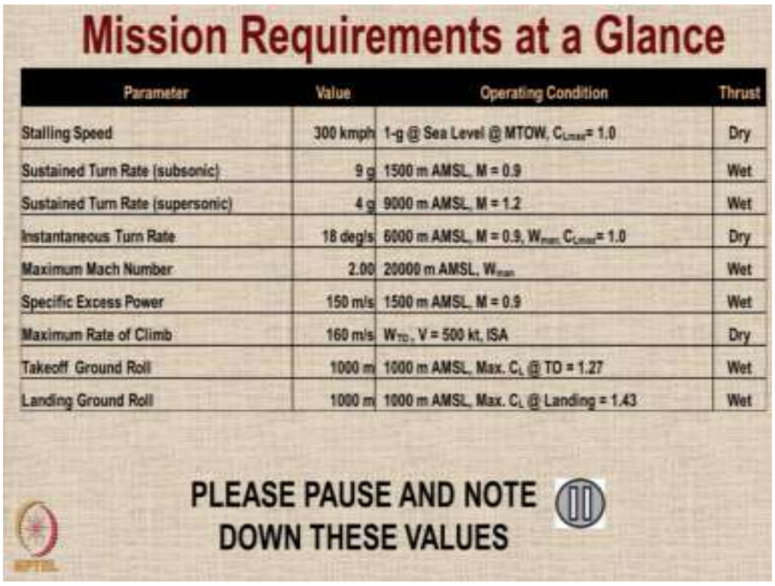

# Mission Requirements at a Glance

| Parameter | Value | Operating Condition | Thrust |
|---|---|---|---|
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Now, here are the requirements at a glance. And at this point of time, I would like you to please pause and note down these values on a piece of paper or in any other location where you want to, because you will be required to use these numbers when we do the calculations. So, what we will do is we will do 9 calculations for each of these constraints, try to then see how the diagram looks like and then pick up the design point.

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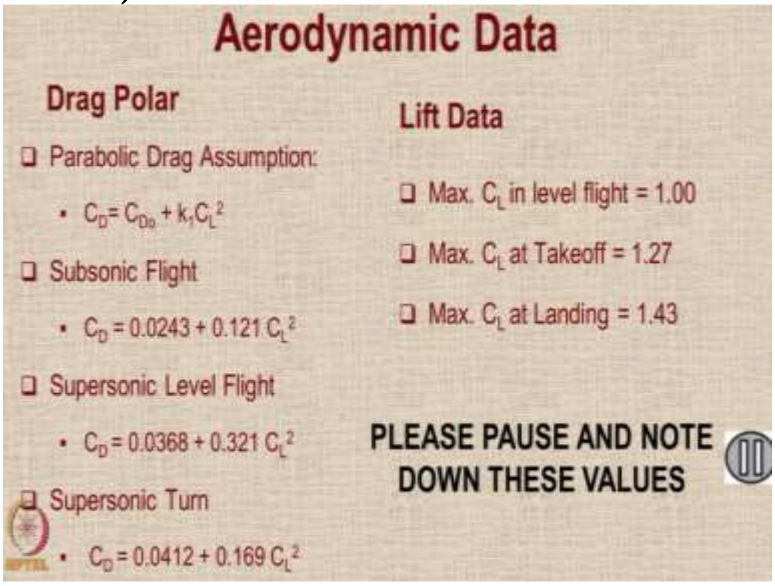


Now, as I mentioned to you, the aerodynamic data related to this aircraft under various conditions is expected to be known at this condition. So, either it is at given data or it is estimated by the methods that have been already discussed in the aerodynamic estimation techniques. So, I just want you to quickly have a look at what the data is assumed to be. So, we assume first of all that this aircraft follows a parabolic drag polar.

So, in subsonic flight, the value of $C_{D_0}$ is expected to be 0.0243 and the value of $k_1$ is 0.121 in level flight in supersonic conditions, the value of $C_{D_0}$ increases because of the wave drag and also the value of k increases from 0.121 to 0.321 this is because of the reduction in the aerodynamic efficiency. In supersonic turn, we expect a further degradation in the value of $C_{D_0}$, but improvement in the value of $k_1$, the lift data is expected to be as follows in level flight without any flaps, etc.

It would be 1.0 the maximum aircraft $C_L$ at takeoff with flaps at takeoff condition it would be 1.27 landing, the $C_{L_{max}}$ would be 1.43. So, again, I would request you to note down these numbers because these numbers are going to be used in the calculations that we are going to carry out.

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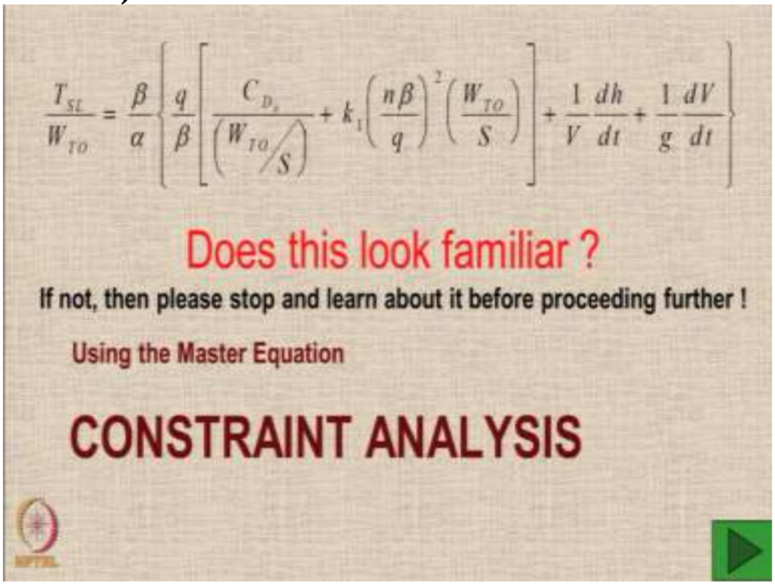


Let us look at now the constraint analysis using the master equation. So, just to refresh your memory, here is the master equation. And I am sure it looks familiar to you if you have already learned or gone through the lecture on constraint analysis for military aircraft. But if it does not look familiar to you, and if you feel that you do not appreciate what it is, then again this is a point where you should stop and no point proceeding further because you will not be able to appreciate what is being discussed. So, assuming that you know this equation and you are familiar with it, I would like to proceed further with the analysis.

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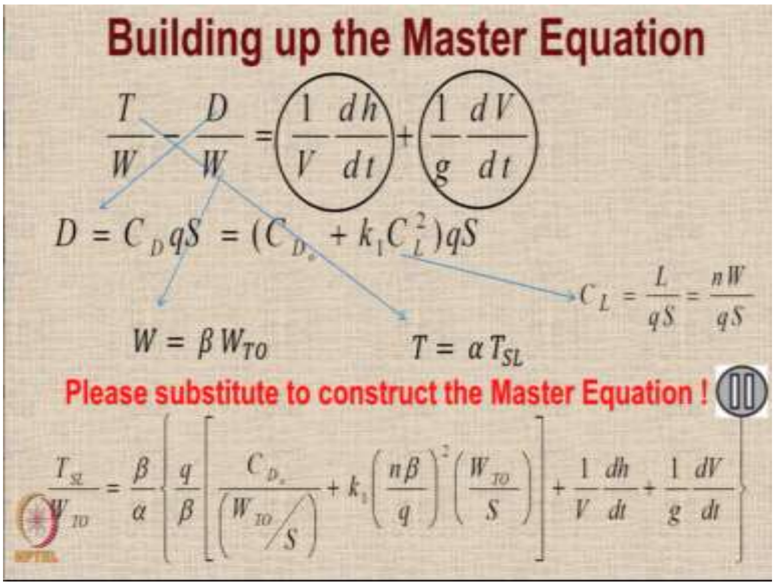


But just to refresh your memory, let me just show you how this equation has come about. It has come about from a very simple expression that the specific excess power that is excess of the thrust over the drag divided by the aircraft mass so (T – D)/ W. If you multiply this with velocity you get the specific excess power, but (T – D)/W is the specific excess thrust and that is equal to

$$\frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$$

where dh/dt stands for the steady climb and dV/dt stands for the acceleration where V is the forward velocity.

Now, you can use your excess thrust or specific excess thrust either to increase your dh/dt that means to climb or to increase your speed that is to accelerate with dh/dt = 0 or a combination of the 2. So, you could go for a steady climb with dV/dt = 0 but dh/dt present or you could go for a steady acceleration which dh/dt = 0 and dV/dt present or you could go for a mix of the 2. Now, let us see how we can expand this into the master equation.

So, the D or the drag can be replaced by $C_D qS$ where $C_D = C_{D_0} + k_1 C_L^2$. So, wherever you see $C_L$ you can replace by $\frac{nW}{qS}$ or $\frac{n}{q}\frac{W}{S}$, the W is the weight of the aircraft and that does not remain same throughout the mission.

So, for any condition at which the constraints are calculated the weight is the local weight and that weight is equal to beta times $W_{TO}$ where beta is the weight parameter beta = 1 when you are at a take-off condition beta is equal to let us say 0.8 or 0.7 when you come into land and beta can be any other value depending on the condition at which the constraints are being evaluated. So, replace all W's by beta W takeoff.

Similarly, the thrust is also not going to be the same in all the mission segments or in the segments wherein wherever you carry out the constraint analysis. So, in general, we can say that T will be equal to alpha times $T_{SL}$ where alpha is the thrust lapse factor this number can be less than 1 as you go for higher altitude from sea level or it can be more than 1 for example, if you put an afterburner it may become more than 1.

So, it all depends on the operating condition. So, all the T terms have to be replaced by alpha $T_{SL}$. So, if you do this, that means, if you just do this yourself at this point, I suggest that you take a pause. So, wherever you see this particular symbol in this tutorial, where there are a circle with 2 vertical lines, that is an indication for you that this is the time for you to pause and to do some calculations, you can pause you can mute you can pause the video and do the calculations and then when you finish the calculations you can resume.

So, for example, if you replace all the values as shown in the master equation, and then you just rearrange you can come up with this expression for the master equation, which gives you an equation between $T_{SL}/W_{TO}$ and $W_{TO}/S$ as a function of the various aircraft operational requirements and other performance parameters.

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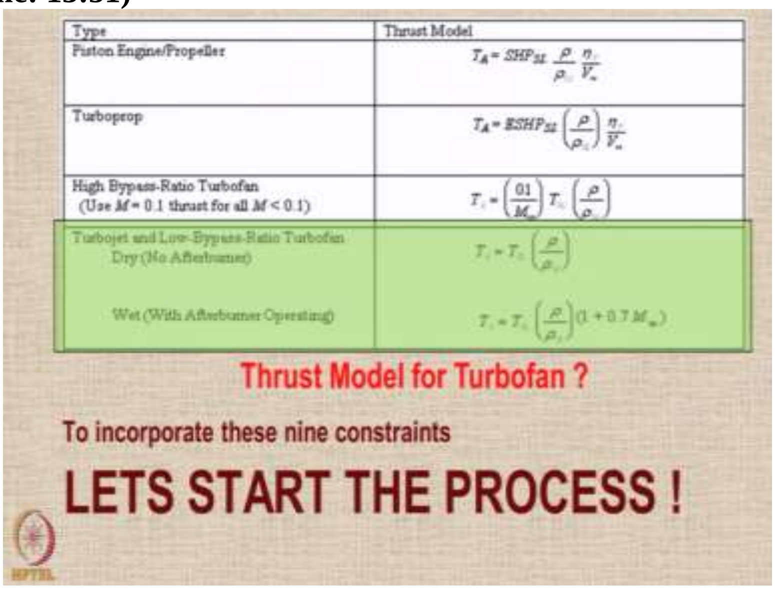


So, let us start the process of constraint analysis to incorporate these 9 constraints. Now, before we go ahead, we need to understand that this aircraft is powered with a single turbofan engine.

And so, we need to know how to get the value of alpha or the thrust lapse parameter. So which model will be used for the turbofan if you recall, I have just taken an image from the lecture on constraint analysis where various thrust models were suggested for types of aircraft like piston prop, turboprop, high bypass turbofan.

And turbo jet with low bypass turbofan so, in our case, the aircraft is a fighter aircraft, so, it will be having a low bypass ratio turbofan engine or a turbo jet engine. So, therefore, under the condition of dry that is no afterburner or no reheat the thrust at any condition will simply be equal to the thrust at sea level into the density ratio but if you are going to put an afterburner then you have to add a multiplicative factor of 1 + 0.7 times Mach number and with that you will get the value of T. So, T available upon T sea level is the value of alpha.

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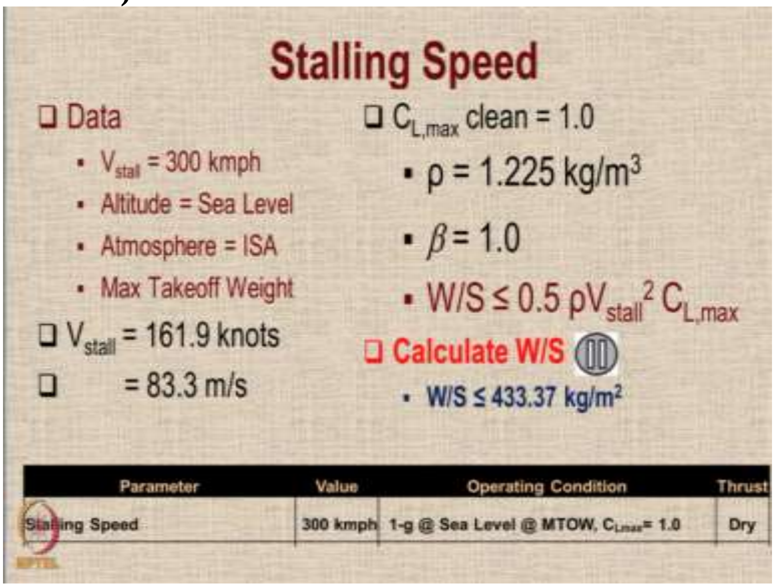


So, the first constraint is the stalling speed constraint, this is one of the simplest constraints to incorporate. So, I am just reproducing here the requirement for the stalling speed. So, as you can see from the data in the table on the bottom line, we are already given the requirement of stall speed of less than or equal to 300 kilometers per hour at sea level under ISA conditions with maximum takeoff weight the conversion of the speed 300 kilometers per hour into knots results in 161.9 this you get by simply dividing by 1.853.

And then if you convert that into meters per second, you get 83.3 meters per second. So, you can see that the stalling speed requirement is less than or equal to 83 meters per second, we are also told that the $C_{L_{max}}$ is 1.0 in the clean condition when it is in a 1 g stall condition. So, therefore, and at sea level the density is a standard value which all of us are remembering, we are going to assume beta = 1 because we are assuming that this happens just after takeoff. So, therefore, there is no loss in the weight of the aircraft.

So, from simple balance of forces lift is equal to weight and it is n = 1 condition. So, $W = \frac{1}{2}\rho V^2 S C_L$ and from there you can get that to maintain a stall speed of less than 300 Kmph, we need to have during loading less than equal to $\frac{1}{2}\rho V^2 S C_{L_{max}}$. So, with this you can straightaway calculate the value of W/S by inserting the values of $\rho$ which is 1.225 $V_{stall}$ which is 83.3 $C_{L_{max}}$ which is 1 and if you do that, it will take.

So, if needed you can pause the video and do the calculation the number will come out to be 433.37 kg per meter square. In other words, to meet the stalling speed requirement of 300 kilometers per hour or less, the aircraft has to have a wing loading of up to 433.37 or less.

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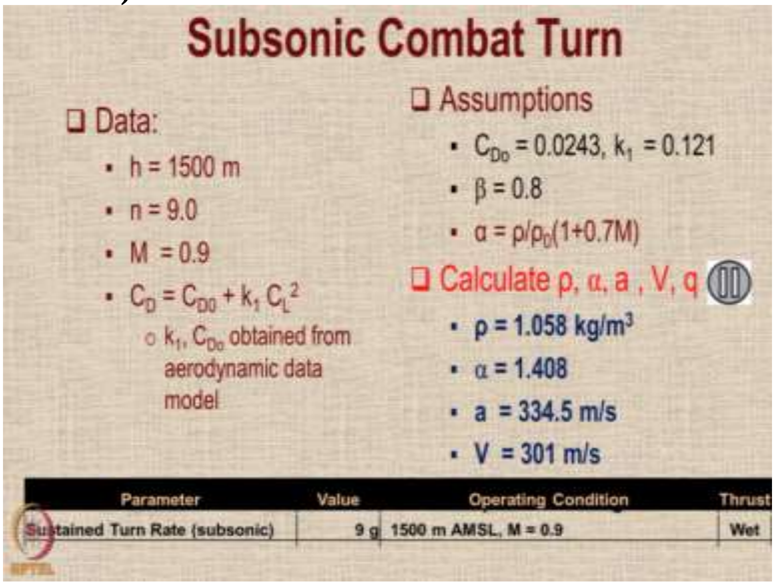


The next constraint is the subsonic combat turn. And for this the requirement reproduced from the table shown earlier is given on the bottom line. So, what we see here in the data is that the height is 1500 meters above mean sea level the Mach number is 0.9 the load factor in the turn is 0.9 sorry is 9 g and the value of aerodynamic coefficients $C_{D_0}$ and $k_1$ have to be obtained from the aerodynamic data.

We know from our information that these values are 0.0243 and 0.121 for the subsonic or the transonic condition. And we assume that the beta is pointed so some amount of fuel has gone during in the process of climbing and reaching out to the place where the combat turn happens.

So, you need to calculate the values of parameters $\rho, \alpha \; and \; a$ is sonic speed to get velocity V and dynamic pressure q. So, once again, I would like you to pause the video and do these calculations and work out the values of these parameters. Once you get these numbers, you will see that the parameters that you calculate are as shown on the screen. So, note down these parameters, I am sure you must have calculated them. If not, you can just note down these parameters because we will need them in the next slide when we go in for the calculation

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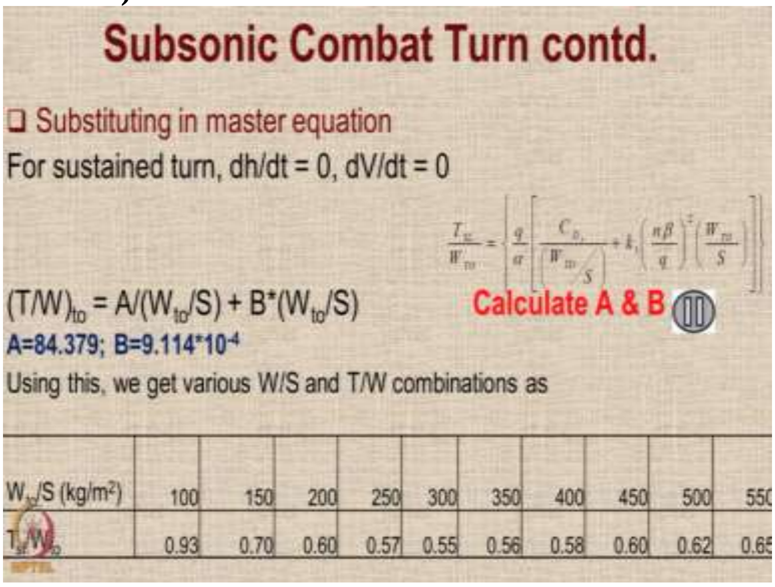


To continue on subsonic combat turn what we do is in the master equation, because it is a subsonic combat turn we have no change in the altitude no change in the velocity this is a sustained turn. So therefore, we cannot change the velocity we cannot change the altitude. So, what you do is you just put dh/dt and dV/dt=0. So, the last term of the expression vanishes, and you get a truncated master equation.

So, you can calculate the values of A and B, and then we will get a simple expression between $T_{SL}$ and $W_{TO}$ which can be plotted.

So, again, I would like you to pause at this point do the calculations for A and B. And once you have done the calculation, you can match with the values which I will show you. So, the values of A and B turn out to be as shown in the screen inserting these values in the expression for T/W as a function of W/S, you can actually get a table and you can notice that if you have a wing loading of say 300 kg per meter square, the thrust to weight ratio required for this condition comes out to be 0.55.

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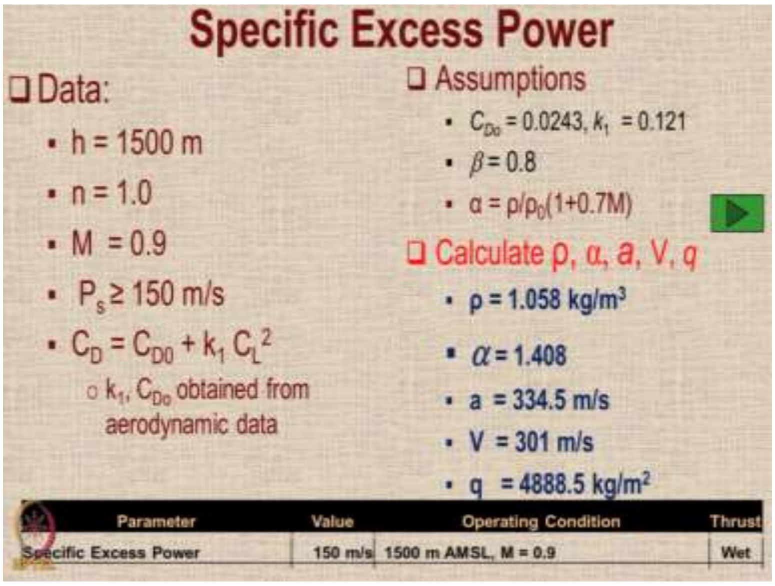


Moving ahead let us look at the specific excess power which is again specified in the constraints as shown in the line in the bottom. So, what do we see, we see that the height is 1500 meters, this is the level flight so, n = 1 the Mach number is transonic 0.9 and in this condition we want the specific power to be 150 meters per second or more the aerodynamic data is to be obtained from the information available.

So, moving ahead, we see that the $C_{D_0}$ and $k_1$ values are obtained from their dynamic information. Now, let us see what happens so, beta again is assumed to be 0.8 it is a similar assumption that this requirement is there when the aircraft has lost some amount of fuel and it is gone for combat and since it is a wet condition again the value of alpha will be obtained using this standard expression at this point.

We can calculate the values of rho the density of the ambient air at 1500 meters alpha using the expression shown on the screen once you know $\rho_0$ and the Mach number which is given as 0.9 then you can calculate the value of sonic speed a because you know the value of temperature at 1500 meters you can calculate the V because you know the Mach number and you know $a$ and since you know the values of dynamic  pressure q.

So, once again I would like you to know if you can but you know understand one thing this calculation is not needed, why because in the previous calculation, the conditions were identical we had the same altitude same Mach number and the same engine condition. So, what we can do is we can just reproduce those values here, because the values of $\rho, \alpha, V, q$ will not be changing because the conditions are the same that is why I have taken this particular constraint out of the sequence for it ease and calculation.

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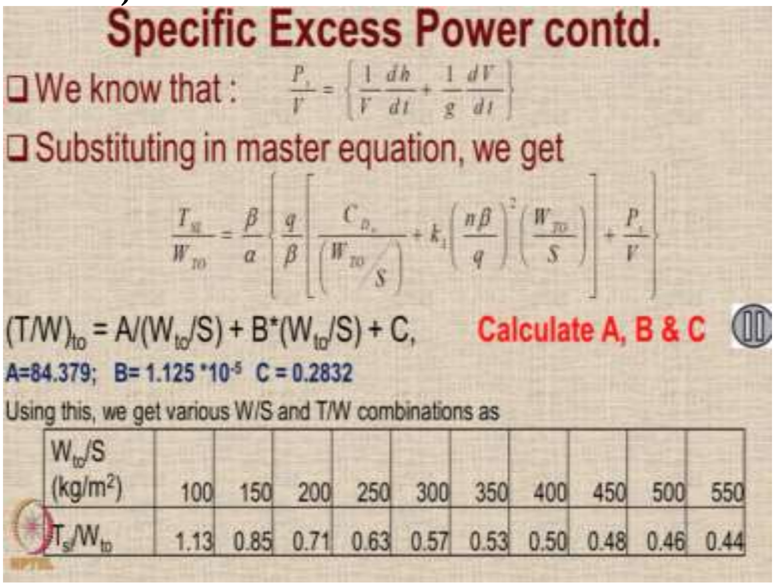


So, once again, let us continue ahead once you have the value of these parameters, now, the specific excess power can be shown to be equal to this expression as shown. So, if you just take you get this expression. So, what you can do is in the master equation.

Now, you have to just insert the values of the parameters $\beta, \alpha, q, C_{D_0}, k_1, n$ and by that you will be able to get the values of the 2 constants A, B as earlier, but now, there will be another constant C. So, please take a pause and calculate the values of parameters A, B and C so, here are the values that have been calculated.

And using these you can get an expression between T/W and W/S. And with that you can get a table which can allow you to calculate the value of the thrust to weight ratio for any wing loading. For example, for a wing loading of 400 kg per meter square, you need a T/W value of 0.5.

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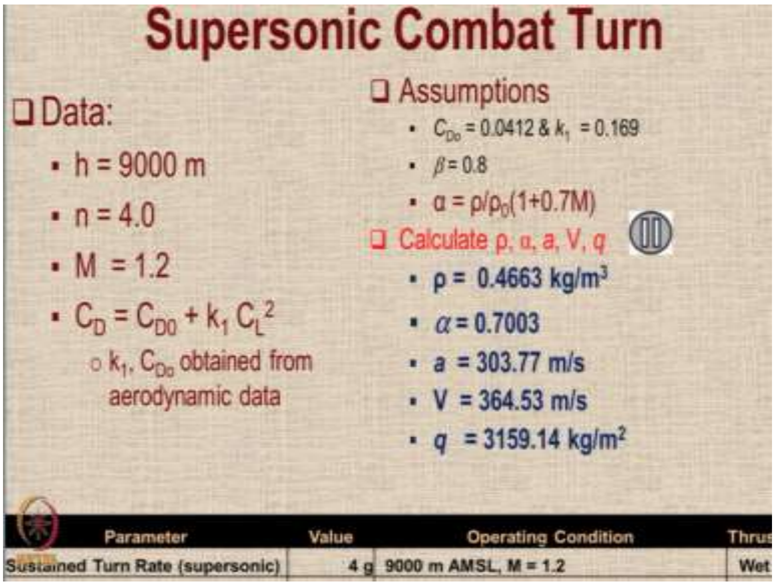


Let us take the supersonic combat turn, which is as mentioned in the requirements, it is a sustained turn. And it happens with value of 4g at 9 kilometers above mean sea level with afterburner working and Mach number equal to 1.2. In this case, the data is just reproduced from the requirements and the aerodynamic data will come from our previous assumptions as we have mentioned the value of beta and alpha will also be the same as assumed earlier.

So, therefore, this is now a time for you to pause and calculate. So, using the atmospheric tables or using simple formulae that are available, you have to calculate the values of rho at 9 kilometers in ISA conditions with that you will get the value of alpha and since you know the temperature at 9 kilometers, you can get the value of sonic speed a since, you know the Mach number as 1.2 you can get the value of V.

So, please pause the video at this point do these calculations for $\rho, \alpha, a, V$ $and$ $q$. So, values obtained are as shown on the screen.

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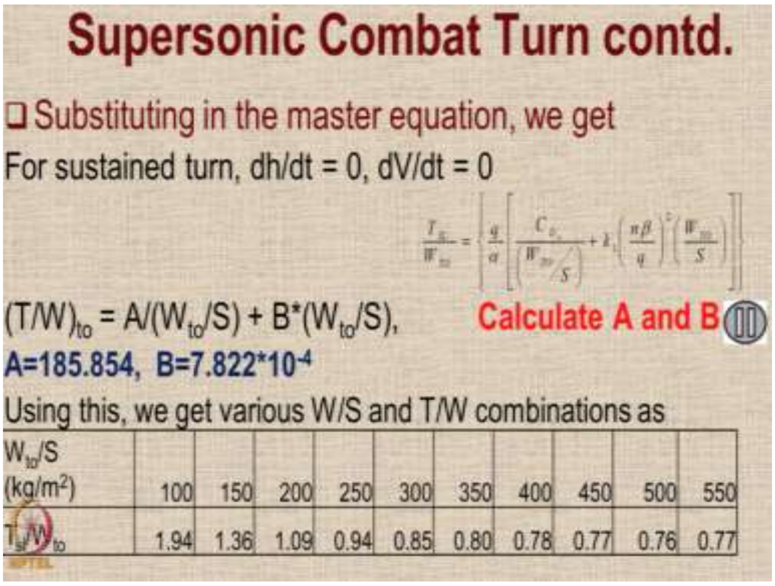


And moving ahead if you substitute in the master equation for a supersonic combat turn, which is a sustained turn, therefore, there will be no terms on the right hand side $\frac{1}{g}\frac{dV}{dt}$ etc will cancel out. So, you will get a simple expression. And with that, you should be able to again calculate the values of A and B. So, once again I will take a pause here, you should pause the video calculate the values of A and B.

And then match with the values which are shown here on the screen. And with that this particular table can be created. So, as this table shows, if you want to have a wing loading of 400 kg per square meters, then the thrust to weight ratio requirement increases to a value of 0.78.