Introduction to Aircraft Design
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Lecture - 64
Constraint Analysis - Military Aircraft

Let us have a look at the procedure to be followed for carrying out the constraint analysis of multi role military aircraft.

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First let us have a look at the typical constraints which are specified on a military aircraft mainly they come from the customer requirements, they could be constraint like sustained turn rate at specified Mach number and altitude sustained turn rate means, the ability of an aircraft to turn at a particular degrees per second in the yaw plane and to maintain that particular turn rate without any loss in speed or loss in altitude. Then you have instantaneous turn rate this is similar to the sustained turn rate.

But in instantaneous turn rate we are permitted to sacrifice either speed or altitude to achieve a turn rate. So, this is a momentary turn rate which cannot be sustained. Hence, it is called instantaneous whereas, a sustained turn rate is something that you need to maintain. Then maximum Mach number at a specified altitude is the ability to fly at that particular Mach number at a given altitude.

One important performance attribute of an aircraft is specific excess power that is the excess power over power required divided by the weight of the aircraft or it is also called as SEP, this

SEP can be traded to achieve either acceleration or climb or both. So, at a given Mach number and a given altitude, sometimes a specific excess power that needs to be possessed by an aircraft is explicitly specified. Climb rate while operating at a given altitude is another performance parameter the stalling speed of the aircraft in level flight.

So it is 1 g stalling speed at a specified weight and altitude. Takeoff and landing ground roll under ISA conditions or under ISA+X or the off ISA design conditions. Absolute and combat ceilings once again under either ISA conditions or under specified ISA+X conditions. These are the typical requirements which the customer specifies. There are also some airworthiness requirements which may be present and that is a function of the regulatory bodies or the airworthiness agencies, which are having a sovereign control over the military aircraft or the domain in which it is operating. So, there is no specific requirement it is as specified.

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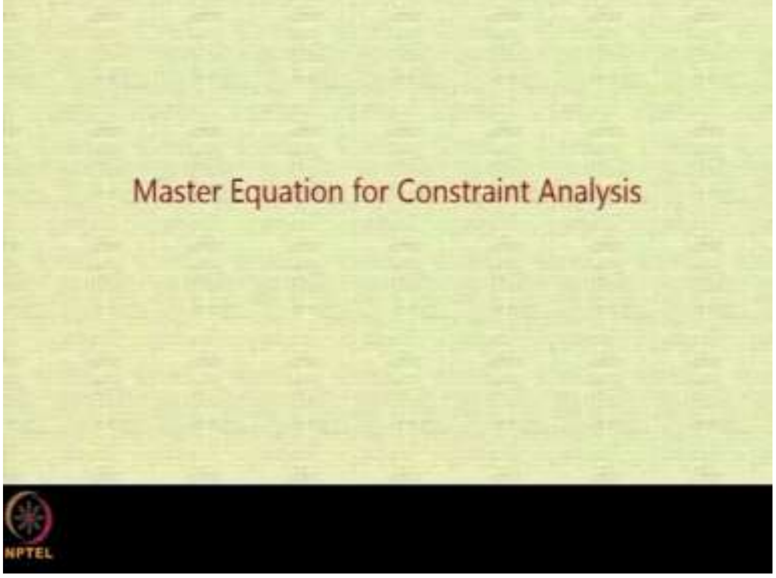

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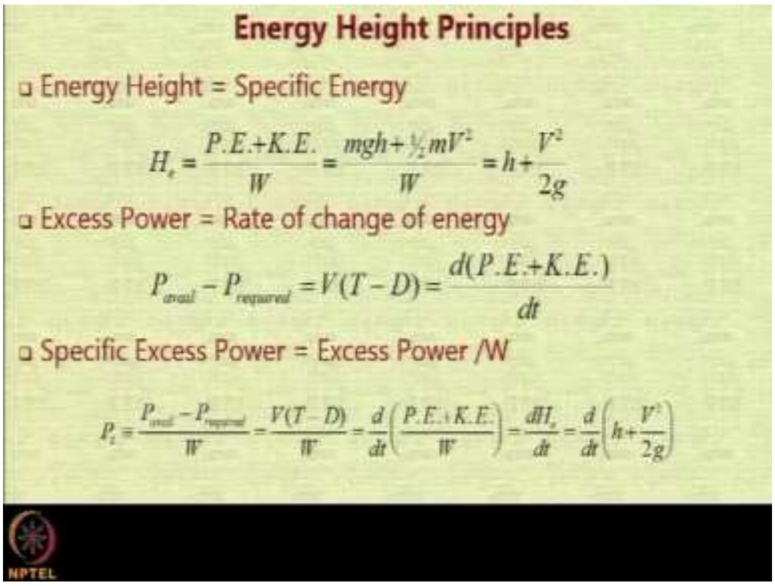

For a military aircraft it is easier to look at a master equation for constrained analysis. And this master equation comes from the energy height principles. So, the energy height basically is specific energy or the excess energy divided by the aircraft weight. The energy that an aircraft possesses is the summation of the potential energy and the kinetic energy that is $mgh \text{ and } \frac{1}{2}mV^2$, when you divide that by aircraft weight, you get specific energy. Now, W is equal to basically mg.

So, if you know do a simple algebraic manipulation, the specific energy turns out to be

$$H_e = h + \frac{V^2}{2g}$$

Now, V is in meter per second and g is in meters per seconds square. So, you know, if you look at the units of this specific energy, they will come in meters. So that is why we call it as energy height. So, the energy height, $H_e = h + \frac{V^2}{2g}$. Now, the excess power that an aircraft possesses is basically equal to d / dt of the energy or the rate of change of the energy.

So, the excess power that is $(T - D)V$ where T into V is the power available and V into D is the power required. So, the excess power is the rate of change of the total energy and hence, the specific excess power will be excess power divided by the weight of the aircraft. In other words, P_s which is the symbol used for specific excess power it will be

$$P_s = \frac{P_{available} - P_{required}}{W} = \frac{(T-D)V}{W} = \frac{d}{dt}H_e = \frac{d}{dt}\left(h + \frac{V^2}{2g}\right)$$

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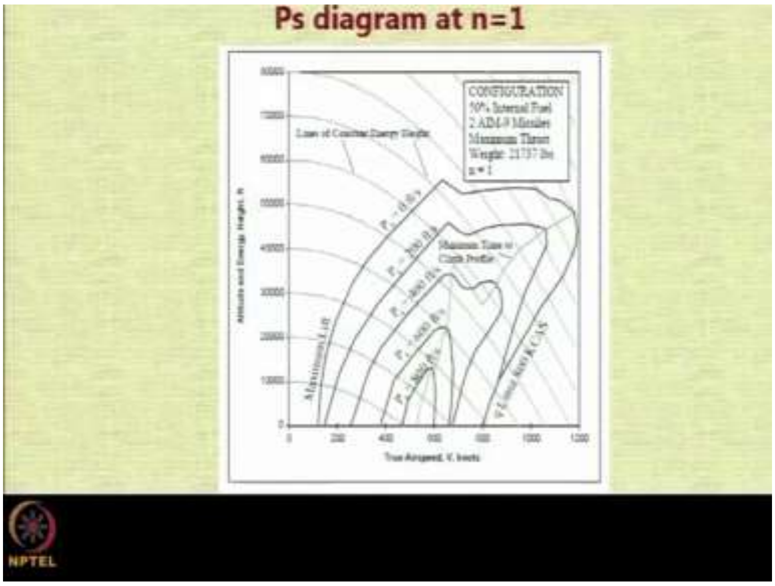

So, using this expression, one can actually draw a diagram like this for various values of P_S. So, this particular diagram is a P_S diagram at n = 1 and it kind of gives you some maneuvers limit that the aircraft has, we will come back to this in more detail a little bit later.

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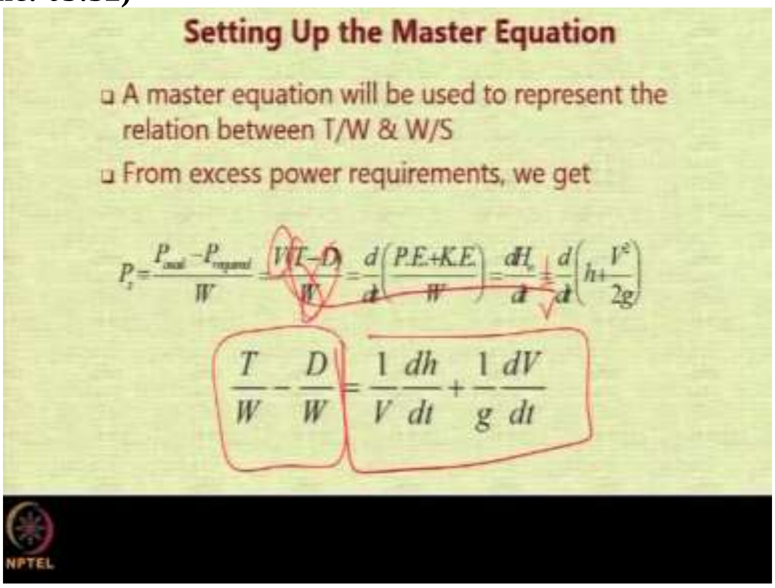

So, let us see how we can set up the master equation, a master equation is an equation which will be used to represent the relationship between T/W and W/S because ultimately in a constraint diagram, we are going to plot T/W on the y axis and W/S on the x axis. So, if we can get an equation between these 2 parameters, then we can use that equation directly to plot the constraint diagram.

So, you know it can be shown that if you basically take this V in the denominator you know, you will get

$$\frac{T}{W} - \frac{D}{W} = \frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$$

So, by cancellations you get this expression so, this is the basic master equation. Now, we have to also keep in mind that this equation has got terms like T, W, D, V dh/dt so, let us see one by one.

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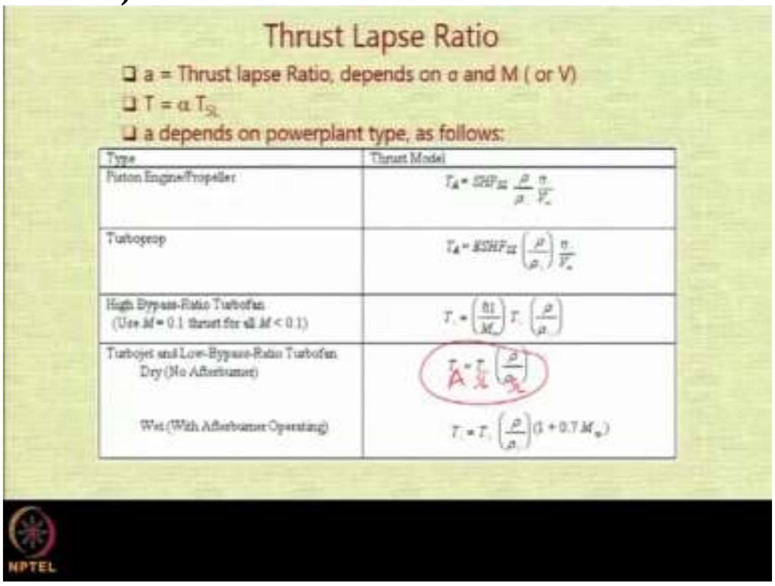

Let us first look at T, T is the thrust and the thrust is not constant at all conditions. So, in general you can say that $T = \alpha T_{SL}$ where α is the thrust lapse ratio this represents the change in the thrust compared to the sea level static thrust value it depends on the density ratio sigma and the Mach number or velocity as the case may be. So, at any condition T will be equal to $T = \alpha T_{SL}$ note that alpha can also be more than 1 we in case we are using reheat or afterburner that is normally used in military aircraft during takeoff.

So, it is not that the value of a will be always less than 1 in general the value of a is going to be less than 1 higher altitudes. So, this particular chart or this particular table helps us in determining, which is the appropriate equation to be used to calculate the value of T_A or the value of, so, T_A/T_{SL} will be α. So, if we have for example, if we look at high bypass turbofan, we have this expression and you know you can there is a 0.1 here.

If you look at a turbojet for example, it just shows that for turbo jet or a low bypass turbofan So, it just goes by the density ratio. So, this is a simplification of what can be seen in real life and it helps you to find out the value of the parameter.

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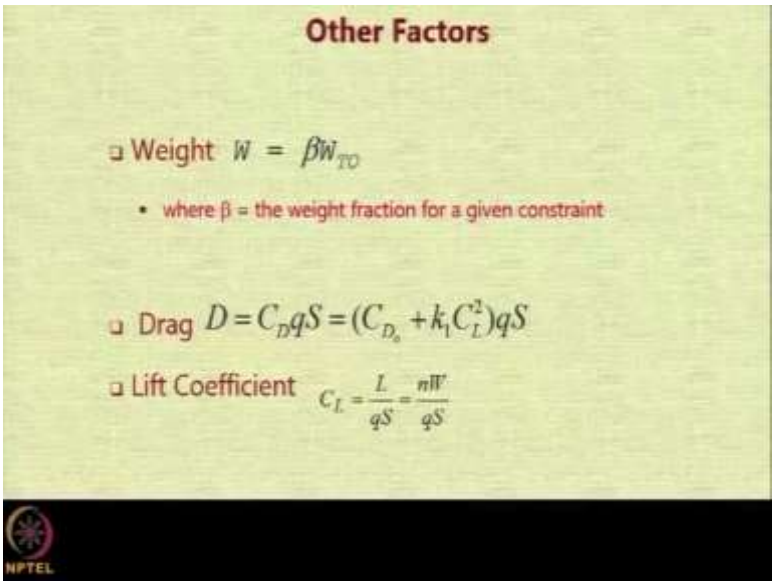

So, we take care of thrust by multiplying it by α. Now, we look at weight now, weight of the aircraft W is always going to be changing as we go into the flight profile. So, in general weight at any segment at any point in the performance of the aircraft will be beta times W_{TO} where β is the weight fraction for a given constraint regarding drag.

And if we assume parabolic profile for the drag polar, Now, the lift coefficient is going to be n times W.

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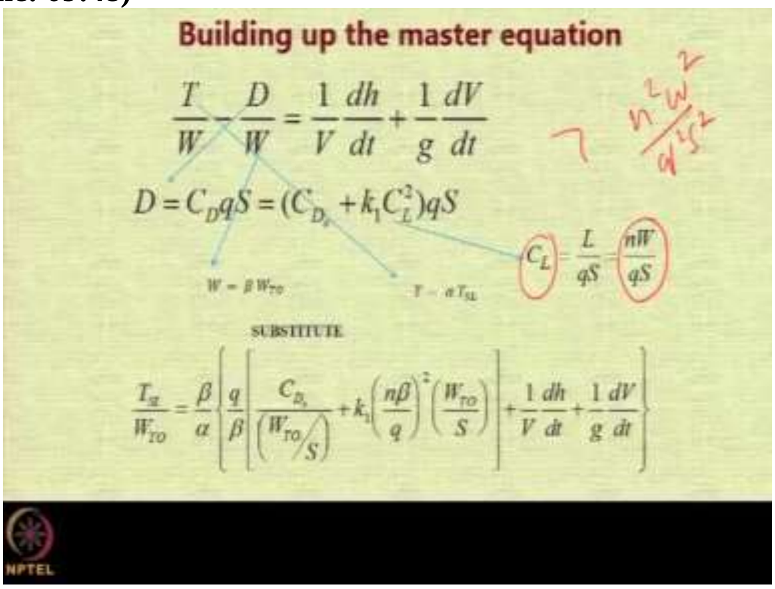

So, this is the basic master equation that we have derived. Now, what we can do is we can replace the D with $(C_{D_0} + k_1 C_L^2) qS$.

Now, since $C_L = \frac{nW}{qS}$, the $C_L{}^2$ here can be replaced by $\left(\frac{nW}{qS}\right)^2$. so, long story short if you substitute in the above expression all the values. And if you take out the terms common you get a final expression.

So, one advantage of this particular expression is that there is no need now for you to worry about converting the values from the ones operating or being present at the constraint value to the ground level. Because now, you are getting directly in terms of T_{SL}/W_{TO} and W_{TO}/S through the values of β, α and this expression can then be used for you know calculating any of the constraints specified here except a few.

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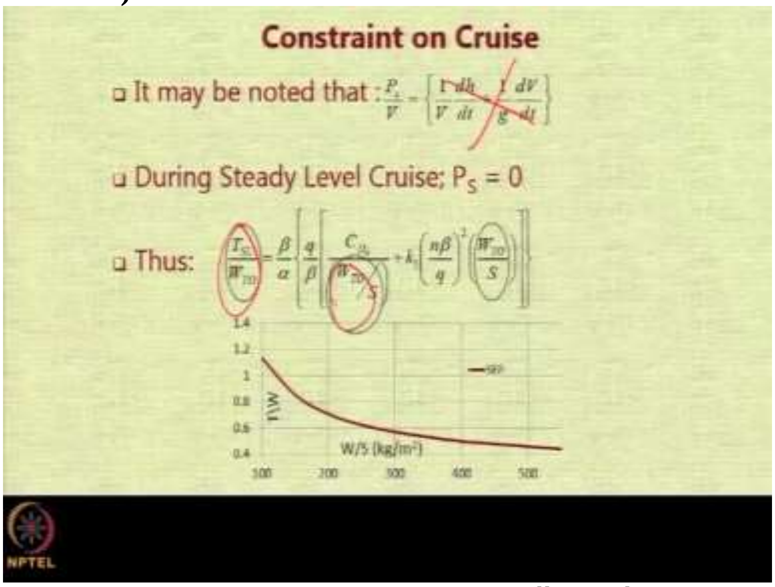

Now, it may be noted that the expression on the RHS $\frac{1}{V}\frac{dh}{dt} + \frac{1}{g}\frac{dV}{dt}$ this is nothing but $\frac{P_S}{V}$. So, therefore, during steady level cruise we do not have any excess power so, this whole term is going to be 0. So, therefore, the master equation for cruise in flight will convert into an expression as shown here the terms on the LHS are going to be 0.

So, if you now plot you can see it is a quadratic relationship. And you can see that on the y axis you have T/W on the x axis you have W/S for one particular condition. And the line that you see represents the relationship between the T/W and W/S for some particular values of β, q, α and etc.

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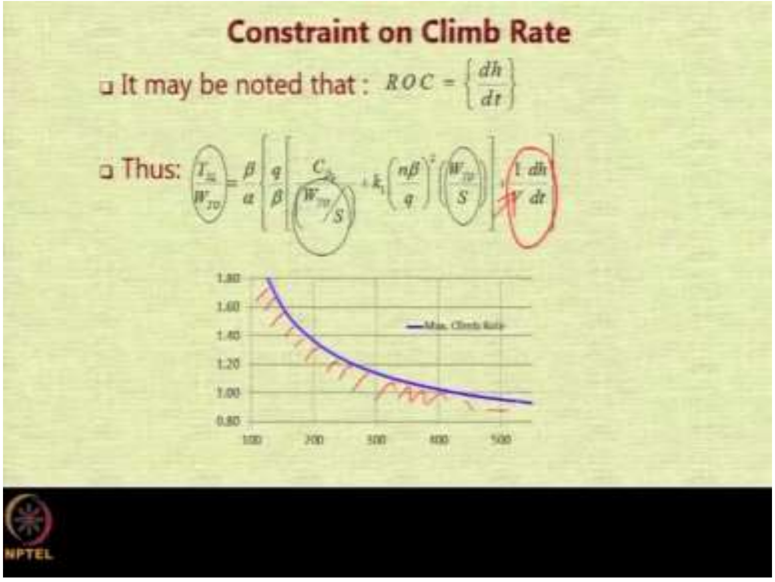

Let us look at the constraint on the climb rate. The rate of climb is nothing but dh/dt. And therefore, if I have a steady climb, then there will be 0 value of dV/dt. So, the specified value of dh/dt at the specified climb velocity can be given in this expression to get a link between T that T_{SL}. So, once again we have a curve where a max climb rate constraint can be shown and this form and the area below this line is going to be the infeasible area the area above this is going to be feasible.

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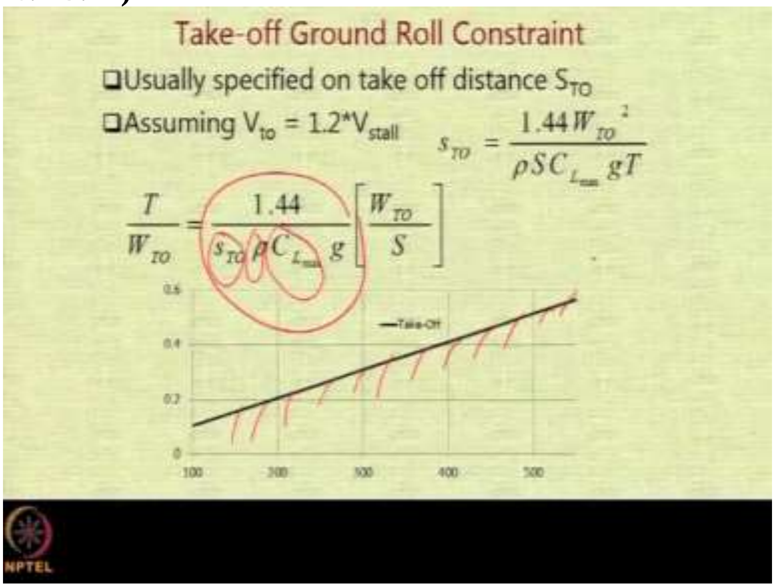

Similarly, let us take a constraint on takeoff ground roll now, usually specified in the terms of takeoff distance S_{TO}. So, if you assume the V takeoff to be 1.2 times V_{stall} which is the regulatory requirement, then S_{TO} is equal to because V_{TO} is 1.2 times V_{stall} you have a factor of 1.2 square or 1.44. So, therefore, what will happen is that the T/W_{TO} will be coming as a straight expression these items are going to be so, for a given S_{TO} for a given altitude for a given $C_{L_{max}}$ in takeoff.

You should be able to get a direct link between the T/W and W/S. So, this is going to be a linear line and the area below this line is going to be infeasible because that will not meet the specified takeoff value.

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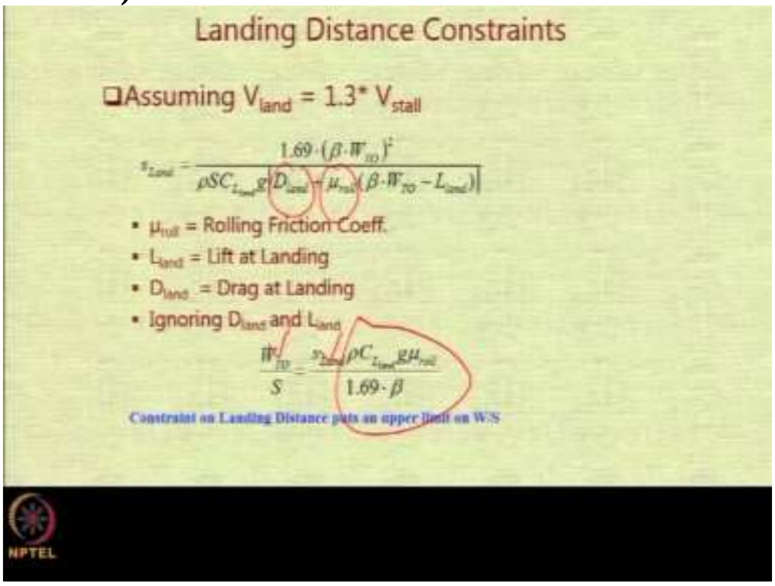

Let us look at the landing distance constraint the landing speed at which you touchdown is normally expected to be around you know 30% higher than the stalling speed. So, with that an expression for landing distance can be obtained. This 1.69 comes from 1.3 square and because during landing there is going to be some friction, the rolling friction and some drag during landing these terms μ_{roll} and D_{land} also appear in the calculation.

But in some cases, one can ignore D_{land} and L_{land}. So you get a much simpler expression. And this simpler expression simply allows you to look at landing distance as a function of only the W/S. So it will be actually a straight line because all the other quantities can be assumed to be constant for a given operating condition. So it is basically W/S is a function of the landing distance so it puts a upper limit on W/S.

If I want to have a landing distance below a particular number, I cannot have wing loading higher than a particular number. Thanks for your attention we will now move to the next section.