Introduction to Aircraft Design
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Lecture - 63
Tutorial on Constraint Analysis of Transport Aircraft: Part-2

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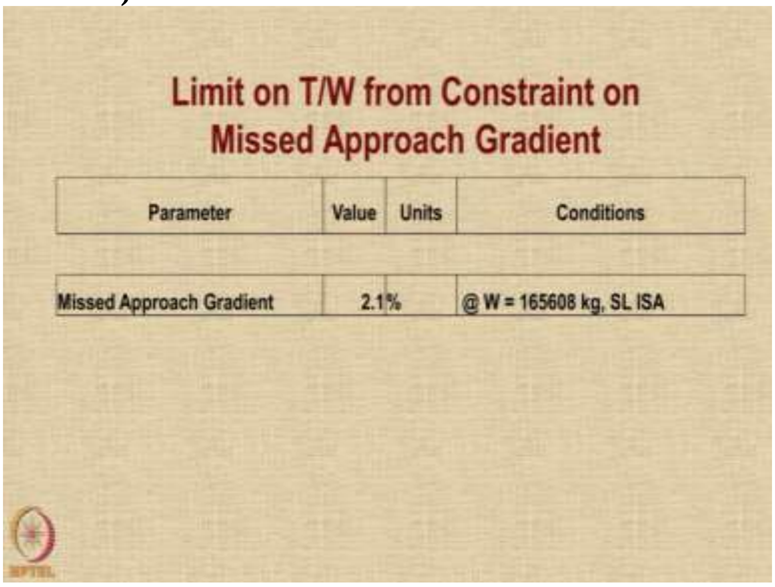

Let us look at how we can get the limit on the thrust to weight ratio from the constraint on the missed approach gradient. As we can see from the table, the specified value of missed approach gradient is 2.1%. This happens at the maximum landing weight of 165608 kilograms under sea level ISA conditions.

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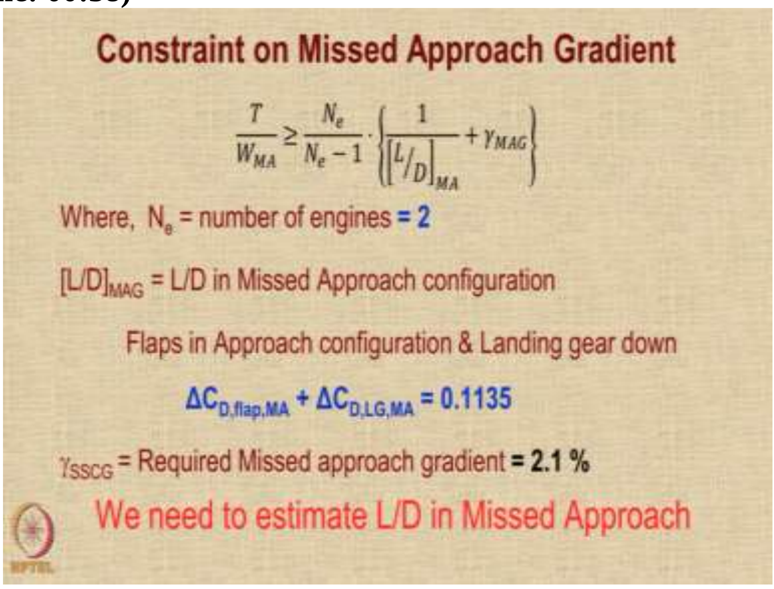

So, once again this is the constraint the formula for constraint on the missed approach gradient only thing is I have inserted the term MA wherever there is missed approach and MAG stands for the missed approach gradient. Now, here N_e stands for number of engines, which is 2, $(L/D)_{MAG}$ is the L/D of the aircraft in the missed approach configuration. As you know, during the missed approach, the flaps are in the approach configuration.

So, they are at a higher angle than in takeoff and the landing gear is down. So, from the aircraft data related to this particular aircraft, we have got this number of 0.1135. So, the ΔC_D because of the deflection of flaps and the landing gear is a very large number 0.1135. And the requirement on the missed approach gradient as specified is 2.1%. Let us now calculate L/D in the missed approach, just like we did it for the previous case of second stage climb gradient, the procedure is identical only the numbers will change slightly.

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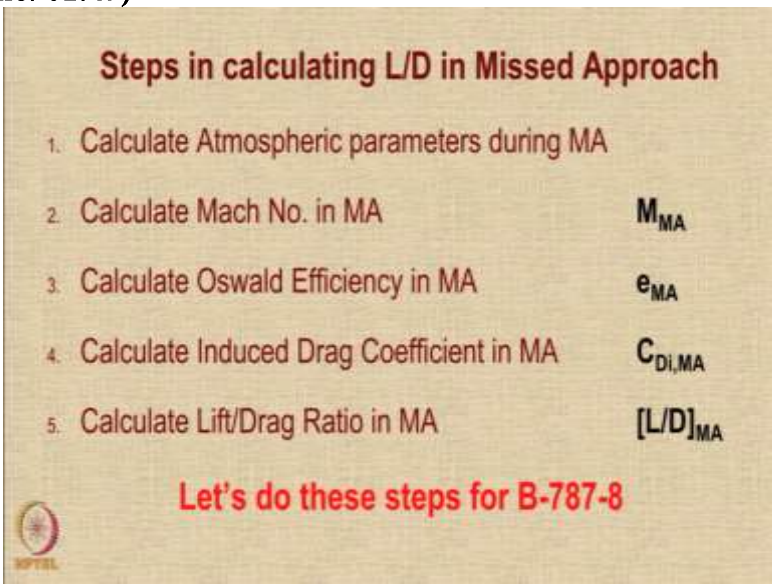

So, the steps are very similar, first we calculate the atmospheric parameters during MA. In this case, we do not have to do the step because the missed approach is happening under ISA conditions. So, therefore, we already know the values of density and temperature and sonic speed under ISA conditions then we calculate the Mach number of the aircraft because once you know the speed and the sonic speed, you can get the Mach number.

Once you get the Mach number, then you can get the Oswald efficiency. Once we have Oswald efficiency, we can get the induced drag coefficient, once we have the induced drag coefficient, we can get the total drag coefficient we already know the lift coefficient. So, therefore, we can get the L/D and let us do these steps for Boeing 787 now.

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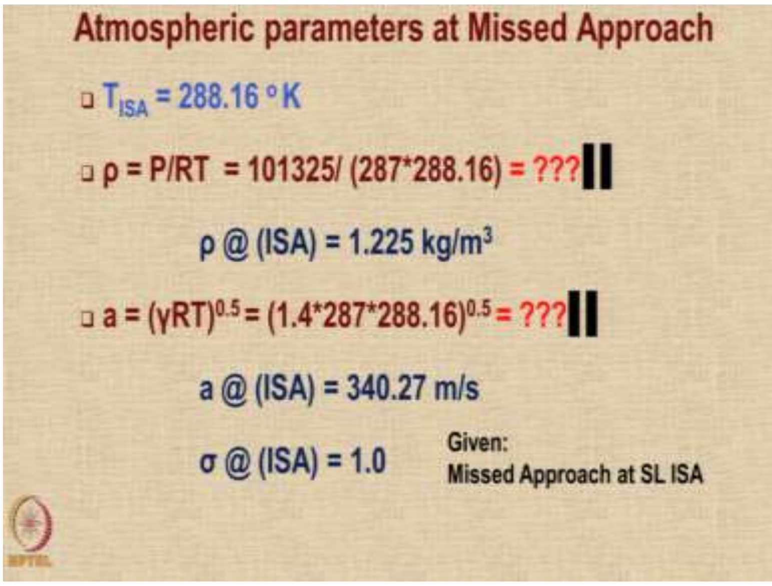

It is specified that the missed approach occurs at sea level ISA conditions. So, therefore, the parameters of the atmosphere are very well known T=288.16; ρ= P/RT which is 101325/(287*288.16). Although all of us know this value, but still it makes sense to calculate it, the number comes out to be 1.225 kg per meter square the sonic speed will be obtained by $\sqrt{\gamma RT}$ for these standard values.

Once again this number is known to us, but still it is important for us to calculate it and assure ourselves that we are on the right track. Now, the value of σ will be equal to 1 because ρ and ρ_0 are the same in this case.

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Now, to calculate the Mach number, we first use the fact that the V_{stall} landing as specified in the requirements is 102 knots EAS at ISA, now at under ISA conditions at sea level EAS =

TAS, because sigma = 1. Therefore, a and also we have just calculated that a = 340.27. So, just convert the value of speed 102 knots into meters per second.

Now, during missed approach, we assume that the speed is not 1.2 but 1.3 times V_{stall}. So, that will come as 1.3 times 52.47. Pause the video and please calculate this value, the value is 68.22 meters per second. And hence, since we know the value of V_{MA} and we also know the value of sonic speed a we can get the Mach number by taking a simple ratio please pause the video calculate the value, the value comes out to be 0.20.

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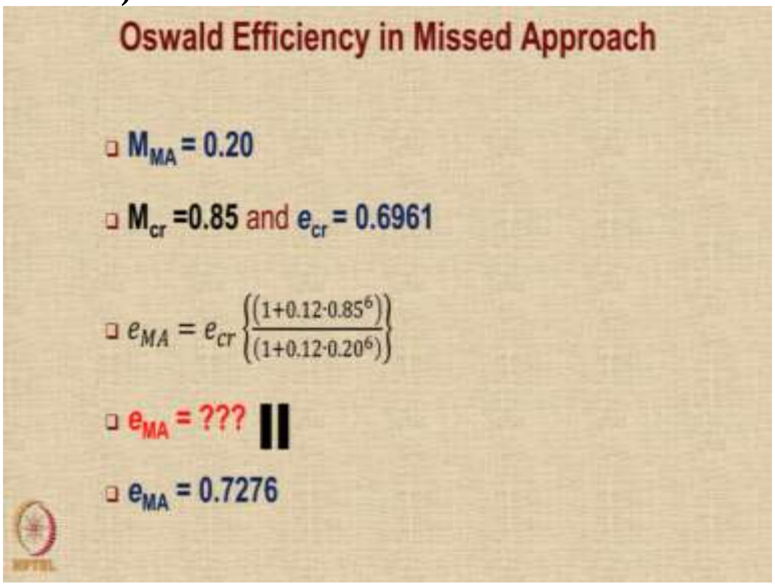

Once we know the Mach number we can calculate the Oswald efficiency factor using the information that is available already using the same formula that we have used earlier for the value estimation of the value of e during the second stage climb. So, calculate the value of e missed approach. Please pause the video here and calculate the value by this simple expression, the value comes to be 0.7276.

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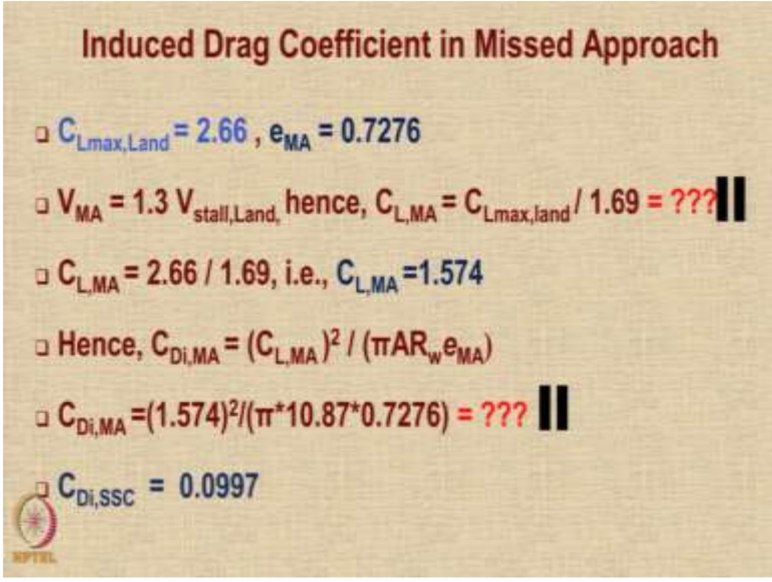

So, with this we can now get the induced drag coefficient in missed approach. So, we know the value of $C_{L_{max}}$ landing as 2.66 is specified in the aerodynamic data. And we have just calculated the value of Oswald efficiency in the missed approach e_{MA} as 0.7276 we know that in missed approach condition, the stall velocity is 1.3 times V stall hence, $C_{L_{MA}}$ will be $C_{L_{max}}$ landing by 1.69 please calculate this value it will be 2.66 / 1.69 that is 1.574.

Once the value of C_L is known, therefore, you can get the induced drag coefficient because now, we have all the information we have $C_{L_{MA}}$, and we have the value of aspect ratio 10.58 and e_{MA} 0.7276. So, by including these values in the formula, now, you can pause the video and calculate the value of $C_{Di_{MA}}$ the value comes out to be 0.0997.

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Moving ahead, now we can calculate the lift to drag ratio in the missed approach. For this, we know from the previous calculations the value of $C_{D_{issc}}$ and $C_{L_{MA}}$. And now we also know that we have to add additional term because of the extra drag of the flaps and the landing gear during this condition and that will be 6.965. Let us look at the T/W missed approach.

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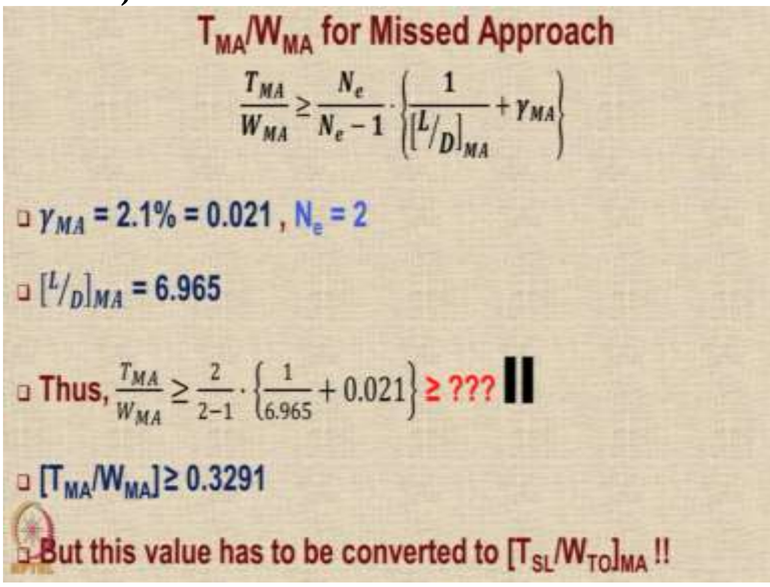

For that, we insert the corresponding values of T/W and L/D and the γ in the formula. Now, we have already been given by the requirements that the missed approach gradient is 2.1%. And we know that this aircraft Boeing 787-8 has 2 engines. The L/D missed approach has been calculated previously as 6.965. So, therefore, once you insert these numerical values in the expression, it becomes very easy for us to calculate.

So, pause at this stage and calculate the values we see that the value of T_{MA} / W_{MA} has to be more than 0.3291 during the missed approach condition. But remember that the missed approach condition occurs during landing whereas we are interested in the value of T/W under the sea level static conditions and for maximum takeoff weight. So therefore, we have to convert this value. And to convert this value we will have to incorporate some factors.

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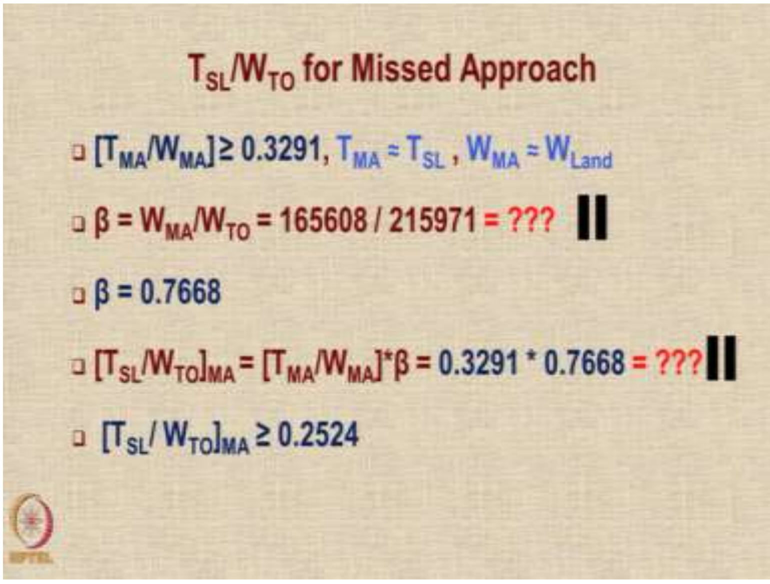

So, to get the value of missed approach T/W limitations at sea level static conditions and max takeoff weight we just reproduced from the previous slide the values that we know we also know that the thrust during missed approach is equal to thrust at the sea level condition because the missed approach constraint is supposed to be met at sea level conditions. And also we can assume that the weight of the aircraft during the missed approach is almost equal to the weight at landing.

So, keeping these 2 in mind, we can first calculate the value of beta which is the ratio of weight of the aircraft during the missed approach condition and the max takeoff weight. These 2 numbers are known to us and hence it can be easily estimated please pause the video and calculate this ratio, the ratio turns out to be 0.7668. So, now, we know that

$$\frac{T_{SL}}{W_{TO}} = \frac{T_{MA}}{W_{MA}} * \beta$$

So, you just multiply this number by β and you will get the value $\frac{T_{SL}}{W_{TO}}$ missed approach will be more than or equal to 0.2524. This is one mistake which many students make they forget to convert the values obtained for a constraint to the standard conditions of sea level static thrust and weight of the aircraft at takeoff so be very careful.

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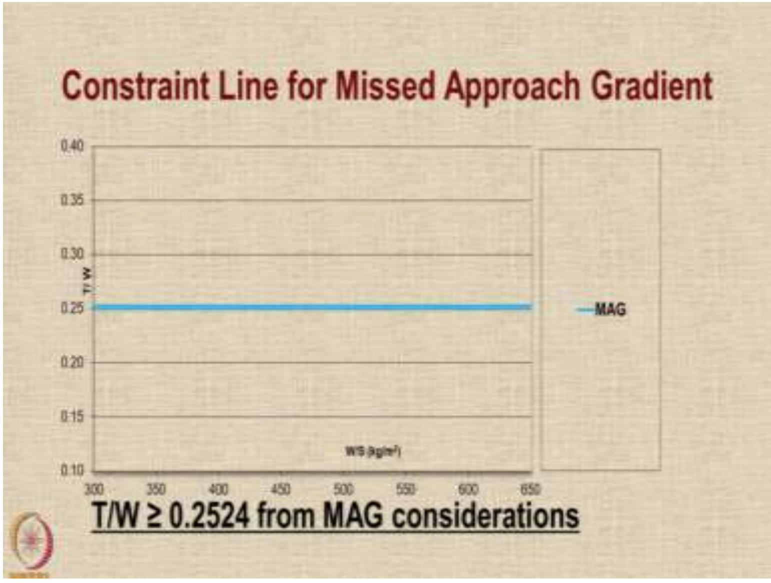

Let us see how this maps into the constraint diagram as a line because the constraint was directly on T/W there is no role of W/S here. So therefore this will appear as a horizontal line in the graph as shown. So, we also arrive at an interesting conclusion that the thrust to weight ratio of the aircraft cannot be less than 0.2524 from the constraint on the missed approach gradient.

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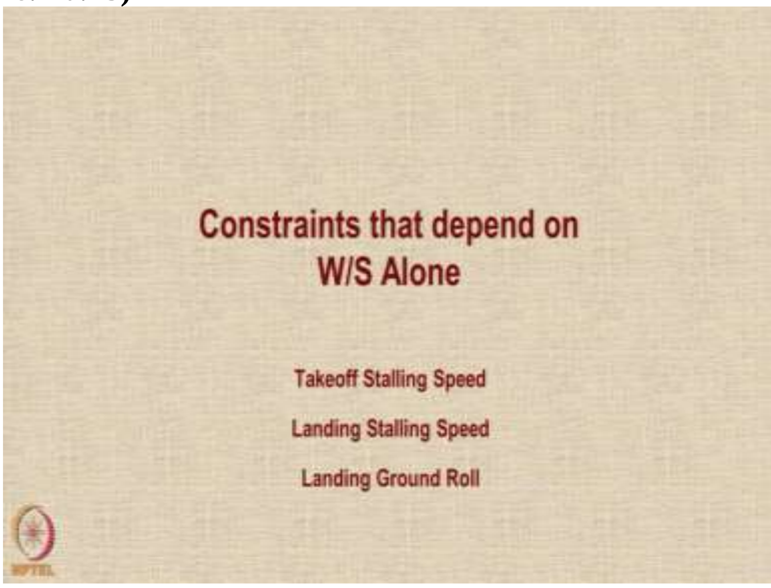

Let us look at the constraints that depend on wing loading alone. In our table of constraints, there are 3 such constraints the one on takeoff stalling speed, landing stalling speed, and the landing ground roll. So, let us see how these 3 appear in the constraint diagram. So, since they depend on W/S alone, it is clear that they will appear as vertical lines.

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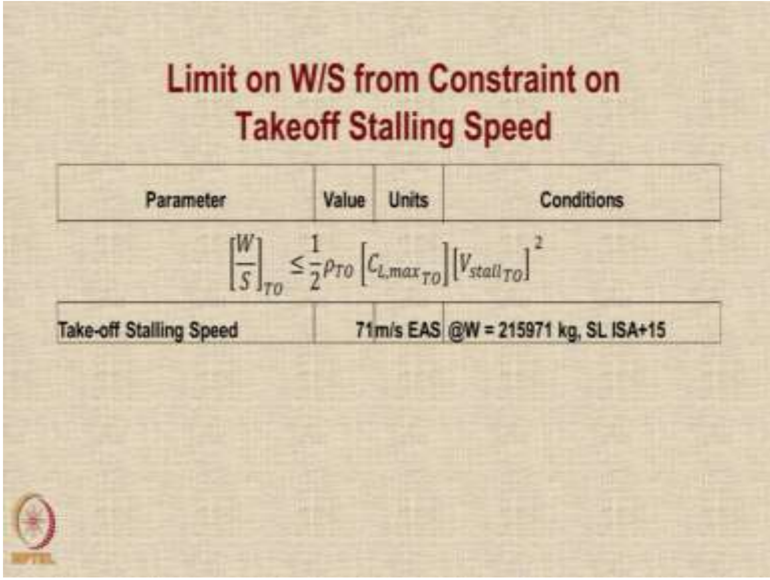

First let us look at the limit on wing loading from the constraint on takeoff stalling speed as per the specified requirements, the value has to be 71 meter per second equivalent airspeed or less at the max takeoff weight under sea level conditions, but under ISA+15 conditions this is the requirement which you have to meet the formula that we will use is very straightforward, we know that lift is equal to weight during a 1 g stall.

So, from there you can easily get an expression between the wing loading required versus the density of the air at takeoff condition the maximum value of C_L at take-off condition and the specified value of stalling speed square.

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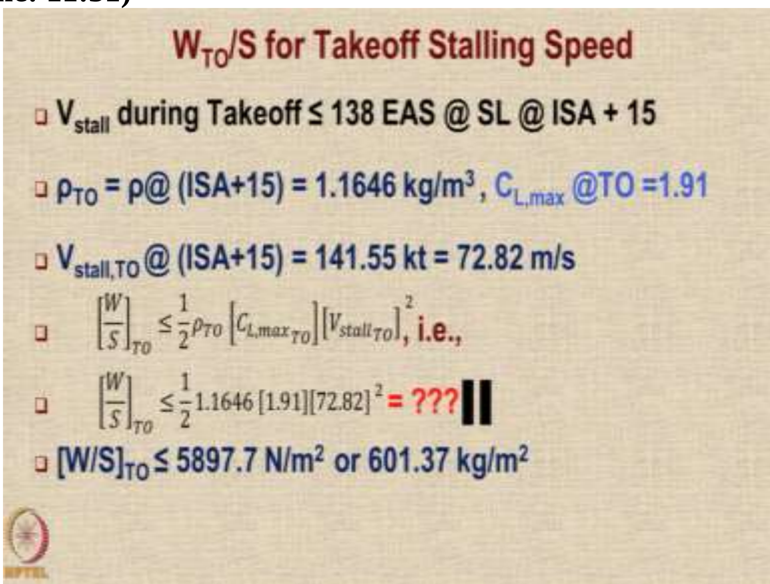

So, let us see how this constraint can be incorporated. First of all, there is a specified requirement that the V_{stall} should be less than 138 EAS which I have already converted to meter per second. Now, previously, we already calculated that the density of air at ISA+15 at sea level is 1.1646 kg per meter cube. And we also know from the data that $C_{L_{max}}$ of the aircraft at takeoff conditions is 1.91. So, therefore, you can get the value of the stall speed at ISA+15 conditions.

And once we obtain the value, we just have to insert the numbers in this formula, which means we have to replace rho takeoff by 1.1646 $C_{L_{max}}$ takeoff by 1.91 and V_{stall} take off by 72.82 pause at this stage and calculate the value of wing loading required we see that the wing loading has to be less than or equal to 5897.7 Newton per meters square this is what you get by directly inserting the numbers in this formula.

But remember that the answer you get would be in Newtons per meter square which has to be converted into kg per meter square. So, that we can by dividing with g, so that we can have consistent units in our constraint diagram.

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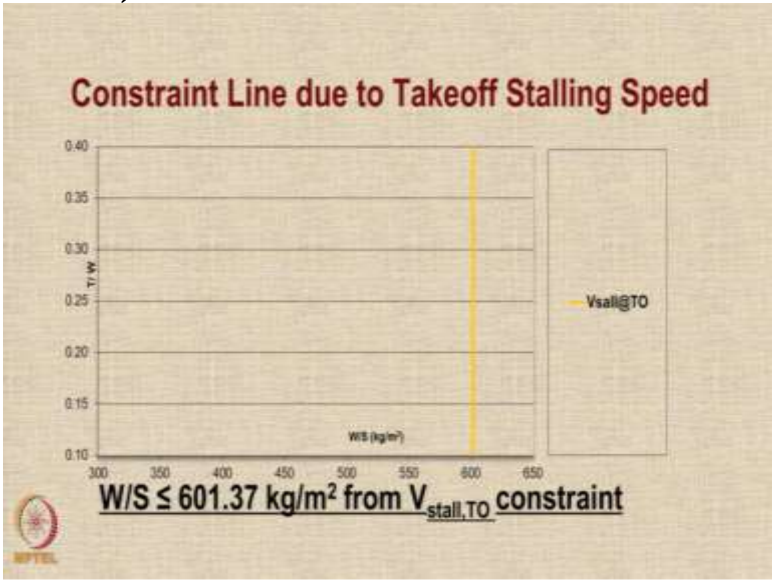

So, with this, let us see how the constraint line appears on the diagram because of takeoff stalling speed, as I said, it will be a vertical line which is stationed at 601.37 kg per meter square and the area on the left of this line is going to be feasible because the wing loading cannot be more than this number it has to be less than or equal to this number. So the area on the right of this line is infeasible on the left is feasible.

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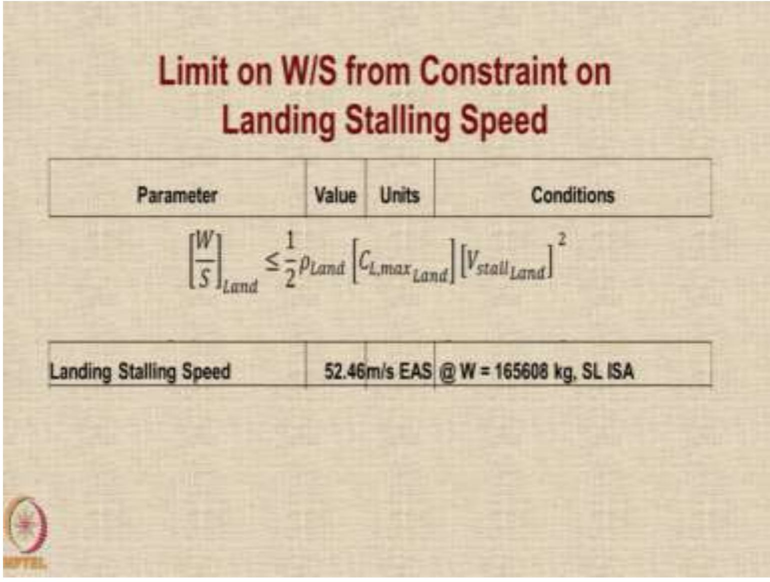

Moving on let us look at the limit imposed on wing loading from the constraint on landing stalling speed. Here the requirement is to calculate at the landing weight at sea level ISA conditions there will be no need to convert the densities we can use the standard sea level values and the units are also the value is specified as 52.468 into a 102 knots EAS which can be converted to meter per second using a simple formula. Once again we will use the standard formula just like we used for the takeoff condition the only difference is that we will put subscript land for wing loading density $C_{L_{max}}$ and V_{stall}.

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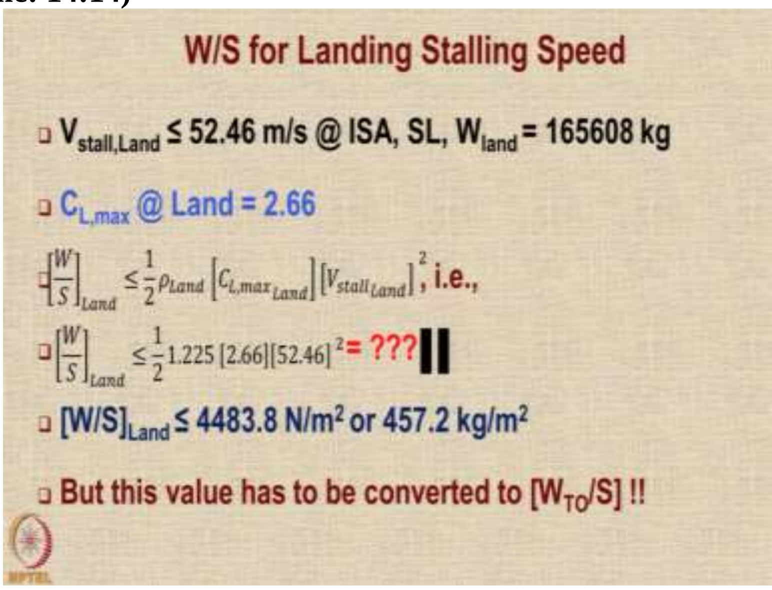

So, let us see how this number pans out. This is the requirement which I am repeating here for ease in memory $C_{L_{max}}$ at landing is given as 2.66 from the aerodynamic data for this aircraft. So, you have a simple expression and now if you substitute in it the values you get 1.225 in place of rho Land $C_{L_{max}}$ Land is replaced by 2.66 and V_{stall} Land by 52.46. So, pause at this stage and please do the number.

We find that the value is 4483.8 Newtons per meter square or 457.2 kg per meter square. So this is a much stricter limit, but remember, again this is at landing so it has to be converted to the values corresponding to W take off.

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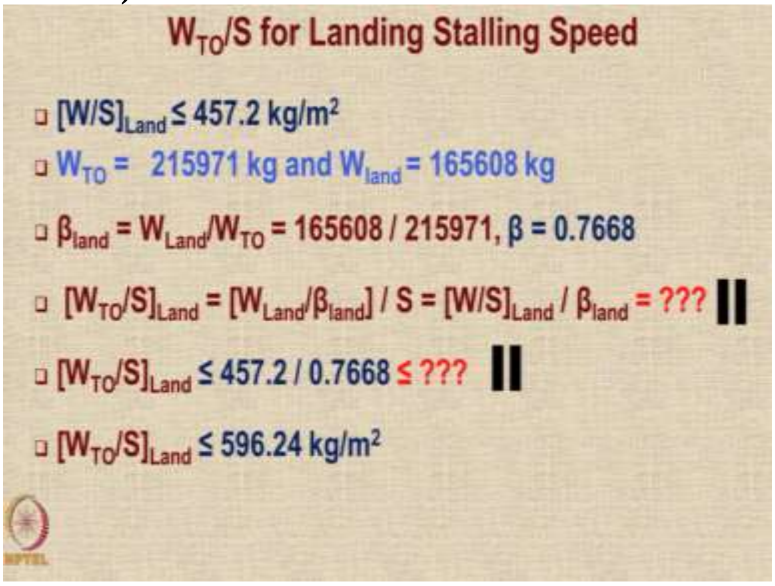

So, ${W_{TO}}/{S}$ is what is our on the constraint line. So, during landing condition it is supposed to be less than 457.2 and we know that the takeoff weight is 215971 and W_{land} is 165608 specified. So, we can calculate the value of beta which we got in the last expression also as 0.7668. So, with this beta you can easily estimate what will be the value of W_{TO}/S at landing it will be

$$\left(\frac{W_{TO}}{S}\right)_{land} = \frac{W_{land}/\beta_{land}}{S}$$

So, we know these values insert the values pause the video and try to get the constraint limit as shown. So, get this number this number is going to be less than 596.24 kg per meter square. So, it turns out that the landing stalling speed constraint is slightly more harsher than the constraint due to the takeoff stalling speed. But that is only for this particular example it is not a general statement.

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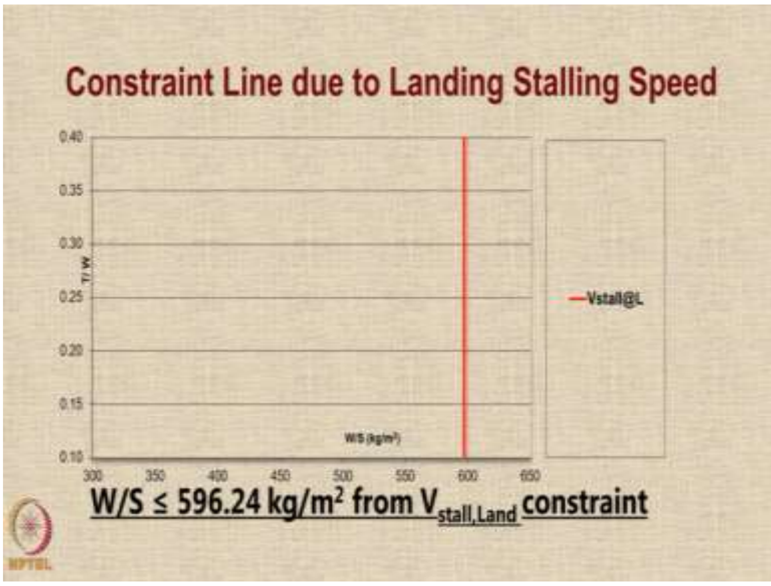

So, once again we see that the line will map on the constraint diagram as a vertical line with the area on the left of the line as feasible and the area on the right as infeasible.

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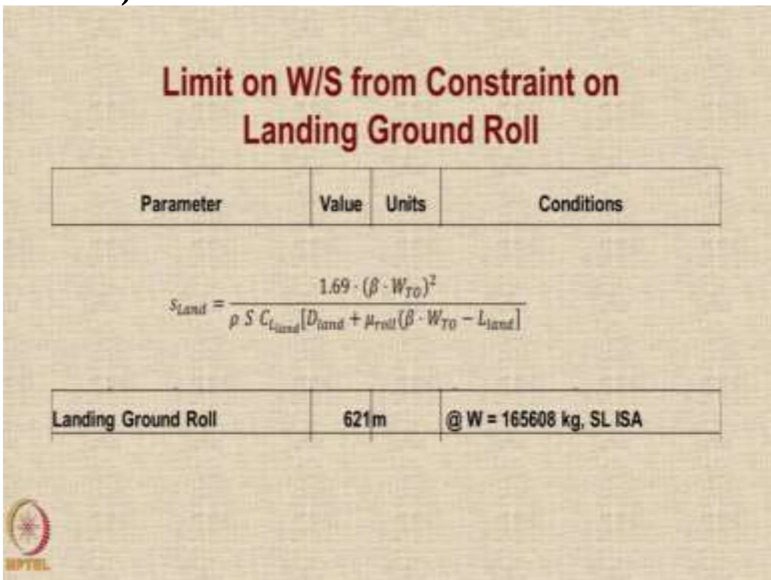

Moving ahead, let us look at the third constraint which depends only on W / S that is the landing ground roll. The requirements specified that the ground role should be less than 621 meters at maximum landing weight of 165608 kg under sea level ISA conditions. So, for that, we will use this particular formula. If you are not familiar with this formula, I would request you to go back and look at the video of clips of the lecture on constraint analysis for transport aircraft where all these formulae have been explained so, this formula will be used by us.

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So, the specifications are that the atmosphere is ISA the altitude is sea level and the required distance is less than 621 meters. The aerodynamic data is that the $C_{L_{max}}$ is 2.66 we have to assume a few values in this case we do not assume any after burner. Therefore, $\alpha = 1$ we assume that the operations are from some standard airports therefore, the μ_{roll} is going to be 0.4.

Since the thrust force you know at landing is such and also just after landing during the ground roll, we put the spoilers therefore, we can assume that the lift at landing is completely killed and hence we also neglect the drag at landing. So, and the sea level condition density is 1.225. So, with this what we need to now do is just calculate β is already known actually as 0.7668 we have done it just a few minutes ago. So, we can just remember that value.

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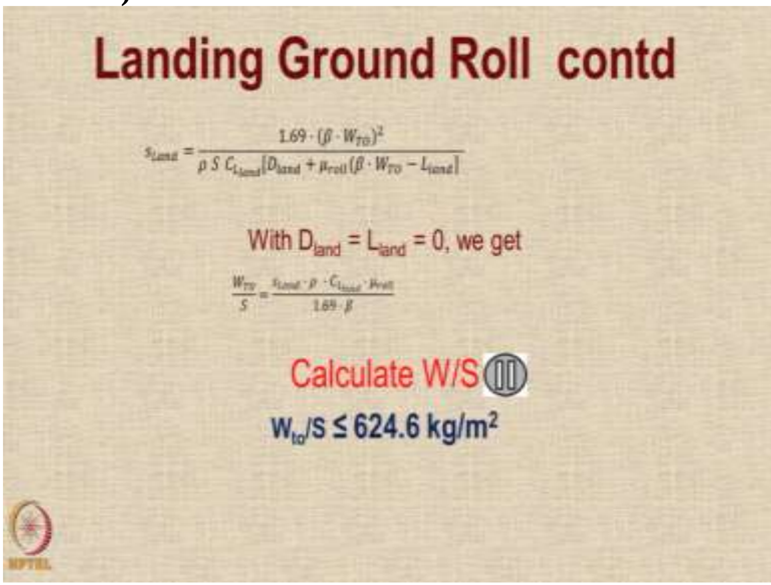

So, this is the formula that I mentioned. Now, if you look at neglecting the values of D_{land} and L_{land}, then the formula simplifies and with the simple rearrangement of W / S on the left hand side you can see that it is just a formula in terms of landing distance as land density C_L land and μ_{roll} and β. So, inserting these values we can calculate the W/S constraint. So please pause and insert the values we all know these values from the last slide. So the number turns out to be less than 624.6 kg per meter square.

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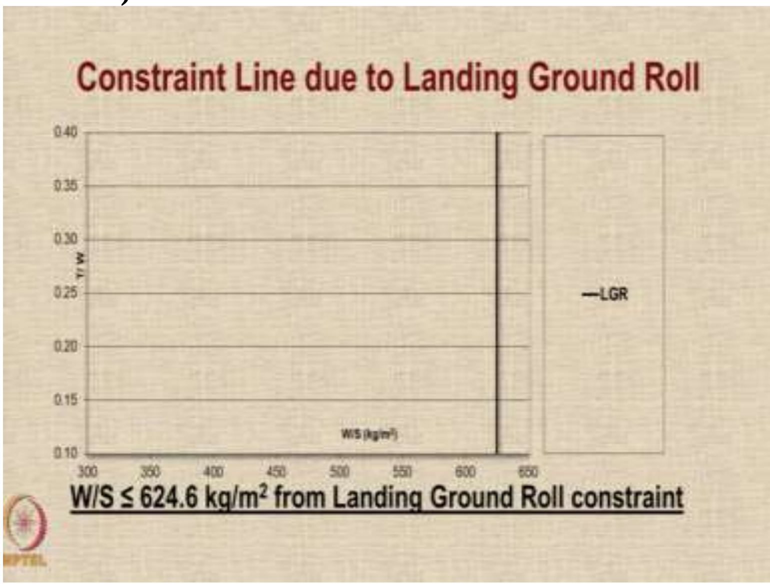

So, it is interesting to note that the landing ground roll constraint is not really that harsh in this case. And also please remember that we have not worried about converting it to take off conditions because already in the formula we have included those parameters in the form of beta. So therefore, the constraint because of the landing ground roll turns out to be 624.6 kg per meter square and appears as a vertical line with the area on the left being feasible and area on the right being infeasible.

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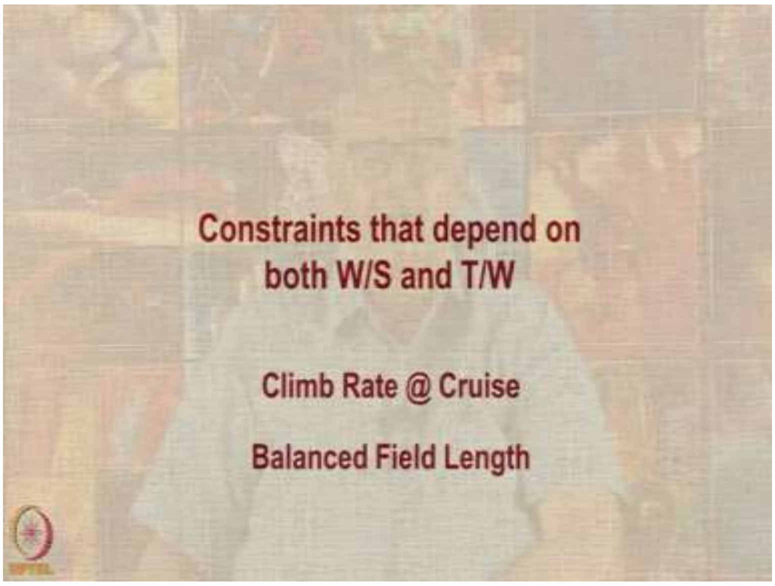

So this is how we looked at the constraints that depended only on W/S. Let us look at constraints that depend on both, W/S and T/W there are 2 such constraints in our table. The first one is the constraint due to the climb rate at cruise conditions and the second is the balance field length.

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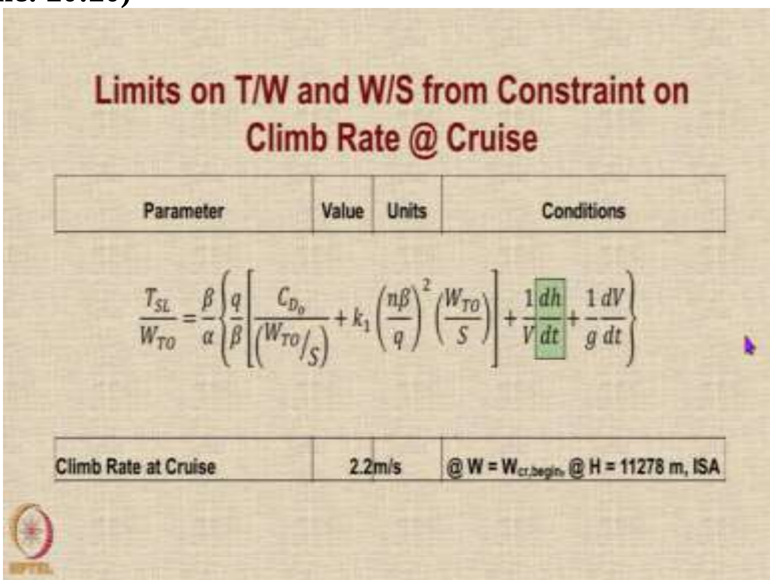

So let us see the first one, the one which is a constraint due to the climb rate at cruise, this particular requirement is 2.2 meters per second or 429 feet per minute, which is to be calculated when the aircraft is just at the beginning of the cruise and the altitude is the cruising altitude of 37,000 feet or 11 to 78 meters under ISA conditions. For this, we will use this master equation.

And if you have if you do not understand this equation, or if it seems very unfamiliar to you, I would request you to go back and have a look at the video clips of the lecture on constraint analysis for military aircraft, because we have used master equation mostly in that particular application, but we will use it also for the current requirement. So, in this large expression, which relates the T_{SL} / W_{TO} and W_{TO}/S for several operating conditions, we notice that the climb rate is actually dh/dt.

So, we are given the value of dh/dt and we have to calculate the link between or the relationship between T_{SL} and W_{TO}/S for, we have to calculate the relationship between T_{SL}/W_{TO} and W_{TO}/S for a given value of dh/dt and the other parameters will be obtained from either the known value or from the specified constraints.

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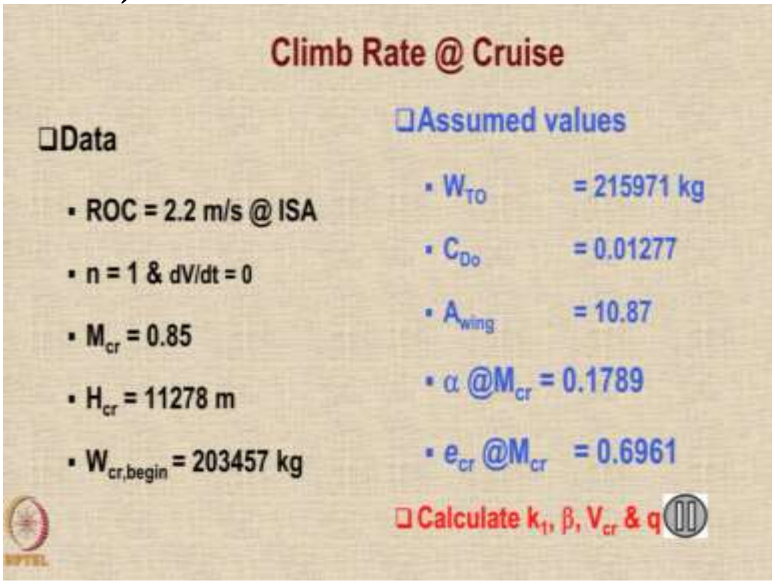

Let us see how it is done. So, the data for this requirement is that ROC is 2.2 meters per second under ISA conditions, since it is a level flight n = 1 and also it is a steady flight. So, therefore, the there is no acceleration. So, the aircraft is in a steady level flight the cruise Mach number is 0.85 and the cruise altitude is 37,000 feet or 11 to 78 meters and we have obtained some information about the fuel consumed in the warm up taxi out, takeoff, climb etc.

So, through that we come to know that the aircraft weighs approximately 203457 kg when it comes in at the beginning of the cruise, the condition at which this climb rate is to be estimated there are some values which we will assume first of all we will assume that the takeoff weight of the aircraft is 215971 this is given in the aircraft data. We also know from their dynamic information that the C_{D_0} of this aircraft is 0.01277 the wing aspect ratio is 10.87.

And from the engine data we come to know that the value of alpha at this cruise Mach number is 0.1789. And we have also calculated earlier that the value of the Oswald efficiency at cruise Mach number is 0.6961. So, with this information, we now have to calculate few parameters which are required in the master equation. These are the induced drag coefficient k_1, the value of aircraft to weight ratio β, the cruise speed and the dynamic pressure during cruise. These can be straightaway calculated using standard formula.

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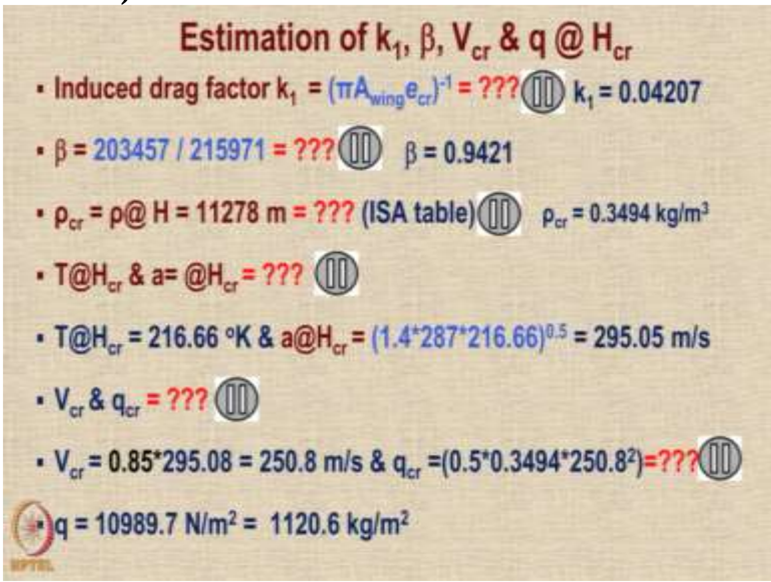

Let us see how it is done, first of all, for the induced drag factor. So A_{wing} is known e_{cr} is known, so pause the video and calculate the value this value turns out to be 0.04207. The next information we need is β which is the ratio of the weight of the aircraft at the cruise condition or beginning of the cruise divided by the weight at the maximum takeoff condition or the maximum takeoff weight this also can be easily calculated the number is 0.9421.

Then the cruise velocity at the cruising altitude can be obtained either by a numerical calculation or you can also pick it up from the ISA table because it is an ISA condition and that number is 0.3494 kg per meter cube. We can also calculate the temperature at the cruising altitude and the value of sonic speed. The temperature we all know temperature at cruising altitude is already known because it is a standard value it is 216.66 at an altitude between 11 to 25 kilometers under ISA conditions.

And hence the value of sonic speed can be easily calculated as $\sqrt{\gamma RT}$, where gamma is 1.4, R is 287 joules per kg degree Kelvin and T is 216.66 degree Kelvin. So, with that the value comes out to be straight away as 295.05. Now, since we know the value of cruise Mach number and now, we know the cruise or the sonic velocity at cruise conditions, you can calculate the value of V and once you know V you know rho and you know V.

So, I would request you to pause and calculate comes out to be 1120.6 kg per meter square. So, we now have all the numbers that we need k_1, β, V_{cr}, q and H_{cr} which we need in the master equation.

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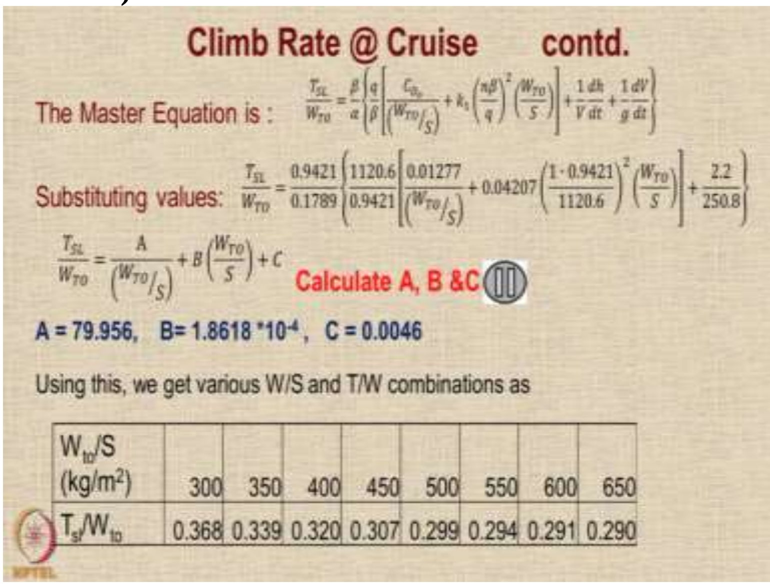

So, we can go ahead the master equation is written there on the on the top of the screen. So, we know that dv/dt is 0, therefore, the last term will vanish we know that dh/dt is 2.2 and V is 250.8. So, with that, you can actually substitute the values and you can get a long expression. And here what we have done is we have inserted the values of $\beta, \alpha, q, C_{D0}, k_1, n, V$ and $\frac{dh}{dt}$ in the expression so, this can be easily solved.

In fact, if this turns out to be an expression like T_{SL}/W_{TO} is equal to this whole constant this whole number divided by W_{TO}/S and then this whole number divided by into a W/S plus a constant C which will be the terms outside the bracket this these terms and these terms. So, I think we should you should pause the video at this stage and calculate the values of these 3 constants A, B and C the values come out to be as shown on the screen.

Now, with these values you can replace now in this equation, so, you will get relationship between T_{SL} /W_{TO} because you can put A here you can put B here you can put C here and for various numerical values of W_{TO}/S ranging from 300 kg per meter square to 650 kg per meter square, you can see how it pans out into the diagram. So, this constraint on climb rate at cruise depends both on W_{TO}/S and T_{SL}/W_{TO} therefore, it will come up as a line in the constraint diagram.

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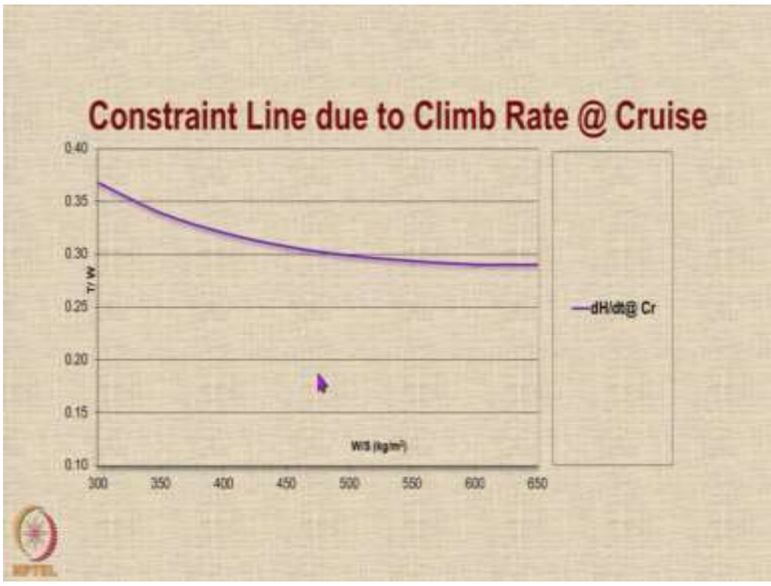

So, if you draw this line in the constraint landscape, we see that it appears as a curved line with the area below the line as being infeasible.

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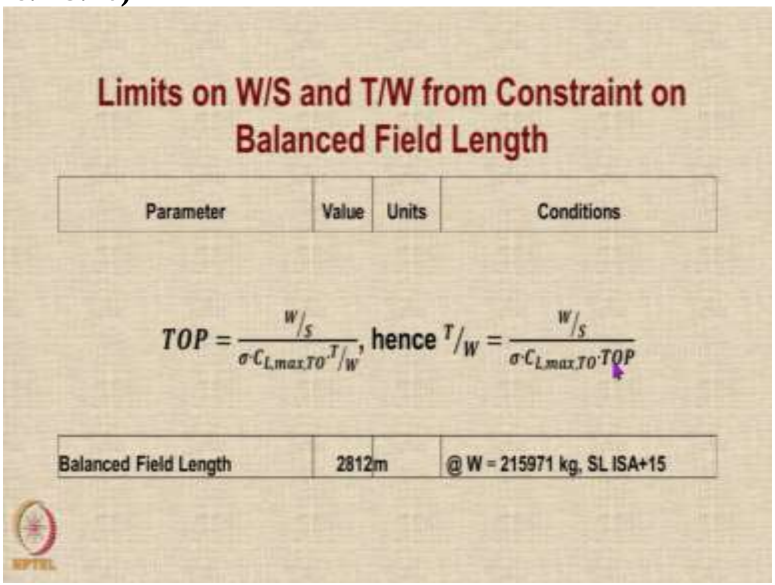

Moving ahead to the last constraint which is on balance field length, the requirement given is that the balance field lengths should be to 2812 meters at maximum takeoff weight and under sea level and ISA+15 conditions for this we are going to use the takeoff parameter as explained in the textbook by Daniel Raymer. So, the takeoff parameter is a parameter that is obtained from a graph and from there you can get a direct link between T/W and W/S.

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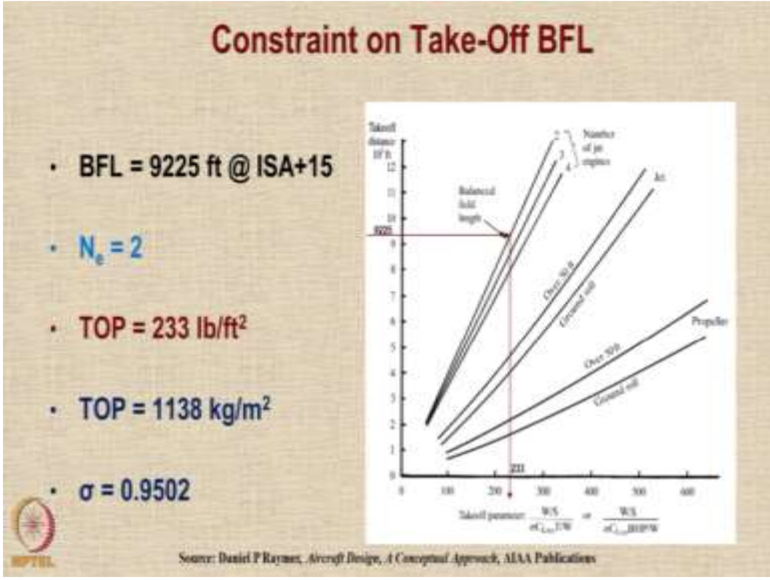

Let us look at the constraint on takeoff balance field length. This is an image taken from the famous textbook by Daniel Raymer which shows the correlation between various types of takeoff distances and the takeoff parameter. So, we know from our specifications that the BFL has to be 9225 feet at ISA+15. And the data is that there are only 2 engines in this aircraft.

So, with that if you proceed from the y axis with 9225 feet and hit the line for BFL or balanced field length for twin engine jet aircraft, you can go down and obtain the value of takeoff parameters as 233 pounds per square feet. This has to be converted into SI units. And after that the value of sigma at ISA+15 has already been calculated it is 0.9502.

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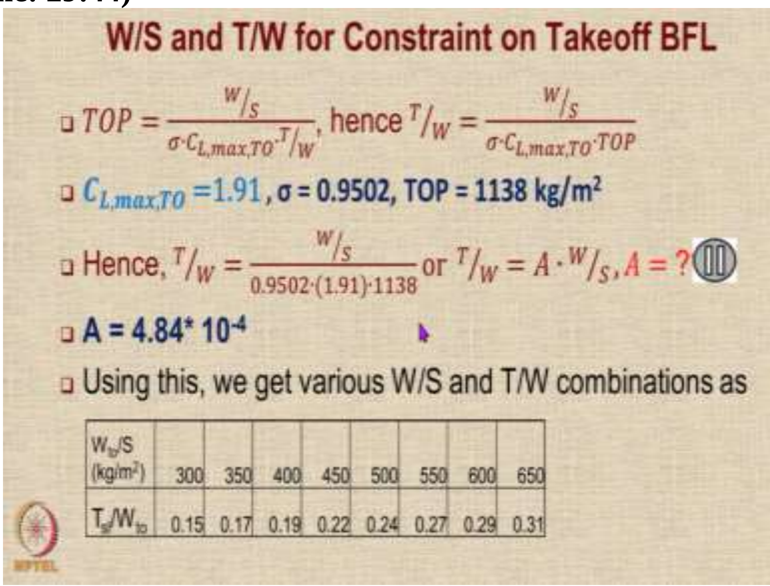

So, let us look at how this constraint can be calculated.

So, I think you can pause the video and calculate the value of A, A turns out to be $4.48 * 10^{-4}$.And with this you can get various combinations between W/S and T/W as shown.

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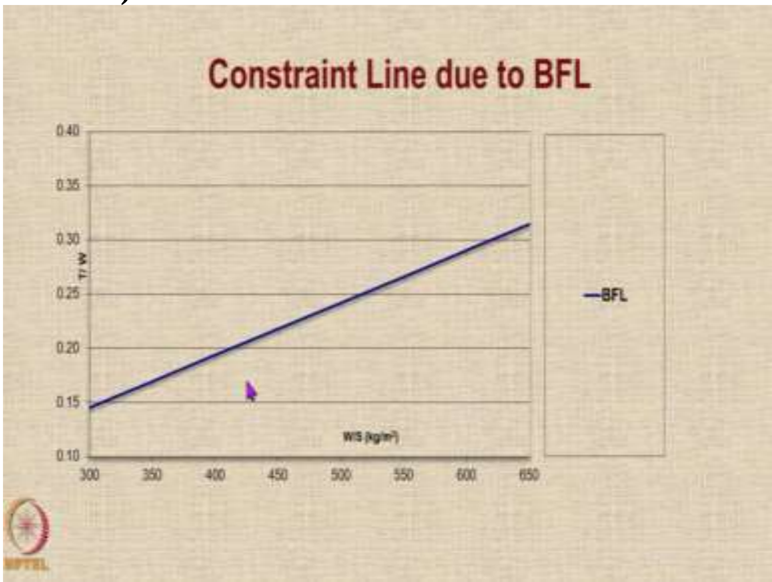

So, this pans up into the constraint diagram as a straight line. And this line, the area below this line is infeasible and above the line is feasible.

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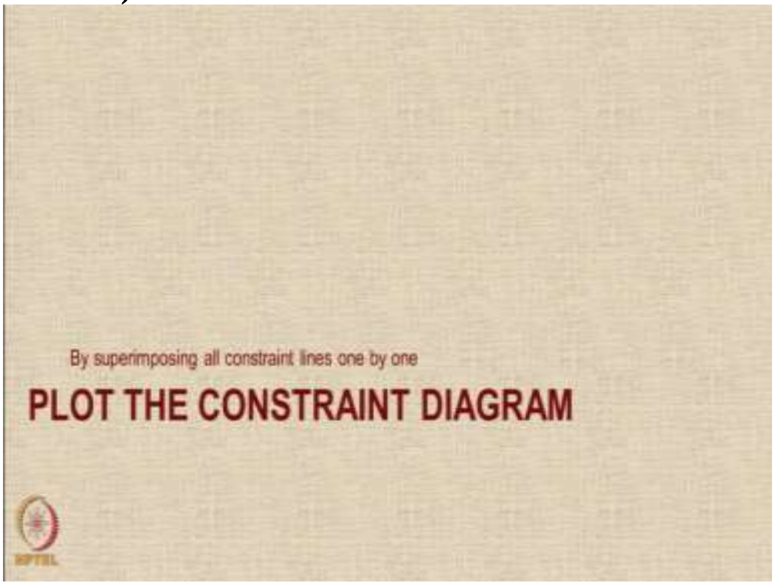

So, now let us look at how do, we plot the constraint diagram by superimposing all the constraint lines one by one.

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First, let us look at the constraint that depend on T/W alone, which are the second stage climate radiant and the missed approach gradient.

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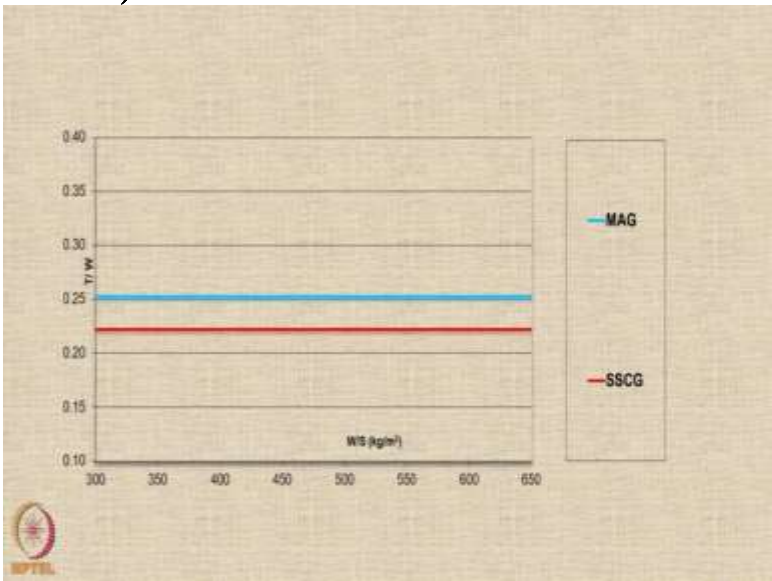

These are appearing as horizontal lines, the first one is SSCG, and the second one is MAG. So for this problem, we see that the missed approach gradient is more severe than the second stage climb rate of constraint.

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Then let us look at the constraint that depend on W/S alone there were 3 constraints in our diagram, takeoff stalling speed, landing stalling speed and landing ground roll, these will appear as vertical lines.

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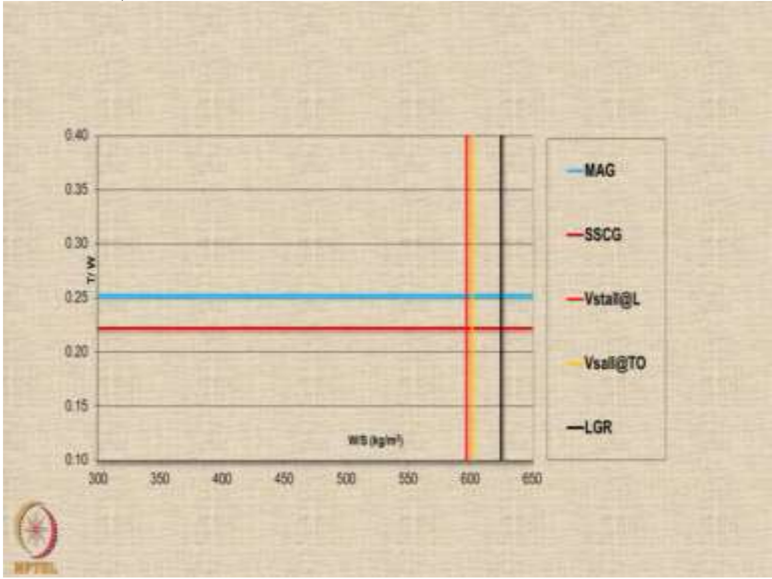

So, the first one is the one on stall takeoff. Then, the next one is on stall at landing and the third one is because of the landing ground roll area on the left of these lines is feasible. So we notice here that the constraints because of the stalling speed at takeoff and landing ground roll are weaker than the constraint on the stalling speed at landing.

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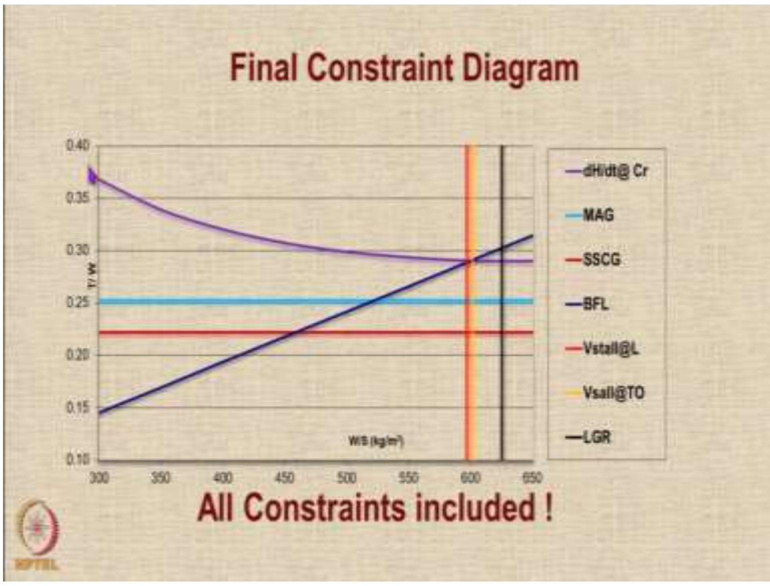

Finally, let us look at the 2 constraints that depend on both W/S and T/W which are the climb rate at cruise and balance field length, these will appear as curves or lines in the constraint diagram. The first one that we bring here is the line due to the climb rate, which is a curved line and the next one is the constraint due to the landing as due to the balance field length.

So, the intersection of all these lines this is the final constraint diagram. So this area above this purple line and on the left of this red line, this is going to be the feasible area, because the area below this line is infeasible, the area on the right of this line is infeasible. So, therefore, this area is the one which is the feasible area.

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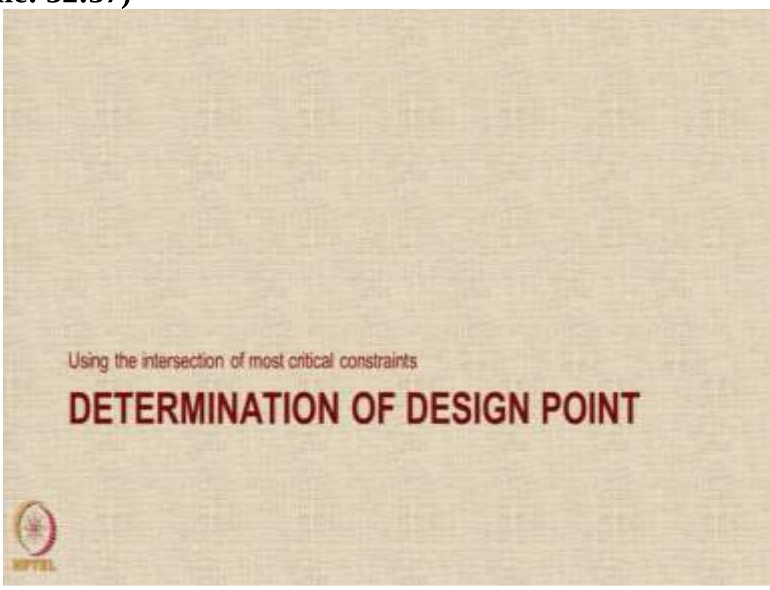

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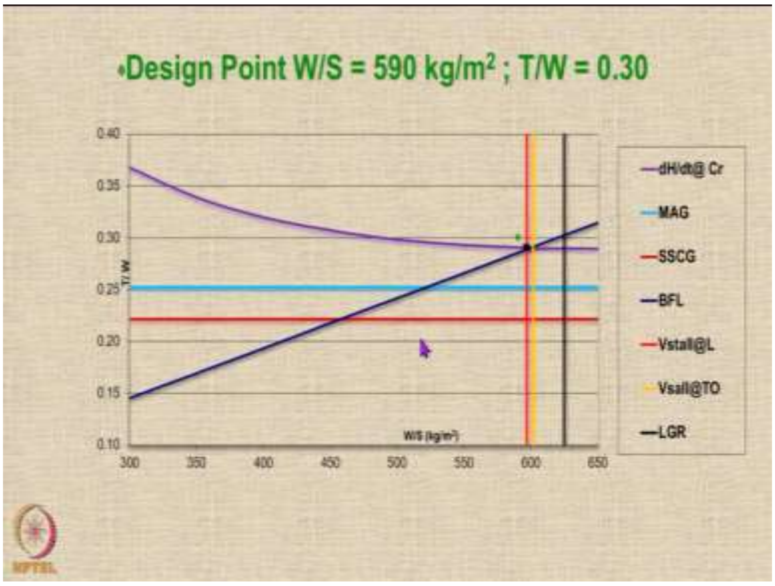

So, now moving on, let us see how to find the design point. This is obtained by taking the intersection of the most critical constraints which is as shown in the screen 596.6 kg per meter square and T/W of 0.291 that design point will be taken slightly above and slightly on the left of this constraint, these intersection points because of the requirement to give some margin for growth of the aircraft and for changes in the design as the analysis procedure progresses.

So it is always good to give some margin so what I have done is I have given a little bit of margin on the left hand side and literally margin on the vertical side so the design point.

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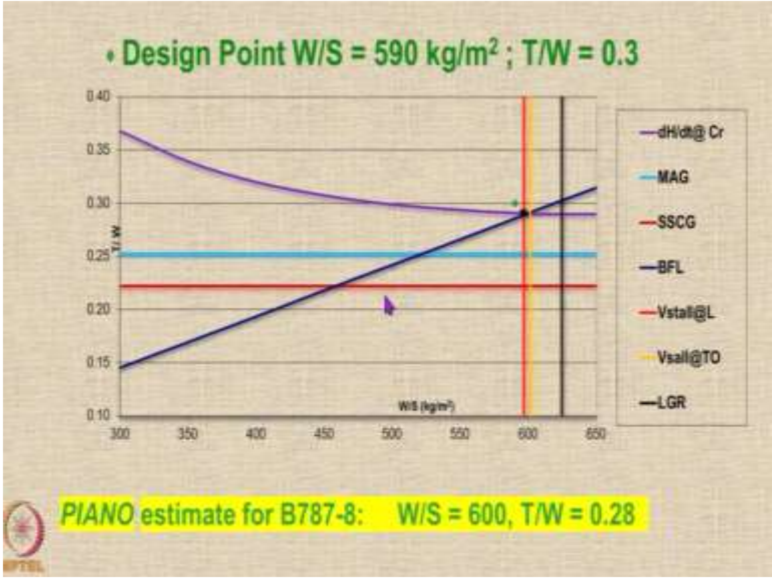

Now let us see how far we are from reality let us just check, so this is our constraint diagram. And the values that are given in PIANO for Boeing 787 are 600 and 0.28. So what we see is that we are very much near the values which are available in PIANO. So it shows that our constraint analysis calculations are very much near reality. Hope your calculations were matching with this. Thanks for your attention.