Introduction to Aircraft Design
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Lecture - 62
Tutorial on Constraint Analysis of Transport Aircraft: Part-1

Hello friends, today we will look at the constraint analysis of Boeing 787-8 Dreamliner aircraft.

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This aircraft is a very path breaking aircraft. And the picture that you see is of the first 787, which was delivered to air Nippon Airways, they placed an order for 50 aircraft in 2009. And that is how the journey of this aircraft started.

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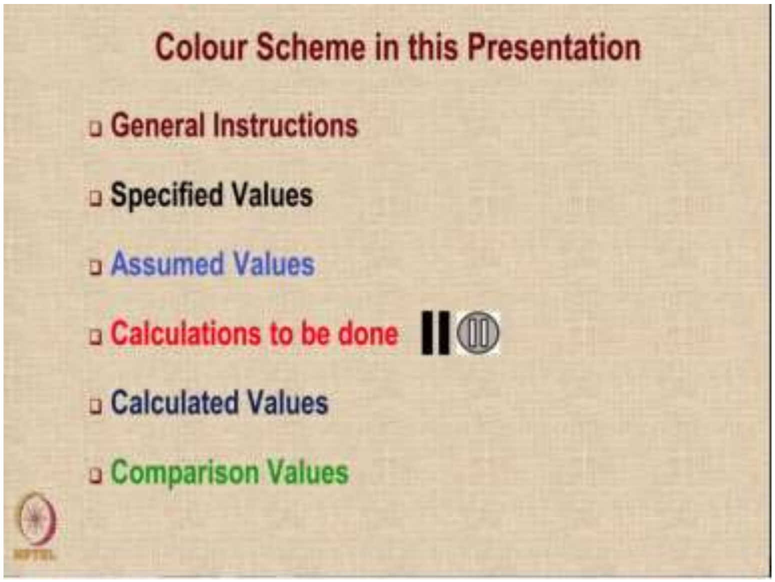

I am going to follow a particular color scheme in this presentation which I would like you to make note of the general instructions regarding calculations are going to be given in this brown color which is the basic background color wherever there are values specified as the parameters of the constraints those values will be shown in black color. Any values which are assumed based on past information or data about the aircraft will be shown in light blue color.

At places where I would like you to do some calculations, I am going to use the red color and followed by 2 vertical lines either 2 black lines or a pause button. The purpose of this is to alert you while watching the video that at that stage, you have to pause the video and you have to do some calculations and then resume. Finally, the calculated values that you obtained using the formula will be shown in the dark blue color you have to match the values obtained by you with these values.

And in the end, I have some comparison between our estimated values, calculated values and the values which are prescribed for Boeing 787 only at one place in the whole presentation towards the end, we will show that data in green color. So I hope you appreciate this color scheme and it will help you in understanding the scheme of things.

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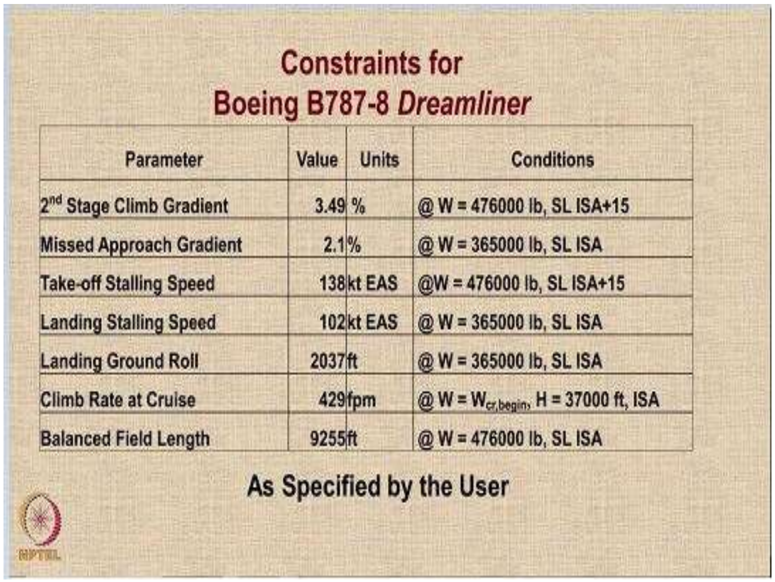

Constraints for Boeing B787-8 *Dreamliner*

Parameter	Value	Units	Conditions
2nd Stage Climb Gradient	3.49	%	@ W = 476000 lb, SL ISA+15
Missed Approach Gradient	2.1	%	@ W = 365000 lb, SL ISA
Take-off Stalling Speed	138	kt EAS	@W = 476000 lb, SL ISA+15
Landing Stalling Speed	102	kt EAS	@ W = 365000 lb, SL ISA
Landing Ground Roll	2037	ft	@ W = 365000 lb, SL ISA
Climb Rate at Cruise	429	fpm	@ W = $W_{cr,begin}$, H = 37000 ft, ISA
Balanced Field Length	9255	ft	@ W = 476000 lb, SL ISA

As Specified by the User

Let us look at the list of constraints for the Boeing 787 Dreamliner that we will look at today. You can notice there are 7 constraints in all as specified by the user. And you can also observed that the value specified are in the British system of units. So, this is the reality in aviation, most of the aviation experts still work in the FPS system. And hence I have decided to reproduce the values as it is. But as you know, we are supposed to work in the SI system. So the values in this table have been converted into the SI system in this table.

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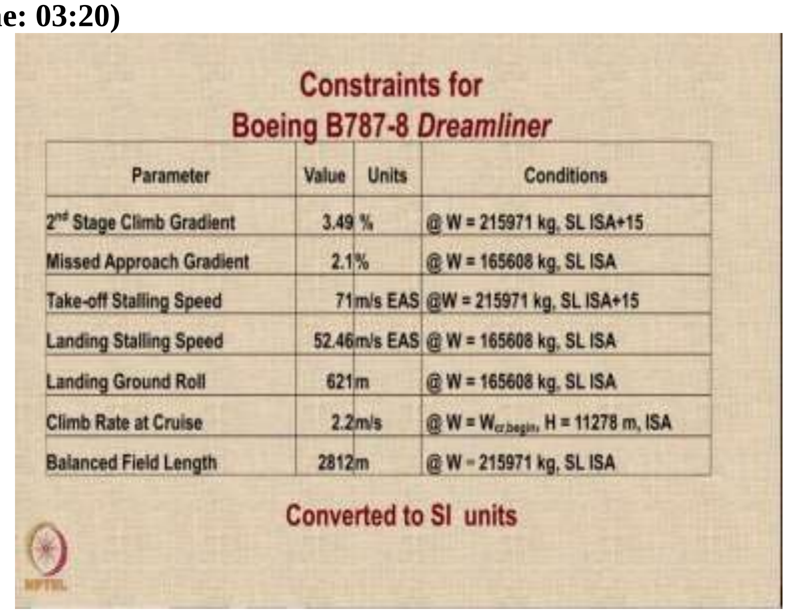

Constraints for Boeing B787-8 *Dreamliner*

Parameter	Value	Units	Conditions
2nd Stage Climb Gradient	3.49	%	@ W = 215971 kg, SL ISA+15
Missed Approach Gradient	2.1	%	@ W = 165608 kg, SL ISA
Take-off Stalling Speed	71	m/s EAS	@W = 215971 kg, SL ISA+15
Landing Stalling Speed	52.46	m/s EAS	@ W = 165608 kg, SL ISA
Landing Ground Roll	621	m	@ W = 165608 kg, SL ISA
Climb Rate at Cruise	2.2	m/s	@ W = $W_{cr,begin}$, H = 11278 m, ISA
Balanced Field Length	2812	m	@ W = 215971 kg, SL ISA

Converted to SI units

Now, the numbers are not rounded, but they are all in the SI units. So I think it is important for you to note down these values in fact, we will use them we will need them as we go ahead so we will show them before we do the calculations.

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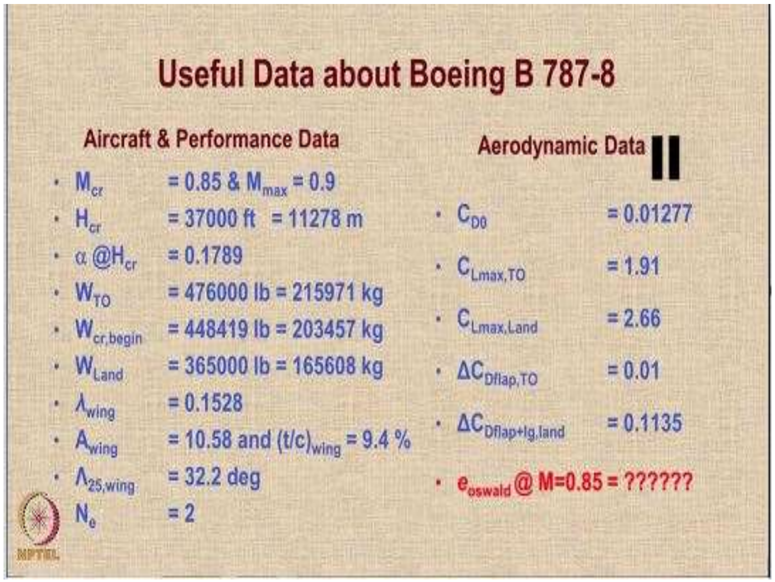

Let us have a look at some useful data about Boeing 787 it will be assumed that either this data is supplied to you as input or you have obtained these numbers prior to this tutorial through previous tutorials and calculations. Let us first look at the aircraft and performance data. This aircraft cruises at a Mach number of 0.85 and the maximum Mach number in cruise is 0.9 the cruising altitude is 37,000 feet which becomes 11278 meters.

The value of thrust lapse ratio α at the cruising altitude is 0.1789. This information is not readily available. We have obtained this information by looking at some specific data regarding the Boeing 787. The maximum takeoff mass of the aircraft is 476000 pounds, which converts to 215971 kilograms. The mass of this aircraft when it ends its climb and begins the cruise is 448419 pounds which converts to 203457 kilograms. This information also is not easily available, but it has been obtained by us by looking at some data.

The maximum landing mass of the aircraft is 365000 pounds, which converts to 165608 kg the taper ratio of the wing is 0.1528 the aspect ratio of the wing is 10.58 and the thickness to chord ratio of the wing is 9.4%. In reality, the taper ratio the thickness ratio of the wing varies from root to mid span to tip, but we have taken some medium value. In fact, I have taken the value at the break the quarter chord sweep of the aircraft is 32.2 degrees and this aircraft has 2 engines this information is related to the aircraft and its performance.

Now, let us look at some aerodynamic data. The zero lift drag coefficient of this aircraft when it is in cruising flight is 0.01277 we can assume it to be the same in all phases of flight as long as we do not go at a very high Mach number. The $C_{L_{max}}$ at takeoff is 1.91 with flaps deflected and the $C_{L_{max}}$ at landing is higher it is 2.66 the additional drag due to deflection of flaps at takeoff is 0.01. And the additional drag due to deflection of flaps during landing plus the landing gear that is 0.1135 we assume that during the initial climb stage the landing gear is retracted.

So, there is no additional drag coefficient because of the landing gear. Now, what is remaining is an important parameter which is the Oswald efficiency and that too, we would like to calculate the value at various Mach numbers. But to start with, we would like to get the value at Mach number of 0.85 which is the cruise Mach number. So, at this stage, I would request you to please note down this data, or you can take a picture of the screen and use it while you do the calculations.

Because these blue numbers that you see are going to be required by you in the calculations that follow. So, let us start our first calculation by trying to obtain the value of the Oswald efficiency of this aircraft.

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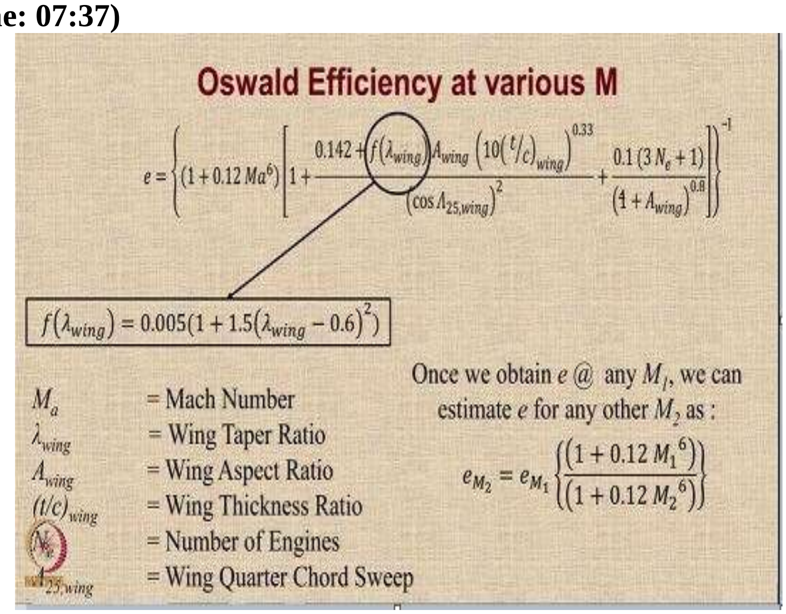

Now, there are many methods available for estimating the value of Oswald efficiency. As you know, it is a very important parameter. And there is a huge amount of literature available on which formula to use. The one that is very popular and most suitable for transport type aircraft is the one that you see on the screen given by Professor Dennis Howe of Cranfield University. So, this particular formula is quite detailed.

And you can see it has several parameters related to the aircraft let us look at it one by one. The first and the foremost parameter that you need is the Mach number M_a because the value of e depends upon the sixth power of Mach number. The next parameter is the taper ratio. And once you know the taper ratio, you have to calculate a function of taper ratio called $f(\lambda_{wing})$ then we have to also use the wing aspect ratio and then the $\left(\frac{t}{c}\right)_{wing}$.

So, here the $\left(\frac{t}{c}\right)_{wing}$ will be not expressed in percentages, but if it is for example, if it is 10% it will be 0.10 Ne as you know stands for number of engines and $\lambda_{25,wing}$ is the quarter chord wings sweep. So, all these parameters are needed before we can estimate the value of the Oswald efficiency. As I mentioned, we need to calculate a function related to lambda wing it is called as $f(\lambda_{wing})$ and the value of $f(\lambda_{wing})$ is as shown in the screen.

Now, using this formula, once we obtain the value of e for any Mach number, we can estimate the value at any other Mach number simply by dividing e_{M_2}/e_{M_1}. Now, you can notice that the quantities inside the square bracket are all constant with respect to Mach number therefore, if you want to obtain the Oswald efficiency at some other Mach number M_2 and you know it at M_1, it is very easy e_{M_2} will be e_{M_1} times just the ratio of the first term.

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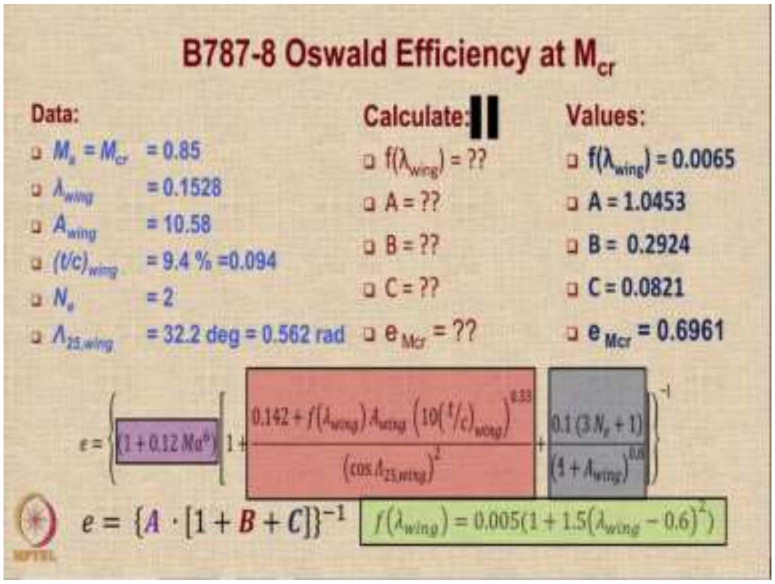

Let us proceed now this is the formula that we will use, let us try to calculate the Oswald efficiency factor of Boeing 787 at the cruise Mach number, using this formula, we will divide this formula into some 3 components and do it slowly. So first of all the data, so M_a is M_{cr} which is 0.85 $\lambda_{wing}$is 0.1528 Aspect ratio of wing is 10.58 t/c wing is 9.4% or 0.094. And $N_e = 2$, and the quarter chord wing is 32.2 degrees.

But remember to convert it into radians, because in the formula that you see on the screen, where you calculate the cos in the denominator, you have to use the value of quarter chord wings with in terms of radians. So what we can do is we can divide this formula into 3 parts. So e will be equal to

$$e = \{A * [1 + B + C]\}^{-1}$$

where A, B and C are highlighted as shown in this screen. So therefore, first thing we need to do is we need to calculate the value of f wing, which is given as per the formula shown in the bottom in the green background.

Then we need to calculate the values of A, B, C. And once you know A, B and C, you will be able to calculate the value of $e_{M_{cr}}$. So at this point, please pause the video and do these calculations. And watch ahead only after you have done the calculations so that you can match. If you really want to learn aircraft design, you need to solve the tutorial along with me and not just watch the video because then you will not learn.

So please pause the video do these calculations. Once you are ready, then you can play it again. Assuming that you have obtained these values by calculation, let us compare. So the $f(\lambda_{wing})$ is 0.0065. The value of the first parameter A is 1.0453. The value of parameter B is 0.2924. And the value of parameter C is 0.0821. So using these values of A B C in the Formula

$$e = \{A * [1 + B + C]\}^{-1}$$

we can estimate the value of e or the Oswald efficiency to be 0.6961.

Remember, this value is only for the cruise Mach number. And for any other Mach number you need to use the formula as I described earlier to estimate the value.

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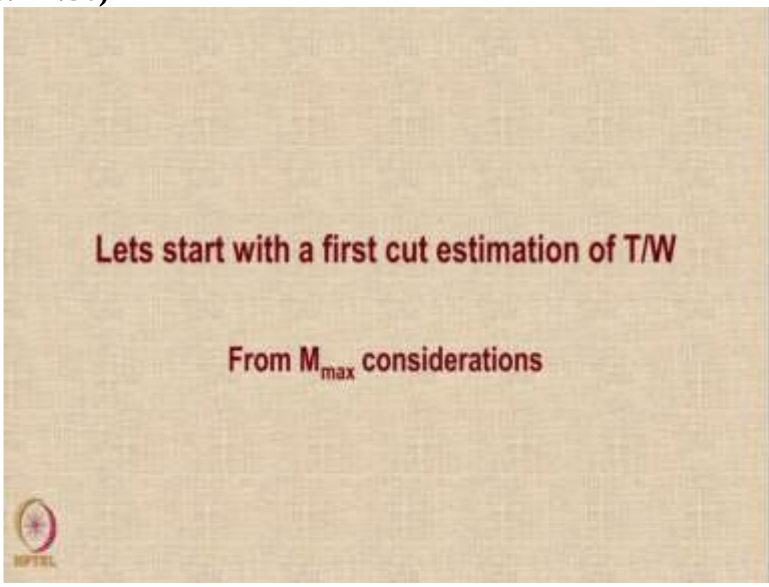

Moving ahead, let us start our constraint analysis in which we determine the values of T/W and W/S that meet all the requirements. But to begin with, we will have a first cut estimation of thrust to weight ratio. And for this we will use the considerations for maximum Mach number.

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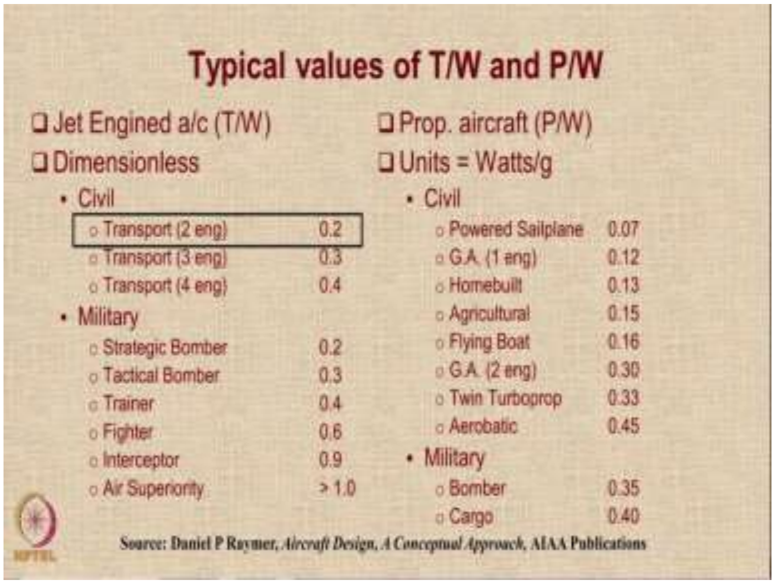

The typical values of T/W and P/W for various types of aircraft, we need to have an idea about it. So let us first look at jet engine aircraft or aircraft whichever turbofan or turbo jet engines, in these cases, we calculate the thrust to weight ratio T/W. This ratio this is a dimensionless parameter in case of jet engines because both thrust and weight are in the same units. So for civil transport aircraft, it is very easy to remember if it is a 2 engine aircraft it is around 0.2 if it is 3 engine, it is around 0.3 if it is 4 engine, it around 0.4.

But when you go to the military aircraft, then depending on the type of the aircraft, whether it is a strategic bomber, or a tactical bomber, or a trainer or a fighter or interceptor or air superiority aircraft, the value comes as shown in the screen. So, for an aircraft like f 16, the value of T/W is approximately 1. If the air craft is powered by a turboprop or a piston prop then we calculate power to weight ratio.

Because there is nothing like thrust to weight ratio defined in that because turboprop and piston prop engines they develop power and that power is then extracted in the case through a propeller to generate the thrust force. So the engine has only P/W which has to be converted to T/W. The units in this case would be watts per gram. This is the standard SI unit for P/W where P is the power in watts and W is the mass in grams. So for civil aircraft, again, depending on the type, there are various values, which are the ballpark or recommended or first cut values.

And if you are, looking at the military aircraft, the values tend to be slightly on the higher side. This data shows that for our case, which is a twin engine aircraft, we can expect the value to be nearly 0.2 or between 0.2 and 0.3. And this data has actually come from the textbook by Daniel Raymer.

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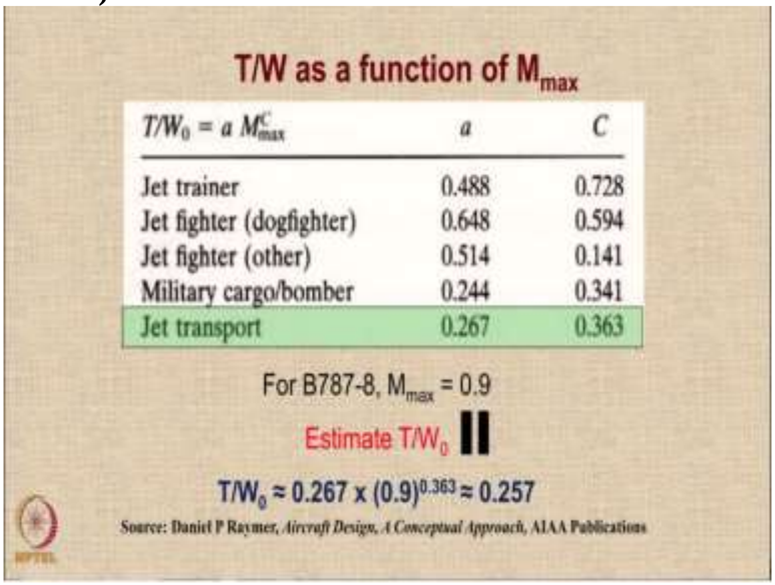

Moving ahead, let us also estimate based on some suggestions by Daniel Raymer, in his book on how to estimate or get a better estimate for thrust to weight ratio as a function of the maximum Mach number because we are looking at a jet engine aircraft. So, Daniel Raymer has given a table there is one table for aircraft powered with jet engine aircraft and there is another one for prop engine aircraft, we are looking only at the one with jet engines and in our table our aircraft is a jet transport aircraft.

So, therefore, the T/W_0 that is the thrust produced at sea level static conditions divided by the design gross weight W_0 is expected to be

$$\frac{T}{W_0} = aM_{max}^c$$

where M_{max} is the maximum Mach number and a and C are constants to be determined from this table for example, for jet transport aircraft Raymer recommends to use a as 0.267 and C as 0.363. So, for Boeing 787 dash 8 the maximum Mach number as specified is 0.9. And therefore, if you can estimate the T/W_0 using this particular simple approximation, so, let us do that calculation.

So, the calculation will be very simply is just

$$\frac{T}{W_0} = 0.267 * 0.9^{0.363}$$

the value comes out to be 0. 257. So, please remember as per the previous slide, the approximate value was 0.2 between 0.2 and 0.3 because it is a twin engine aircraft and now, because we have the maximum of Mach number information available we get a slightly better estimate. So, if you get your answer very far away from this number suppose you get the answer as 0.4 or 0.5 etc. or 0.1.

You should start distrusting your calculations if you get a value near 0.257 then you can be quite confident that the answer is correct. Moving on. Now, let us start the process of doing constraint analysis.

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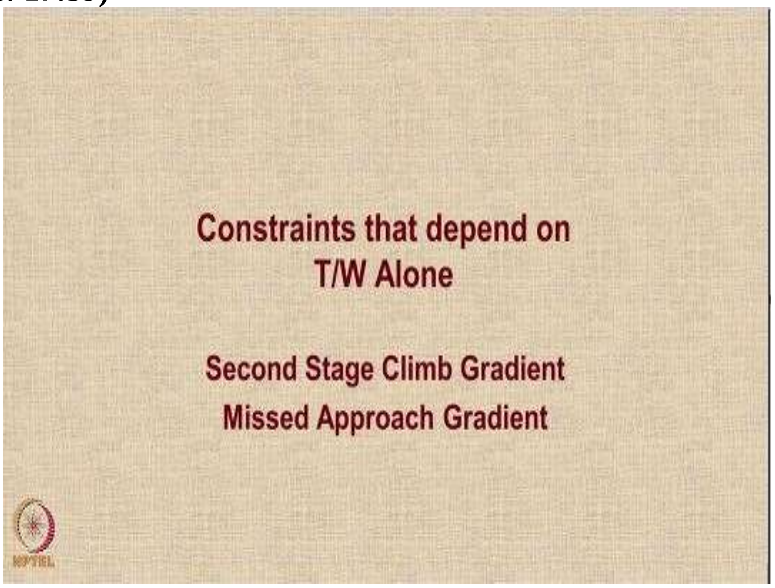

So, what we will do is first we will look at constraints that depend only on thrust to weight ratio. There are 2 such constraints as we know in the list the first one is the second stage climb gradient and the second one is the missed approach gradient.

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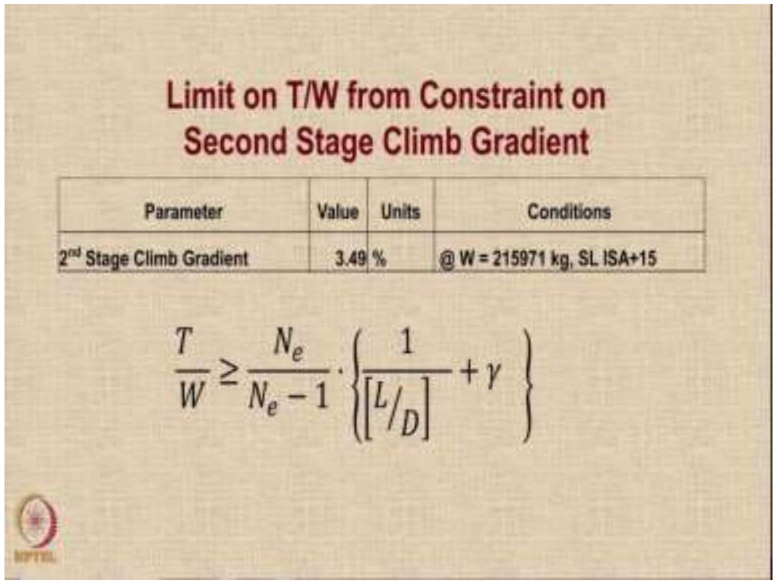

And we will start by looking at how the thrust to weight ratio is limited by constraints on these parameters. So, the first one is the second stage climb radiant as specified in the constraint table. The second stage climb gradient is expected to be 3.49% minimum when the aircraft is at the maximum takeoff weight of 215971 kilograms under ISA + 15 conditions at sea level. And to calculate this we will use this particular formula if this formula seems unfamiliar to you.

I would request that you stop here go back and watch the lecture on constraint analysis for civil aircraft where we have explained how this formula is obtained and what it stands for. So, proceed further only if you are familiar with the formula and you have watched the lecture regarding constraint analysis procedure for civil transport aircraft otherwise, you will not be able to appreciate the numbers.

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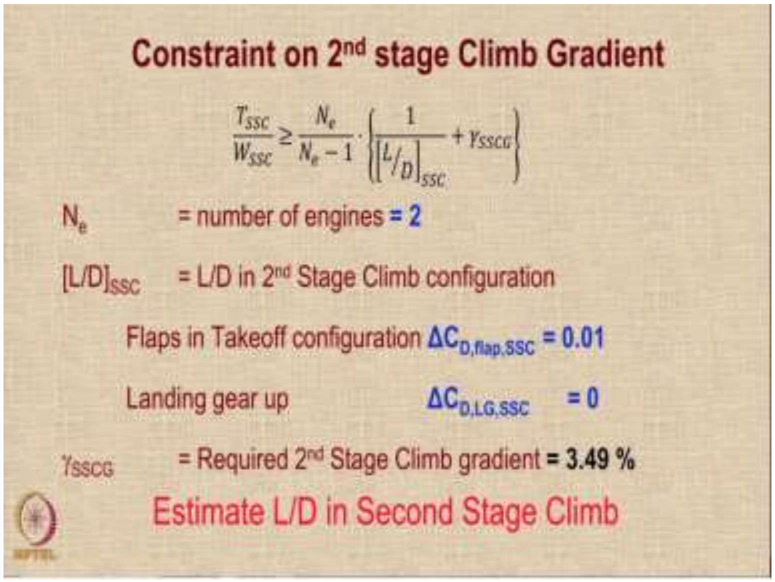

Moving on, when we are looking at the second stage climb gradient then we are using the subscript SSC for second stage climb and SSCG stands for second stage climb gradient. So, the formula which I just flashed, I have just inserted the corresponding values for the thrust for the weight for the L/D and for the γ in this particular formula. So, here N_e is number of engines which is specified are given as to the L/D in the second stage climb is the L/D to be calculated for the second stage climb configuration.

Now, recall from our previous lecture that during the second stage of the climb flaps are in the takeoff configuration and for that we have already been given that the additional drag because of the deflection of flaps in the takeoff configuration for this aircraft can be assumed to be 0.01 and also the landing gear is up or retracted and therefore, there will be no additional drag because of landing gear deflection.

So, the only additional drag which will occur would be only because of the flaps the required second stage climb radiant as specified in the constraint diagram constraint table was 3.49%. So, with this information, now, we need to calculate the L/D in the second stage climb. So, that is our next step.

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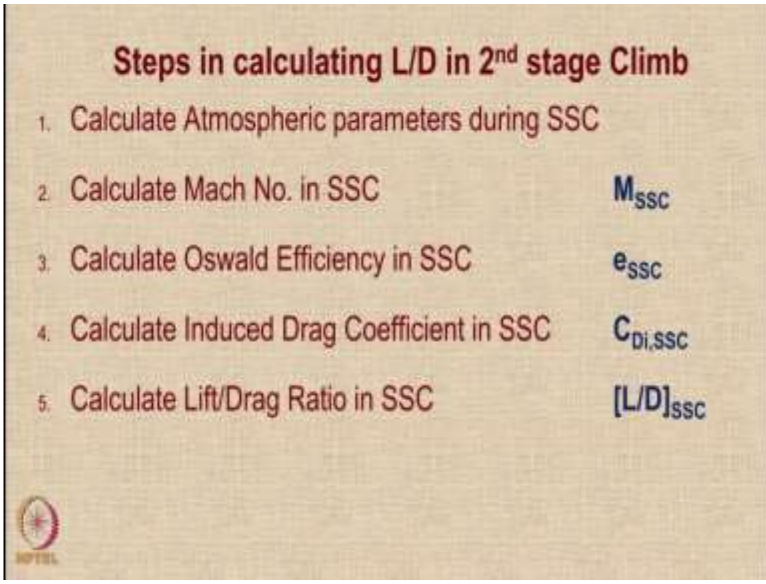

And let us see how we can calculate it. So, this calculation happens over I think 5 steps first, because the second stage climb is happening at ISA + 15 degrees, we need to do some atmospheric parameter calculation, then we calculate the Mach number in the second stage climb which we call it as M_{SSC}. Now, recall we have seen a formula to get the value of Oswald efficiency as a function of Mach number.

So, the next step will be to calculate the Oswald efficiency in the second stage climb, call it as e_{SSC} once you know the value of e_{SSC}, then the induced drag coefficient can be estimated. So, that is the next step. Now, once you calculate the induced drag coefficient, you already know the value of C_{D_0} and you can add the additional drag coefficient because of the flap deflection.

So, with that, you can get the value of C_D and once you get the value of C_D, you already know the value of C_L therefore, you can get the value of L/D, let us do these calculations for Boeing 787-8.

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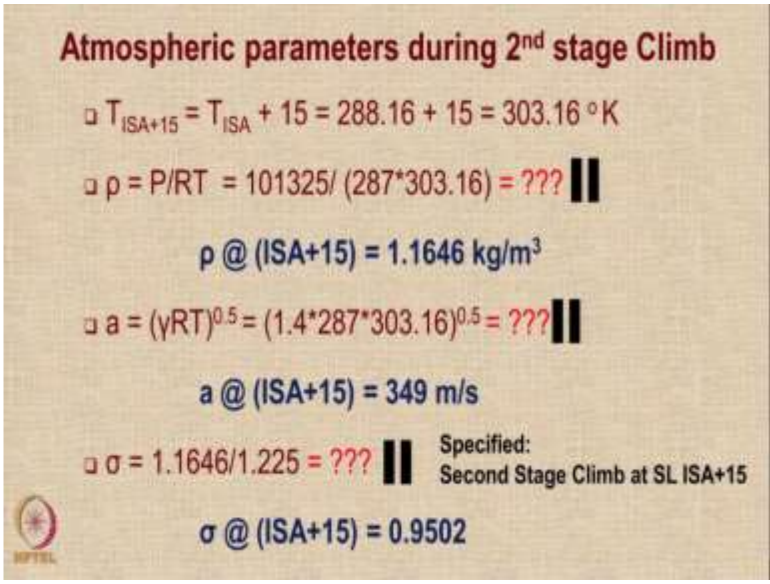

It is specified that the second stage climb occurs that ISA + 15 degrees. So, first thing to do is to calculate the value of the temperature at ISA + 15 degrees centigrade that is 288.16 which is the ISA temperature at sea level plus 15 degrees. And using this information we can calculate the value of density.

So, pause here and do the calculations for ρ the value comes out to be 1.1646 kg per meter cube. Next, we need to calculate the sonic speed at this particular condition So, calculate this value by pausing the video the values turns out to be 349 meters per second.

And the value of σ which is the density ratio would be the density under these conditions divided by the standard sea level density of 1.225 pause the video do the calculation the value of σ turns out to be 0.9502.

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The next step is to calculate the Mach number in the second stage climb for this the V_{stall} is already specified as 138 knots AS under ISA conditions. So, first what we do is we convert this into TAS. So, it will be 138 upon the value of σ which has been obtained in the previous slide as 0.9502.

Please get this value pause the video and let us continue the value comes out to be 141.57 knots or 72.83 meters per second to convert knots into meters per second, you should multiply by 1852 and divide by 3600 or you can also multiply by 0.514444. Now, we assume that the velocity of the aircraft in the second stage climb would be 1.2 times V stall to keep some safety. So, that would be 1.2 times 72.83 please calculate this value by pausing the video the value turns out to be 87.4 meter per second.

And since we know the velocity now and we also know the sonic speed obtained in the previous slide, we can calculate the value of the Mach number in cruise which is 87.4 divided by 349 or 0.2504. So, the Mach number in the climb for this the Mach number in the second stage climb is 0.2504.

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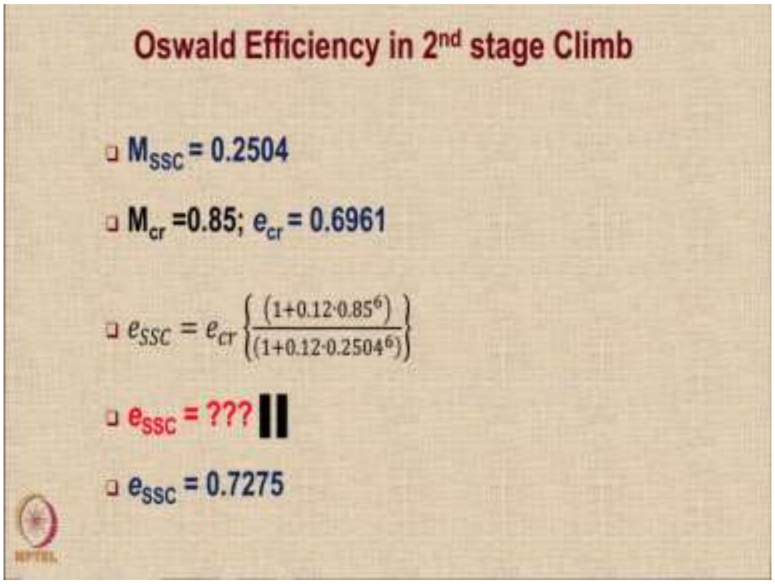

The next step would be to calculate the value of the Oswald efficiency in the second stage climb, you know that the Mach number is 0.2504 during the second stage climb, we also know that the cruise Mach number is 0.85 and the value of e at cruise was derived earlier as 0.6961 we have a formula which can be used to calculate the value of e at any other condition, we need to calculate the e at the second stage climb. So, we can use this formula the simple ratio of 2 terms multiplied by the value of e at the cruise condition calculate the value it turns out to be 0.7275.

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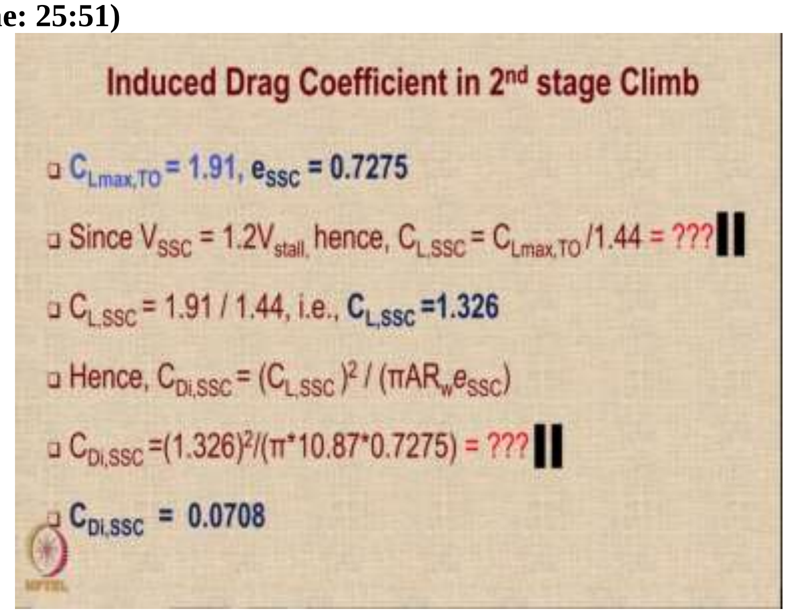

Now, once you know the value of e you can go ahead and calculate the induced drag coefficient during the second stage climb because $C_{L_{max}}$ takeoff is known as 1.91 specified e_{SSC}, we have just now calculated as 0.7275. Now, since V_{SSC} is 1.2 and V_{stall} therefore, C_L in SSC will be $C_{L_{max}}$ by

1.44 which will be 1.91 / 1.44 that would be 1.326. I hope you are pausing the video whenever you see this 2 vertical lines and doing the calculations yourself.

Once again, I would like to repeat that otherwise, you will not be able to really get the hang of this procedure, you will simply nod your head and go ahead and you will not be able to appreciate so please spend some time, wherever you see 2 vertical lines like this, you should pause the video, do the calculation that has been asked and then check your values in the next, so now once we know $C_{L,SSC}$ we can get the value of C_{D_i}.

So I put the values in the formula for you for your recent calculation, but please pause the video and calculate the value of $C_{D_i,SSC}$ the value comes out to be 0.0708.

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Now, finally, once you know the lift to drag ratio, once you know the values of these parameters, and you know that the additional drag because of flap deflection is 0.01 and additional delta drag because of the landing gear is not to be considered. So therefore, you know that the

$$C_{D,SSC} = C_{D_0} + C_{D_i,SSC} + \Delta C_{D_{flap},SSC} + \Delta C_{D_{LG},SSC}$$

please calculate this number by putting in the corresponding values C_{D_0} is 0.0201277 C_{D_i} is all known.

So with all these values, you can get the value of $C_{D,SSC}$ as 0.0936. So L/D would be $C_{L,SSC}/C_{D,SSC}$. Both values are known to you. So therefore, please pause the video and calculate L/D_{SSC}. The

L/D_{SSC} turns out to be 14.17 compare this with the L/D of approximately 19 to 20 that you get in a cruise condition. So we see that during the second stage climb, because the landing gear is up and only the flaps are deflected, you lose a little bit from 19 to 20, you just come to around 14.

So there is definitely a reduction but it is not too much. Later on when I show you the calculations for the missed approach gradient, you will notice that the L/D is going to fall to a much lower value.

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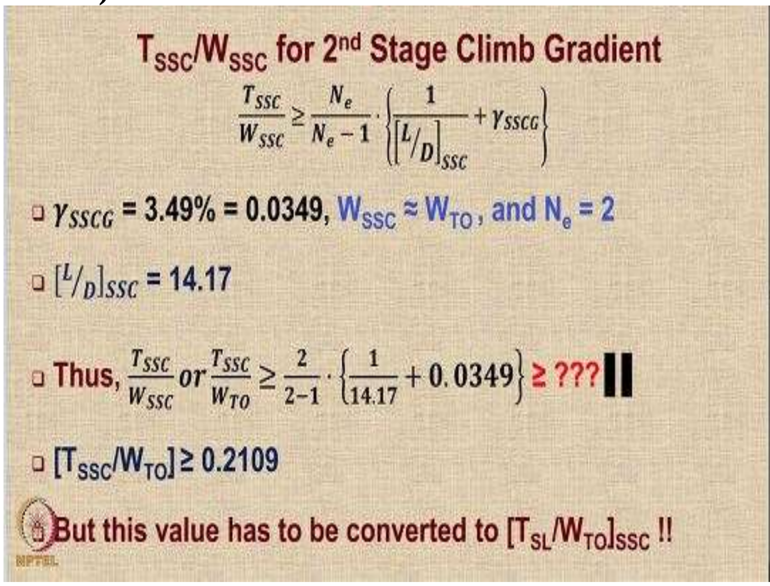

So with this, we can calculate the thrust to weight ratio for second stage climb gradient again, I am just flashing the old formula, but now we have all the values available SSCG is specified as 3.49% we can assume that during the second stage the weight is equal to almost the takeoff weight. So, therefore, the weight parameter beta will be almost equal to 1 the number of engines is equal to 2 L/D_{SSC} is just calculated.

So therefore, the value of T/W in SSC condition or T_{SSC} / W_{TO} can be easily calculated using this formula, this value pause the video and calculate please the value comes out to be 0.2109. However, this is not the answer, because this is T_{SSC} / W_{TO} what we want is T_{SL} /W_{TO}. So, this is at an ISA + 15 condition whereas, we need to get the values at ISA T_{ISA} upon W_{TO}.

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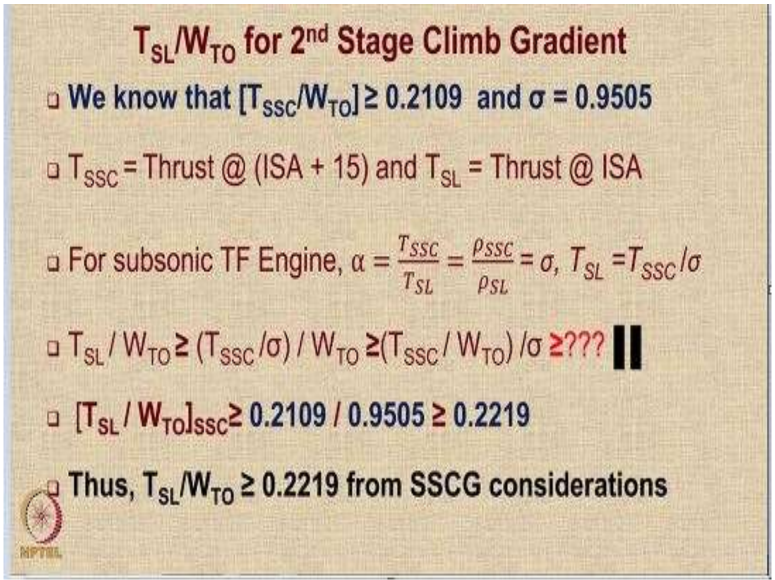

So, for this we need to use the value of alpha that is obtainable, which I will show you shortly. So, what you do is because we know the value of T_{SSC} / W_{TO} and we also know value of sigma we can always say so, the T_{SSC} is basically the thrust under ISA + 15 condition and T_{SL} is the thrust under ISA conditions and we know from our basic engine model that for a subsonic turbofan engine, the value of thrust lapse ratio alpha when there is no afterburner.

So, you can show that T_{SL} / W_{TO} which is what we want to be more than equal to T_{SSC} by sigma divided by W_{TO}. But, since both are in denominator you can exchange them and you can get T_{SSC} / W_{TO} divided by sigma. Now, T_{SSC} /W_{TO} is known as 0.2109 and σ is known as 0.9505. Therefore, please pause the video and calculate the value of T_{SL} / W_{TO} it will be 0.2219.

Hence, we come to the first conclusion that thrust to weight ratio of the aircraft should be more than 0.2219 from second stage climb gradient consideration. So, now, we are going to do similar calculations for other constraints, let us look at how we can calculate the constraint line due to the second stage climb gradient.

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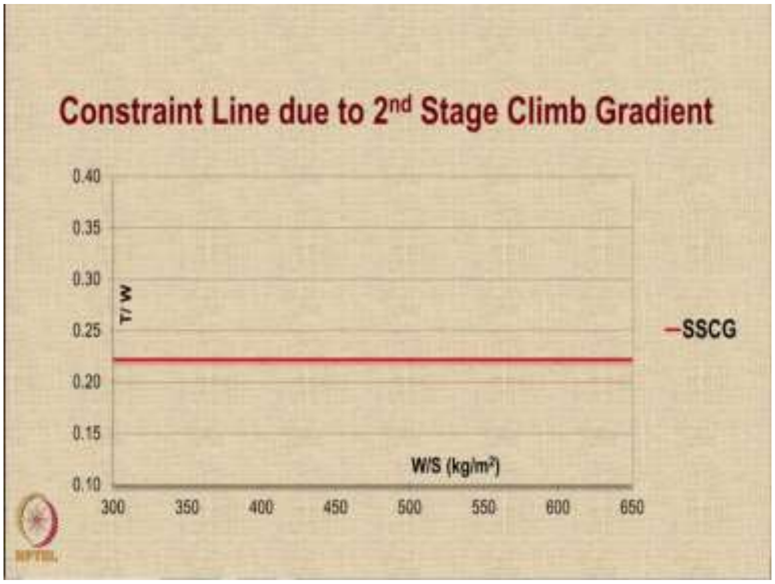

So, this appears as a single horizontal line because for there is no role of including in this case we got a direct constraint on T/W. So, the T/W value has to be always more than 0.2219. So, therefore, this is the horizontal line. So, the area which is shaded below this line is infeasible and the area above this line is feasible.

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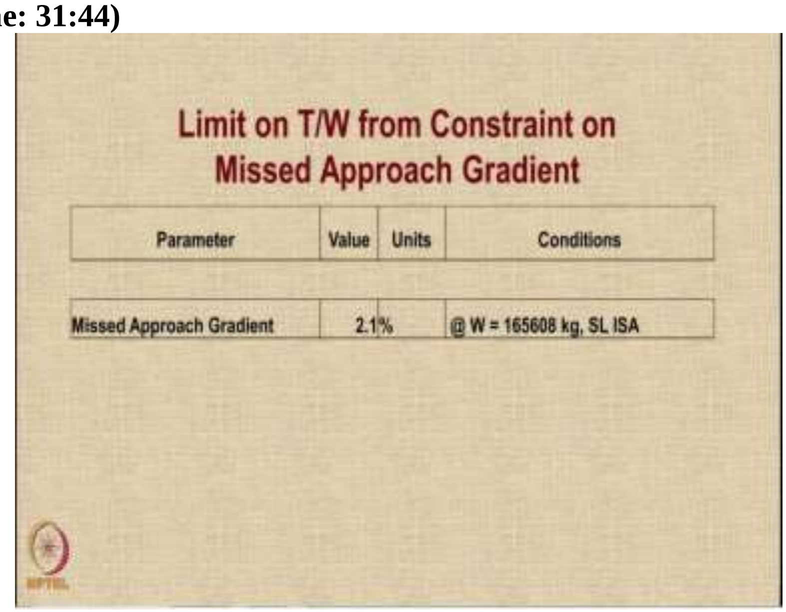

In the same way we can obtain the lines on the constraint diagram due to several other constraints. Thank you so much for your attention. We will move on to the next section now.