Introduction to Aircraft Design
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Lecture - 61
Constraint Analysis- Transport Aircraft- Part-02

Let us look at the second approach in which we estimate the wing loading from specified constraints first and then use that number to back calculate the required $\frac{T}{W}$ from other constraints.

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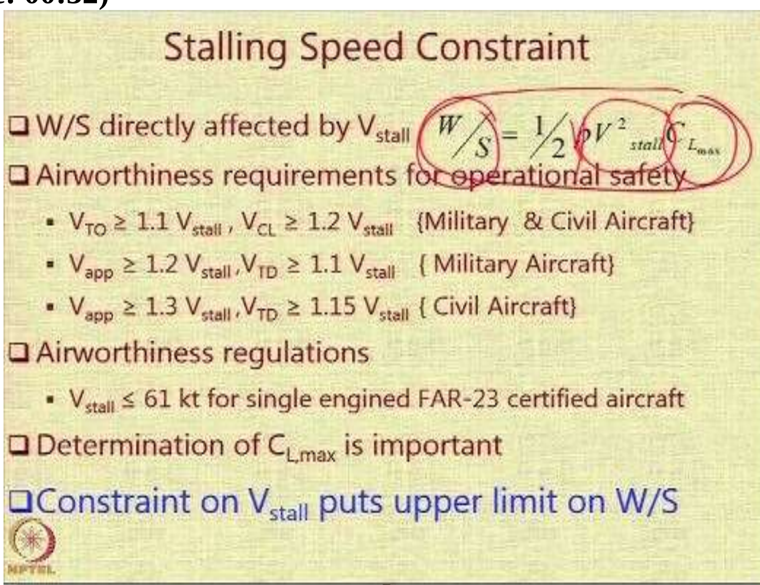

For this, there are 3 or 4 constraints, which depend only on wing loading, I remember them as the 3 things and the first thing is the instantaneous turn, ceiling, stalling and climbing. So, stalling speed constraint is directly affected by V_{stall}. So, the $\frac{W}{S}$ value directly is a value of the V_{stall} that is specified because

$$\frac{W}{S} = \frac{1}{2}\rho {V_{stall}}^2 C_{L_{max}}$$

where V is V_{stall} and C_L is $C_{L_{max}}$. So, from there one can easily get the value the airworthiness requirements for operational safety are specified in some cases especially in FAR 23.

So, for instance for FAR 25 if we look at military and civil aircraft, the V takeoff has to be 1.1 times V_{stall} and the V_{Climb} has to be 1.2 times V_{stall} or more these are as specified approach speed has to be at least 20% higher than V_{stall} and the touch down velocity has to be at least 10% higher than V_{stall} both for military aircraft and in the case of civil aircraft, the margins are slightly higher 30% for approach speed and 15% for touch down speed respectively.

So, the regulations are also sometimes very explicit regarding the stalling speed for example, if the aircraft is to be designed for FAR 23 then if a single engine aircraft is used, then V_{stall} has to be less than 61 knots. So, once you know the value of $C_{L_{max}}$, then you know you can easily get value of the upper limit on the $\frac{W}{S}$ which can correspond to that particular V_{stall} as you can see in this equation, if the value of $C_{L_{max}}$ is known, and assuming that the density is density at sea level or the altitude at which the velocity is given higher the highest value of permitted V_{stall} will directly give you the highest value of the wing loading.

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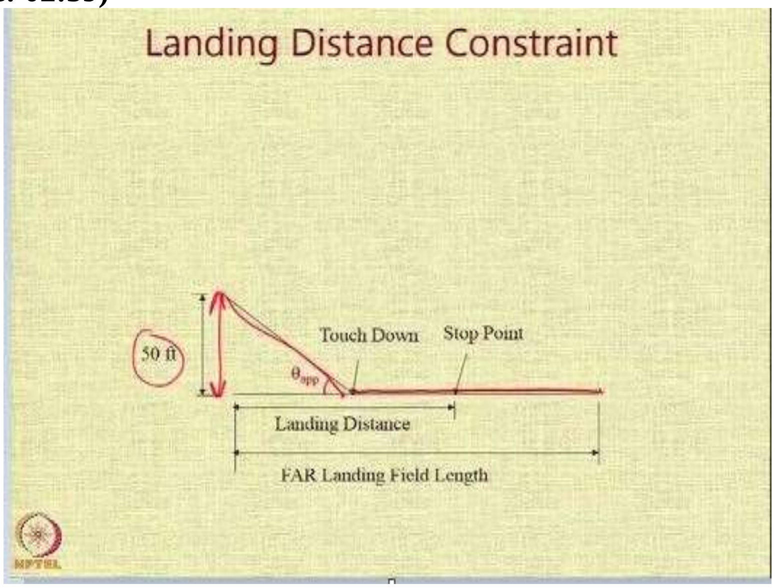

Landing distance constraint this is how we define the landing distance the obstacle height is considered to be 50 feet as against the obstacle height of 35 feet for takeoff. So, the aircraft is supposed to be about the runway when it reaches the obstacle height and then it is supposed to approach the runway at a particular approach angle usually this angle is 3 degrees but it can vary slightly depending on the setting of the instruments installed by the airport authority at various airports. Once it touches down, it is supposed to decelerate and stop.

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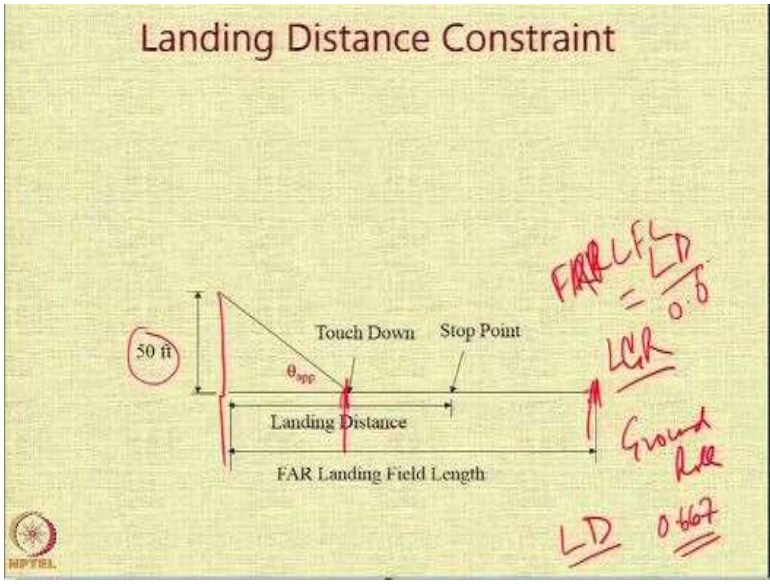

So, here there are interesting differences. So, this distance which is from touchdown to stop this distance is called as the landing ground roll or ground roll at landing ground roll. But from the point where the obstacle height is reached to the point where you stop that distance is called as the landing distance or LD landing distance. So landing distance is going to be the landing roll plus the approach distance though this will be the approaching distance, this will be the landing distance and the total of these, now the FAR field length.

The FAR landing field length includes a factor of 0.667 which is a dividing factor. So, the FAR landing field length will be the landing distance divided by 0.667.

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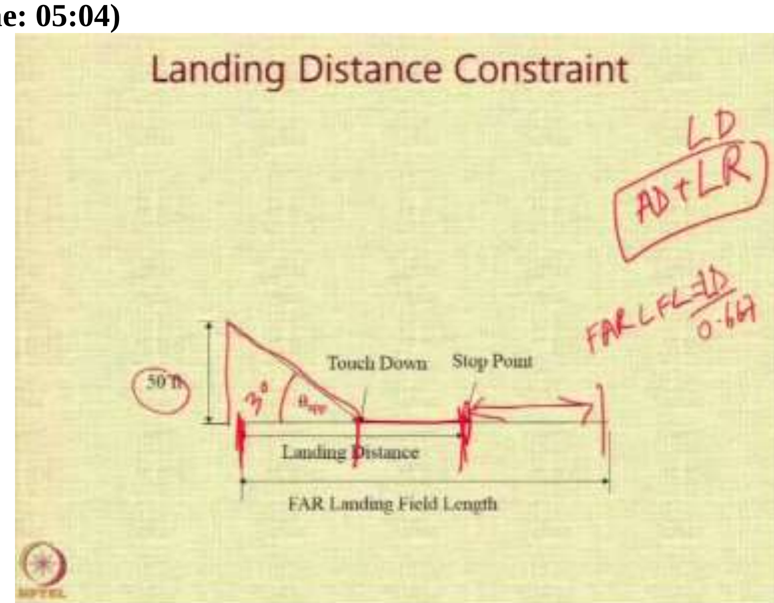

So, that is because they want to specify a margin in the, they want to specify some margin to take care of the pilot technique to take care of the weather conditions etc. So, this is how we define the landing distance this angle is the approach angle. So, the aircraft is expected to approach at a angle specified by each airport this angle is approximately 3 degrees but it can vary slightly the obstacle height in the case of landing distance calculation is specified as 50 feet in contrast with the obstacle height of 35 feet specified for takeoff.

So, when the aircraft is above the obstacle height on the runway and approaches touches down and then comes to a stop this distance from the starting point where the obstacle height is reached to the stop point is called as the landing distance or LD. So, LD will include the approach distance which is from here to here and the landing roll which is here from here to here. Now, the distance after touchdown to the point where you stop is called as the landing run or landing roll.

So, landing distance is equal to the approach distance plus the landing role and FAR regulations specify a safety factor above this as the landing field length. So, the landing field length or FAR landing field length will be equal to the landing distance which is from the obstacle height to the stop point divided by 0.667 this margin of 0.667 in the in the denominator increases the landing distance and this extra length takes care of differences in pilot approaching or flooring techniques, weather conditions, runway conditions etc.

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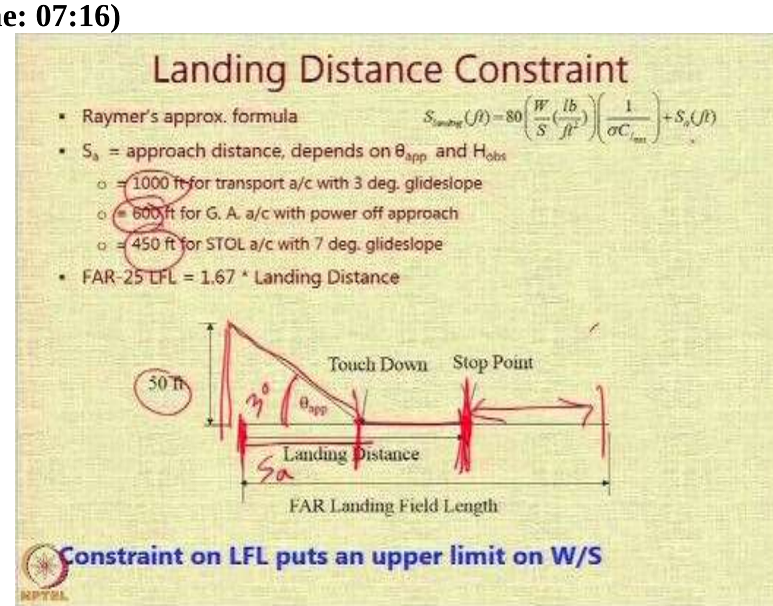

So, let us see Raymer has given a generic formula for estimating the landing distance in feet in terms of the wing loading in pounds per square feet sigma density ratio $C_{L_{max}}$ and S_a the approach distance. So, this distance approach distance which is called as S_a is a function of the obstacle height and the angle by simple trigonometry. So, if you have 3 degree glide slope, this distance S_a can be approximately 1000 feet which is the case for most transport aircraft.

If you have a general aviation aircraft that comes in with a power of approach, you can allow a larger angle of theta and therefore, this larger angle theta would lead to a shorter distance and if you have a STOL aircraft which comes at a much larger glide slope say 7 degree glide slope then you know, you can have further reduction in the approach distance. So, depending on the condition, depending on the type of the aircraft the approach distance varies slightly, so you can estimate the landing distance using this particular formula approach distance using the angle.

And then you can see that the constraint on landing field length will put an upper limit on wing loading. So, the constraint on landing and the constraint of stalling both of them put the upper limit and then we pick up whichever is more critical from that. Let us now look at the third category of constraints, which affect directly both $\frac{W}{S}$ and $\frac{T}{W}$.

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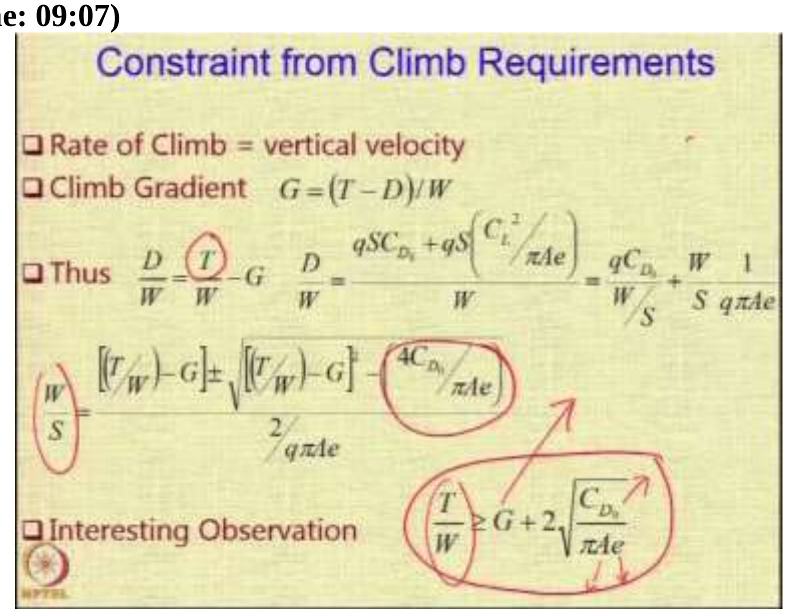

So, the constraint from climb requirements is one such example. The rate of climb of an aircraft is nothing but $\frac{dh}{dt}$ the vertical velocity and the climb gradient as defined is the excess thrust upon weight or the specific excess thrust $\frac{T-D}{W}$. So, from here you can easily show that

$$\frac{D}{W} = \frac{T}{W} - G$$

and then this T is equal to D and D will be essentially

$$D = \frac{1}{2}\rho V^2 S C_D$$

and

$$C_D = C_{D_0} + KC_L^2$$

So, from there you can derive this expression as shown on the screen.

So, in short what you get is a quadratic in $\frac{W}{S}$ because $\frac{W}{S}$ appears in the denominator as well as in the numerator. So, if you do simple geometrical simple algebraic calculations, it can be shown that the wing loading would be obtained by the expression of $\frac{W}{S}$ and with this you get a condition this particular quantity cannot be more than this quantity. So, from there you can get a condition and the condition is that $\frac{T}{W}$ has to be always more than G plus this particular quantity mentioned in square root in the equation and this can easily give you can easily the actual value of W will come from here, but quick estimate on the lower limit of thrust to weight ratio can be given by the expression shown on the bottom of the screen. So, if the gradient is specified let us say 2% or 4% if it is not is known, if the aspect ratio is known as e can be estimated, then some minimum value of $\frac{T}{W}$ will automatically be estimated using this particular observation.

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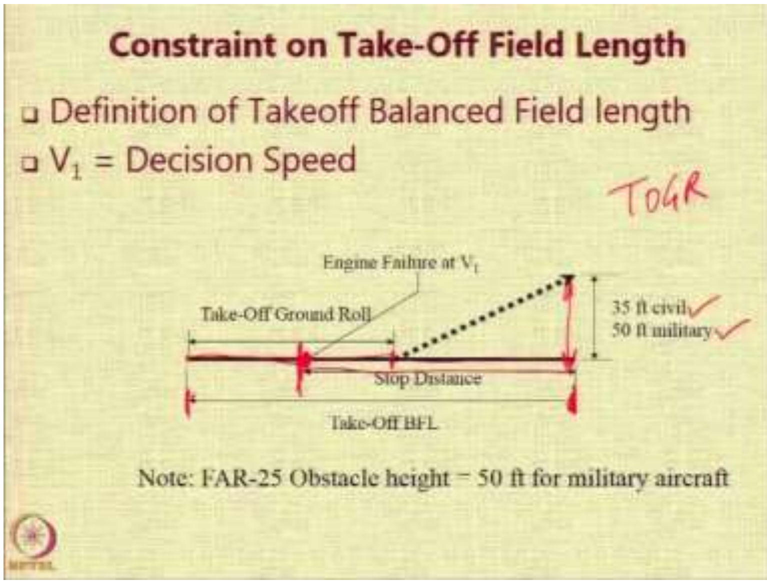

Take off field length now, in takeoff field length we have a similar thing like we saw in the landing field length. So, you start from where you are addressed and when you proceed to a place where you have liftoff in case all engines are working and there is no incident that particular distance on the ground from start to the place where you leave the ground is the take of ground roll.

And then the aircraft starts climbing and when it starts climbing at some distance on the ground, it will clear the obstacle height this obstacle height is 35 feet for civil aircraft and 50 feet for military aircraft. So, the total distance from on the ground from where you start to where you can achieve the obstacle height clearance that will be the takeoff and there is a concept called as a takeoff balance field length in which the total takeoff distance with 1 engine not 1 engine becoming un-operational at a particular speed called a speed V_1.

Even when this happens and the pilot continues with only other engines working the distance that you cover on the ground is same as the distance that you cover if you come to a stop if that is the case, then that distance from the ground is called as the takeoff balance field length. So, whether you are specified the takeoff ground run roll or the takeoff distance or the takeoff balance field length you can use simple formula to calculate the constraint values.

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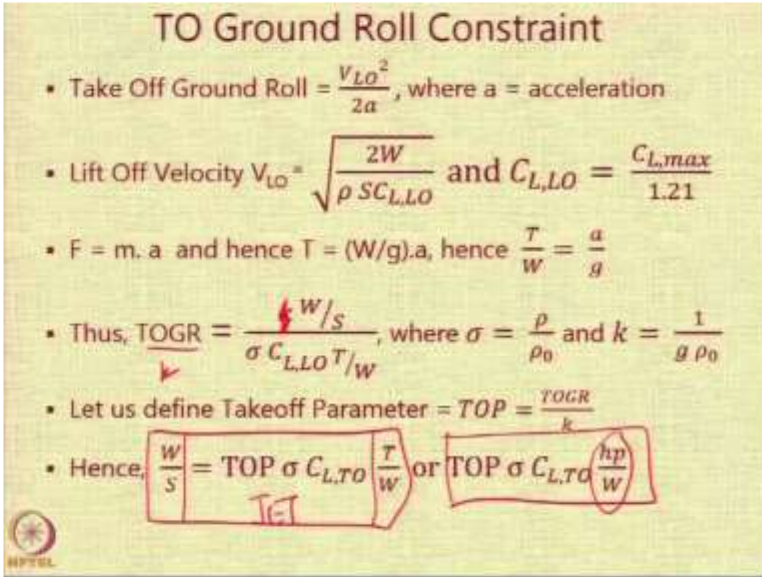

So, takeoff ground roll is basically from $V = 0$ to $V = V_{lift-off}$. So, using

$$V^2 = u^2 + 2aS$$

the S will be coming as

$$S = \frac{{V_{LO}}^2}{2a}$$

where a is the acceleration the lift of velocity is basically the velocity at which you achieve lift equal to weight with C_L as $C_{L_{LO}}$ which is slightly lower than $C_{L_{max}}$. So, by putting the value of $C_{L_{LO}}$

$$V_{LO} = \sqrt{\frac{2W}{\rho S C_{L_{LO}}}}$$

And hence the simple forces equal to mass into acceleration formula tells you that the force that is acting is the thrust force

$$T = \left(\frac{W}{g}\right) a$$

So, from here you can get

$$\frac{T}{W} = \frac{a}{g}$$

So, with this you can get an expression. So what will happen is the $\frac{W}{S}$ will be in the numerator and $\frac{T}{W}$ is the denominator and if you simplify this then you can easily show that take off ground run is basically

$$TOGR = \frac{k\frac{W}{S}}{\sigma C_{L_{LO}}\frac{T}{W}}$$

So, you can say that take off ground run is equal to

$$TOGR = \frac{A * \frac{W}{S}}{\frac{T}{W}}$$

Where

$$A = \frac{k}{\sigma C_{L_{LO}}}$$

So, we can define something called as a take-off parameter which is the value of this TOGR upon k so, if you take this k on the denominator you bring the k here. So, you will have take-off parameter is TOGR divided by k.

$$TOP = \frac{TOGR}{k}$$

So, this is a simple expression we get now, this expression correlates wing loading with thrust to weight ratio this is for jet engine aircraft and a similar expression can be obtained for the turboprop or piston prop engine aircraft.

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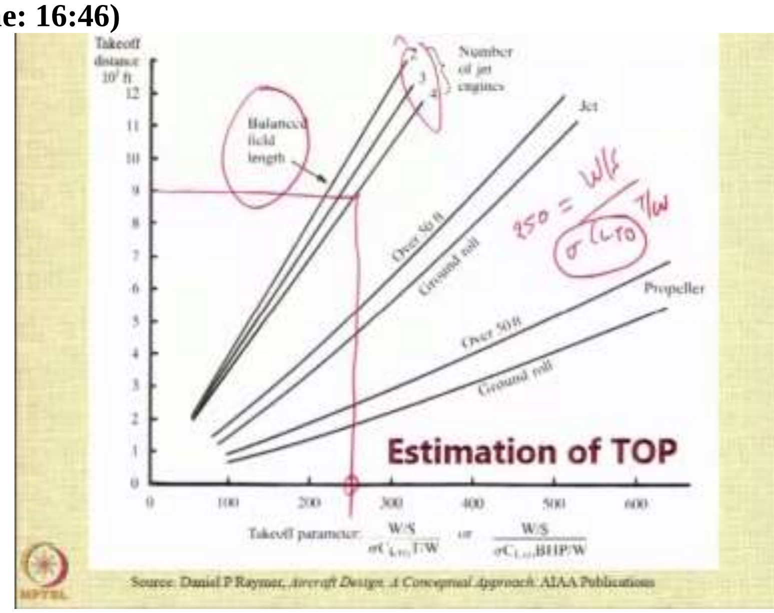

So, using this expression, we can get an estimate for the take-off parameter and through that link between W/S and T/W. So, this graph given by Raymer is very interesting graph because in one graph he has given us either depending on which requirement is specified. So, for instance if you have been given the balance field length and you have been told that this is for an aircraft with some n number of jet engines where minimum would be 2 because you cannot have less than 2 engines in the passenger aircraft.

So, depending on for which kind of aircraft the BFL or balance Field Length have specified what you can do suppose, we are given that the balance field length is approximately 9000 feet for an aircraft with 4 engines, so, you go from 9000 to this line and then you come down and you get the takeoff error to be approximately 250. So, then what you get is that 250 is equal to W/S divided by sigma C L take-off T/W sigma and C L take-off are known numbers. So, you get a direct link between W/S and T/W.

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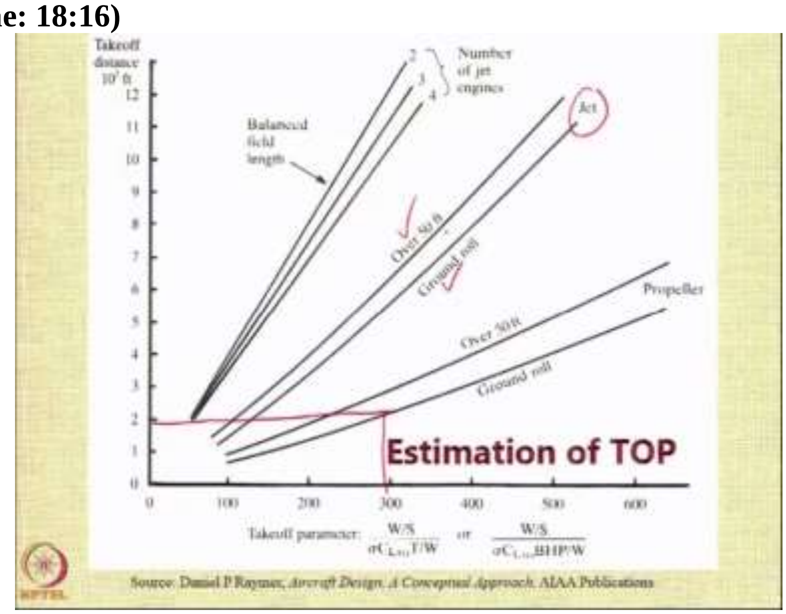

Similarly, if you are given the condition that for an aircraft which is powered by a propeller, the take-off ground roll is 2000 feet. So, then you start from here you come on this particular line this is the line for a propeller driven aircraft and this is for ground roll you get the take-off parameters around 300 or if you have given the similar requirement for jet engine air craft either the distance are 50 feet or distance on the ground roll.

That means either the takeoff distance or take-off roll you can use the corresponding value corresponding line and get the estimation of takeoff parameter. And the same graph is working for both the piston prop and turboprop engine aircraft. So, with this one graph, you can get the takeoff parameter and you can use that to draw the lines.

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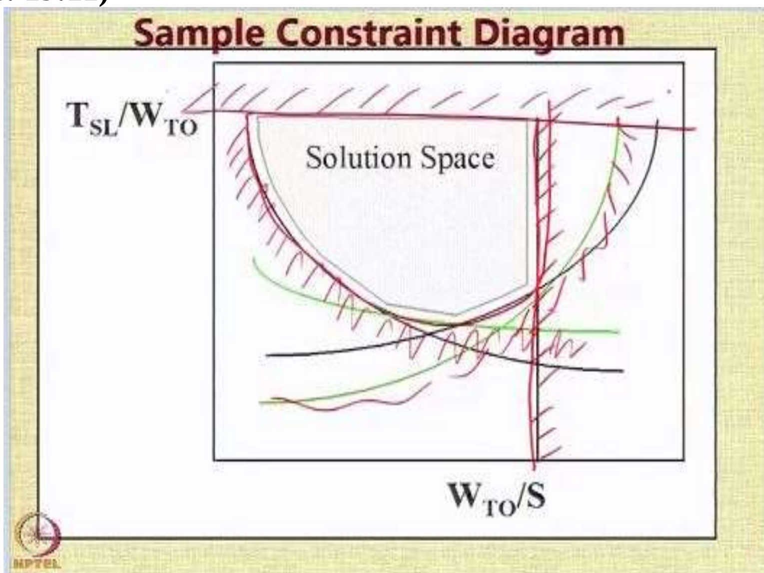

So, a constrained diagram essentially is a diagram in which on the y axis we have something like the design thrust to weight ratio. So, this is the T_{SL}/W_{TO} and on the x axis you have the wing loading the design wing loading that is wing loading of the aircraft when it is at take-off divided by the wing area. So, here once again I would like to reiterate that the numerical values that you will get for various segments of the mission are for various constraints are actually going to be applicable for the wing loading and thrust loading at that particular point.

But you have to bring it down or you have to scale it up or scale it down to the values at the pre take-off conditions because this graph is going to be only one for all the constraints. So, this could be a line specified from say the stalling consideration which is an upper limit so, you cannot have any loading more than this and you know this could be a line which is specified from some other constraint and there is another line.

So, the area that is feasible is going to be only above these lines and one has to always put some limit on the T/W it cannot be unlimited on the top. So, there will be some constraint on the top which will say that T/W cannot be more than a particular number. So, you end up with the solution space. And now, you have a choice to pick up any point in this space and that would be a feasible point,

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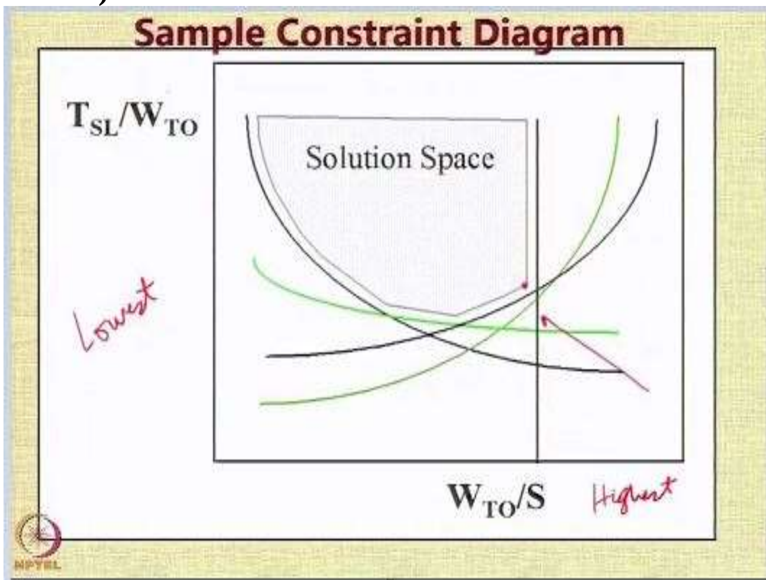

But when you are going to pick up any point in this feasible space, it is important that you pick up that point which gives you the highest wing loading and lowest thrust to weight ratio. So, which would be either this point or this point or this point some people would pick up this point citing that thrust to weight ratio is the lowest. But, this is another good candidate to be picked up because with a slight increase in thrust to weight ratio, we are getting a phenomenal increase in the W/S value we would like W/S to be as high as possible.

Because that gives you a smaller wing and we would like T/W to be as low as possible because that gives you the smallest engine and the least fuel consumption. So, between any point in this design space, the corner points are optimal. So, we would choose this point as our design point, because this will represent as I said the best condition of highest possible wing loading and the lowest possible W/S correspondingly also notice that we are not picking up this point we are leaving a small margin.

And that margin is meant for allowing the growth because as we mentioned in the beginning the requirements are going to always creep and also the aircraft is going to pick up excess weight as its life progresses. So, designing the aircraft for the absolute optimum at this point, it may be disastrous, because very soon the aircraft may actually go out of the feasible space. So therefore, we always give some margin so a little bit of margin on W/S and a little bit of margin.

So a little bit excess T/W a little bit lower W/S is always given and hence the design point is taken to be slightly lower along this direction from the absolute intersection point. Thanks for your attention. We will now move to the next section.