

Introduction to Aircraft Design
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Lecture - 60
Constraint Analysis - Transport Aircraft - Part-01

We will also start with approach number 1 that is, we estimate Thrust to weight ratio from the specified constraints and then we get the wing loading as a back calculation.

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Estimation of T/W

- T/W directly affects aircraft performance
 - High T/W
 - Higher V_{cr} , acceleration, ROC, sustained turn rate
 - Higher Fuel Consumption, heavier aircraft
- T/W keeps changing during mission
 - Lower W due to fuel consumption
 - T (or P) changes due to changes in H, V and η_p
- Design T/W
 - ISA, SLS, at W_{gross} , max. throttle setting
 - Calculated T/W to be adjusted to Design T/W

✳ *Most Common error!*

Now, estimation of $\frac{T}{W}$.

$\frac{T}{W}$ is an indication of how much thrust the engines are producing as a ratio of how much is the aircraft weight and this parameter which is normally considered non-dimensional it directly affects the aircraft performance, higher thrust to weight ratio means, you will have a higher cruising speed you will have a high acceleration you will be able to climb faster and you will be able to turn much faster and maintain that turn.

However, you are going to have higher fuel consumption and in general it will lead to a heavier aircraft because engines which have a high thrust to weight ratio also are generally heavier. The second point to be kept in mind is that the thrust to weight ratio is not constant both the numerator that is the thrust and the denominator that is the weight they keep changing during the mission. So,

throughout the mission and we are looking at a transport aircraft, we are assuming that the only loss of weight is because of fuel consumption throughout the mission.

The aircraft weight is reducing and during the mission even the thrust is not remaining constant. So, because of the changes in the altitude changes in the forward velocity and in the case of a piston prop aircraft, changes in the efficiency η_p because of changes in these parameters and density you know the value of T or P will be changing. So, therefore, it is important for us to consider one norm and that norm is called as a design thrust to weight ratio.

So, in the design thrust to weight ratio, we assume that the atmosphere is the international standard of atmosphere we assume that the thrust is at the sea level static condition, we assume that W is W gross and we assume that we are looking at a maximum throttle setting. So, what you do is the thrust to weight ratio required for individual mission segments it has to be adjusted to the design thrust to weight ratio. In other words, you may get the value of required thrust to weight ratio for let us say the cruise condition.

So, in cruise, that thrust weight ratio should be let us say 0.2, but that is not a design thrust to weight ratio, because the weight of the aircraft in cruise is not same as the weight of the aircraft before takeoff or the Gross weight. Similarly, the thrust produced by the engine is not the same as the thrust at the sea level static condition with full throttle. So, this is something that has to be done every time you calculate the thrust to weight ratio required in a particular mission segment.

You have to say the thrust to weight ratio at the design value should be such that when the aircraft comes to that particular mission, the thrust to weight ratio is what is needed to meet this requirement. And I want to highlight that this is the most common error that many students make when they do constraint analysis. They calculate thrust to weight ratio required from various performance requirements or various mission segments, but they forget to convert it to the design thrust to weight ratio value. This point we will highlight further when we do some numerical exercises.

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Typical values of T/W and P/W

<ul style="list-style-type: none"> □ Jet Engine a/c (T/W) □ Dimensionless 		<ul style="list-style-type: none"> □ Prop. aircraft (P/W) □ Units = Watts/g 	
<ul style="list-style-type: none"> • Civil ○ Transport (2 eng) 0.2 ○ Transport (3 eng) 0.3 ○ Transport (4 eng) 0.4 		<ul style="list-style-type: none"> • Civil ○ Powered Sailplane 0.07 ○ G.A. (1 eng) 0.12 ○ Homebuilt 0.13 ○ Agricultural 0.15 ○ Flying Boat 0.16 ○ G.A. (2 eng) 0.30 ○ Twin Turboprop 0.33 ○ Aerobatic 0.45 	
<ul style="list-style-type: none"> • Military ○ Strategic Bomber 0.2 ○ Tactical Bomber 0.3 ○ Trainer 0.4 ○ Fighter 0.6 ○ Interceptor 0.9 ○ Air Superiority > 1.0 		<ul style="list-style-type: none"> • Military ○ Bomber 0.35 ○ Cargo 0.40 	

From historical data we have got typical values of thrust to weight ratio for turbojet and turbofan powered aircraft and the power to weight ratio for the piston prop turboprop aircraft. If you have a jet engine aircraft, then it is dimensionless. And it is easy to remember 2 engines 0.2, 3 engines 0.3, 4 engines 0.4 these are typical values the values could be more than this if there are certain requirements which are a little bit critical.

For a military aircraft also one can look at the values and notice that for an air superiority aircraft, the thrust to weight ratio normally that you see is more than 1. In other words, the total thrust produced by the aircraft could be even more than the aircraft weight and hence the aircraft should be able to actually go vertically upwards if needed for a propeller driven aircraft we have a power to weight ratio and this is normally expressed in terms of watts per gram.

And there are certain values which have been suggested to be taken as the baseline or the starting values depending on the typical aircraft type. For a military aircraft powered with turboprop or piston prop aircraft, you have the values given as 0.35 and 0.4 for bomber and cargo respectively because, normally you use a turboprop engine aircraft or a piston prop aircraft only for these 2 applications in a military aircraft. So, these are numbers to be used only for verification purposes and to get an idea they are not to be taken as sacrosanct and to be used blindly.

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T/W or P/W as a function of M_{\max} or V_{\max}

$T/W_0 = a M_{\max}^C$	a	C
Jet trainer	0.488	0.728
Jet fighter (dogfighter)	0.648	0.594
Jet fighter (other)	0.514	0.141
Military cargo/bomber	0.244	0.341
Jet transport	0.267	0.363

$P/W_0 = a V_{\max}^C$: hp/lb or (Watt/g)	a	C
Sailplane—powered	0.043 {0.071}	0
Homebuilt—metal/wood	0.005 {0.006}	0.57
Homebuilt—composite	0.004 {0.005}	0.57
General aviation—single engine	0.025 {0.036}	0.22
General aviation—twin engine	0.036 {0.048}	0.32
Agricultural aircraft	0.009 {0.010}	0.50
Twin turboprop	0.013 {0.016}	0.50
Flying boat	0.030 {0.043}	0.23

Source: Daniel P. Raymer, *Aircraft Design: A Conceptual Approach*, AIAA Publications

Raymer has given another table in which he has related the thrust to weight ratio with maximum Mach number for turbojet, turbofan engine aircraft and the maximum speed for the piston prop and turboprop aircraft. So, this is like bringing in a little bit more accurate information related to the maximum Mach number rather than having only one number for aircraft type we have inclusion of the maximum Mach number or the maximum speed respectively, which is the parameter that actually affects the thrust to weight ratio.

So, let us understand how we can calculate thrust to weight ratio from the climb gradient considerations.

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T/W from Constraint on Climb Gradient

□ Climb Gradient = Excess Thrust / Weight = SET

□ If desired Climb Gradient in climb is specified then

$$\left(\frac{T}{W}\right)_{cl} = \frac{1}{(L/D)_{cl}} + \frac{V_{ver}}{V}$$

□ The two major climb gradient constraints are

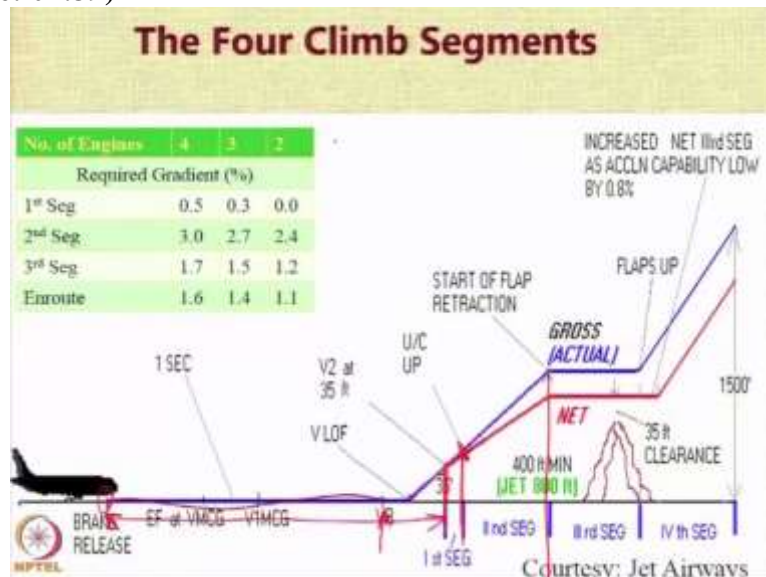
- Missed Approach
- 2nd Stage Climb

So, before that we need to really understand what is meant by climb gradient, climb gradient is basically excess thrust divided by weight or you can also call it as a specific excess thrust or SET. So, if the desired climb gradient is specified by any requirement maybe regulatory bodies or by the performance requirement, then thrust to weight ratio in the climb can be

$$\left(\frac{T}{W}\right)_{climb} = \frac{1}{\left(\frac{L}{D}\right)_{climb}} + \frac{V_{ver}}{V}$$

Now, 2 major climb gradient considerations have to be included based on the regulatory bodies. One is the missed approach gradient and second is the second stage climb gradient. Let us have a look at the climb gradients, because this is not something very easily available. Therefore, it is important for us to understand what they are.

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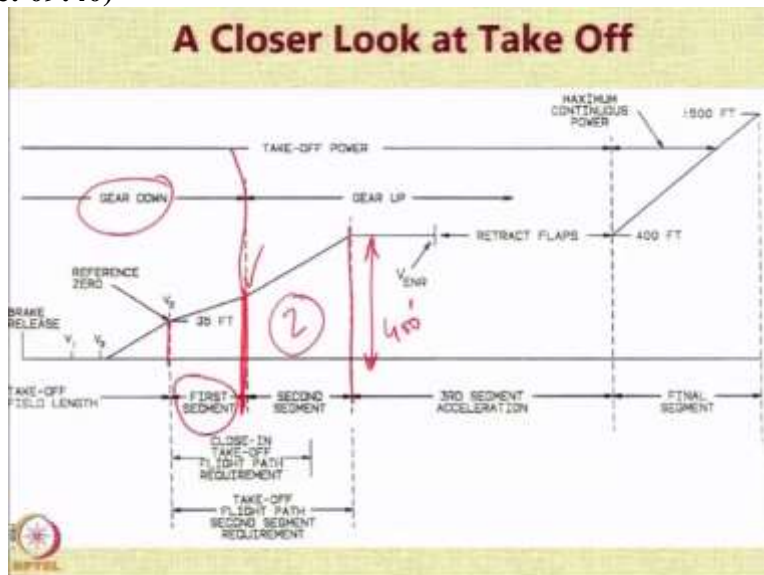
This graph has come courtesy from Jet Airways and it shows the typical climb procedure or sequence that is followed. So, from the brake release, when the aircraft starts moving on the ground, you reach a speed called v rotation at which you start rotating the aircraft soon after that, you reach a speed called as V liftoff at which the aircraft can now be lifted up and then you start climbing. So, you reach this height of 35 feet the obstacle height.

So, the distance from the ground where you start to the place where you clear the obstacle height that can be called as the all engine operating takeoff distance. After that, there are some segments which have to be followed as we will see in more detail as we go ahead. So, the first segment is a segment where you clear the obstacle height and keep on continuing in your path till you so at the

end of the takeoff when you have cleared the obstacle height you start retracting the landing gear and it takes some time.

So that is the first segment. And then you have the second segment in the second segment that flaps the landing gear is retracted but the flaps are still in the deployed condition. So, you start slowly retracting the flaps that takes time and that is the second segment and this segment is the one which becomes an important regulatory requirement. So we are going to look at the climb gradient in this particular segment. In this segment, the aircraft configuration as follows. The landing gear is retracted and the flaps are still being retracted.

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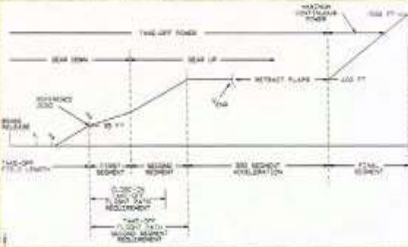
So let us see this information in little bit more detail. So this is the first segment which start from the obstacle height achievement till this particular point so, till here the landing gear is down. So, when the second segment starts the landing gear is up and now you are climbing with the flaps in the climbing configuration till you reach a height of approximately minimum value of 400 feet, but this number varies from airline to airline.

For example, we saw that Jet Airways follows this 800 feet as the altitude during this particular segment, the second stage climb rate segment certain gradient has to be maintained as per the requirements.

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The Four Climb Segments

- **First Segment:**
 - From the end of the takeoff distance to the point the landing gear is fully retracted. (Speed = V_L)
- **Second Segment:**
 - From the point where the landing gear is retracted to an altitude of at least 400' (obstacle dependent). (Speed = V_L)
- **Third (Transition) Segment:**
 - The horizontal distance required to accelerate at a constant altitude to facilitate flap/slat retraction and acceleration to final climb speed.
- **Final Segment:**
 - End of third segment to at least 1500' (obstacle dependent) with flaps/slats retracted, max. continuous power, and final climb speed.




The diagram shows a graph of speed and altitude versus time. The top graph shows altitude increasing through four segments: 1. Gear down, 2. Gear up, 3. Flaps retract, 4. Flaps up. The bottom graph shows speed increasing through the same segments: 1. Gear down, 2. Gear up, 3. Flaps retract, 4. Flaps up. Key speeds V_L and V_{LO} are marked. A small circular logo is in the bottom left corner.

So, this particular slide explains this whole thing in more detail and we describe all the segments. (Refer Slide Time: 10:48)

FAR 25 requirements

Segment	Engine	Thrust	LG	Flaps	Speed	Weight	Altitude
1 st	OEI	Takeoff	Down	Takeoff	V_L	Takeoff	35 ft
2 nd	OEI ✓	Takeoff	Up	Takeoff	$1.2V_L$	Takeoff	35 – 400 ft
3 rd	OEI	Max. Cont.	Up	Up	$1.25V_L$	End of Takeoff	400 ft
4 th Enroute	OEI (2) 2EI (3/4)	Max. Cont.	Up	Up	Any	End of Seg 3	Clear Obstacles
Approach	OEI	Takeoff	Up	< Landing	$1.5V_L$	Landing	
Landing	AEO	8 sec after idle to Takeoff	Down	Landing	$1.3V_L$		



The diagram is identical to the one in the first slide, showing speed and altitude profiles for the four climb segments. A small circular logo is in the bottom left corner.

So, for us the important point is that there are requirements specified for the second segment which have to be met on to the; so, the conditions are as follows in the second segment, I mean one engine inoperative I mean that is the requirement. So, when you calculate the required gradient the climb gradient you have to assume that one engine is not working out of many engines the thrust is not takeoff condition landing gear is up flaps are in takeoff condition and during that condition when the weight is equal to takeoff weight you have to calculate the gradient.

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Second Segment Climb Gradient

□ **FAR-25 requirement:**

- Sufficient thrust must be installed in the aircraft so that in the event of an engine failure the following minimum gradient may be sustained, with flaps in the take-off position, but with the landing gear retracted.

γ_{SSCG}	No. of Engines
3.0 %	Four ✓
2.7 %	Three ✓
2.4 %	Two ✓

(2.4%) (2x)

So, the requirement is that in the event of an engine failure the gradient must be sustained. So, flaps are in the takeoff position and landing gear is retracted. So, what are the requirements if you have 4 engines, it is 3% if you have 3 engines it is 2.7% and if you have 2 engine which is there in most cases it is 2.4%. So, this 2.4% for a twin engine aircraft is normally the one that is specified as the critical one of course, if you have more engines you have to have a higher value of the gradient.

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Missed Approach Gradient

□ The situation in which the aircraft is on final approach to a landing but does not land for one of several reasons; instead, power is applied and the aircraft climbs, usually to circle the airport and initiate another landing approach.

□ **FAR-25 Regulation:** Sufficient thrust for the aircraft to climb at a specified gradient (γ_{MA}) with one engine inoperative and at maximum landing weight.

$\gamma_{MA} = 2.7\%$	for four-engine aircraft
$= 2.4\%$	for three-engine aircraft
$= 2.1\%$	for two-engine aircraft

Where, $\gamma_{MA} = (T - D)/W$ during the climb

Let us also look at the missed approach gradient. Now, what is meant by missed approach the missed approach is that situation in which you are cleared to land you are in the final approach, but when you come into land because of many reasons, you are now asked to abort your landing instead you apply a power and you climb and the aim of this is either to just go up to a circle and

come back and land or maybe if you are told that the conditions are not going to be favorable or safe, you are asked to divert to some other airport.

So, the regulation says that you must have sufficient value of thrust in the engine. So that with one engine inoperative and maximum landing weight you should have the following gradients you should have you know you should have the value as 2.7 for 4 engine 2.4 for 3 engine and 2.1 for 2 engine aircraft.

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Constraint on Missed Approach Gradient

$$\frac{T}{W} = \frac{N}{N-1} \cdot \left\{ \frac{1}{L/D} + \gamma_{MA} \right\}$$

Where, N = number of engines
 L/D = aircraft L/D during missed approach
 Flaps in approach configuration
 Landing Gear Down
 γ_{MA} = required Missed approach gradient

Constraint on γ_{MA} puts a lower limit on T/W

So, the constraint on the missed approach gradient can be expressed very simply as

$$\frac{T}{W} = \frac{N}{N-1} \left\{ \frac{1}{L/D} + \gamma_{MA} \right\}$$

this is a specification this is specified. And in the missed approach remember the flaps are in the approach configuration and the landing gear is down. This particular constraint is going to put a lower limit on $\frac{T}{W}$.

$\frac{T}{W}$ cannot be less than the specified cannot be less than a given value. So, that the specified requirement is met.

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Constraint on 2nd stage Climb Gradient

$$\frac{T}{W} = \frac{N}{N-1} \cdot \left\{ \frac{1}{L/D} + \gamma_{SSCG} \right\}$$

Where, N = number of engines

L/D = aircraft L/D in 2nd Stage climb configuration

Flaps in Takeoff configuration

Landing gear up

γ_{SSCG} = Required Missed approach gradient

Constraint on γ_{SSCG} puts a lower limit on T/W

On the second stage climb gradient the same formula except now, we have to keep in mind that the numerical value of this particular second stage climb gradient is different. But flaps are in takeoff configuration and the landing gear is up and the requirements is that you should be able to have thrust to weight ratio sufficient to meet the specified requirements of γ_{SSCG} . And again, this constraint puts a lower limit on $\frac{T}{W}$. Let us have a look at whether since both these gradients are going to put a lower limit which one of them is going to be more important.

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γ_{MA} v/s γ_{SSCG}

γ_{MA}	γ_{SSCG}
<ul style="list-style-type: none"> <input type="checkbox"/> Aircraft is Lighter <input type="checkbox"/> Flaps deflection is larger <input type="checkbox"/> Landing Gear Down <input type="checkbox"/> Drag is larger <input type="checkbox"/> L/D is lower <input type="checkbox"/> Required γ_{MA} is lesser 	<ul style="list-style-type: none"> <input type="checkbox"/> Aircraft is heavier <input type="checkbox"/> Flaps deflection is lesser <input type="checkbox"/> Landing Gear Up <input type="checkbox"/> Drag is smaller <input type="checkbox"/> L/D is larger <input type="checkbox"/> Required γ_{SSCG} is larger

$$\frac{T}{W} = \frac{N}{N-1} \cdot \left\{ \frac{1}{L/D} + \gamma \right\}$$

Not known which one will impose higher constraint !

So this is the general formula for the missed approach as well as the second stage climb gradient, the numerical value as I told you differ, now, this expression is the same for both the aircraft the value of gamma is different. But the $\frac{L}{D}$ of the aircraft also is not the same during missed approach

and the second stage climb gradient. So let us do a comparison very quickly between the γ_{MA} approach and the γ_{SSCG} .

During missed approach the aircraft is lighter, because this condition occurs when you are coming into land. The second stage climb gradient occurs when the aircraft is just after takeoff so it is heavy. And the flap deflection during landing is larger the flaps are deflected to nearly 30 to 60 degrees during landing. Whereas in the case of climb, after takeoff, the flaps are deflected to between 15 to 20 degrees. So, the takeoff flap deflection is lower.

The landing gear is down when you come into approach, but the landing gear is up in fact, the second segment starts when landing gear is up and the flaps are in the takeoff configuration. So, therefore, the value of drag acting on the aircraft is going to be larger because flaps are larger in deflection landing gear is down drag will be smaller because flat deflection is lesser and landing gear is up. So, the aircraft is going to have a higher value of L/D when it is in the climb consideration and the L/D will be lower.

However, you have to keep in mind that the aircraft the required value of γ_{MA} is lesser it was for example, it was 2.1% for a twin engine aircraft compared to the required value for γ_{SSCG} , which was 2.4%. So, the L/D in missed approach is lower. So, L/D is less, but the value of gamma also is less the L/D will be larger, we are not very sure because see, the value of L is going to be more compared to the value of L here.

The value of L will be the takeoff weight the value of L here will be the landing weight and the landing weight is approximately 85% of takeoff weight. So, you cannot tell in advance which of these 2 requirements are going to be driving you have to calculate the value and then decide so not known which one will impose higher constraint. Therefore, every time we do constraint analysis, we have to calculate the T/W required because of γ_{MA} and T/W required because of γ_{SSCG} and then take the higher of these 2 as the lower limit. Thanks for your attention. We will now move to the next section.