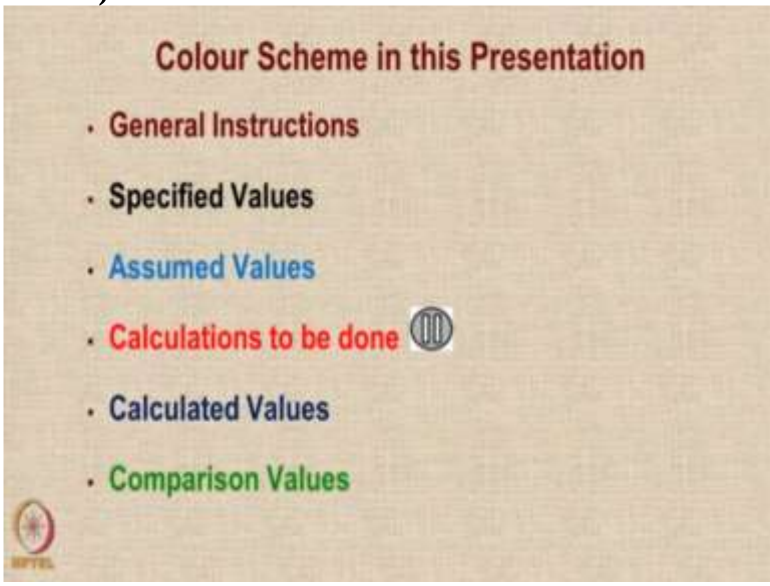
Introduction to Aircraft Design
Prof. Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology – Bombay

Lecture – 58
Tutorial on Lift Coefficient Estimation of Military Aircraft

Hello, today we will look at how to estimate the lift coefficient of a military aircraft. And as an example, we have taken F16-C Fighting Falcon to illustrate the procedure. Before you actually go ahead and watch this tutorial, I would advise you to have a look at the procedure for lift coefficient estimation that we have already discussed. Because tutorials are meant for students who have already seen the lecture they know the procedure and here we only show you how that procedure is implemented for a practical aircraft.

So, if you have not watched the lecture on lift coefficient estimation for military aircraft, I would recommend that you first watch those lecture clips and then only do this tutorial.

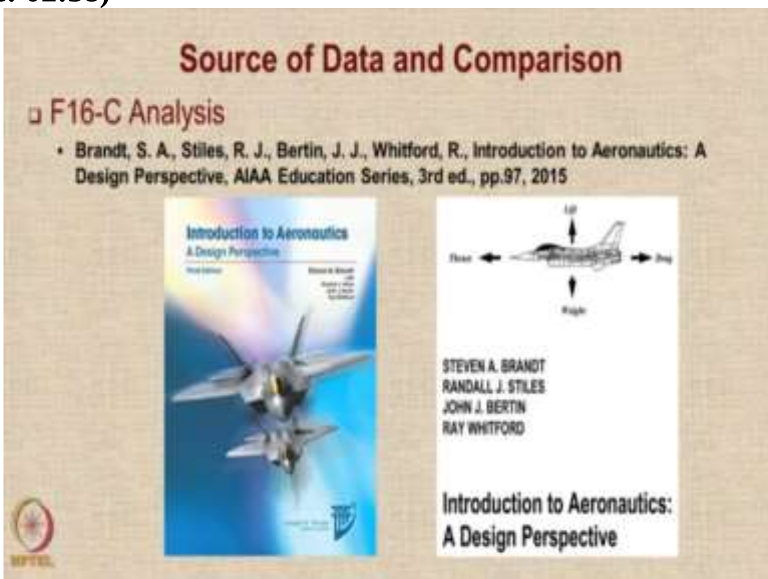
(Refer Slide Time: 01:12)



We are going to follow a color scheme in this presentation the general instructions are going to be given in brown color which is the basic theme of this presentation. If there are some values which are specified either online or in the specifications of the aircraft, they will be shown in black color. There are certain values which we will assume based on literature or any other source book, those will be shown in this blue color. The places where you have to do some calculations you will be alerted with a red color question marks.

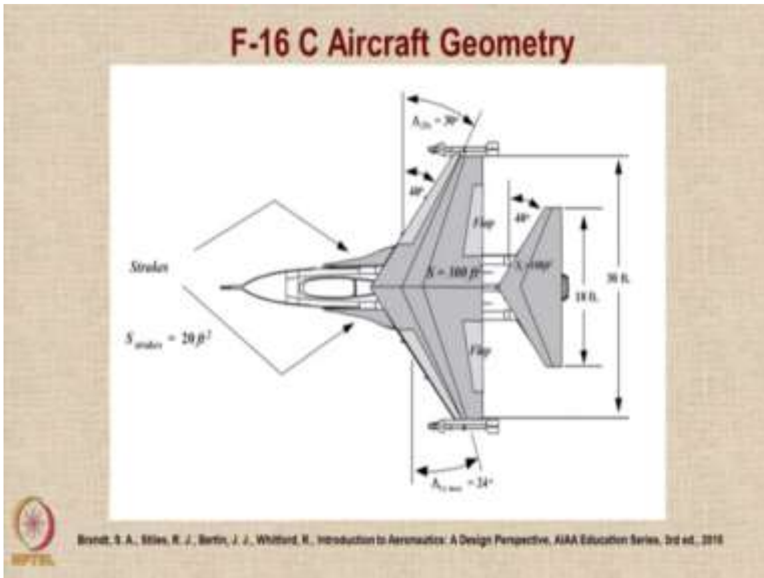
And when you see this pause button, I would recommend that you actually pause the video at that stage and do the calculations and then only proceed further. I would like to reiterate that aircraft design is learned best by doing the calculations just by listening to a video and nodding your head you will not be able to really appreciate the calculations and hence you will not get a feel for aircraft design. The values which are calculated will be shown in this dark blue color. And towards the end we will compare the values that we have obtained with some values mentioned in documents or open sources. So those will be shown in the green color.

(Refer Slide Time: 02:38)



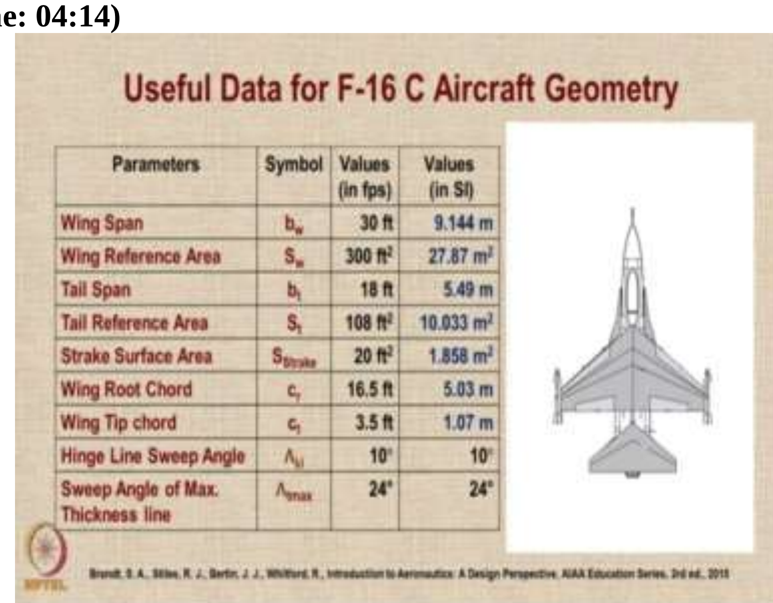
Let us look at the source of the data and comparison for F 16-C analysis. Our principal source for this particular tutorial is this excellent textbook by Brandt, Stiles, Bertin and Whitford it is strongly recommended to go through this particular textbook, it is a very wonderful textbook, which gives a general introduction to aeronautics, but with a design perspective. So, we have looked at the third edition of this book. And there is an example of a whole aircraft drag estimation and lift estimation. So, for this particular tutorial, we are following the lift estimation procedure explained by the authors in this particular textbook.

(Refer Slide Time: 03:27)



This is the top view of the F 16-C aircraft. And this information is presented in the textbook in the form of geometrical data. So, we see that the leading edge sweep of the wing and the tail is 40 degrees, but the quarter chord sweep is 30 degrees, this aircraft has a wingspan of 30 feet and the other numbers are already all mentioned there. So, in the textbook, what they have done is the geometry has been presented in this way in the textbook, the data is given in the FPS system and we are going to use SI system in our tutorial.

(Refer Slide Time: 04:14)



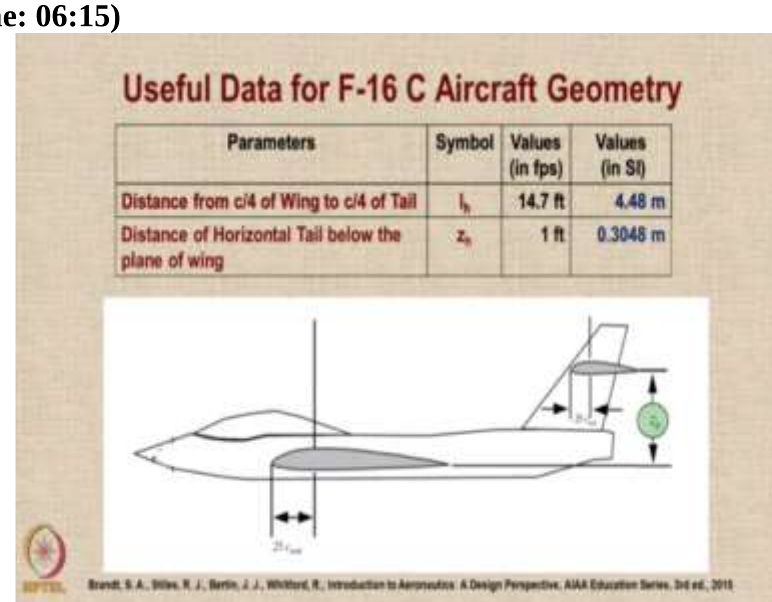
Parameters	Symbol	Values (in fps)	Values (in SI)
Wing Span	b_w	30 ft	9.144 m
Wing Reference Area	S_w	300 ft^2	27.87 m^2
Tail Span	b_t	18 ft	5.49 m
Tail Reference Area	S_t	108 ft^2	10.033 m^2
Strake Surface Area	S_{Strake}	20 ft^2	1.858 m^2
Wing Root Chord	c_r	16.5 ft	5.03 m
Wing Tip chord	c_t	3.5 ft	1.07 m
Hinge Line Sweep Angle	Λ_{HL}	10°	10°
Sweep Angle of Max. Thickness line	Λ_{tmax}	24°	24°

So, let us have a look at some of the useful parameters which we will utilize in this particular tutorial. First is the wing span which is given us 30 feet or 9.144 meters. The wing reference area which is the plan form area in the top view, including the area inside the fuselage as you can see in the figure is 300 square feet which is 27.87 square meters. The tail span or the distance between the tips of the 2 tail to horizontal tails is 18 feet 5.49 meters.

The tail reference area or the area of the 2 trapezia that represent the tail is 108 square feet or 10.033 meter square. This aircraft is fitted with leading edge strakes near the wing root wing fuselage junction and these strakes are a total of 10 square feet each. So, that is 20 square feet area 1.858 square meters the root chord of the wing on the fuselage centerline is 16.5 feet which is 5.03 meters, the tip chord is 3.5 feet or 1.07 meters and the hinge line of the flaps are swept at an angle of 10 degrees.

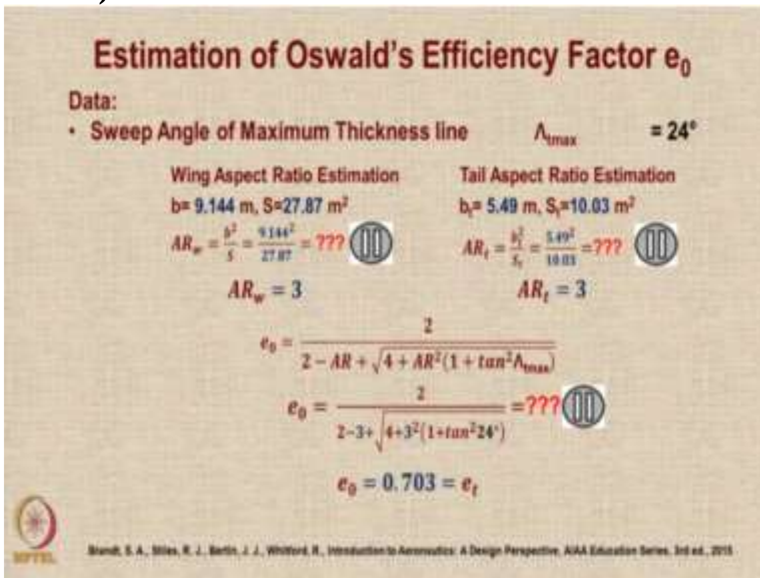
And also the maximum thickness line the line along which we have the maximum thickness of the airfoil approximately 40% of the chord, the angle of that line is 24 degrees. So, these are some of the parameters that we will need in our calculation. So, it will be a good idea for you to either take a picture of this slide and keep it with you, when you do the calculations or you can even note down these values on in a small document.

(Refer Slide Time: 06:15)



Similarly, there is some information that we need regarding the side view of the aircraft for example, the distance from the quarter chord of the wing and to the tail the so, called tail arm, this is 14.7 feet or 4.48 meters. And the location of the horizontal tail below the plane of the wing or above the plane of the wing as in this case is 1 feet or 0.3048 meters. So, this is the other geometrical information that we will need.

(Refer Slide Time: 06:51)



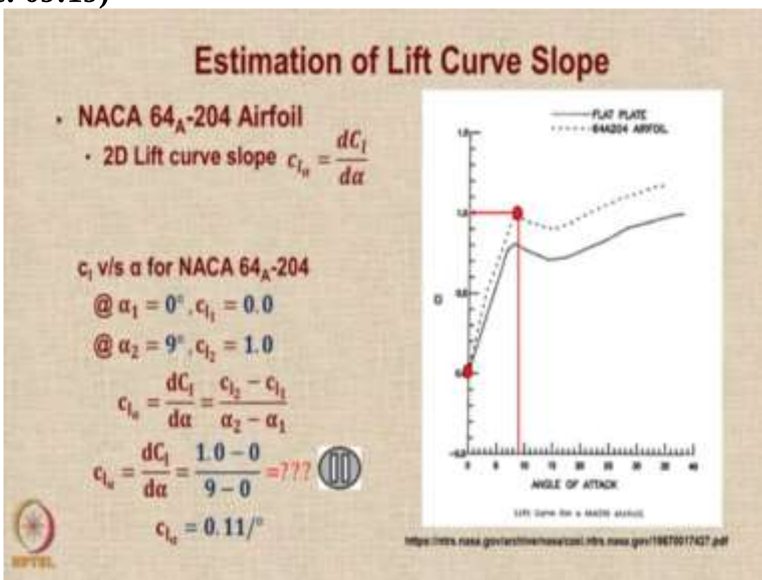
The first thing we will do is we will estimate Oswald's efficiency factor is e_0. For that we will use some data for example, the sweep of the maximum thickness line. First let us calculate the wing aspect ratio. As you know aspect ratio is defined as Span Square by area. So, we know the span 9.144 we know the area 27.87. So, therefore, aspect ratio is Span Square by area. This is the first place where you have to stop the video and do these calculations, it is very straightforward 9.144 square divided by 27.87 this number comes out to be 3.

Similarly, for the horizontal tail, we need to calculate the aspect ratio for that we know that the tail span is 5.49 meters and tail area is 10.303 square meters. Once again aspect ratio is 5.49 square divided by 10.03. The second place to pause do the calculations turns out that the tail aspect ratio also is 3. So, in this aircraft, the tail is nothing but a scaled down version of the wing with similar geometry and, same aspect ratio.

Now, here is a formula from the book by Brandt et al for calculating the Oswald efficiency factor. There is a much more elaborate formula which we have used for transport aircraft. But for a military aircraft since we are following a standard reference, we would like to calculate the value of e_0 also using that same formula. So as for this formula, you need to know the aspect ratio of the wing or the tail and the sweep of the maximum thickness line these numbers are known to us.

So therefore, when we insert these numbers in the formula, now have to pause the video and calculate the value of e_0. So do it first for the wing. The value turns out to be 0.703. And that is also applicable for the tail because the tail in this case has the same aspect ratio and the same sweep of the maximum thickness line. So both wing and tail have the same Oswalds efficiency factor of 0.703.

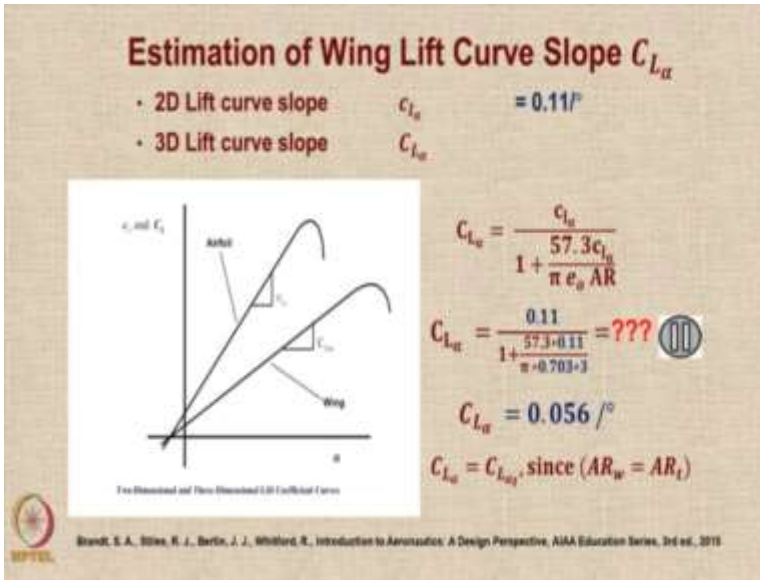
(Refer Slide Time: 09:19)



Let us estimate the lift curve slope now. We are told that this aircraft is fitted with NACA 64 A-204 airfoil in the wing as well as in the tail, the 2 dimensional lift curve slope of the airfoil which is c_{l_α} or also $\frac{dC_l}{d\alpha}$. That is the definition. Here is a figure from one of the NASA archive that talks about the aerodynamic data of the flat plate and the 64 A-204 airfoil. So from this figure, we see that angle of attack of 0, we have c_{l_1} equal to 0.

And at the angle of attack of 9 degrees, we have c_{l_2} equal to 1. So with these 2 points if we fit, if we want to get the linear lift curve slope, we just fit a straight line between these 2 points. So, it is a simple calculation, 1 divided by 9. But it is a good idea to pause and do the calculation yourself the number is 0.11. So, the lift curve slope of the airfoil is 0.11 per degree.

(Refer Slide Time: 11:05)



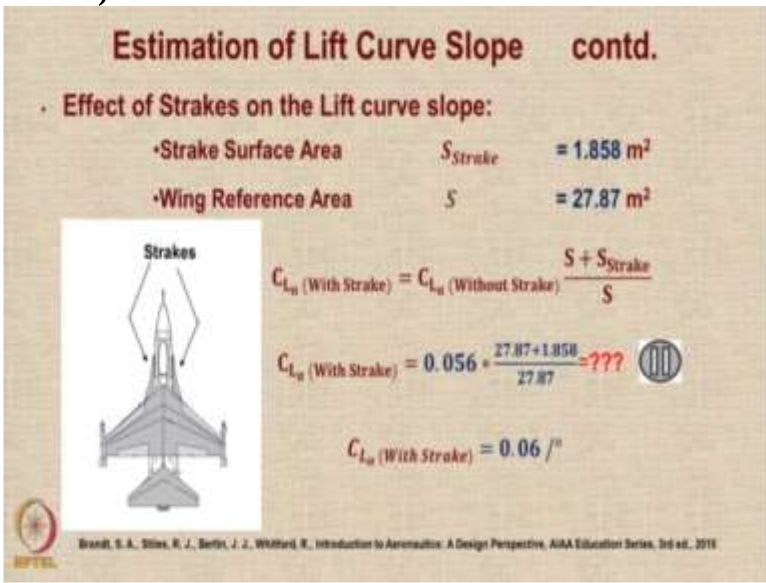
Let us now use this information to calculate the lift curve slope of the entire wing the so called 3D effects. So, the 2D lift curve slope we have just calculated was 0.11 per degree, the 3D lift curve slope will be called as C_{L_α}. And here is a graph that shows how the C_{L_α} of the airfoil and wing are different we notice that the wing has in general a lower lift curve slope compared to the airfoil.

And another thing we observe is that the angle at which the airfoil attains the maximum C_L, which was 9 degrees in our case, is lower than the angle at which the wing will obtain its highest value. So, the highest angle at which the wing will obtain the maximum C_L, is not 9 degrees, but probably around 15, 16 degrees, but there is a usable value and that number for this aircraft can be assumed to be 14 degrees.

So, remember, we will use this number when we go ahead. So, therefore, the formula to be used as given in the book by Brandt et al for the wing lift curve slope is equal to the airfoil lift curve slope divided by a quantity that takes care of the Oswald efficiency factor and the wing aspect ratio. So, putting in the numbers for our case, c_{l_α} for the airfoil is 0.11, the aspect ratio is 3 and Oswald efficiency factor is 0.703.

So, therefore, we can get the value of C_{L_α} for the wing, please pause the video and do this calculation. The value comes out to be 0.056 for C_{L_α} for the wing. And in this case, it will be the same for the tail also, because the tail aspect ratio is the same as the wing aspect ratio. So therefore, it will have the same geometry and hence it has a similar same value of the lift curve slope.

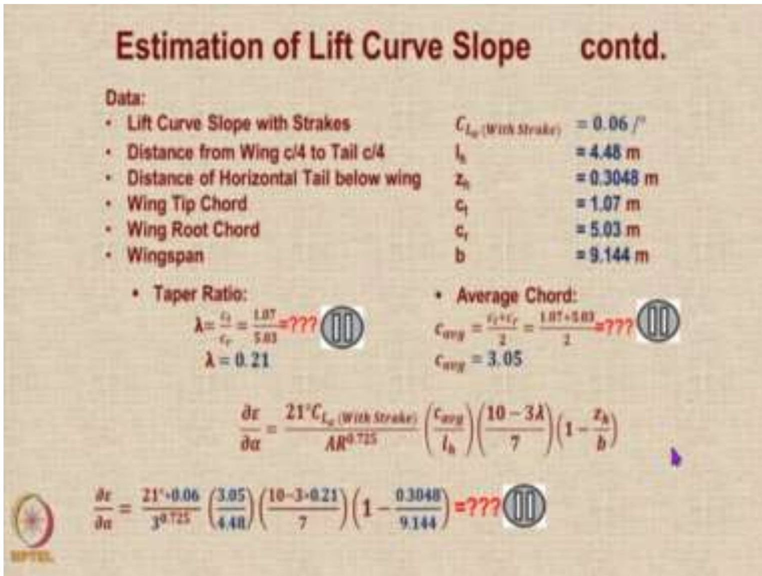
(Refer Slide Time: 13:22)



Let us continue ahead, we will like to look at what is the effect of strakes on the lift curve slope. Because as I mentioned, this aircraft is fitted with 2 strakes which are mounted at the wing fuselage junction ahead of the wing. These 2 strakes total area is 1.858 square meters, it was 20 square feet, and the wing reference area was 300. So it is 27.87 square meters. So the book by Brandt et al gives a simple formula for calculating C_{L_α} of an aircraft with strake.

If you know the value of C_{L_α} without the strake is nothing but scaling up the area of the wing in terms of the additional area of the strake, In our case, we know that the lift curve slope of the wing was 0.056. And we just have to multiply it by the ratio of the wing reference area plus strake area divided by the reference area or we just scale up the value. So we find that the C_{L_α} with the strake becomes 0.06 per degree higher than 0.056 because of the presence of 20 square feet of those 2 strakes.

(Refer Slide Time: 14:43)

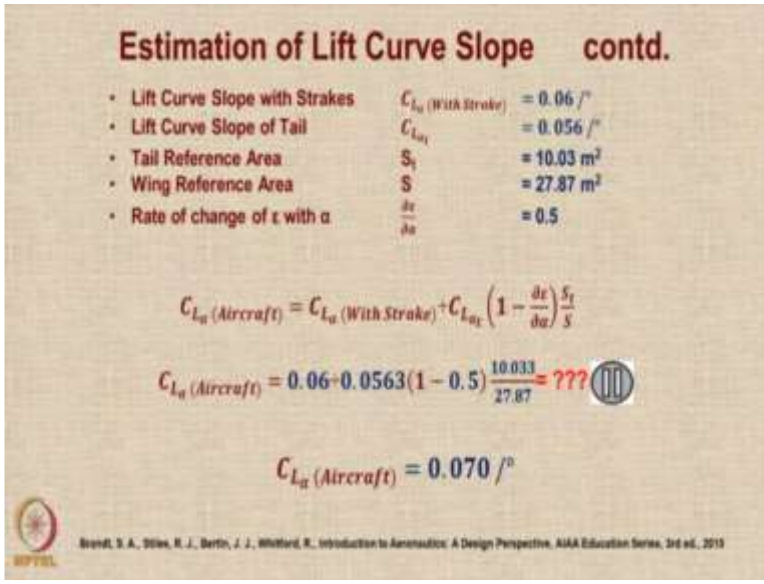


Let us look at the effect of the tail on the lift curve slope. So for that we use the data that the lift curve slope with strakes we are just calculated is 0.06. And these 2 distances l h of 4.48 the distance between the 2 quarter chords wing and the tail and the distance of the horizontal tail below or above the wing. So, this information will be useful for us to calculate of course, we also need wingtip called wing root called wingspan. So, first we calculate the wing taper ratio which is nothing but the tip chord by the root chord or 1.07 divided by 5.03 it is a good time to pause the video and calculate this number, the answer is 0.21.

For the average chord, we just take the average of these 2 numbers again you can pause here the value is 3.05. So, the parameter $\frac{\partial\varepsilon}{\partial\alpha}$ which is the rate of change of the downwash angle with the change in the angle of attack, it can be estimated using this formula given in the book by Brandt et al. So, what we do is we just insert the values of the parameters that we know.

So, once you do all these calculations, you can get $\frac{\partial\varepsilon}{\partial\alpha}$ please pause the video at this stage calculate this value turns out to be 0.5.

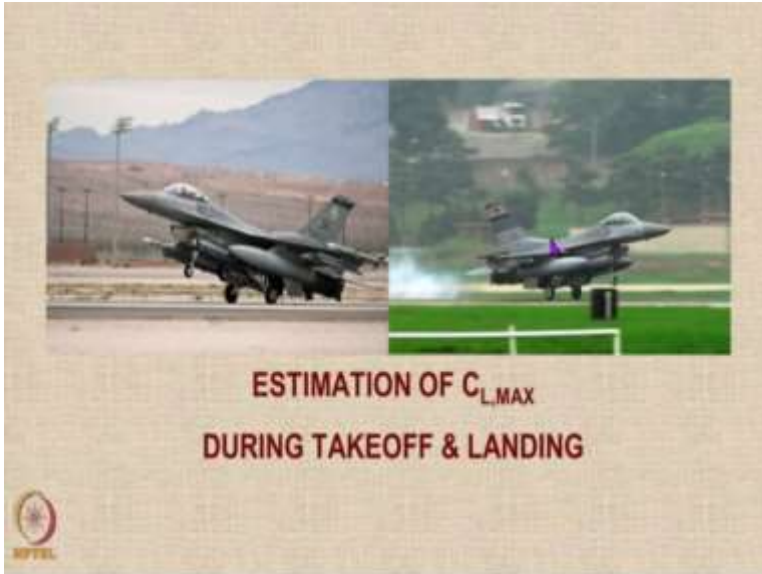
(Refer Slide Time: 16:29)



Moving ahead, so, we know the lift curve slope which takes as 0.06 we have just calculated the lift curve slope of the tail also as 0.056 the tail reference area was 10.03 meter cubes and we also know the wing reference area, and we are just calculated $\frac{\partial \varepsilon}{\partial \alpha}$ of 0.5. So, therefore, C_{L_α} for the aircraft will be C_{L_α} with the strake which we have already calculated

Because, this is a in a way a measure of the tail efficiency how much of the tail is actually efficient. So, putting in the numbers we get this expression 0.06 remember was the value of lift curve slope with strakes and 0.0563 was the slope of the tail lift curve of the tail so pause at this stage and calculate this value, we see that the C_{L_α} for the aircraft is 0.070. So, in other words, contributions to C_{L_α} have been taken from the tail from the strakes and from the wing, the net value is 0.070 per degrees.

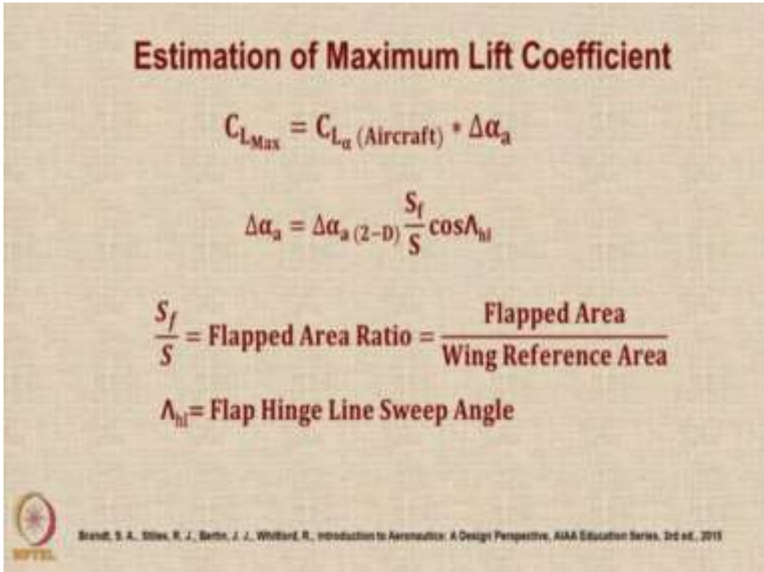
(Refer Slide Time: 16:29)



Now, we are ready to do the estimation of the $C_{L_{max}}$ of this aircraft during takeoff and landing. Takeoff is shown in the figure on the left and landing is shown in the figure on the right if you notice carefully the angle of the aircraft both during takeoff and during landing seems to be very similar. So, this is actually the maximum usable angle of the aircraft. Now, this angle is a function of at what is the maximum angle you can deflect the aircraft along the main wheels during takeoff or landing without hitting the tail on the ground.

This number is approximately 15 degrees, but we keep a 1 degree margin. So, it will be 14 degrees and that will be called as the absolute maximum value of the absolute angle of attack of the aircraft 14 degrees, and it will be the same in case of both takeoff and landing in this case,

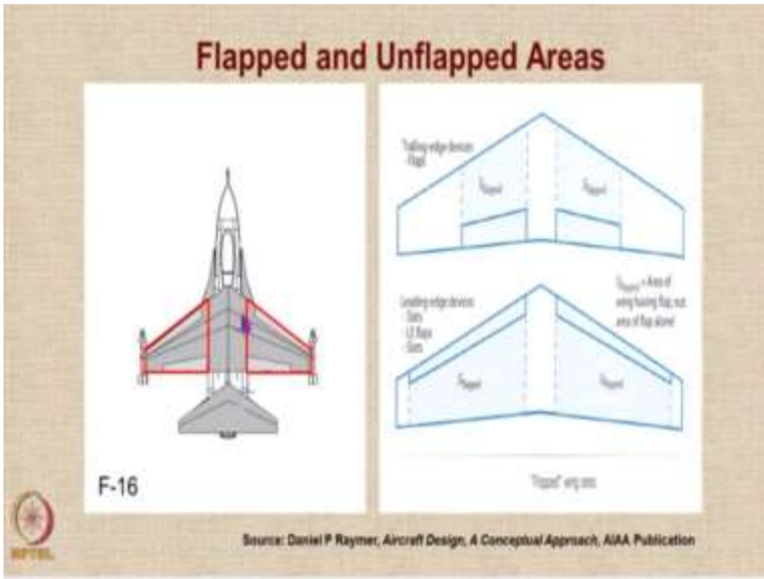
(Refer Slide Time: 18:52)



Estimation of maximum lift coefficient can now be attempted $C_{L_{max}}$ will be basically the C_{L_α} of the aircraft into the change in the absolute angle of attack which is attained during the flight. Now, the change in the absolute angle of attack $\Delta\alpha$ is a function of what changes available if you use only the airfoil or the 2 dimensional value, then you multiply it by the flap area to the wing area or the flapped area ratio and then cos of the sweep of the hinge line.

So, we noticed that if you do not use full span flaps, only a part of the wing is affected with the flaps. And hence that part is the one that gets a improvement in the effective absolute angle of attack increase. And if you have a hinged line which is swept then that causes a reduction in its efficiency.

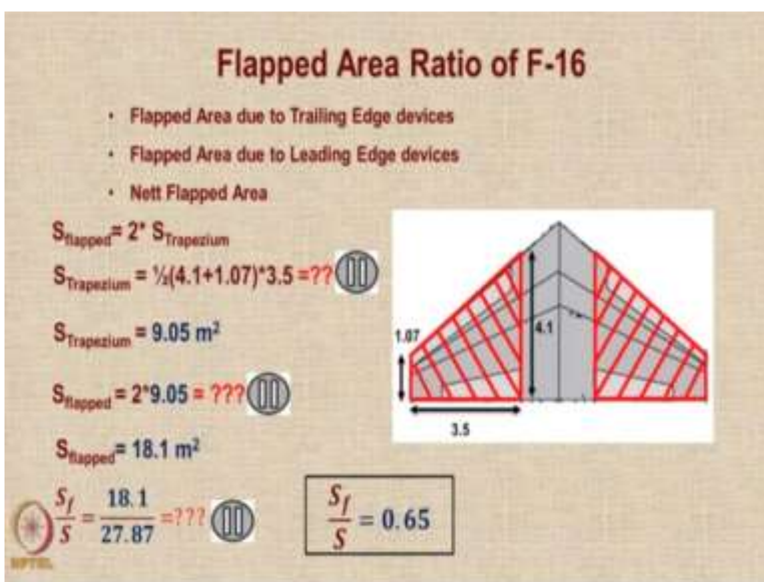
(Refer Slide Time: 20:12)



So, first let us have a look at the flap areas and unflapped areas. So, all the area of the aircraft which is under the influence of the flaps, whether leading edge flaps or trailing edge flaps that area the total area that is under the influence of both of them is called as the flapped area. And as far as F 16 is concerned you know this would be the total flapped area because the trailing edge flaps start from almost at the junction of the wing and the fuselage you can see almost.

And they go up to this place with the sweep of 10 degrees for the hinge line and there are leading edge flaps which start almost near the wingtips and go right up to here. So, actually if you take the area, which is influenced under both of these flaps, you will get these 2 trapezia

(Refer Slide Time: 21:03)



And that is going to be the flapped area. So, let us calculate the flapped area ratio for F-16 C, this is the area that we have to calculate this is the area which is under the influence of the trailing edge devices. And this is the area which is under the influence of the leading edge devices. So the net area is the one that is hashed. And looking at the geometry of the aircraft, we have determined that the dimensions of the flap area are that of a trapezium of the larger side 4.1the parallel side 1.07.

And the distance between the 2 sides as 3.5 meters, , so the area of the flaps will be the twice the area of each of these trapezium, each of these trapezium. So the area of one trapezium would be half of the sum of the 2 parallel sides into the distance between them. So please pause the video and calculate the value of each trapezium area that is going to be 9.05 square meters. So, each of these areas is 9.05 square meters, the total flat area is actually going to be double of this. So 2 into 9.05 please calculate the value the answer is 18.1 square meters.

So, the flat area is 18.1 square meters and remember that the total area is 27.87 square meters. So, with this we can get the ratio of S_{flap}/S or the flapped area ratio. Again, I would suggest you pause the video and calculate this number the number turns out to be 0.65. Hence, for this aircraft, the flapped area ratio is 0.65 only 65% of the whole wing is actually under the influence of the flaps, 35% of the flaps is not under the influence of the flaps.

(Refer Slide Time: 22:54)

And that area is mostly the area inside the fuselage actually over the fuselage. So the flapped area issue is known as 0.65, hinge line of the flaps is swept at an angle of 10 degrees. From literature given in the textbook or you can say back calculated from some number in the textbook, we have determined that the angle of attack 2D at takeoff is 1.5 degrees. And as I mentioned to you the maximum usable angle of attack at takeoff is 14 degrees.

So an aircraft lift curve slope was determined as 0.07 per degree. So, we put the numbers here for your convenience. And now you should pause the video and do the calculations for the value of delta alpha during takeoff that number is 4.8 degrees. So, what it basically means is that the presence of the flaps of this particular area ratio of a particular area.

And a particular sweep of the hinge line effectively results in increasing the angle of attack of the aircraft by 4.8 degrees that is the meaning of this calculation effectively when you deflect the flaps, you get the increment in C_L corresponding to what you would get if the aircraft had gone for angle of attack increase of 4.8 degrees. So therefore, the $C_{L_{max}}$ flap will be the slope of the lift curve slope of the aircraft $\frac{dC_L}{d\alpha}$ times whatever is the maximum angle of attack.

That you can operate plus what is the additional effective angle of attack created because of the flap deflection. as I mentioned is the maximum angle which is usable during takeoff and landing. Both in takeoff and landing. And 4.8 is the effective increase in the angle. So the $C_{L_{max}}$ is going to be 1.32 in the case of the takeoff condition.

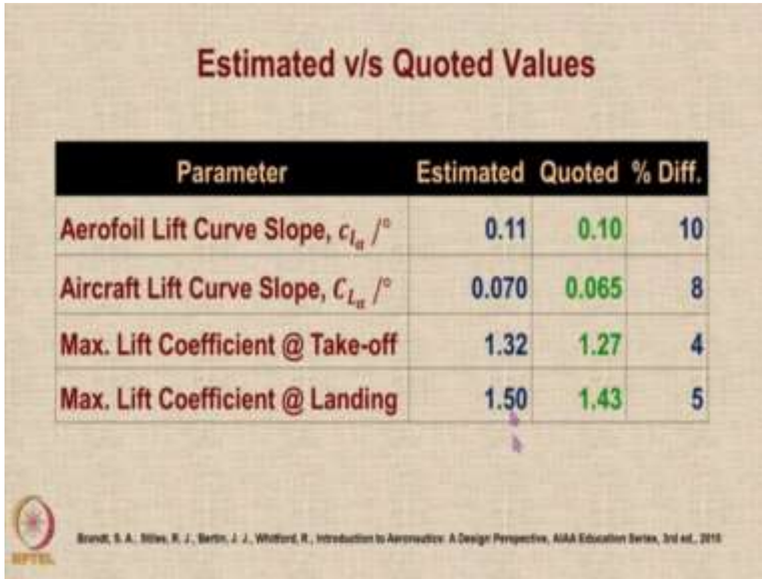
(Refer Slide Time: 25:05)

Let us repeat the same calculations for the landing condition during landing the geometrical parameters actually remain the same, the only change that happens is that the angle of attack 2D at landing because of the flaps in landing configuration is slightly higher because flaps are more effective in landing.

This number was 7.5 earlier now it is 11.5. So you can just do this calculation or multiplying these 3 terms the value comes out to be 7.4 degrees, I hope you are pausing the video and doing the calculations yourself. Once again the $C_{L_{max}}$ of the flap aircraft is going to be this lift curve slope of the whole aircraft into the maximum usable angle of attack.

And the additional angle created by the flap deflection. So 0.07 is the lift curve slope of the entire aircraft with the effect of strake, effect of tail, effect of wing included, 14 is a maximum angle of attack absolute angle of time that is available and 7.4 is the increment in the angle of attack effectively due to landing. So with this the value of $C_{L_{max}}$ during landing comes out to be 1.50.

(Refer Slide Time: 26:37)



Estimated v/s Quoted Values

Parameter	Estimated	Quoted	% Diff.
Aerofoil Lift Curve Slope, c_{l_α} /°	0.11	0.10	10
Aircraft Lift Curve Slope, C_{L_α} /°	0.070	0.065	8
Max. Lift Coefficient @ Take-off	1.32	1.27	4
Max. Lift Coefficient @ Landing	1.50	1.43	5

Now let us just see, what is the difference between our estimates and the data which is quoted in the source. So, for comparison purposes, we have utilized the data given in the book by Brandt, Stiles, Bertin and Whitford. So, the first parameter is the lift curve slope, they have quoted the value of that as 0.1 whereas we have estimated the value to be 10% higher of 0.11. This is for the airfoil itself. Now, if there is a difference in the airfoil lift curve slope itself, obviously, the numbers are going to change for the parameters that follow.

So it is no surprise that the aircraft lift curve slope also comes out to be around 8% higher than the quoted value of 0.065. This is the value that is the actual value for the aircraft as quoted in the textbook. Then the maximum lift coefficient we have estimated 1.32 whereas the quoted value is 1.27. So, there is a 4% difference in the estimated value and the quoted value and at landing we have estimated the lift coefficient to be 1.5 whereas the value given is 1.43.

So there is a 5% error. In other words, if there is a fundamental error in the basic airfoil, lift curve itself then we expect these errors to come. But still we are actually very much comparable to the value quoted in the literature.

(Refer Slide Time: 28:07)

I would like to acknowledge the items I have borrowed from the book by Brandt, Stiles, Bertin and Whitford. I also would like to thank Daniel Raymer for his seminal textbook on aircraft design from where also we have drawn some information. And last but not the least; I would like to sincerely thank my teaching assistant Nouman Uddin has been for help in creating this tutorial. Thank you.