Introduction to Aircraft Design
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Lecture – 57
Tutorial on Lift Coefficient Estimation of Transport Aircraft

Hello, let us look at how we estimate the lift coefficient for a long range transport aircraft.

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Using Boeing 787 Dreamliner as an example. So here is a picture of this beautiful aircraft belonging to ANA airways.

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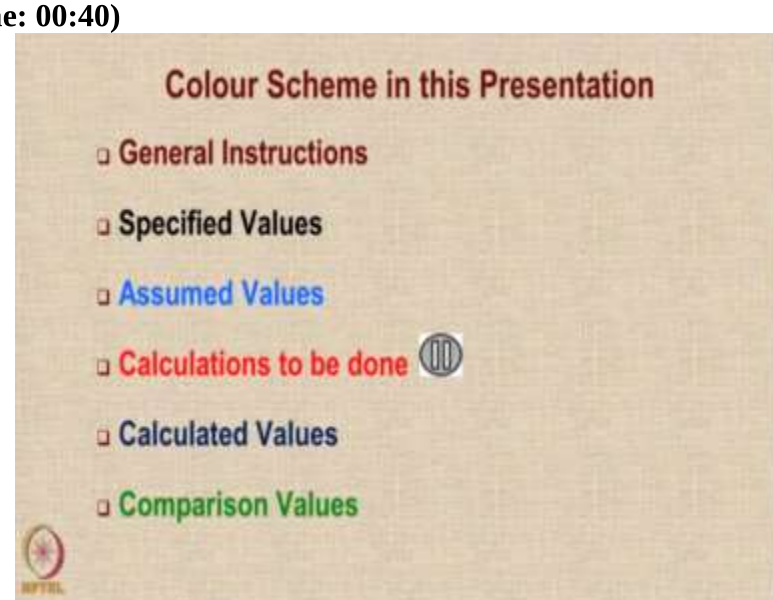

Let us look at the color scheme in this presentation. The general instructions are going to be given in brown color, the specified values if there are any, would be shown in the black color. The values which are assumed based on the existing literature, or any online source, which we follow as a constant data or a given data will be in this light blue color. The values that have to be calculated would be shown in the red color with this symbol.

So wherever you see this symbol in the screen, it is a hint for you that you have to pause the video do some calculations and then proceed further. The calculated values will be shown in this dark blue color. And in the end, if we do a comparison with any data, which is reported, then those comparison values will be shown in the green color.

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Useful Data about Boeing B 787-8

Cruise Mach Number	M_{CR}	0.85
Wing Reference area	S_{ref}	359.53 m^2
Thickness Ratio at Wing root	$(t/c)_{wing\,root}$	9.4%
Taper ratio of the wing	λ_{wing}	0.1528
Aspect Ratio of the wing	AR_{wing}	10.58
Sweep at quarter chord line of wing	$\Lambda_{25,wing}$	32.2°
Hinge Line Sweep of inboards flap	$\Lambda_{hl,inboard}$	2°
Hinge Line Sweep of outboard flap	$\Lambda_{hl,outboard}$	23°
Max. Lift Coefficient at Take-off	$C_{L,max,TO}$	1.91
Max. Lift Coefficient at Landing	$C_{L,max,Land}$	2.66

Here is some useful data about Boeing B 787-8, which we will use in this particular analysis. Notice that the bottom 2 entries in this table are the maximum lift coefficient at takeoff also called as $C_{L_{max,TO}}$ 1.91 and the maximum lift coefficient at landing $C_{L_{max,Land}}$ 2.66. These 2 values are the ones that we are going to determine in this tutorial. So, we will compare our calculated values with these 2 numbers in the last slide, along with another parameter called as the wing or Oswald efficiency e.

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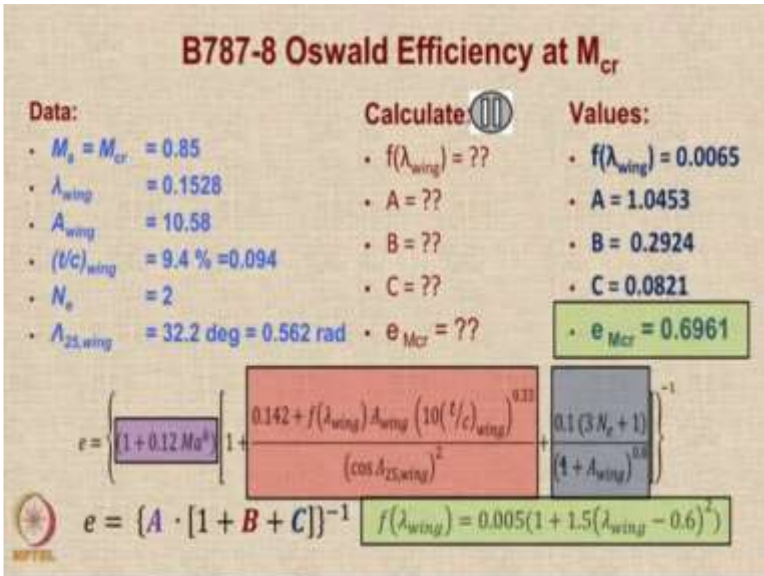

So, talking about Oswald efficiency there is a formula that can be used to estimate the value of Oswald efficiency for an aircraft while operating at any Mach number M this formula taken from the textbook by Professor Dennis How, it is a long formula which has several aircraft related parameters, let us look at these parameters one by one. The first parameter to be considered is the Mach number shown here by M_a. The formula shows that the overall efficiency depends as the sixth power of Mach number.

And then we have λ_{wing} which is the wing taper ratio, we are going to define a function called as $f(\lambda_{wing})$ which we will calculate. A_{wing} is the wing aspect ratio $(t/c)_{wing}$ is the wing thickness ratio. Now, an aircraft has several thickness ratios, there is one at the root one at the tip, maybe there is one at the mid break. In this case, we are going to use an average thickness ratio value and N_e stands for number of engines, $\lambda_{25,wing}$ which stands for the quarter chord wings sweep.

So, once we have the value of these parameters, we can use this equation to calculate the value of e. Now, the function $f(\lambda_{wing})$ is calculated in terms of the wing tapered ratio λ. So, this is the formula and now let us calculate the values for Boeing 787-8 for that we need to recall some data. So it is given to us that the cruise Mach number is 0.85, the λ_{wing} is 0.1528, the taper ratio wing aspect ratio is 10.58. The $(t/c)_{wing}$ is 9.4%, number of engines is 2.

The sweep at quarter chord is 32.2 degrees. So with this, what we can do is we can divide this formula into 3 separate terms. And these terms are color coded as the purple color, the term representing A. The second term is the term B, which is in the center, the large term highlighted in the brown colored box. And then there is a term C, which is highlighted in the grey colored box. So what we need to do is, we need to actually calculate the values of various parameters.

So as you can see, there is a pause button on the screen. So I would request you to pause the screen and calculate the value of f lambda wing, where the formula for f lambda wing is shown in the bottom of the screen, with the green colored box. So I would request you to use your calculators and obtain this value for λ_{wing} of 0.1528. After that, you can calculate the value of A, B, and C.

And once you get the value of A, B and C, then e will be

$$e = \{A[1 + B + C]\}^{-1}$$

So let us get the values. so $f(\lambda_{wing})$ is 0.0065, the term A turns out to be 1.0453. The term B in the center turns out to be 0.2924. And the terms C on the right grey colored box turns out to be 0.0821. Putting all these values together, the value of e turns out to be 0.6961. So we need to remember this value because we will compare this value with the quoted value.

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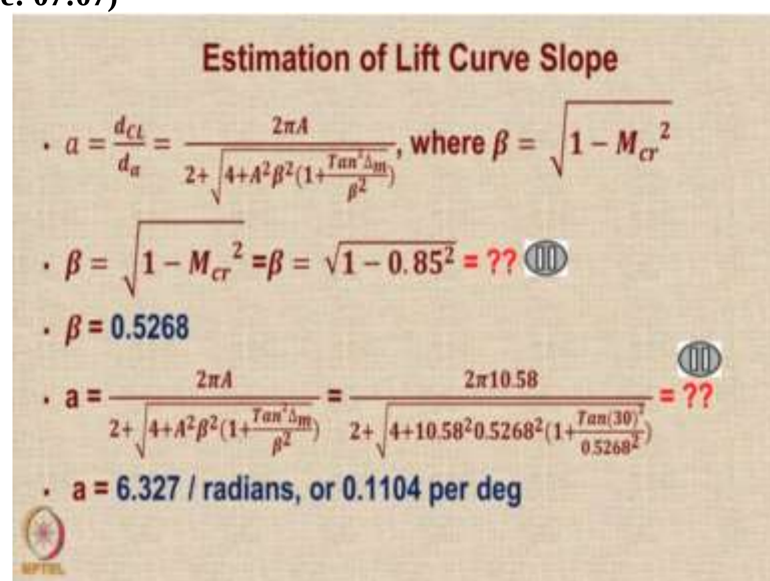

Let us first start with estimation of the lift curve slope for which we will use this formula where a is equal to

$$a = \frac{dC_L}{d\alpha}$$

So the $\frac{dC_L}{d\alpha}$ is estimated in terms of the parameters like wing aspect ratio A, and then the parameter β which is

$$\beta = \sqrt{1 - {M_{cr}}^2}$$

and Δ_m which is the sweep of the maximum thickness line. So, in our case, please pause your screens, calculate the value and write it down.

β turns out to be 0.5268. Next, you calculate the lift curve slope a, in which the values of the various parameters like a, beta and delta m have already been written down for you on the right hand side. So, all you need to do is to just solve this expression and get the value of a the lift curve slope. The answer is 6.327 which will come in per radians. And if you want to convert it into degrees, you will multiply by π and divide by 180. So, that will be 0.1104 per degree.

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Let us now try to estimate the $C_{L_{max}}$ value for this aircraft during takeoff and during landing. We see on the screen 2 pictures of the Vietnam airways Vietnam Airlines Boeing 787 aircraft just after takeoff in a very steep climb, very clearly showing the working of the outboard flaps and the inboard flaps. And here we have the same aircraft when it comes into land.

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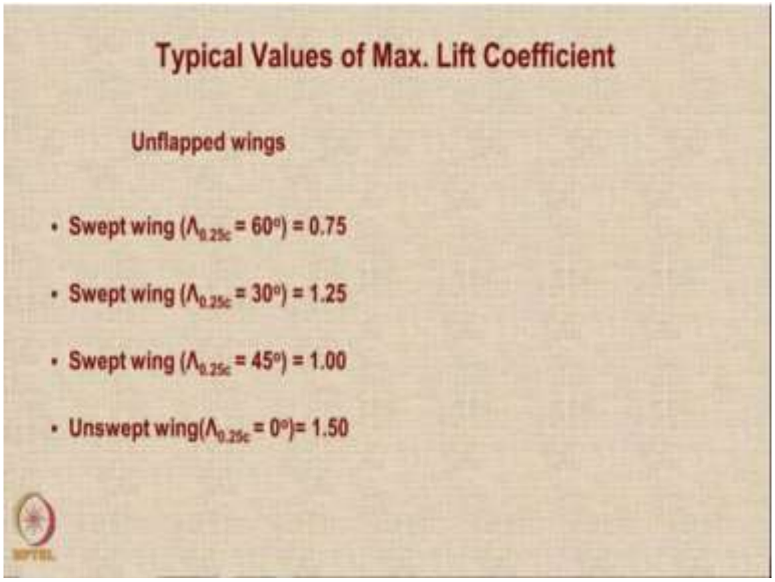

Before we estimate the values, let us get an idea about what to expect by looking at the past literature and searching the typical values of maximum lift coefficient. So if we first look at unflapped wings, then for a swept wing with a very high sweep of 60 degrees. Typically the values of $C_{L_{max}}$ are nearly 0.75. If on the other hand, you have a 30 degree sweep, the value comes out to be nearly 1.25. And for 45 degree sweep, the value turns out to be 1.0. So broadly speaking, if you have un-swept wings, then the value is 1.5.

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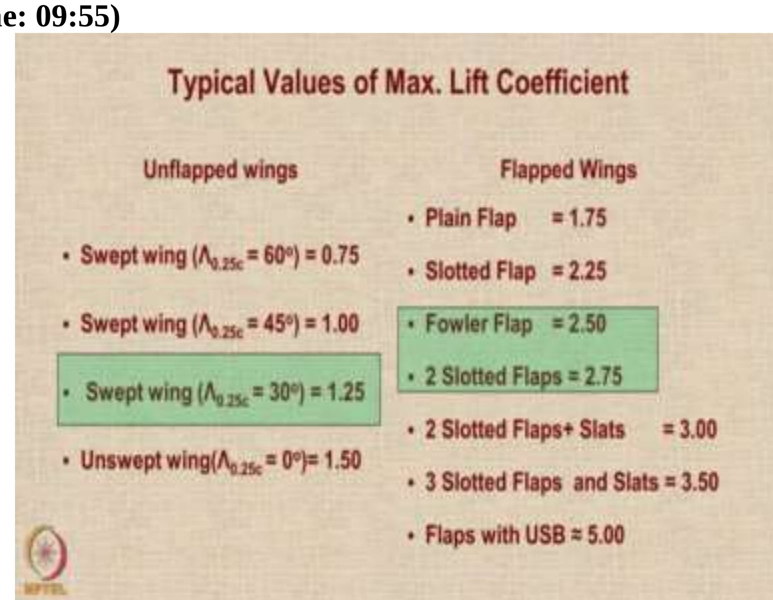

So before we go ahead with calculating the values of the maximum lift coefficient, let us get an idea about what are the typical values based on the literature that is available. If we look at unflapped wings, we see that wings with very large sweep of 60 degrees or so, they have a very low value of $C_{L_{max}}$ of nearly 0.75, if the sweep is reduced to 45 degrees, it comes out to be approximately 1.00, if you reduce it further to around 30 degrees, it comes to 1.25 and if you reduce it to 0 degrees or taken un-swept wing, then you get the value of $C_{L_{max}}$ is 1.5.

So, what we see is that by sweeping a wing to about 60 degrees, you are reducing by 50% the maximum lift coefficient of the base aircraft. As far as we are concerned, our aircraft has a quarter chord sweeps of around 32.2 degrees. So, the value that we expect for $C_{L_{max}}$ will be approximately 1.25. when you have flapped wings, then with a plane flap you get 1.75, if you have slots in the flap, you get a substantial improvement and the becomes nearly 2.25, for a Fowler flap, you have a further improvement because not only you create a slot, but you also have a larger area by moving the flap behind.

If you put 2 slots in the flap, then you get further improvement from single slot to 2.75, of course with additional complications because when you have multiple slots in the flaps, then the mechanical gearing and the design of the system becomes a bit more complicated and complex. On the other hand, if you go for double slotted flats and if you use slats also you get a further improvement and $C_{L_{max}}$ around 3. Similarly, if you have triple slotted flaps with slats, you get up to even 3.5.

If you want to go beyond 3.5 then you need to resort to certain you know special approaches. For example, you can have a flap with upper surface blowing or USB, where you can aim to get $C_{L_{max}}$ values of nearly 5. In our case, our flap is actually a slotted flap, but with some improvements. So the maximum value is expected to be in between the Fowler flap but better than a double slotted flap.

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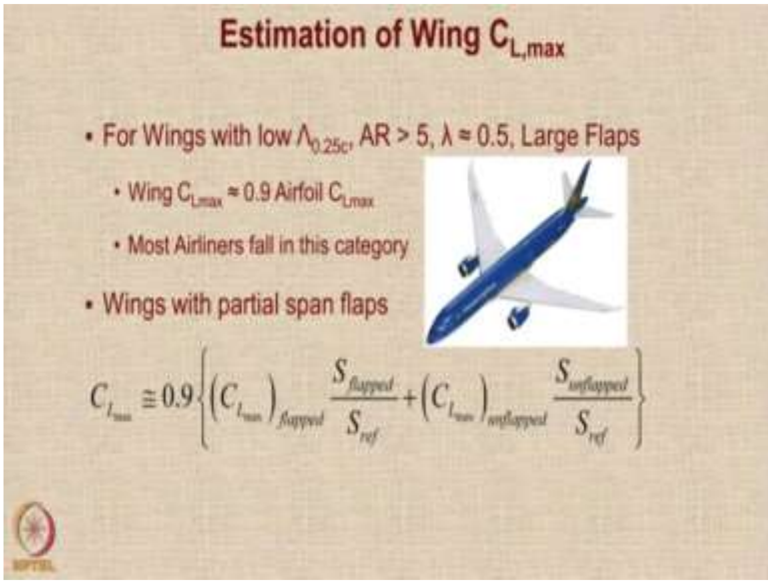

So, let us look at the estimation of the wing $C_{L_{max}}$ for wings with low quarter chord sweep with aspect ratio more than 5 and will taper off nearly 0.5, and those who have very large flaps and that is true for aircraft like Boeing 787-8 which are shown in the screen. So, for these aircraft, the wings $C_{L_{max}}$ can be almost 90% of the airfoil $C_{L_{max}}$. So, the 3D effects because of aspect ratio because of taper ratio, and because of sweep are not going to substantially reduce the $C_{L_{max}}$ value.

If the airfoil value gives you $C_{L_{max}}$ of 0.9 that is a 2D $C_{L_{max}}$, the 3D value will be almost 90% of that. And most airliners that we see they fall in this category, but when you have partial span flaps not covering the entire span, but partial span flaps, then the $C_{L_{max}}$ is calculated as approximately 90% of the $C_{L_{max}}$ of the flap area into the ratio of $\frac{S_{flapped}}{S_{ref}}$ whereas S_{ref} is the total wing reference area plus $C_{L_{max}}$ for the unflapped area and the ratio of unflapped area upon the wing reference area.

So in proportion to how much of the area is a flapped area, you can actually apportion the approximate value of the $C_{L_{max}}$ to that particular portion and just add them together and take around 90% of that that would be a good estimate for $C_{L_{max}}$ of the aircraft of the wing .

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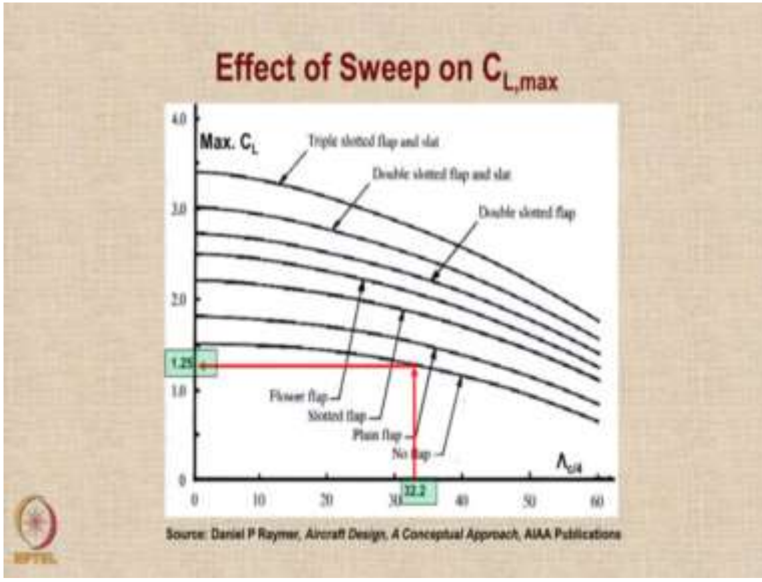

Let us see the effect of sweep on $C_{L_{max}}$. Here is a graph taken from the seminal textbook by Daniel Raymer on aircraft design which showcases the effect of increase in the quarter chord sweep which is on the x axis on the maximum C_L values first of all for an airfoil with an aircraft with no wing with no flaps such as the bottommost line and then you can see how $C_{L_{max}}$ is increasing when you have a plane flap and then you have a slotted flap, then you have Fowler flap etcetera till you go to the double slotted follow flap and slat.

So if you do not have any sweep you can get the values as high as 3.5, 3.4 and with the with the advent sweep back these values are going to only worsen. Now, in our case, our aircraft has a sweep of 32.2 degrees and it has got a basically, if you look at the unflapped wing to get the base C_L value, you just take the sweep and we read off the value the value comes out to be nearly 1.25. So, for us the wing will have the $C_{L_{max}}$ of 1.25 without the effects of sweep and other parameters.

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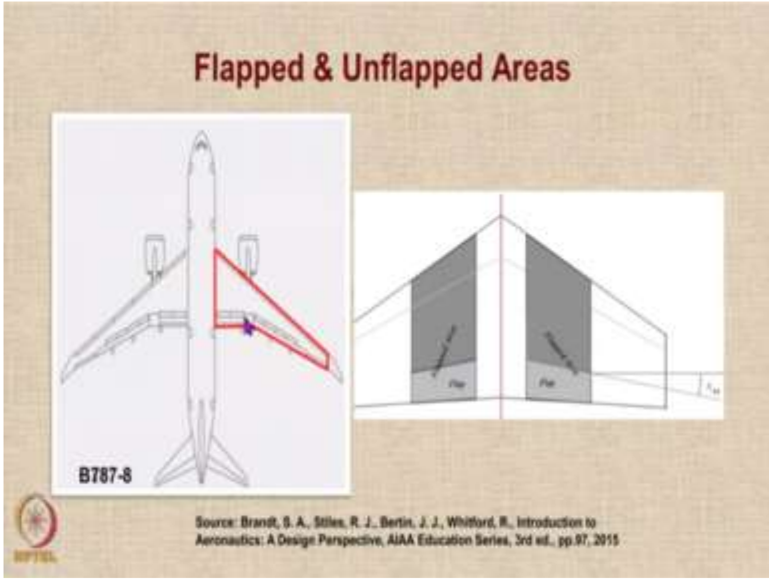

Then we calculate the flap area. So, for that, we need to understand the definition of the flapped area and the unflapped areas. So, whether you have a leading edge flap or a trailing edge flap or both, any area that comes under the influence of flaps whether they are leading edge flaps or trailing edge flaps is called as a flap area. And in case of a trailing edge flap, the area upstream of the flap is also counted. And in case of a leading edge flap, the area downstream of the flap is also counted.

This figure has been taken from the classic textbook by Brandt, Stiles, Bertin and Whitford. This is the top view of the wing of Boeing 787-8. And if you look closely at the wing, we realize that we have both leading edge flaps and trailing edge flaps. So, the leading edge flaps covered from this point to this point and trailing edge flaps covered pretty much from the almost from the root to this point. So, the total area that is flapped area will be the area marked in the red color in this figure. So that would be the flapped area.

And the unflapped area would be only the area inside the fuselage and that in the wing tips are to the outboard of the leading edge flap under , because leading edge flaps are the one which are having the largest distance along the span. So the area marked in red color turns out to be the flapped area.

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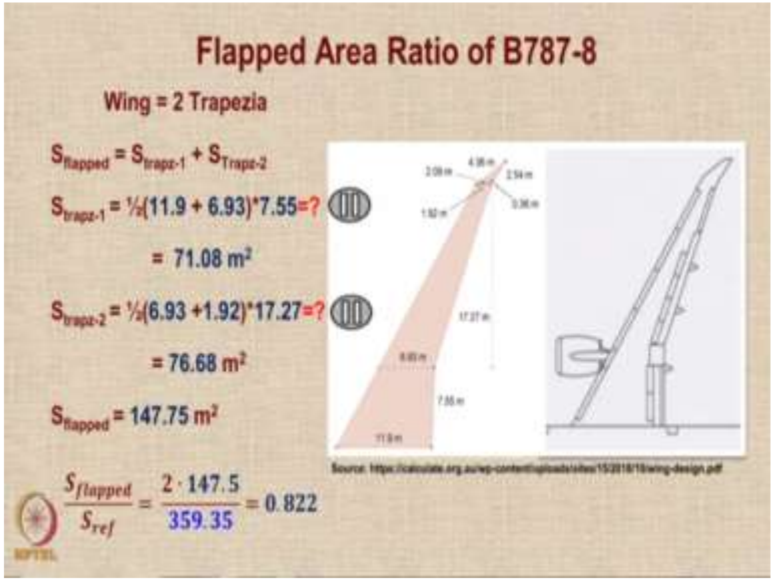

Now let us look calculate the flapped area ratio of Boeing 787-8. So here is the same wing. Once again I show you the wing but only half the wing is shown and this particular wing can be replaced by a series of geometrical constructs. Incidentally, the flapped area would consist of 2 trapezia, the first trapezium would be the one that goes from the root to the mid span break and the second trapezium would go from the mid span break to almost the wing tip portion.

So once you calculate the area of these 2 trapezia, this one and this one, you can get the area of the flap. So $S_{flapped}$ is going to be area of trapezium 1 and area of trapezium 2, the summation of these 2 quantities. Now let us calculate the area of the first trapezium. For a trapezium, we know that the formula for the area is half of the sum of the 2 parallel sides into the gap between the 2 parallel sides. So with that the value of $S_{trapezium\ 1}$ turns out to be half of 11.9 plus 6.93 into 7.55.

Now these numbers for the geometry of the Boeing 787-8 have come from the source as mentioned here. It is a very interesting source which talks about application of mathematics in aircraft design; I would urge all of you to check out this particular website, which has a document containing this description. And along with the document, there is also a small video of about 7 and a half minutes, which explains how the application of mathematics has been found in aircraft design.

So we borrow this figure from that calculation. So you calculate the value of the area of the first trapezium as 71.08 square meters. Similarly, we need to calculate the area of the second trapezium, Please pause the video at this stage and have a look at the values that you obtained and match with the values that we have calculated. The number comes out to be 76.68.

So if you add these 2 numbers together, and then if you multiply by 2 because there are 2 wings in the aircraft, you get the flap total flapped area. So this area into 2. So the ratio as $\frac{S_{flapped}}{S_{ref}}$ will be 2 times the calculated area divided by the wing Reference area which is already specified to us as 359.35. So, hence the value of $\frac{S_{flapped}}{S_{ref}}$ turns out to be 0.822. In other words 82.2% of the wing is under the influence of flaps, whether leading edge flaps or trailing edge flaps.

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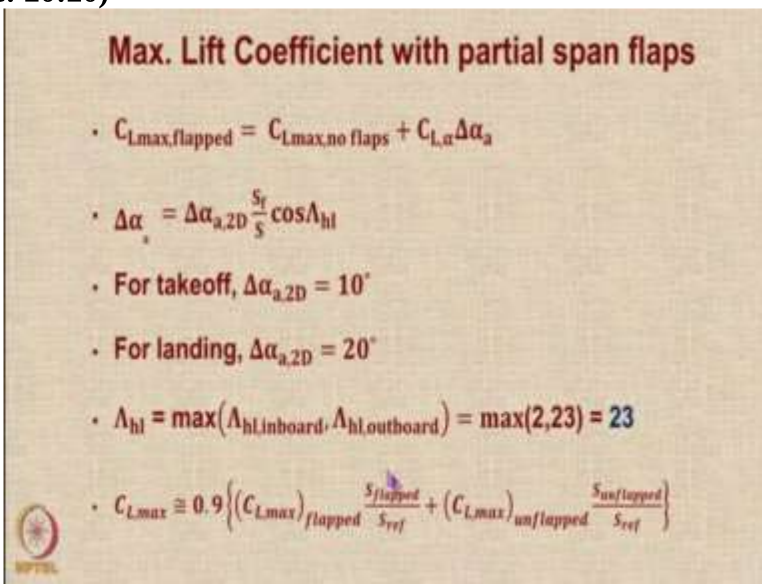

So, let us calculate the max lift coefficient of the aircraft with partial span flaps; we first look at the formula that we will use. So, $C_{L_{max}}$ for the flapped region would be

$$C_{L_{max,flapped}} = C_{L_{max,no\ flap}} + C_{L_\alpha}\Delta\alpha_a$$

Now $\Delta\alpha_a$ is a parameter that can be calculated as

$$\Delta\alpha_a = \Delta\alpha_{2D}\frac{S_f}{S}\cos\Lambda_{hl}$$

So, we have been given in this example, that the takeoff condition you can assume $\Delta\alpha_a$ because of the flap deflection in the 2 dimensions to be 10 degrees and for landing 20 degrees. And for want of any other accurate estimate, we will take the hinge line as a maximum of the inboard and outboard. In any case the outboard flap is very heavily swept; it has a 20 degree sweep. So, therefore, we will ignore this 2 degrees sweep of the inboard portion and take it as 23 degrees.

So, we might error on the side of caution and the actual value may be slightly higher, but that is fine. $C_{L_{max}}$ as I already mentioned is 0.9 times the product of 2 terms the flapped $C_{L_{max}}$ value as obtained by the formula shown in the slide into the ratio of flapped area by the reference area plus the formula $C_{L_{max}}$ unflapped of the base airfoil $C_{L_{max}}$ base wings $C_{L_{max}}$ unflapped $C_{L_{max}}$ into the ratio of unflapped area by the reference area. So, if this area is 0.822, this will be just 1 - 0.822.

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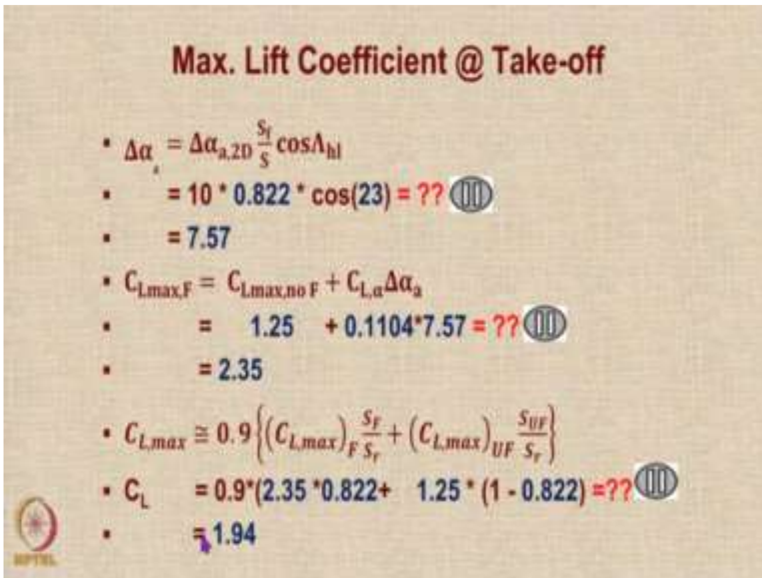

So, using these, let us start calculating the max lift coefficient at takeoff condition. The formula is as shown on the screen; we need to get the parameters $\frac{S_f}{S} \cos \Lambda_{hl}$. So we already know that this number is 10. This ratio we have already calculated as 0.822 and $\cos \Lambda_{hl}$ as well also we took out the maximum of the both, so pause your video here and try to calculate delta alpha the value turns out to be 7.57.

So, the additional lift coefficient because of the flaps is as if the aircraft has an additional angle of attack of around 7.57 degrees. So therefore,

$$C_{L_{maxF}} = C_{L_{max},no\ F} + C_{L_\alpha} \Delta\alpha_a$$

At this stage, I would like you to pause the video and do this calculation. The value turns out to be 2.35. So our estimate is that the $C_{L_{max}}$ from the flapped area is 2.35.

And now you can get $C_{L_{max}}$ for the whole wing with 0.9 times sum of the 2 products. So for this, it is very simple, we just bring in the value of $C_{L_{max}}$ flapped which is 2.35 plus the ratio 0.822 plus $C_{L_{max}}$ unflapped which is the basic wing with no sweep 1.25 and area of unflapped region upon the reference area, which will be 1 – 0.822. So please calculate this value it turns out to be 1.94. So our estimate is that the C_L of the aircraft is 1.94. This is based on the basic calculations.

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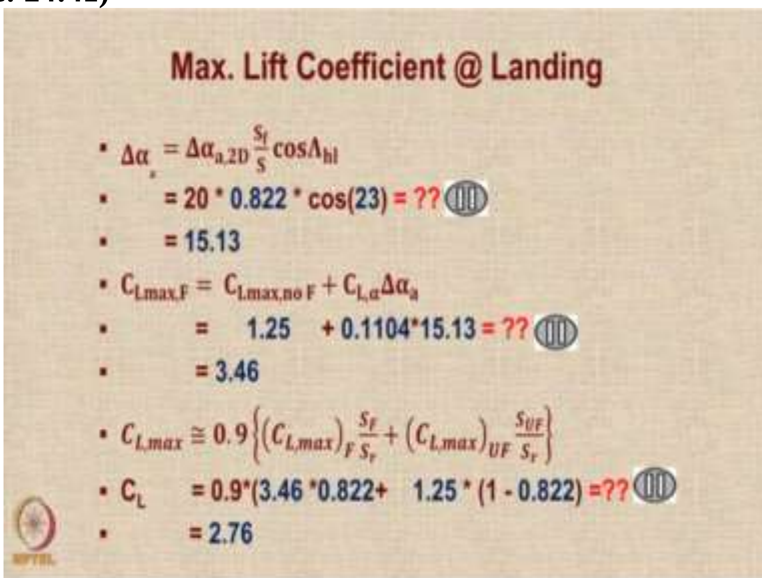

During landing the same thing will be done except the difference is that we will use the value for landing so $\Delta\alpha_{landing}$ is much more 20 degrees because flaps are deflected to a higher angle. But $\frac{S_f}{S}\cos\Lambda_{hl}$ will remain the same. So do calculate this value, it comes to 15.13 degrees.So, at this stage please pause the video and calculate this value it is 3.46.

This is the $C_{L_{max}}$ of the flapped region of the wing and for the entire wing we will again go for 0.9 times the summation of 2 products. So, in other words it is summation of 3.46 is the value obtained for $C_{L_{max}}$ for flapped region $\frac{S_f}{S}$ is already known to you plus 1.25 is the value for the unflapped region and the $\frac{S_{unflapped}}{S}$ is already known to you, it will be 1 – 0.822. So, do calculate this value, and you will get the value of 2.76. So, we estimate that this aircraft has got a$C_{L_{max}}$ of 2.76 with the flap deflected during landing, and 1.94 with the flaps reflected during takeoff.

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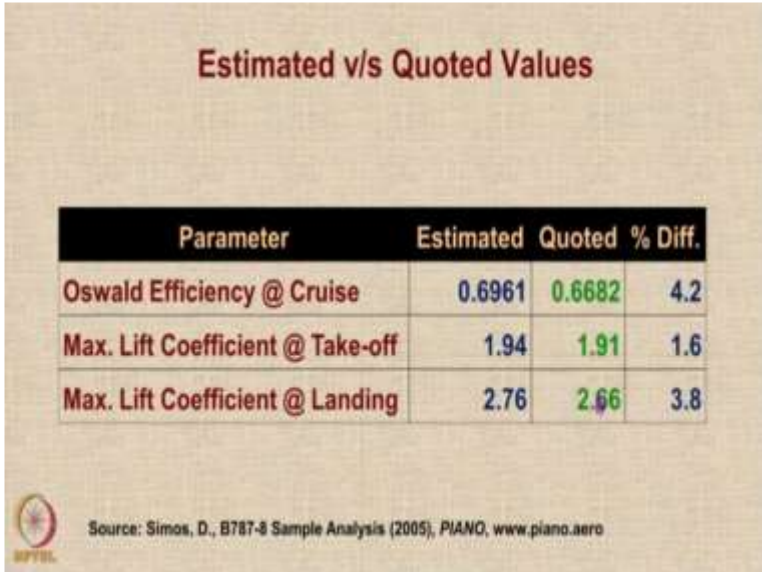

Parameter	Estimated	Quoted	% Diff.
Oswald Efficiency @ Cruise	0.6961	0.6682	4.2
Max. Lift Coefficient @ Take-off	1.94	1.91	1.6
Max. Lift Coefficient @ Landing	2.76	2.66	3.8

Let us see how our estimates compare with the actual values that are quoted. So, for comparison purposes, I have used the values given by Dimitri Simos in his piano documentation available online. So, first we look at the Oswald efficiency at cruise, we have obtained the value as 0.6961. And Simos does not directly quote the value of e but if you look at the non C_{DO} components of the drag at cruise, and if you actually just add all of them together, and as you meet to be the part of induced drag, then you can back calculate the value of e at Mach number given he quotes the C_L to 0.508 and Mach number 2.85.

So, with that you can calculate the value of e as 0.6682. So, we see that we are only 4.2% off; we have our predicted value by approximately 4.2%. The next is the maximum lift coefficient at takeoff, if you recall, we got the value of 1.91. Whereas the value quoted in the literature by piano we got 1.94 and the quoted value is 1.91. There is only about 1.6%. And finally, maximum lift coefficient at landing, we have got the value of 2.76 using our simplistic methods, whereas the actual value quoted is 2.66.

So again, we are off by approximately 4%. So in conceptual design, if you are off by about 4 to 5% it is not really a big deal. So we can say that we have been able to reasonably capture the values and from now on we are going to continue with these values 1.91 and 2.60 for all other practical calculations.

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Before I close, I had like to acknowledge a few people. First of all, Dmitri Simos , whose piano sample data has been very useful in this particular work. I also want to thank the 4 authors Brandt, Stiles, Bertin and Whitford for their lovely book, on introduction to aeronautics, by them on a design perspective. So very nice book and I borrowed 1 figure from that particular book, and Daniel Raymer, as always, for his seminal book on aircraft design, from where I have taken one chart regarding effect of various types of flaps and the sweep angle on to the $C_{L_{max}}$. Thank you so much for your attention.