Introduction to Aircraft Design
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Lecture-53
Tutorial on Drag Polar Estimation of Military Aircraft

Hello everybody let us look at how we estimate the drag polar of a military aircraft. And to illustrate this procedure we have chosen F16-C Fighting Falcon.

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As the base aircraft on which we will do these calculations.

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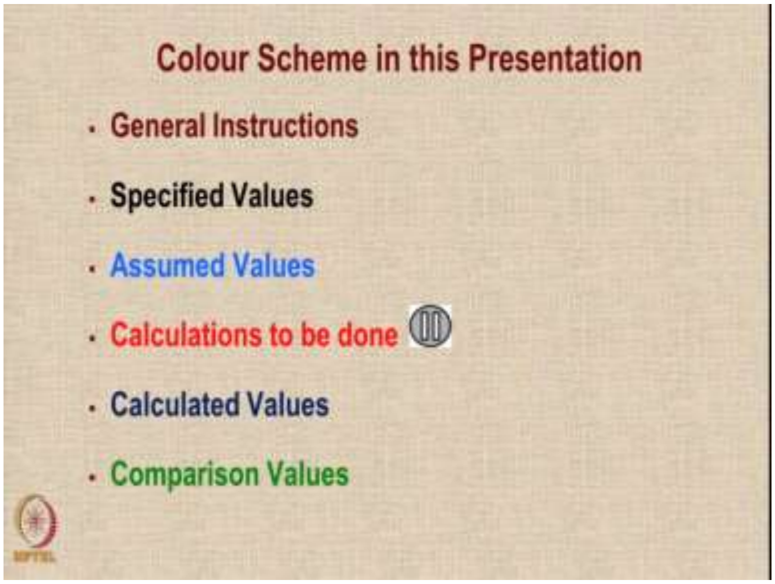

So the colour scheme in this presentation is that the general instructions like these will be given in brown colour. If there are any values which are specified in any reference source they will be shown in black colour any values that we assume will be in blue colour. The places where you should do calculations will be highlighted in red colour with question marks and there will be this symbol the pause button.

So wherever you see this button you have to remember that that is a place where you should stop the video and do some calculations and then match with the values obtained by us. The calculated values will be shown in this dark blue colour. And wherever we compare our numbers with existing aircraft we are going to generally use the green colour.

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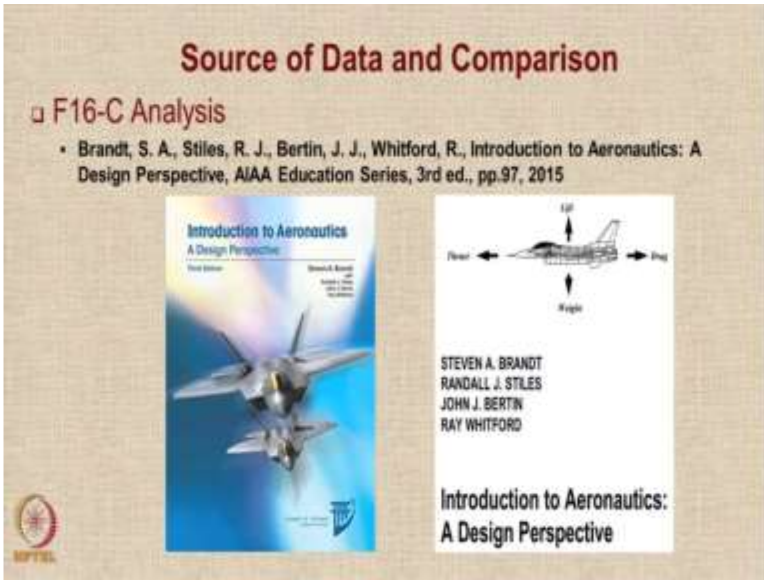

Let us look at the source of the data and comparison for this particular tutorial. As I said we are going to use F16-C and the textbook by Brandt Stiles Bertin and Whitford contains a detailed description of the procedure for estimation of the drag polar and that procedure has been borrowed by us and used in this tutorial.

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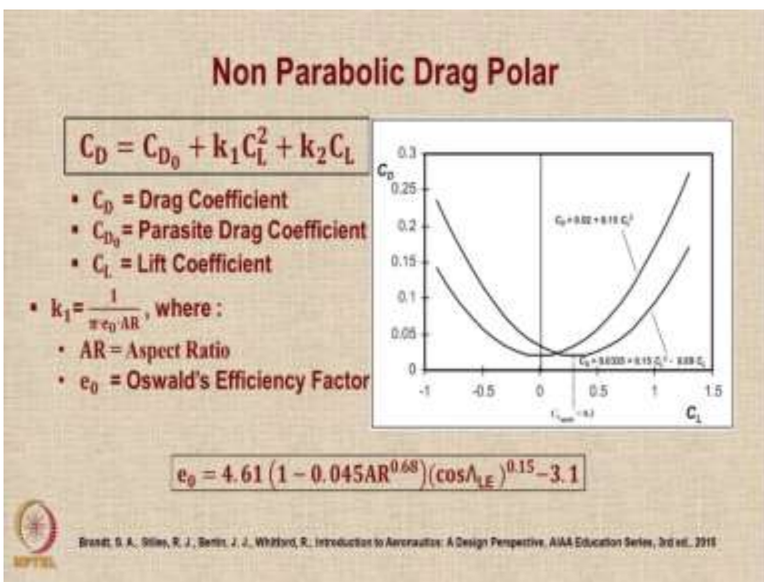

Many aircraft have a non parabolic drag polar here is an example. So the C D for such aircraft can be expressed in terms of

$$C_D = C_{D_0} + k_1 C_L^2 + k_2 C_L$$

There may also be actually some higher order terms but those are normally neglected. So this becomes a non parabolic drag polar because there is a linear term k_2C_L they are the first order term k_2C_L. So we know that here C_D stands for the drag coefficient.

C_{D_0} stands for the parasite drag coefficient also called C_{D_0} and C_L is the lift coefficient k_1 is a coefficient which is obtained in terms of the aspect ratio AR and the Oswald's efficiency factor e_0 which can be determined using this formula in terms of the wing aspect ratio and the leading edge sweep.

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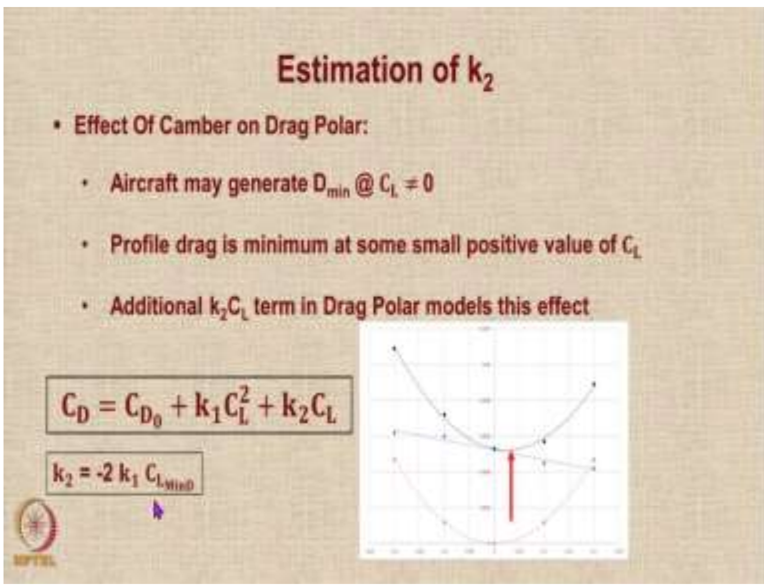

Let us look at the second parameter k_2. Now k_2 is used to model the effect of camber on drag polar among other things there are many aircraft which may generate a minimum drag not at an angle at which $C_L = 0$ but at a slightly different angle. There are many reasons for this because an aircraft is actually not just a wing aircraft is basically wing plus fuselage plus tail. So there are certain situations in which the angle at which the aircraft flies and has the minimum drag does not correspond to the condition where the $C_L = 0$.

So here is an example of an airfoil which has a non parabolic drag polar so that is the top blue line there is a non parabolic drag polar which has got 2 components there is a parabolic component which is modeled by the coefficient k_1 and then there is a non parabolic component that is modeled by this equation. So in such a graph the profile drag is minimum at some small positive value of C_L generally.

So this additional $k_2 C_L$ term in this equation models this particular effect. Otherwise if this term was absent then you would actually get a parabolic drag polar. So we will see today how we can get the equivalent parabolic drag polar also for an aircraft. Now if you look at this expression C_D in terms of $k_1 C_L^2$ and $k_2 C_L$ and let us say if you want to find the value at which the Drag is minimum.

So if you differentiate this expression with respect to C_L take those C_D by those C_L put it equal to 0 then take the second derivative and confirm that that number is negative then you can easily derive the condition that k_2 will be equal to

$$k_2 = -2k_1 C_{L,minD}$$

So that is how the coefficient k_2 can be calculated but to do that we need to get the value of $C_{L,minD}$ that means we need to know k_1 we already know from the previous slide but to know k_2 we need to know the C_L at which the drag is minimum.

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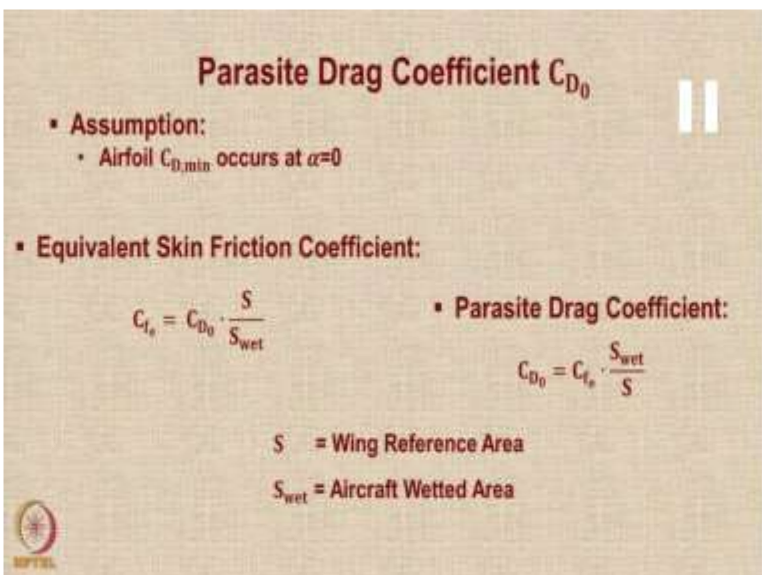

So for that we have to make some assumption. So we assume that this minimum drag occurs the airfoil has a minimum drag at $\alpha = 0$. And we look at the equivalent skin friction coefficient that will be

$$C_{f_e} = C_{D_0} \frac{S}{S_{wet}}$$

and the parasite drag coefficient will be then obtained as

$$C_{D_0} = C_{f_e} \frac{S_{wet}}{S}$$

this just by reverting the formula where S is the wing reference area and S_{wet} is the aircraft wetted area.

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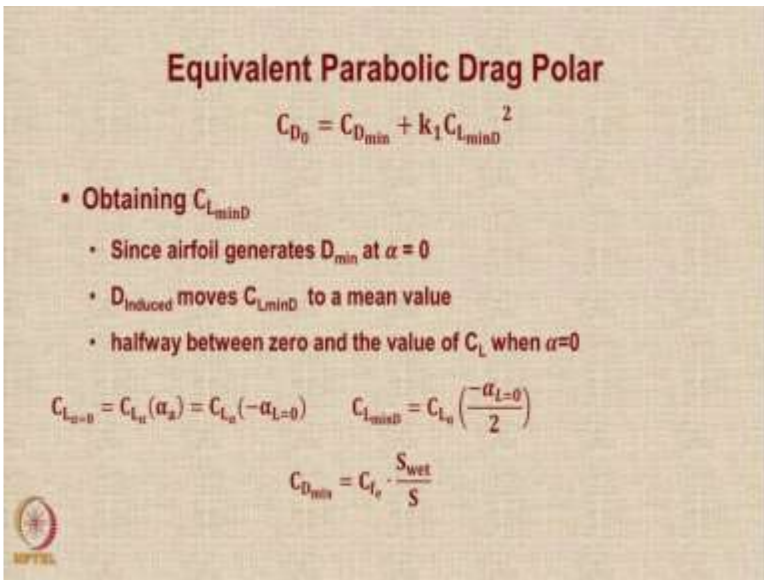

So now let us see how we get the equivalent parabolic drag polar. So this is what we would like to have we would like to have

$$C_{D_0} = C_{D_{min}} + k_1 C_{L,minD}^2$$

this is the equivalent Drag polar. So how do we obtain the value of $C_{L_{minD}}$ that is the question? So what we do is since the airfoil is assumed to generate minimum drag at alpha equal to 0 therefore what we assume is that the induced drag actually does nothing but moves this C_L which gives minimum drag to a mean value.

And this value is assumed to be halfway between 0 and the value of C_L when $\alpha = 0$. So $C_{L_\alpha} = 0$ and half of it if you take that would be the assumption of moving of $C_{L_{minD}}$. In other words we know that the C_L at $\alpha = 0$ will be absolute angle of attack into the lift curve slope and this absolute angle of attack is actually going to be minus of the lift equal to 0. So that means the angle at which lift is equal to 0.

So knowing the airfoil you can get this value and you know also the value of C_{L_α}. In fact we have a separate tutorial on calculating C_{L_α} of an aircraft. So therefore now we can get $C_{D_{min}}$ which is the requirement here as $C_{f_e}\frac{S_{wet}}{S}$.

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Typical Values of Equivalent Skin Friction Coefficient

Aircraft Type	C_{f_e} Values
Jet Bomber and Civil Transport	0.0030
Military Jet Transport	0.0035
Air Force Jet Fighter	0.0035
Carrier Based Navy Jet Fighter	0.0040
Supersonic Cruise Aircraft	0.0025
Light Single Propeller Aircraft	0.0055
Light Twin Propeller Aircraft	0.0045
Propeller Seaplane	0.0065
Jet Seaplane	0.0040

Brandt, S. A., Stiles, R. J., Bertin, J. J., Whitford, R., Introduction to Aeronautics: A Design Perspective, AIAA Education Series, 3rd ed., 2015

Let us look at the typical values of the equivalent skin friction coefficient that is C_{f_e} for jet bomber and civil transport it is point 0.0030 for military jet transporter is 0.0035 for F jet fighter 0.00035 again for a carrier based Navy jet fighter it is slightly higher. For a supersonic cruise aircraft it is lower and for a single seat propeller aircraft it is very much higher. For light twin propeller aircraft it is a little bit lower and for propeller seaplane it is the highest value. So in our case and of course there is also a jet seaplane. So we do have in our case the problem that we are looking at is the Air Force jet fighter. So the typical value of equivalent skin friction coefficient would be 0.0035.

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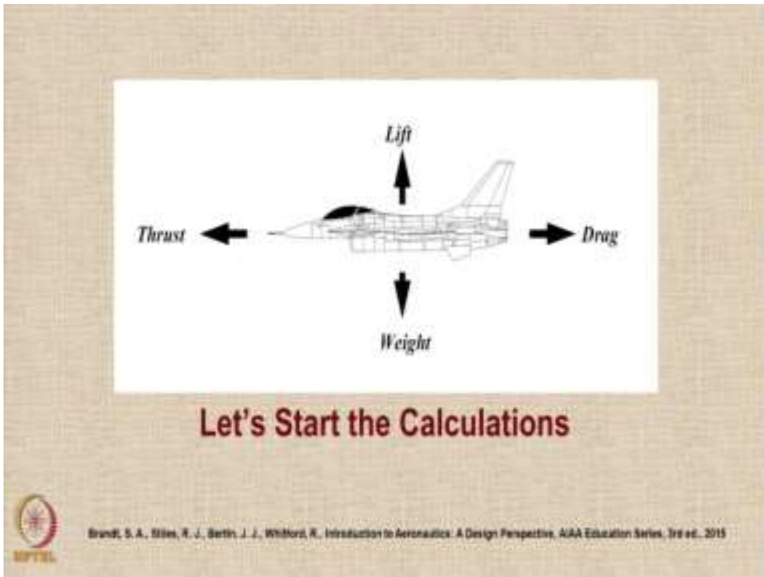

So let us start the calculation for an aircraft in level flight where all the four forces are in balance but while operating at a cruise mach number so we are going to go through calculations at cruise mach number under the ISA.

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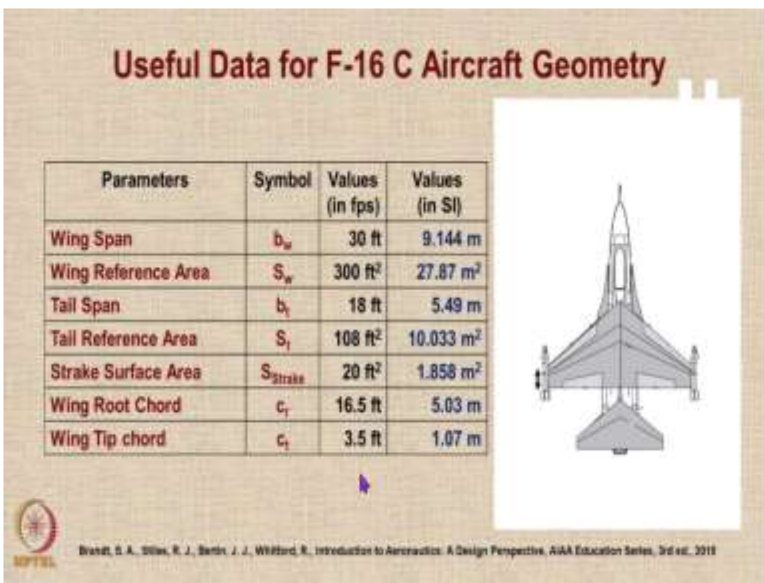

Sea level conditions here is the aircraft geometry and this geometry is then we just note on the value of various parameters. So for this particular aircraft we know that the wing span is 30 feet as specified and the conversion is 9.144 meters. The wing reference area is specified as 300 square feet which converts to 27.87 square meters. Similarly the tail span is specified as 18 feet which converts to 5.49 meters. The tail reference area 108 square feet which converts to 10.033 square meters. Strake surface area 20 square feet strikes 1.858 square meters.

Root chord at the wing at the center of this fuselage 16.5 feet which is 5.03 meters tip chord is 3.5 feet or 1.07 meters

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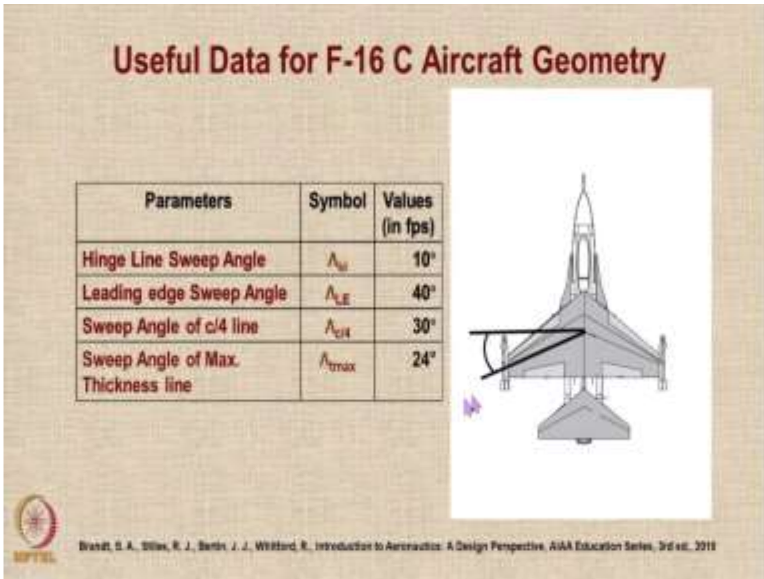

So what we do is we look at some more parameters for example the sweep of the hinge line of the flaps is 10 degrees the leading edge sweep is 40 degrees the sweep of the quarter chord line is 30 degrees and the sweep angle of the maximum thickness line is 24 degrees. So the aerofoil maximum thickness line is at 24 degrees sweep.

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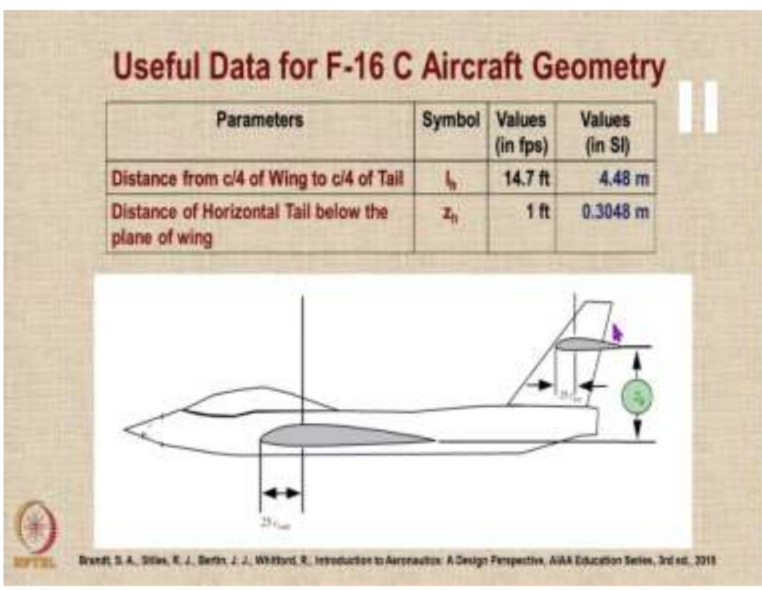

We also need some data from the side view for example we need the distance from the quarter chord of the wing to the quarter chord of the tail the so called tail arm and we also need the value of the lateral or vertical displacement of the horizontal tail from the plane of the wing which is 1 feet 0.3048 meters in this case.

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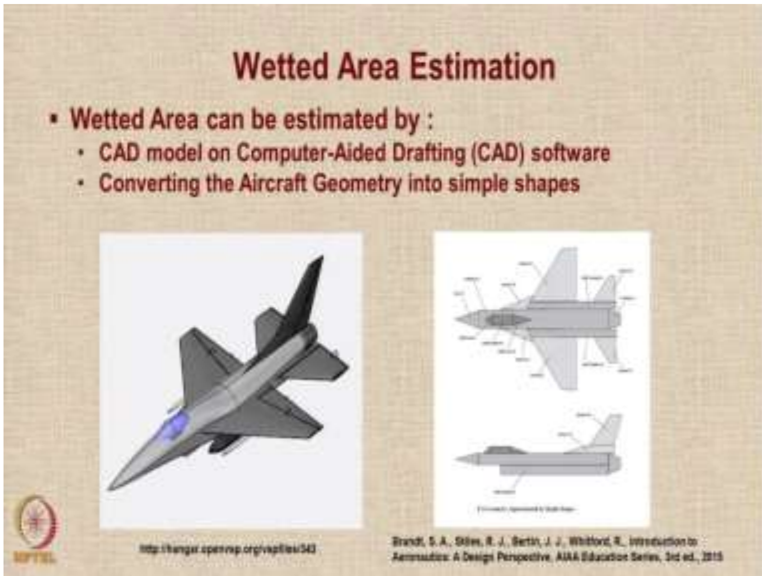

So with armed with this information we can now do the estimation of the wetted area of this aircraft you can estimate this by either making a CAD model and then the CAD software gives you the wetted area and that is what can be done by using a software such as open VSP we have already covered detailed description of the software open VSP and we hope that by now you have already tried our hands on open VSP. The other option that we have is that you convert the whole geometry into some standard and simple shapes like cylinders cones rectangles half cylinders etc and then calculate the wetted area of each component.

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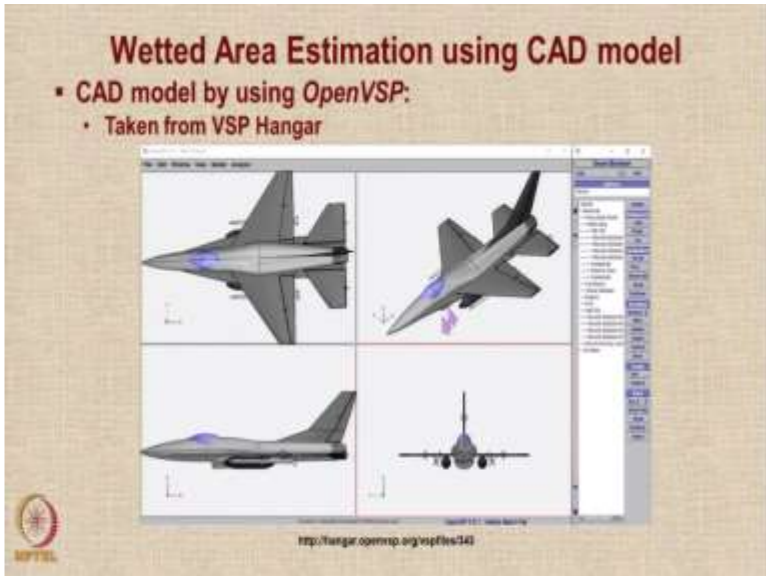

So we have done the same thing. So first we looked at CAD model by using open VSP and we took a model available from the VSP hangar this is that model but if you notice this model has these 2 bombs already loaded and also these 2 additional missiles loaded. So we do not need this. So this is how you render the shape of the model.

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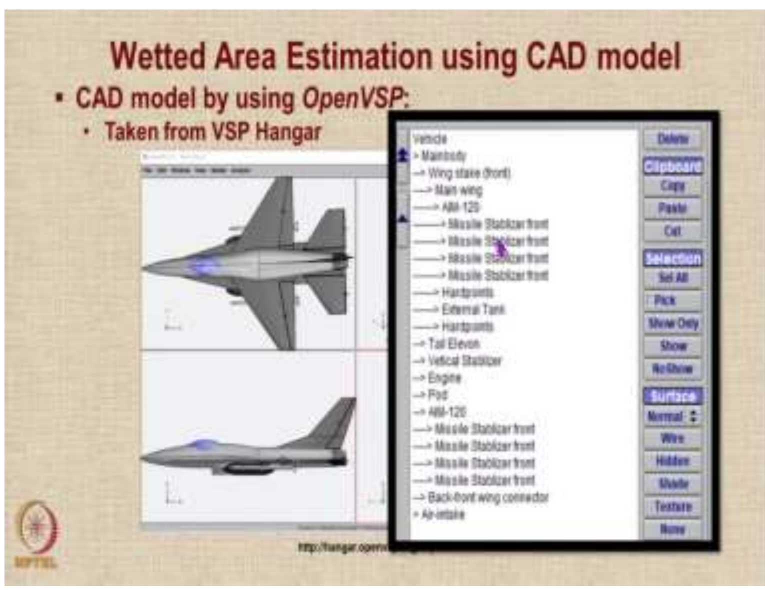

And then you can see there are so many components there are so many components which are there including the missile stabilizer and pods etc.

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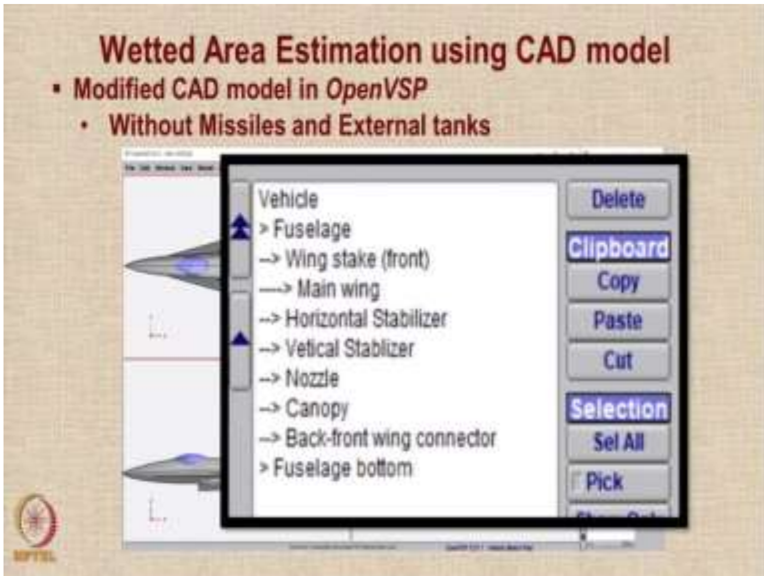

But in our case we have to modify the missile by removing the pods or the external tanks and the missiles. So we get a simple clean model and for this simple clean model we just have these various components which are going to be considered.

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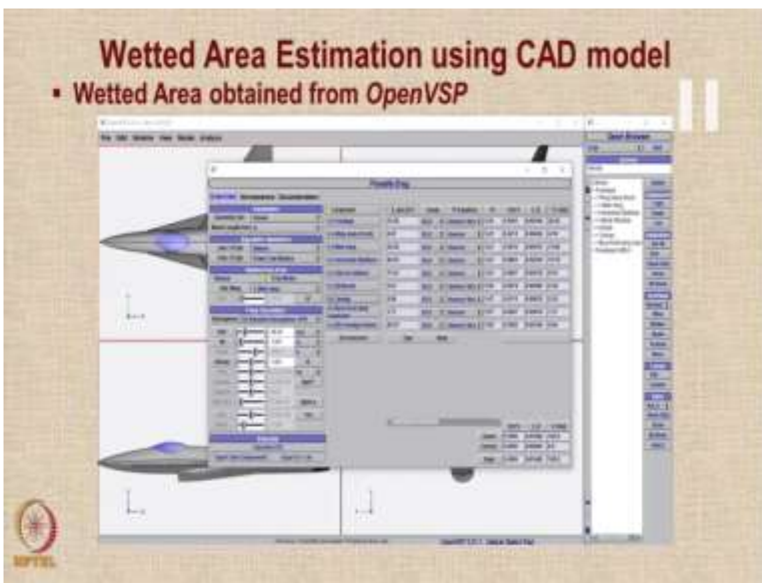

So using this particular CAD model it is possible to actually get the wetted area and also the parasite drag coefficients directly in the software.

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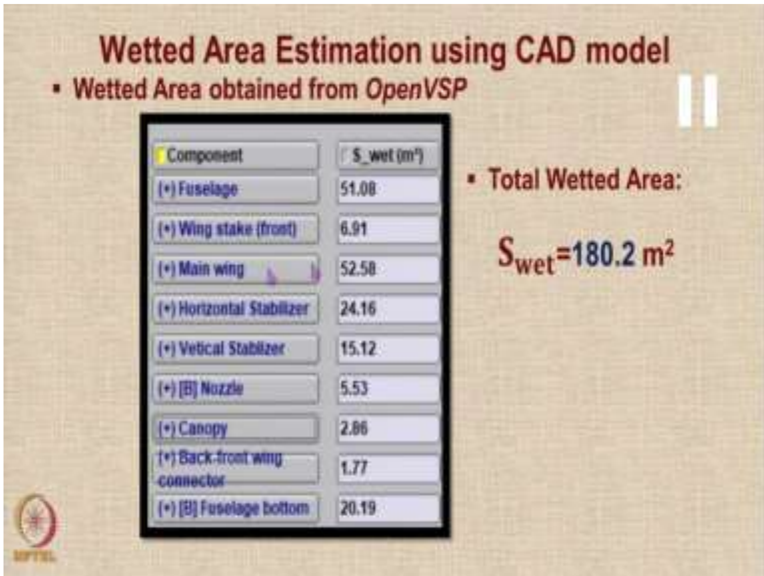

So this is the wetted area of each component as calculated by open VSP and the total wetted area if you add these numbers comes out to be 180.2 square meters.

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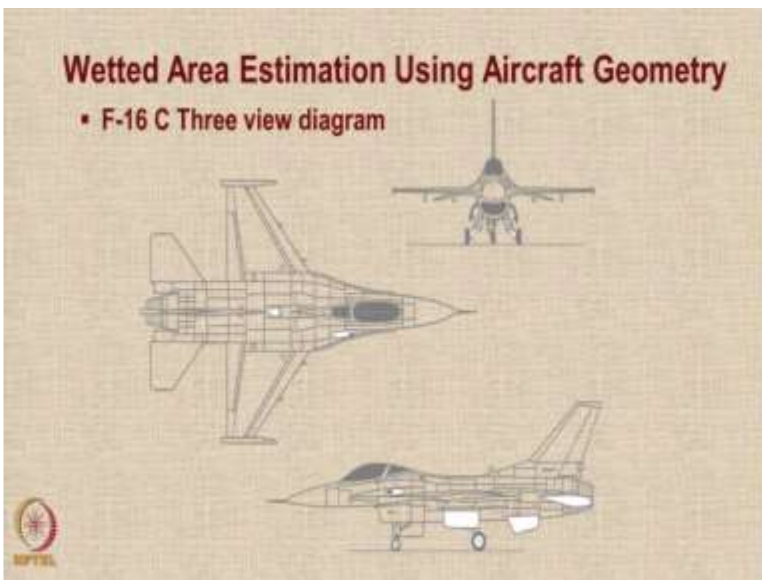

Now let us look at the second method where we look at the aircraft geometry and convert that into standard simple shapes. So here is a 3 view diagram of the aircraft.

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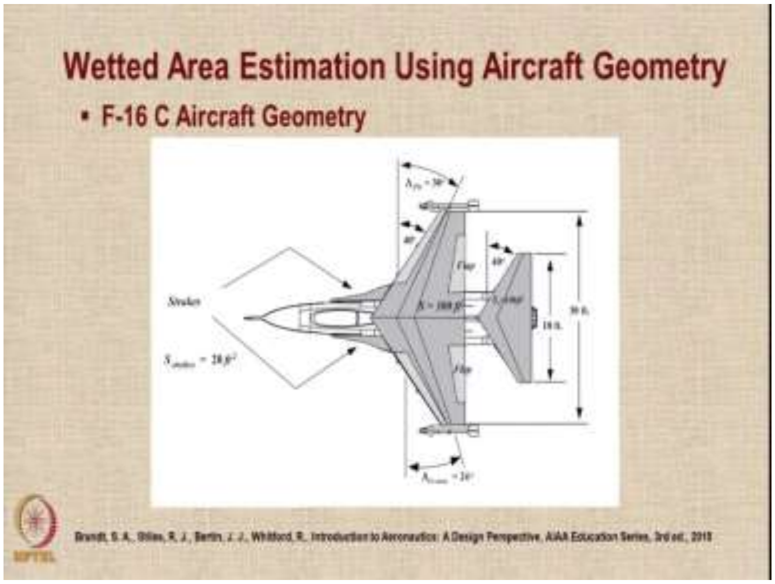

And what we do is we look at the geometrical data as specified in the book by Brandt, Stiles, Bertin
, and Whiftord

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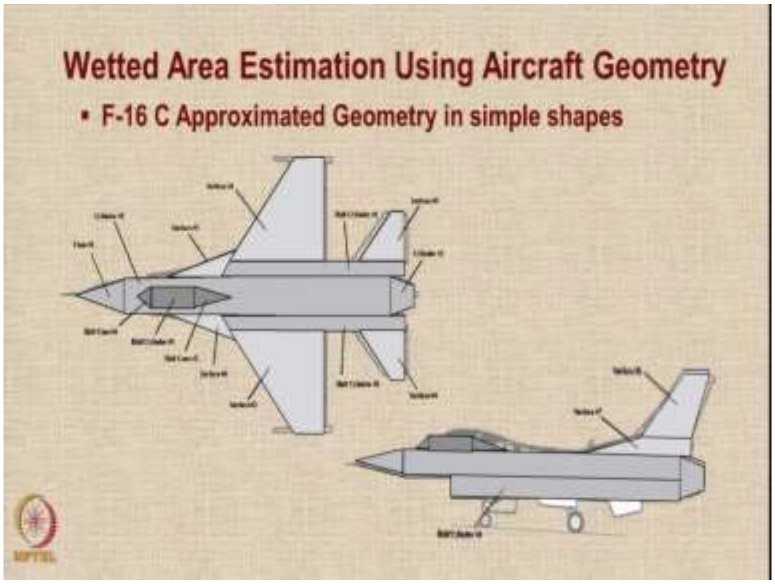

And then what we do is we approximate the geometry in simple shapes. So for instance you have three view of the aircraft. So you can see that in the top view you just create some simple surfaces and also in the side view you also try to create some simple surfaces try our best possible to match the geometry with that given and then you can do rendering of those surfaces to remove the internal details and hence you can get some idea about the various components.

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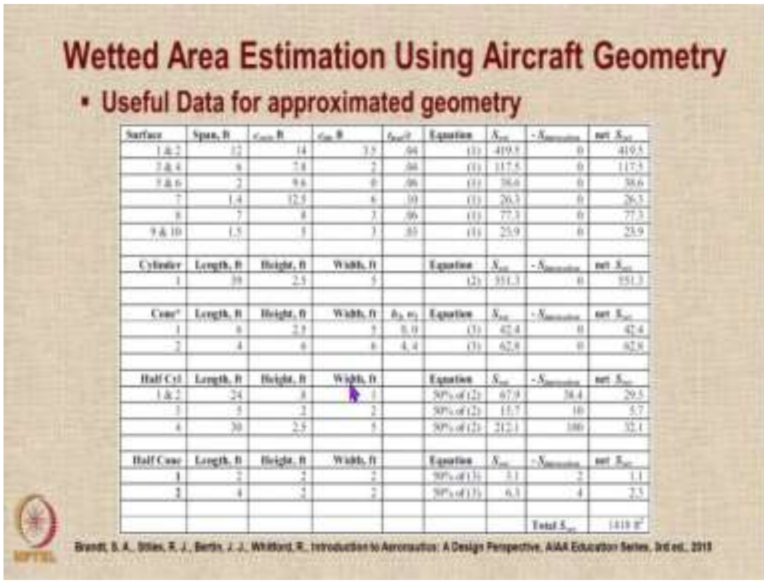

Now what we do is we look at this geometry and calculate the value of S wet for each component using some equations. Now these equations and procedures are already explained in the textbook by Brandt et al the total area comes out to be 1418 square feet but we are going to now explain to you one by one.

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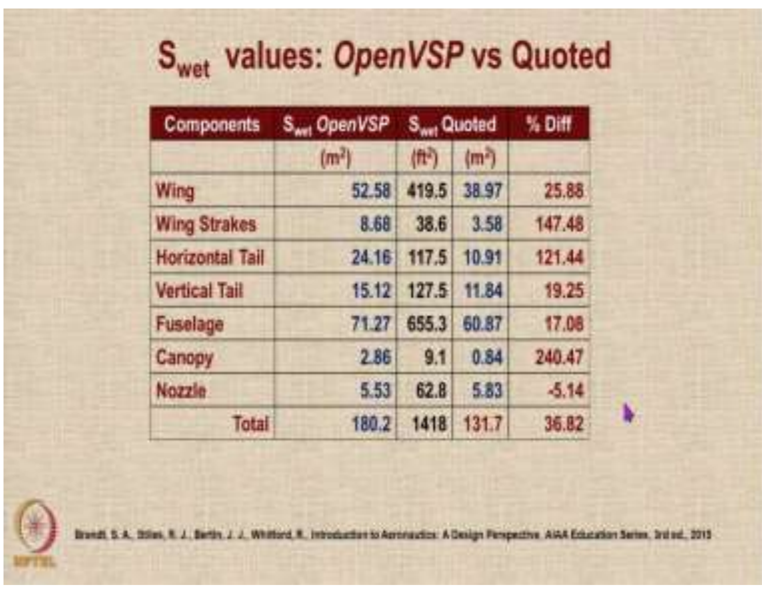

Components	S_{wet} *OpenVSP*	S_{wet} Quoted		% Diff
	(m^2)	(ft^2)	(m^2)	
Wing	52.58	419.5	38.97	25.88
Wing Strakes	8.68	38.6	3.58	147.48
Horizontal Tail	24.16	117.5	10.91	121.44
Vertical Tail	15.12	127.5	11.84	19.25
Fuselage	71.27	655.3	60.87	17.08
Canopy	2.86	9.1	0.84	240.47
Nozzle	5.53	62.8	5.83	-5.14
Total	180.2	1418	131.7	36.82

How this is done. So what we did is that we first tried to compare the values between the numbers that we got using open VSP with the Standard Model available and the values which are quoted in square feet and then in converted in square meters. So we notice that there is a large amount of error around 25% 26% for wings and 19.25% for vertical tail around the same value almost for that but huge variation in being strakes horizontal tail and canopy.

As a result there is a 37% increase in the value of a S wet as against quoted values. So now the problem is that if you continue with this S wet in your calculations then you are going to introduce this much error automatically in.

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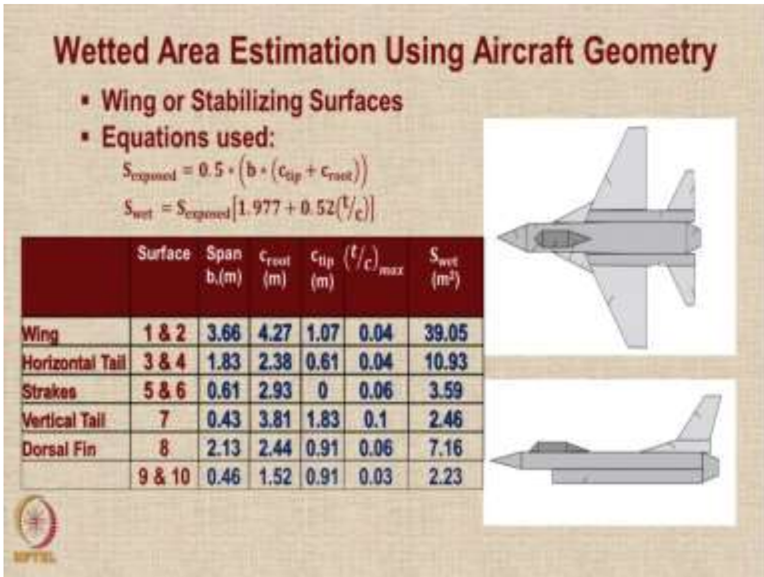

All the calculations. So what we did is we calculated this wetted area using the geometry. So for any surface like wing or any stabilizing surface. So the formula is given here for $S_{exposed}$ and then using that to get the value of S_{wet}. So for the wing which is surface 1 and surface 2 we know the wingspan from the geometry we know the root chord we know the tip chord and we know the t/c max. So we can calculate the value of S_{wet} for each of the trapezium and then multiply by 2.

Similarly for the horizontal tail we have 2 small trapezia which are identical in geometry because the same aspect ratio same sweeps and then we look at the strakes. So these are also simple triangular devices. So maybe you can do half of bass into height once the geometry is known to you vertical tail is also a simple trapezium because the root chord is known to you tip chord is known to you and the span is known to you.

The dorsal fin happens to be the fin which is mounted on the below the vertical tail that is also a standard geometrical construct. And then there are 2 hidden surfaces 9 and 10 which also contribute slightly to the geometry.

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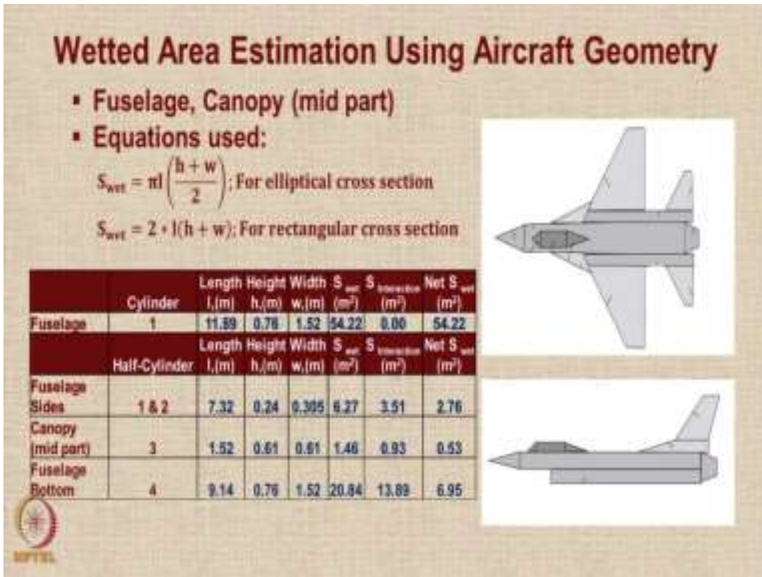

So similarly if we now look at the fuselage and the canopy or the mid part we can replace it with the these we can calculate the wetted area using the standard formula depending on whether the cross section is elliptical or rectangular. So for a fuselage for example it is a rectangular cross section so you can get the cylinder height cylinder width. So it is a mix between it is actually more towards elliptical and then we have fuselage sides these are 2 semi circular cylinders.

Then we have a mid part of the canopy which is a pure cylinder and then we have a fuselage bottom. So all these are the formula which can be used by you to calculate the actual wetted area of the aircraft. So these numbers have been given here only for comparison purposes for you actually you should be doing these calculations yourself.

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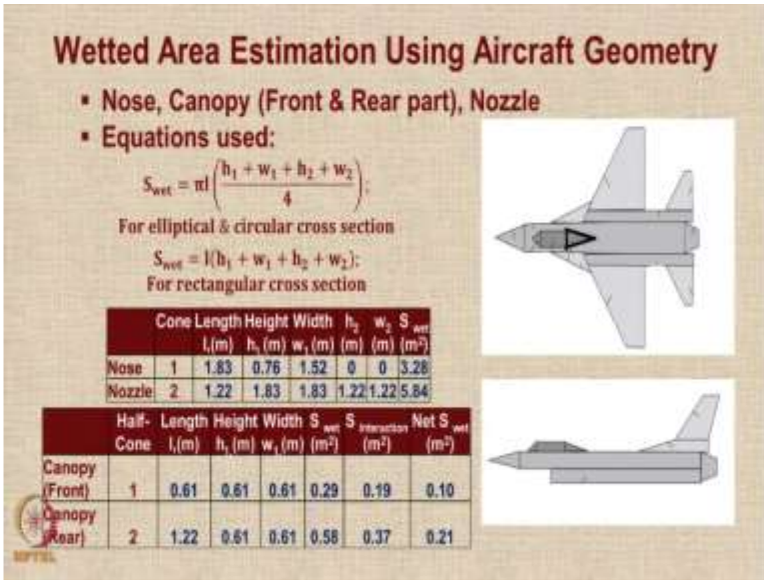

Moving ahead if you look at the nose the canopy front and the rear part because the mid part is a cylinder and the nozzle we get cones or frustrum of the cones and for that we can use standard formula available. So nose can be considered to be conical and with a nose cone of length height and width the end and the rear portion can be a frustum of a cone. So there are 2 cones here with l_1,h_1,w_1,h_2,w_2 then for the canopy front it is half cone or a small cone.

And for the rear can be also it is considered to be conical. So using these numbers you can calculate the areas of various components.

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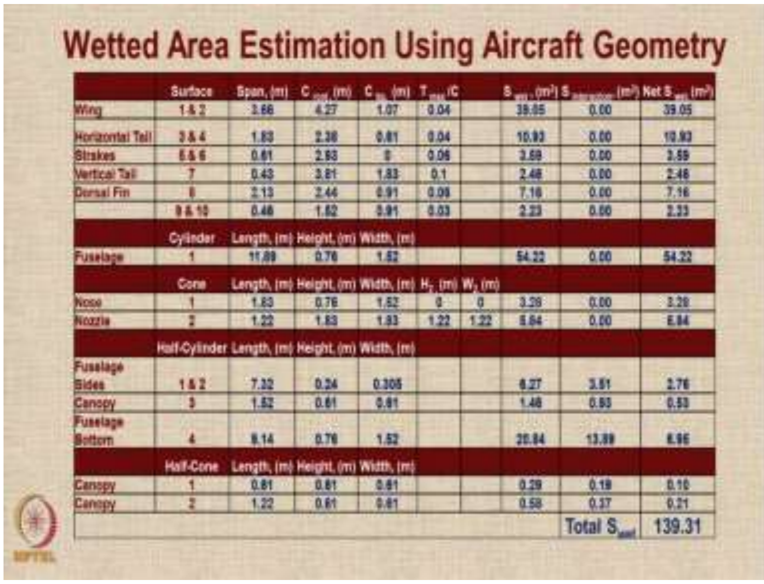

And after that here is the full table and as per this full table if you add these numbers the number comes out to be 139.31 square meters.

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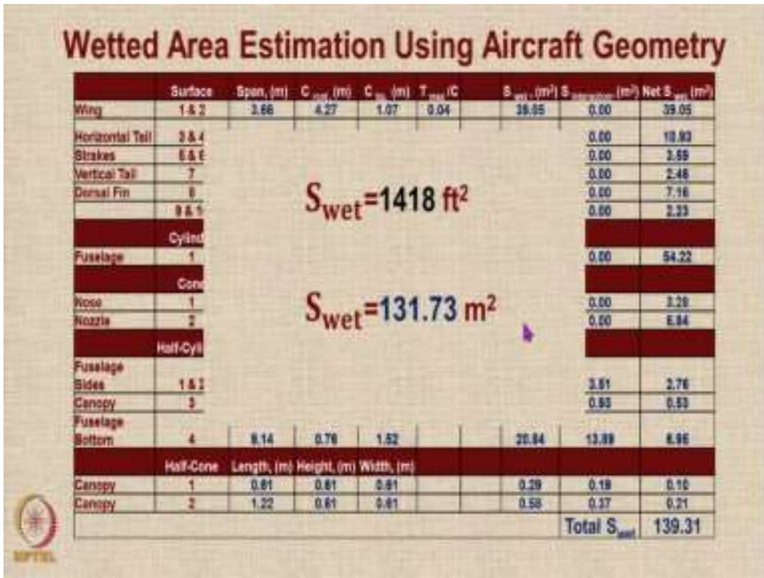

The value of S_{wet} given is 1418 square feet and spread calculated is 131.73 by converting this number. So what has happened is throughout in these calculations we have done rounding off because each quantity given in feet has been converted into meters rounded off and used in the calculation that is why there was an error of approximately you know eight and a half in 130.So we will go ahead assuming this to be the value because we do not want to introduce errors unnecessarily

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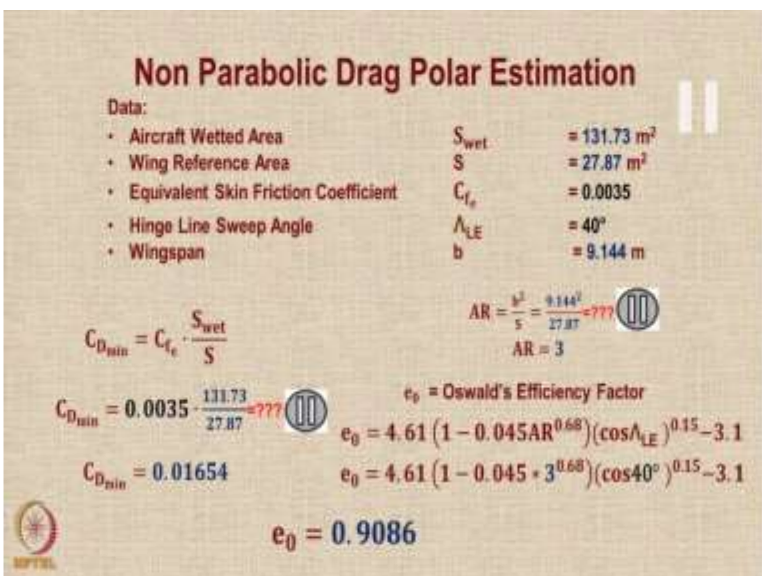

In our calculation. So coming back to the non parabolic drag polar estimation we know now the wetted area of the aircraft is 131.73 and the wing reference area is 27.87 square meter. The C_{f_e} for this aircraft is 0.0035 and the hinge line sweep is 40 degrees wingspan is 9.144 meter. With this information you can now calculate the value of $C_{D_{min}}$ which is $C_{f_e}\frac{S_{wet}}{S}$ and there is a formula for C_{f_e}.

But since we know the value of S_{wet} and S and also the value of C_L are for we can always get the value of $C_{D_{min}}$ which will be 0.0035 that is C_{D_α} C_{f_e} into 131.73 which is the Aircraft wetted area and 37.87 which is the wing wetted area. So in terms of the wing you can get the value of $C_{L_{minD}}$ for the profile drag coefficient of this aircraft to be 0.01654. Similarly the aspect ratio is known as $\frac{b^2}{S}$.

So it is 3 and e_0 is the efficiency factor e which already was described. And explained earlier. So you put the values in this put AR is 3 and Λ_{LE} 40 degrees please calculate these values do not just look at the screen do these calculations yourself and only then you will be able to learn and experience design. So the value comes out to be 0.9086.

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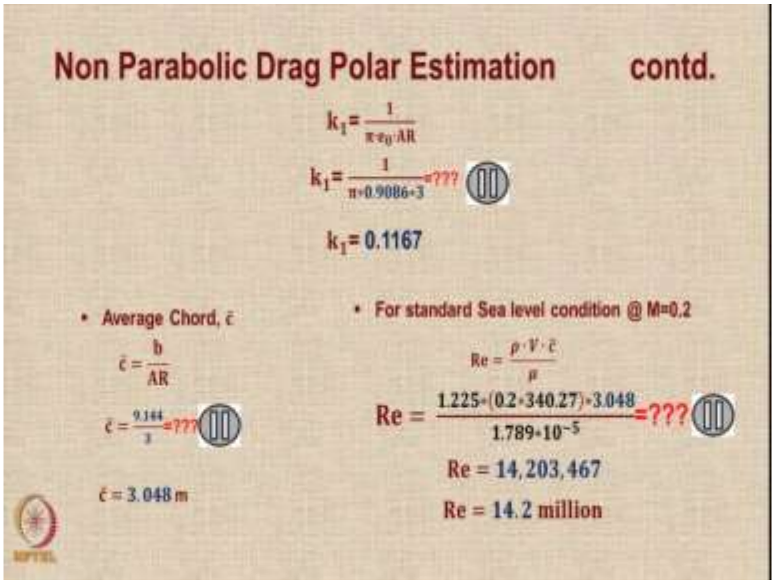

Let us continue with the non parabolic drag polar estimation $k_1 = \frac{1}{\pi e_0 AR}$ now we know the value of e_0 and also the AR. So we know the value of k_1 please calculate this value the value is 0.1167 that is the value of k_1 and average chord will be span by aspect ratio or 9.144/3 that value comes out to be 3.048 meters. So this is for a standard now let us look at standard sea level conditions for M=0.2.

For these conditions that Re number is equal to

$$Re = \frac{\rho V \bar{c}}{\mu}$$

So ρ is the density we is taken as 0.2 times this particular number because 0.2 is a Mach number this is the speed of sound and c bar is the chord upon this is the μ value this number comes out to be 14203467 or 14.2 million. So the Reynolds number at which the aircraft is operating at Mach number 0.2 at sea level is coming to be 14.2 million.

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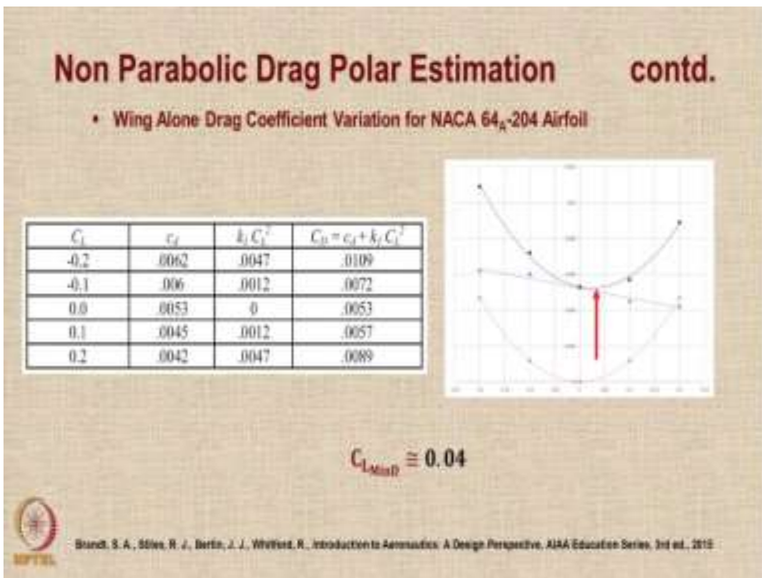

So if you look at the wing alone drag coefficient variation of airfoil NACA 64 A 204 This is the table which shows up and this is the graph that shows up it shows very clearly that the minimum is not at 0 the minimum of the drag is at some point this is the equivalent parabolic polar and this is the K_2 terms the K_2C_L term. So by this by inspection we find that the for the Airfoil the minimum drag occurs at $C_{L,minD}$ equal to 0.04 approximately.

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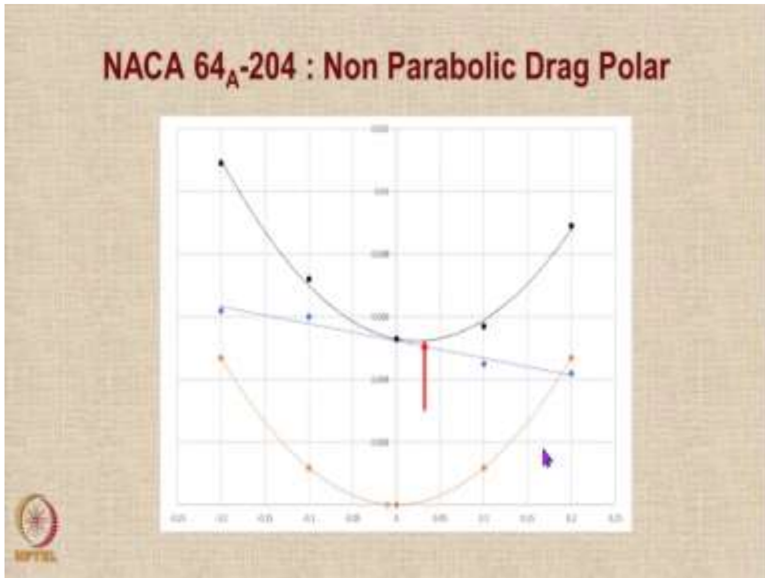

Here is a closer look at the same graph which I just showed you. So here we see that the Aircraft has a non-parabolic drag polar which is not centered at 00. But actually it is centered at some point here because it consists of 2 summations it consists of K_1C_L which is this particular term sorry which is this particular term and $K_2C_L^2$ which this particular term. So $C_D = C_{D_0} + K_1C_L + K_2C_L^2$ sketch it is not constant quantity you get this particular line and from there you can get the minimum as 0.04

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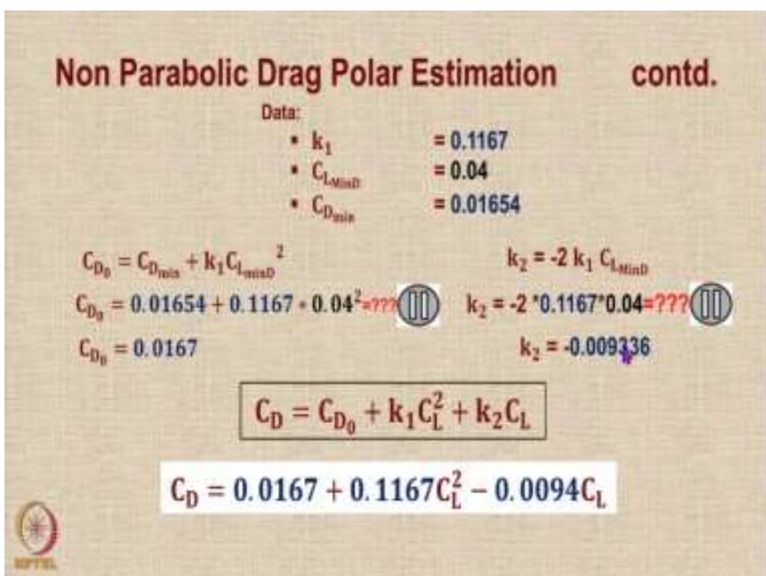

Moving ahead let us look at some data. So K_1 is now known as 0.1167 $C_{L,minD}$ is known as 0.04 C_D for minimum drag is 0. 01654. So

$$C_{D_0} = C_{D_{min}} + k_1 C_{L,min}^2$$

So it is just a simple substitution of the equation there. And C_{D_0} will be called will be known as 0.0167 is C_{D_0} is equal to a function of C_L. So k_2 will become -2 $K_1 C_{L,minD}$ at this point we have calculated a same minimum drag.

And we already know the value of K_2 we should keep in mind that there is a minus sign here by definition of K_2. So K_2 comes -0.009 nearly point 0.1. And this is a formula for C_D as a function of C_{D0} and $K_1$and K_2. So if you put the numbers you can just replace C_{D0} by 0.0167 you can replace K_1 here by 0.1167 into 0.004 that is 0.1167.And you can put the constant term here. So you can notice that the value here is coming more accurate than the value written here that is because we are stopping here at the fourth decimal place.

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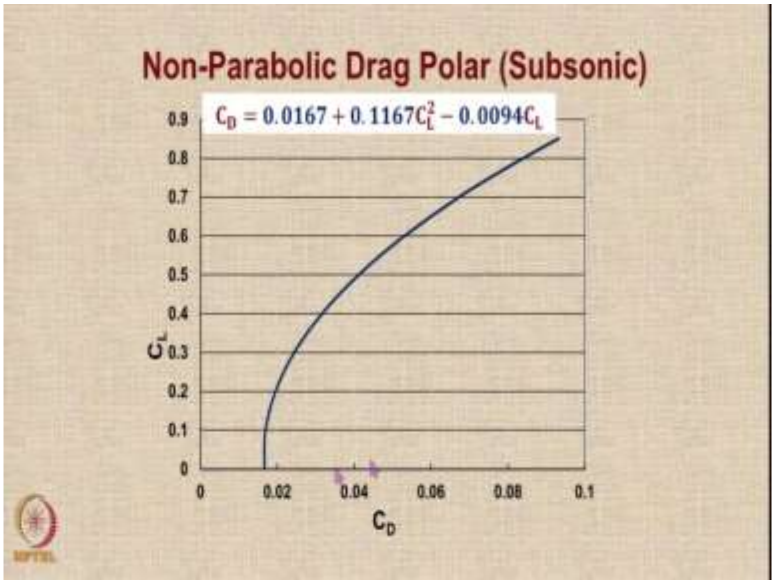

So when you plot this polar this is how you get so not very apparent but it is clear that this polar is not having the minimum at C_L=0. There is a shift in the C_L and this is the formula.

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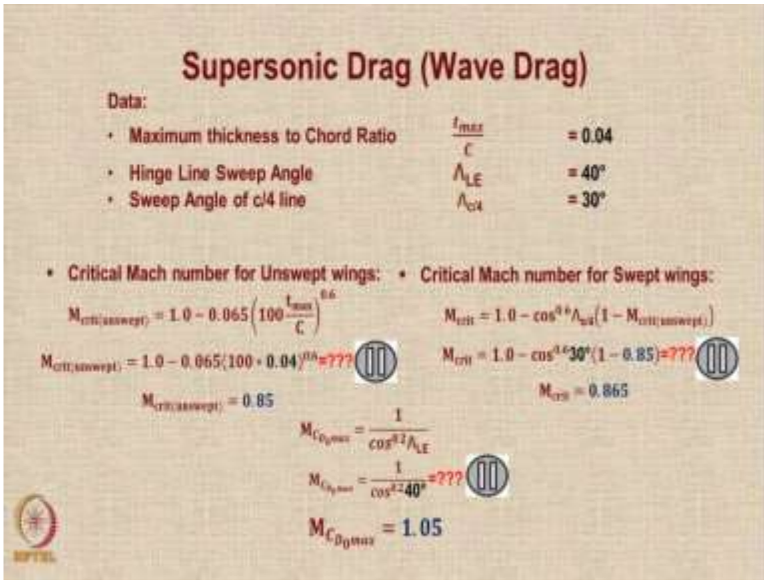

Now let us move on to the last aspect which is the supersonic drag or the wave drag. So for this the maximum t/c is 0.04. Hence we have hinge sweep line is 40 degrees the quarter chord sweep is at 30 degrees. Now we can estimate the critical Mach number for unswept wings using this standard formula. And that number comes out to be so can you please confirm the value what number do you get by taking the claim.

So maximum Mach number for Cruise is occur at some point but that point may not really be of much use to you for your day to day calculation. So the formula is M critical is equal to the same number and just insert the values of the parameters except that here we have voting we have the 1-$M_{critical}$ which is 0.85. So if you calculate this value you should get the value of approximately 1.05 so the maximum mach number will occur the maximum drag will occur at.

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Mach 1.05 then it will fall down again. So for wave drag again we will look at these standard numbers which are available here we have one more new parameter called as a wave dry efficiency parameter. And now we already know the maximum mach number $M_{CD\ 0}$. So in this expression you have to calculate $M_{CD\ maximum}$ taken then subtracted with the cruise mach number and multiply and do this long expression you will get the value of E_{WD}.

So for our case Mach number 1.05 we get the value of $C_{D\ wave}$ as 0. 0261 and then there are some numbers which are calculated.

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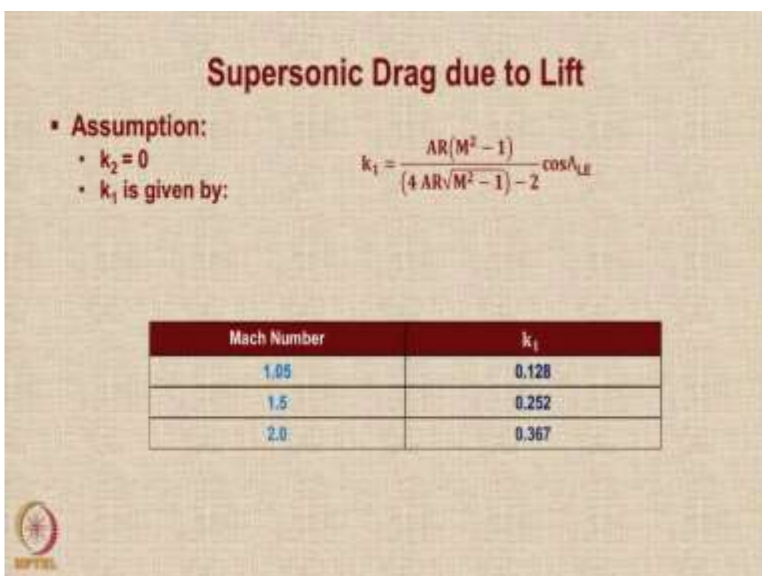

Now let us look at the supersonic drag due to lift our assumption there will be that K_2 and K_1. So with that calculation we are getting the values as follows.

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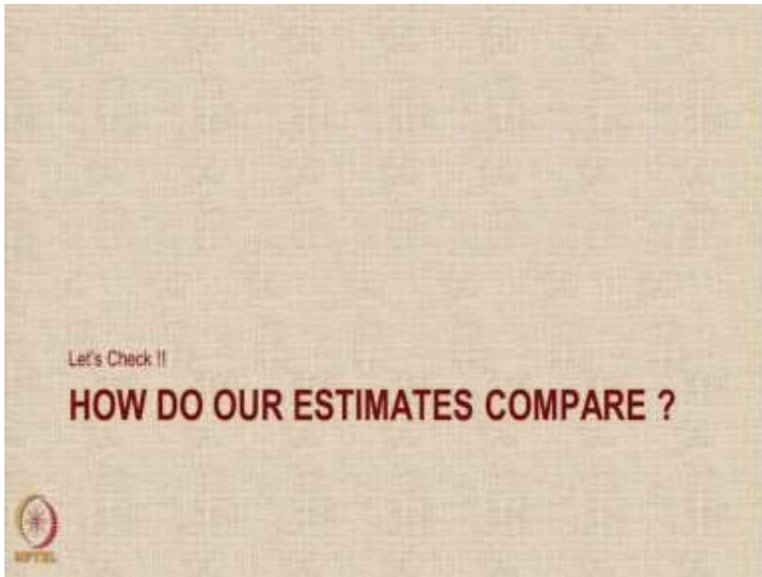

And we need to now know how does our value compared with the actual values. So let us check.

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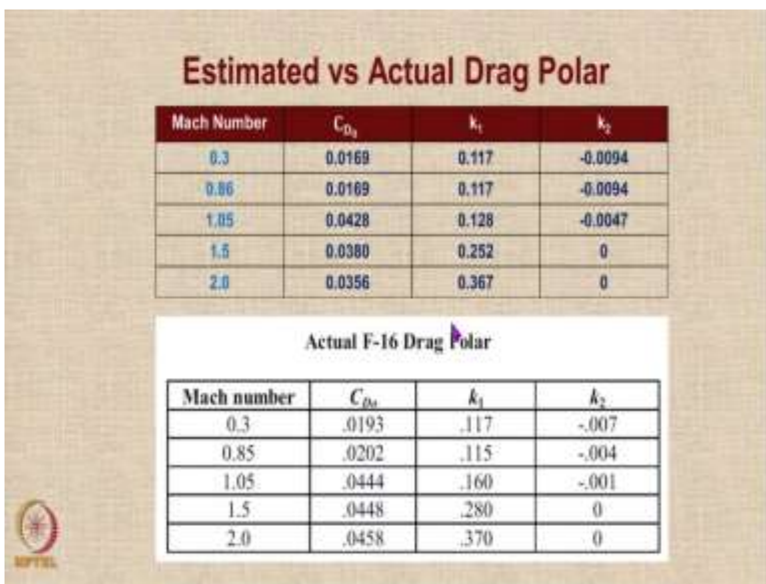

Mach Number	C_{D_0}	k_1	k_2
0.3	0.0169	0.117	-0.0094
0.86	0.0169	0.117	-0.0094
1.05	0.0428	0.128	-0.0047
1.5	0.0380	0.252	0
2.0	0.0356	0.367	0

Actual F-16 Drag Polar

Mach number	C_{D_0}	k_1	k_2
0.3	.0193	.117	-.007
0.85	.0202	.115	-.004
1.05	.0444	.160	-.001
1.5	.0448	.280	0
2.0	.0458	.370	0

So first thing is that here is a table that plots the estimated drag polar for various Mach numbers right from 0.3 to .2.0 with the knowledge that it is maximum at 1.05 and then it comes down and compare this with the data given for the actual aircraft you can see that the numbers are broadly agreeing at least for the values of the initial C_{D0} at the at those conditions.

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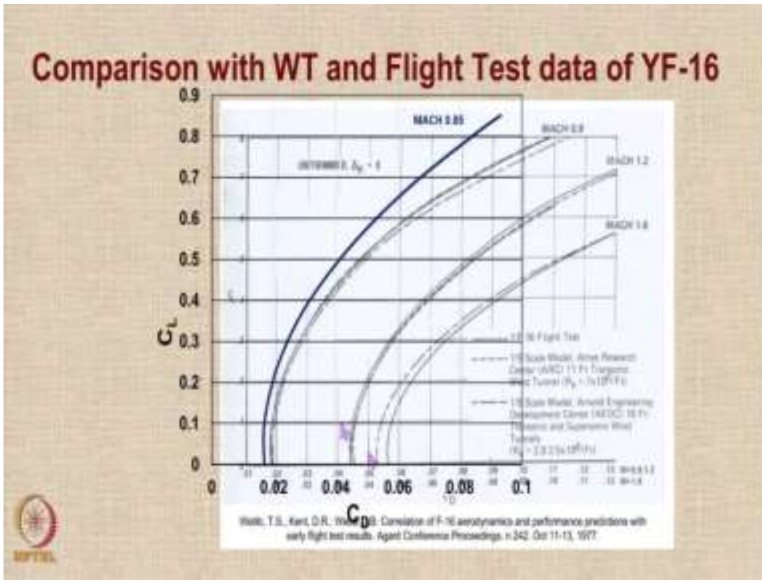

So here is a graph taken from an source which talks about the wind tunnel testing and flight test data of YF-16 which was basically the predecessor of F 16. And on this particular graph if we superimpose our values we see that the graphs are for Mach number 0.9 1.2 and 1.6. And when we calculate our values for Mach number 0.85 we get a curve which is fairly parallel to this aircraft. So this is how we can show that the numbers we have got are also quite reasonable and near to reality. You can notice that the divergence happens more at higher values of C_D or even here we can say also higher values of C_D.

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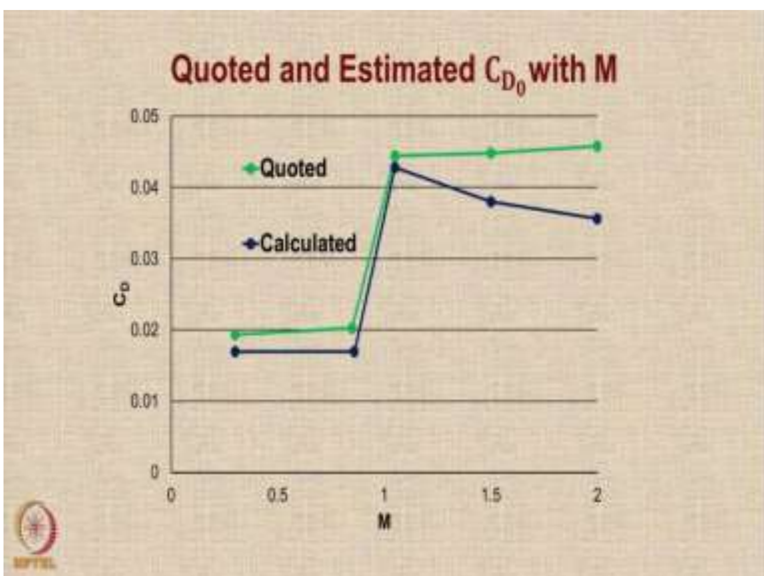

So here is the quoted value of the variation of C_{D0} with Mach number these are the points at which we have calculated the values the graph could actually not be linear like this straight lines but it could be curved but I have just plotted the points which I have calculated. So this is the quoted value. And this is the estimated C_{D0} variation with Mach number. And if you superimpose both of them you see that the values calculated shown in the dark blue line are quite matching with the values which have been available in the open literature.

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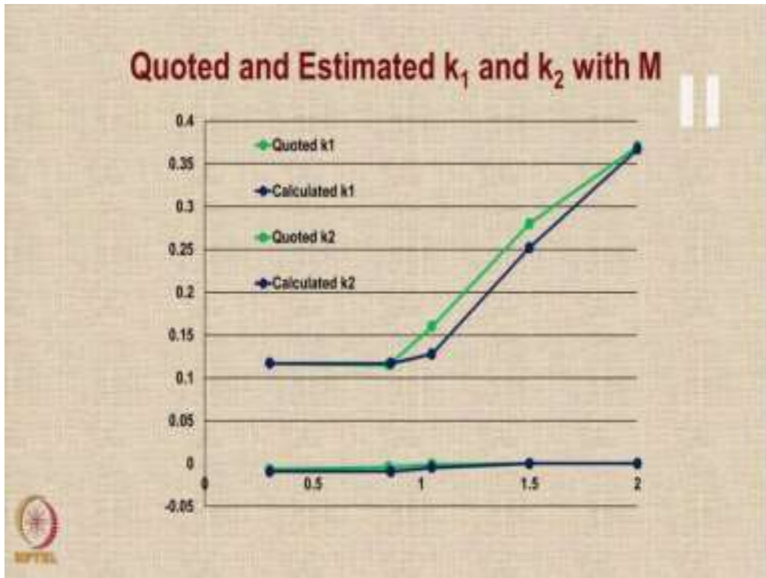

Similarly if you look at the quoted values of K_1 and K_2 there is a Mach number. We notice that K_1 one remains constant up to some mach number I think get to the critical mach number 0.85 and then it starts rising. Whereas the change in the slope of K_2 is far minimum as compared to K_1. So what we did is we also estimated the value of K_1 and K_2 using our methods and plotted them against the same graph and then we superimpose the values. So we realize that we are under predicting we are under predicting obviously our values have to be properly determined.

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So before I close I would like to acknowledge a few people Barndt Stiles Bertin and Whitford for their lovely textbook on introduction to aeronautics design perspective from which I have used a lot of formulae and a lot of data for the existing F-16 aircraft. Daniel Raymer needs to be thanked for his seminal textbook and craft design which we use a lot. And last but not the least I would like to sincerely thank my teaching assistant Nouman Uddin for putting in sleepless nights and creating this tutorial. Thank you.