

Introduction to Aircraft Design
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Lecture - 52
Drag Estimation of Military Aircraft

Hello, let us look at the procedure followed for estimating the drag of a military aircraft. So, when I say drag I basically mean drag coefficient because once you estimate the drag coefficient, then drag is simply a product of half dynamic pressure into the coefficient.

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Military Aircraft

- Military aircraft may fly at:
 - Subsonic speeds
 - Transport or Reconnaissance
 - Supersonic speeds
 - Fighters
- Differences in C_D estimation procedure
 - Wave drag
 - Bluntness of wing / nose
 - Closed / open-nose (due to intake) design

The slide features two images of military aircraft: a transport or reconnaissance aircraft in the top right and a fighter jet in the bottom right. A small IIT Bombay logo is visible in the bottom left corner of the slide.

So, military aircraft can be of various types, they may fly at subsonic speeds, especially aircraft which are used for transport or for reconnaissance or they may be operating at supersonic speeds. So, they cover the entire range of speeds we are not talking about any aircraft which is flying at hypersonic speeds right now. So, therefore, there are some differences in estimation of the drag coefficient for such aircraft as compared to that for the transport aircraft, which mostly fly in subsonic and some in transonic regime.

What are these differences? Let us understand first of all, there is going to be wave drag present when the aircraft flies supersonic. So, we need to include methods to calculate the wave drag coefficient. Secondly, the bluntness of the wing and the nose has a great effect on the drag coefficient of the aircraft. And the intake could be either closed or open nose because of the design. So, that also creates some additional complications in estimation.

Now, the procedure that I am going to describe has been taken from the latest textbooks in aircraft design from the AIAA stable those by Leyland Nicolai and Grant Carichner. Both these gentlemen have a huge amount of experience working on various types of military and transport aircraft and also on unconventional aircraft and airships. So, based on their large experience and database, they have come up with a method which has been described in their textbook. So, a lot of material that you will see in this presentation is also taken from the textbooks from Nicolai and Carichner volume 1.

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Total Drag Coefficient

$$C_D = C_{D_{0wing}} + C_{D_{0body}} + \Delta C_{D_0} + C_{D_L}$$

Zero Lift Drag Lift Induced Drag

The total drag coefficient for a transport can be assumed to be a summation of 4 components,

$$C_D = C_{D_{0wing}} + C_{D_{0body}} + \Delta C_{D_0} + C_{D_L}$$

Let us first look at how we can get the value of $C_{D_{0wing}}$.

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$C_{D_{0wing}}$
**WING ZERO LIFT DRAG COEFFICIENT
(SUBSONIC)**


So, here there will be 3 cases depending on whether you fly subsonic, transonic, or supersonic.

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Wing Zero-Lift Drag (subsonic)

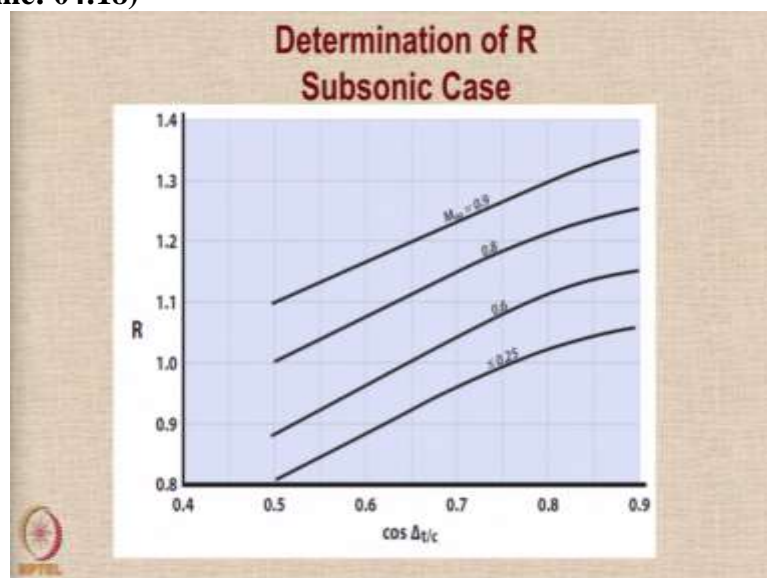
$$C_{D0\text{wing}} = C_f \left[1 + L \left(\frac{t}{c} \right) + 100 \left(\frac{t}{c} \right)^4 \right] R \left(\frac{S_{\text{wet}}}{S_{\text{ref}}} \right)$$

L = airfoil thickness location parameter
 $L = 1.2$ for maximum t/c located at $x \geq 0.3c$
 $L = 2.0$ for maximum t/c located at $x < 0.3c$
 t/c = maximum thickness ratio of the airfoil
 S_{wet} = wetted area of the wing ($2S_c$)
 R = lifting surface correlation factor
 C_f = turbulent flat plate skin friction coefficient



First, let us look at the subsonic condition. In the subsonic condition, the formula is same as that you use for the transport aircraft, which you are very familiar with as shown on the screen. The only 2 small changes here are the value of R the lifting surface correlation factor and C_f the turbulence flat plate skin friction coefficient. So, these 2 parameters, they differ from what we have seen for a transport aircraft. So let us look at how these 2 are determined for a military aircraft.

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So, for a subsonic case, the value of R can be easily obtained based on the \cos of the sweep of the maximum thickness line of the wing and there are these curves for various Mach numbers. So, from the X axis, you proceed up to any value of the Mach number that the aircraft operates and proceed further on the left hand side to get the value of R it is as simple as reading a graph.

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Determination of C_f

Subsonic

$$C_{D_{0\text{wing}}} = C_f \left[1 + L \left(\frac{t}{c} \right) + 100 \left(\frac{t}{c} \right)^4 \right] R \left(\frac{S_{wet}}{S_{ref}} \right)$$


Determination of C_f

Evaluate

Cut-off Reynolds number Re_l (using l/k ratio)

&

Wing Reynolds number $Re_e = \frac{\rho v l}{\mu}$



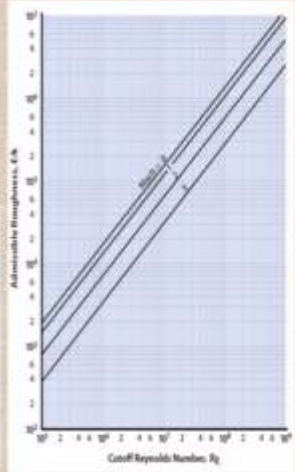

For C_f , you have to follow the same procedure that we follow in transport aircraft that means you have to evaluate the cutoff Reynolds number using the l/k ratio and the wings Reynolds number using $Re = \frac{\rho v l}{\mu}$. So, you calculate both these, you calculate the wing Reynolds number first and then you calculate the cutoff Reynolds number and then you have to choose the one which is smaller.

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Determination of Cutoff Re

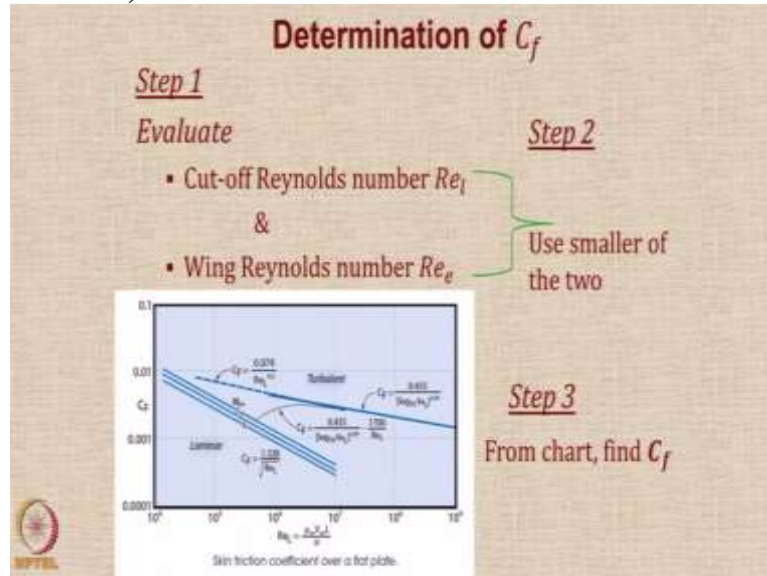
For Wing, $l = \text{chord} = c$
Thus $l/k = c/k$

Roughness Height Values (in Equivalent Sand Roughness)	
Type of Surface	k (in.)
Aerodynamically smooth	0
Polished metal or wood	$0.02 - 0.06 \times 10^{-3}$
Natural sheet metal	0.16×10^{-3}
Smooth matte paint, carefully applied	0.25×10^{-3}
Standard camouflage paint, average	0.40×10^{-3} application
Camouflage paint, mass production	1.20×10^{-3} spray
Dis-polished metal surface	6×10^{-3}
Natural surface of cast iron	10×10^{-3}

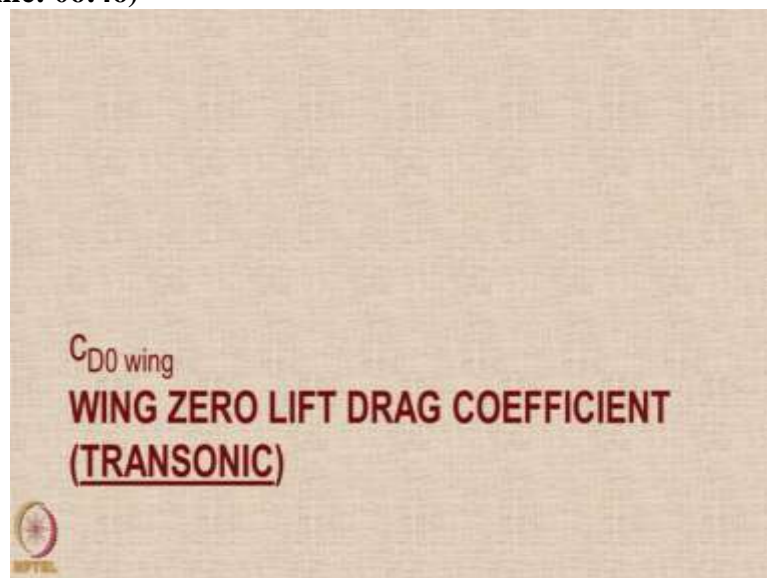
So, for finding the cutoff Reynolds number in case of the wing, the characteristic length would be the chord or the mean aerodynamic chord. So, therefore, l/k would be c/k where c is the mean aerodynamic chord and k is factor that comes from the roughness. So, the table here shows the value of k in inches applicable for various types of surface finishes that you normally encounter on a military aircraft and the graph on the Y axis is basically just a correlation between the cutoff Reynolds number and the admissible roughness l/k . So, for various Mach numbers you can get the values.

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So, for determining C_f there are 2 steps, first step is that you calculate the cutoff Reynolds number Re_l and you calculate the wing Reynolds number Re_e and then you choose whichever is smaller. So, whatever is the smaller value that one you use in this particular graph and there are 2 bunches of lines there is 1 single line for turbulent flow and there are 3 lines for laminar flow. So, depending on the value of the M_∞ , so, you can use the value of M_∞ and calculate the value of C_f .

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So, that was for the calculation of $C_{D0 \text{ wing}}$ for the subsonic case that the procedure was quite similar to what you are used to for transport aircraft, but when it comes to transonic wing.

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Transonic Flow

- Transonic Regime begins at M_{CR}
- Drag Rise starts at M_{DD} i.e., when $\frac{\partial C_{D0}}{\partial M} = 0.1$
- $C_{D_{0wing}} = C_{D_f} + C_{D_w}$
- $C_{D_{0wing}} = C_f \left[1 + L \left(\frac{t}{c} \right) \right] \left(\frac{S_{wet}}{S_{ref}} \right) + C_{D_w}$
- C_{D_f} assumed constant in entire transonic range
= value for $M = 0.6$
- C_{D_w} obtained using Von Karman's similarity law for transonic wings

Then in transonic flow there are a few changes not the transonic flow begins at the critical Mach number and the drag rise starts actually at the M_{DD} drag divergence Mach number and generally it is considered when the rate of change of C_{D_0} exceeds with Mach number exceeds 0.1 that is the point where you can consider to be the drag divergence. So, the

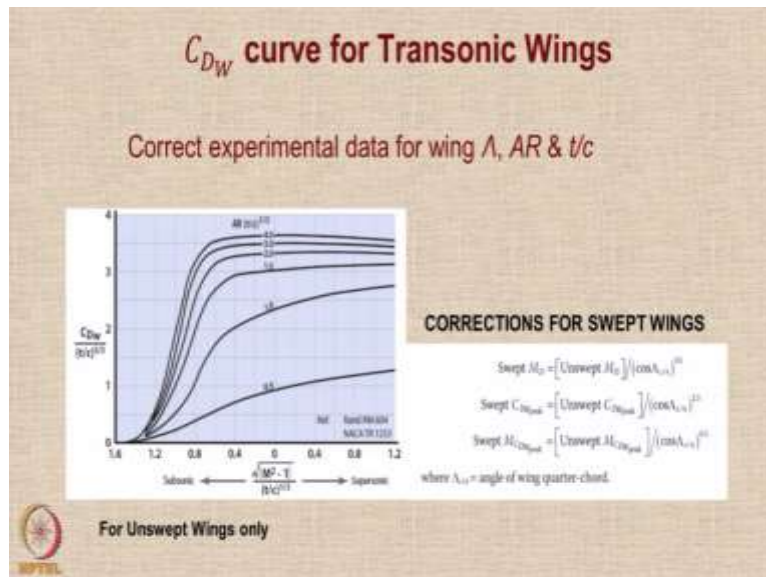
$$C_{D_{0wing}} = C_{D_f} + C_{D_w}$$

and C_{D_f} can be simplified as

$$C_{D_f} = C_f \left[1 + L \left(\frac{t}{c} \right) \right] \left(\frac{S_{wet}}{S_{ref}} \right)$$

Now, this value of C_{D_f} is assumed to be constant in the entire transonic range. So, what you do is you calculate the value for Mach number 0.6 and you assume that that value is applicable in transonic flow. And the C_{D_w} is obtained through Von Karman's similarity law for transonic wings.

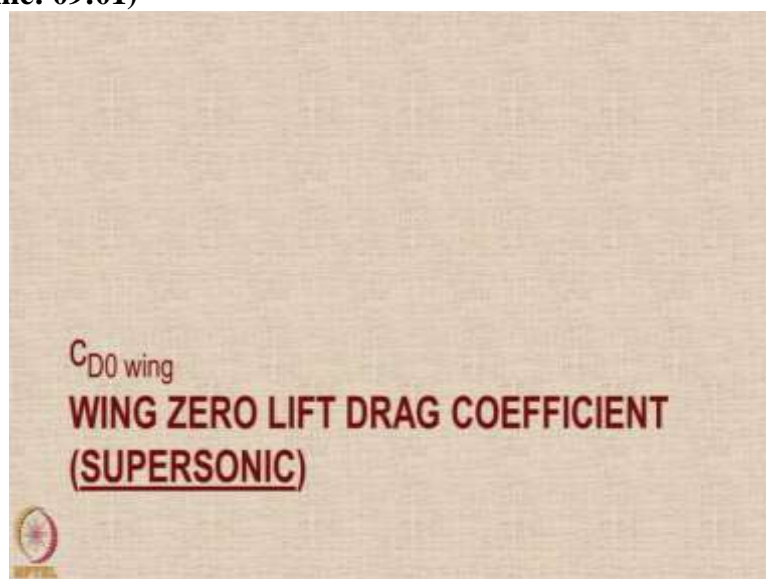
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So, that I will show you so, this is how you calculate C_{D_w} curve for transonic wings, this depends on the usage of the experimental data. So, what you do is you have some experimental data that data has to be corrected for the 3 important parameters the sweep, the aspect ratio and t/c. So, this particular graph is actually applicable only for unswept wings. So, what you do is, you apply the corrections for the 3 parameters which I have mentioned there.

So, the values of M_{DD} , $C_{D_{w_{peak}}}$ and $M_{C_{D_{w_{peak}}}}$ are corrected by using the cos of the quarter chord line cos of the sweep of the quarter chord line. So, these corrections will help you to get the values.

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Now, when it comes to supersonic aircraft, if you want to calculate $C_{D_{0_{wing}}}$ of supersonic aircraft.

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Wing Zero-Lift Drag: Supersonic Flow (C_{Df})

$C_{D_{0wing}} = C_{Df} + C_{D_w}$ (Skin friction + Wave Drag)

where

C_{Df} = Wing supersonic skin friction coefficient
 C_{D_w} = Wing supersonic wave drag coefficient

$C_{Df} = C_f \left(\frac{S_{wet}}{S_{ref}} \right)$ where $C_f = C_{f_i} \times \left(\frac{C_{f_c}}{C_{f_i}} \right)$

$\frac{C_{f_c}}{C_{f_i}}$ = compressibility effect on Turbulent skin friction

$C_{f_i} = \min (Re_l , Re_e)$

Then you need to use this particular procedure. So, the $C_{D_{0wing}}$ will be again the same thing

$$C_{D_{0wing}} = C_{Df} + C_{D_w}$$

So, C_{Df} is the wing supersonic skin friction coefficient and C_{D_w} is the wing supersonic wave drag coefficient. So,

$$C_{Df} = C_f \left(\frac{S_{wet}}{S_{ref}} \right)$$

where C_f is

$$C_f = C_{f_i} \left(\frac{C_{f_c}}{C_{f_i}} \right)$$

This is the compressibility effect on the turbulent skin friction. So, C_{f_i} is calculated based on the minimum values of Reynolds number either the cutoff value or the standard value. So, you can see that the value of $\left(\frac{C_{f_c}}{C_{f_i}} \right)$ this ratio can be obtained for various Mach numbers using this particular line.

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Wing Zero-Lift Drag: Supersonic Flow (C_{Dw}) for sharp nosed airfoils

C_{Dw} = Wing supersonic wave drag coefficient
Obtained from Supersonic Linear Theory

For Sharp nosed airfoils


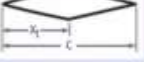
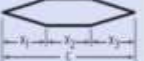
Supersonic leading edge
 $\beta \cot(\Lambda_{LE}) \geq 1$:

$$C_{Dw} = \frac{B}{\beta} \left(\frac{t}{c}\right)^2 \frac{S_e}{S_{ref}}$$

Subsonic leading edge
 $\beta \cot(\Lambda_{LE}) < 1$:

$$C_{Dw} = B \cot^2 \Delta_{LE} \left(\frac{t}{c}\right)^2 \frac{S_e}{S_{ref}}$$

B Factor for Sharp-Nosed Airfoils

Basic Wing Airfoil Section	B	Section
Biconvex	$\frac{16}{3}$	
Double wedge	$\frac{c/\lambda}{1 + \lambda/c}$	
Hexagonal	$\frac{c(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2}$	

Now, for the wings zero lift drag if you look at supersonic flow depending on the shape of the airfoil whether it is sharp nose or blunt, there are different procedures available. So, if you have a sharp nosed aerofoil then you use supersonic linear theory. So, for sharp nosed airfoils basically there is a supersonic leading edge so, beta into cot of the sweep is going to be more than equal to 1. So, from there you can get the value of C_{Dw} and C_{Dw} uses this particular value of capital B the B factor for sharp nose airfoils.

So, this B factor depends on whether the airfoil is biconvex or double wedge or hexagonal depending on the airfoil shape the value of B changes. So, use this particular table to calculate the value of B and after that beta t/c S_e and S_{ref} are already known to you. But, if you have a subsonic leading edge, then the value of $\beta \cot \Lambda_{LE} < 1$. So, here the formula changes and you have you have the value of cot of the sweep of leading edge also coming into picture

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Wing Zero-Lift Drag: Supersonic Flow (C_{Dw}) for blunt nosed airfoils

For Blunt nosed airfoils

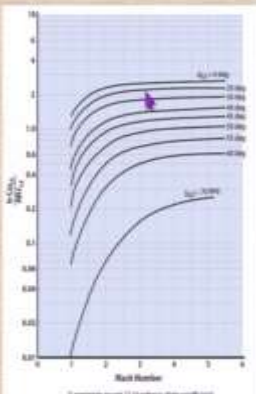
Supersonic leading edge
 $\beta \cot(\Lambda_{LE}) \geq 1$:

$$C_{Dw} = C_{DLE} + \frac{16}{3\beta} \left(\frac{t}{c}\right)^2 \frac{S_e}{S_{ref}}$$

Subsonic leading edge
 $\beta \cot(\Lambda_{LE}) < 1$:

$$C_{Dw} = C_{DLE} + \frac{16}{3} \cot^2 \Delta_{LE} \left(\frac{t}{c}\right)^2 \frac{S_e}{S_{ref}}$$

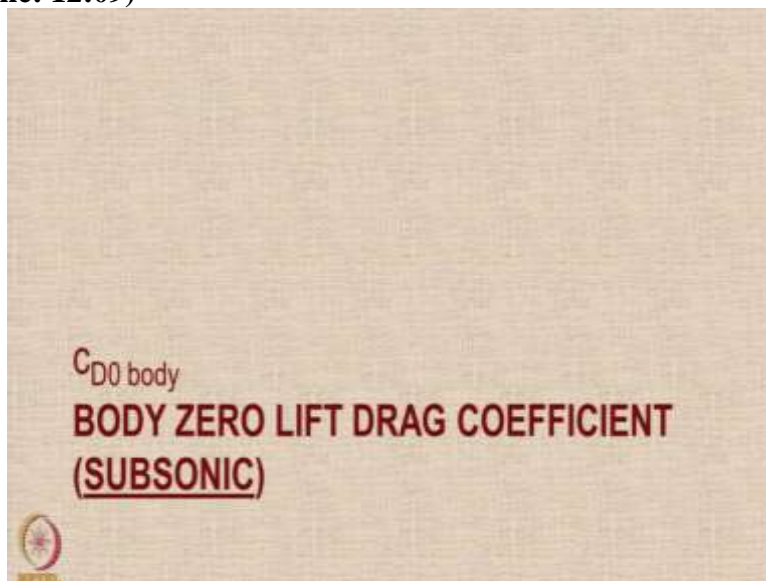
C_{DLE} from relation with M
 r_{LE} = radius of leading edge
 b = wingpan



If you have blunt nosed airfoils, then for supersonic leading edge again the condition remains the same. So, you can get this used this formula can be used to calculate and if it is a subsonic leading edge then the formula changes and the $C_{D_{LE}}$ can be obtained here because it there is a graph that correlates the Mach number with $b C_{D_{LE}}/AR_{LE}$. Now B is the wingspan AR is aspect ratio R is the radius.

So, you know these parameters for an aircraft. So, for various values of the delta leading edge you can use and get the corresponding value and from there you can get the value of C_{D_w} . So, that much is for the $C_{D_{0wing}}$. Now, let us move on to get the $C_{D_{0body}}$ a body coefficient.

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Again there will be 3 cases subsonic transonic and supersonic.

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Body Zero-Lift Drag - Subsonic

$C_{D_{0body}} = C_{D_{fb}} + C_{D_b}$, where

$$C_{D_{fb}} = C_f \left[1 + \left(\frac{60}{(l_B/d)^3} + 0.0025 (l_B/d) \right) \left(\frac{S_S}{S_B} \right) \right]$$

Closed Body

Body Having a Blunt Base

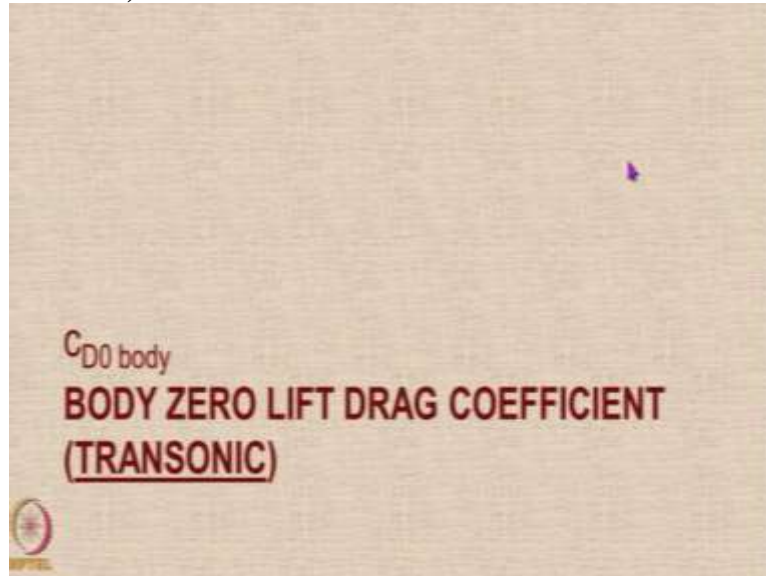
Forebody

Body fineness ratio (body length / equiv. dia.)	l_B/d
Use Equivalent diameter for non-circular bodies	$d = \sqrt{S_s/0.7854}$
Max cross sectional area of body (from geometry)	S_B
Wetted area of body surface (from geometry)	S_S
Base drag term (avoid blunt-base bodies if possible)	$C_{D_b} = 0.029 (d_{base}/d)^2 / \sqrt{C_{D_{fb}}}$

A small circular logo is visible in the bottom left corner of the slide.

In the subsonic case, again the formula is very similar to what you are already used to for transport aircraft. The only difference is that the value of l_B/d you know depending on what type of body is used we have to use various formulae to get the correct expressions in this equation.

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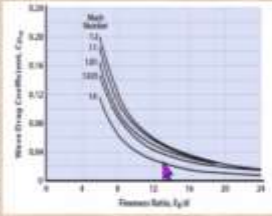


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Body Zero-Lift Drag - Transonic

$$(C_{D_0})_b = C_{D_f} + C_{D_P} + C_{D_b} + C_{D_w}$$

C_{D_f} = Skin Friction drag coefficient = $C_f S_s/S_b$
 C_f = Turbulent skin friction drag coefficient for $M = 0.6$



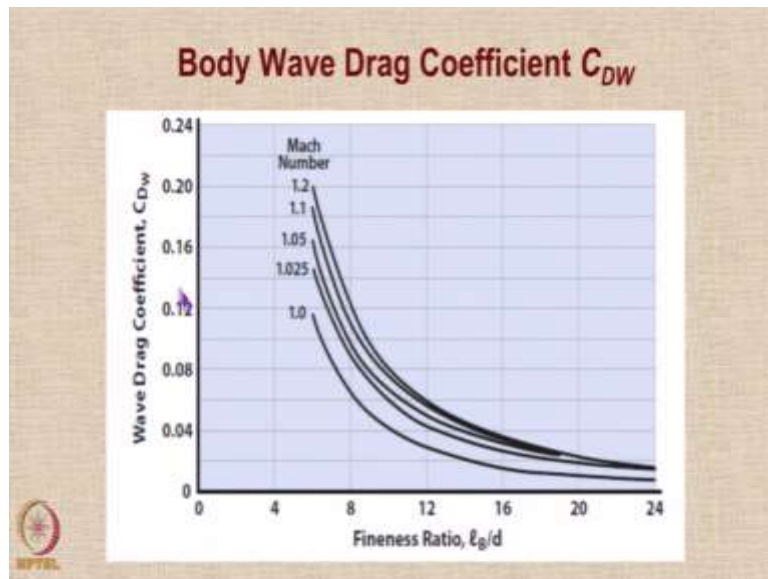
$$C_{D_P} = C_f (M=0.6) \left[\left(\frac{60}{(l_B/d)^3} \right) + 0.0025 (l_B/d) \right] \left(\frac{S_s}{S_B} \right)$$

$$C_{D_b} = -C_{pb} \left[\frac{d_p}{d} \right]^2$$

The graph shows Wave Drag Coefficient C_{D_w} on the y-axis (ranging from 0.04 to 0.28) versus Thickness Ratio t_B/d on the x-axis (ranging from 0 to 24). Several curves are plotted for different Mach numbers: 1.2, 1.1, 1.0, 0.9, and 0.8. The curves show that wave drag increases significantly as the thickness ratio increases and as the Mach number increases.

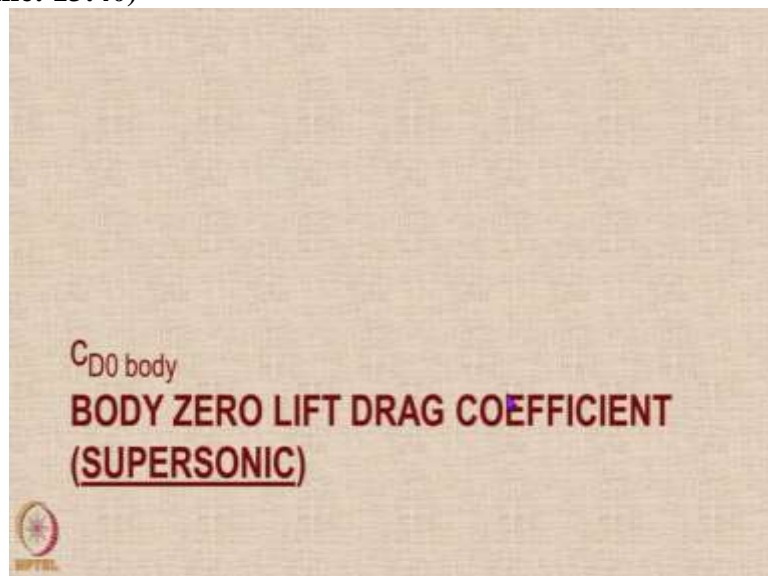
For transonic flow the $C_{D_0\text{ body}}$ would be a sum of 4 components C_{D_f} that is the skin friction drag coefficient. So, with the fineness ratio you can get for various Mach numbers the value of the wave drag coefficient C_{D_w} . So, with this once you get all these 4 parameters, you can get the value of C_{D_0} .

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So, what you need is you need the wave drag coefficient as a function of the fineness ratio. So, the same graph that you saw here for improving the clarity it has been redrawn here in a larger frame, so, that you can easily use it to get the values of C_{D_w} for various Mach numbers given the fineness ratio.

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Drag Build-up : Body Zero-Lift Drag - Supersonic

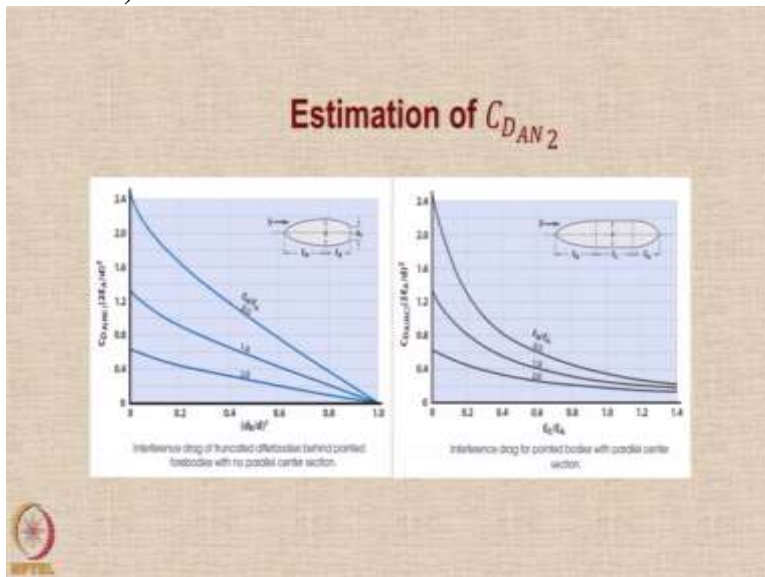
$$C_{D_{0b}} = C_f \left(\frac{S_S}{S_B} \right) + C_{D_{N2}} + C_{D_A} + C_{D_{AN2}} + C_{D_b}$$

- C_f = compressible turbulent skin friction determined in the same fashion as the supersonic wing skin friction
- $C_{D_{N2}}$ = interference drag coefficient acting on the afterbody due to the center body (cylindrical section) and the nose, obtained from Figs.
- $C_{D_{N2}}$ = nose wave drag obtained from Figs.
where f_N is nose fineness ratio l_n/d (see Fig)
- C_{D_A} = body afterbody wave drag obtained from Figs.
where f_b is the afterbody fineness ratio l_a/d (see Fig)
- C_{D_b} = base drag term



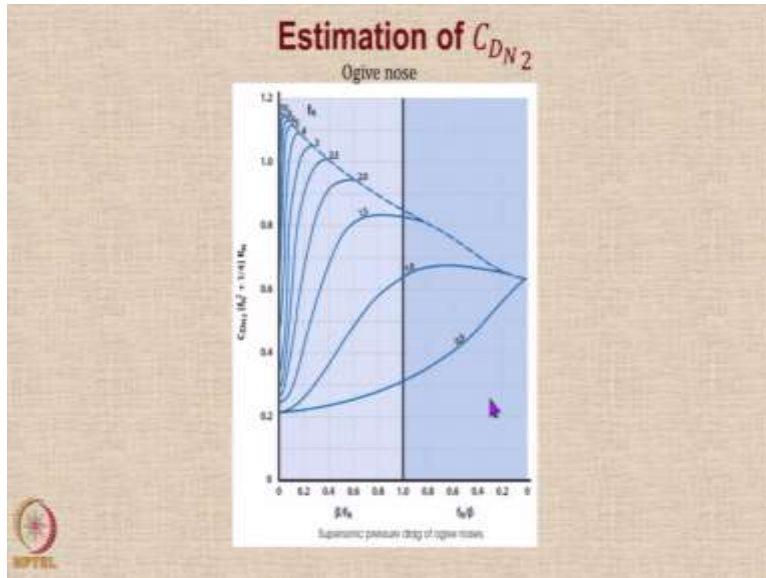
Moving on to supersonic flow for a body so, here again the $C_{D_{0body}}$ would be a sum of these 5 components and each of these components is explained here. And now, we will see the formula for each of them.

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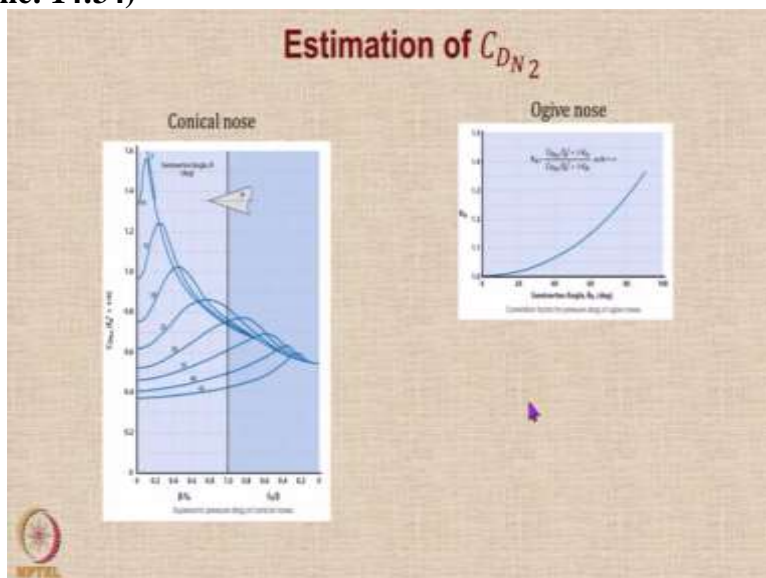
So, first is $C_{D_{AN2}}$, $C_{D_{AN2}}$ is this term body after body wave drag. For that you know you can get the interference drag based on whether the shape has got a blunt body at the end or it has got a closed body at the end depending on that you can use the corresponding graph.

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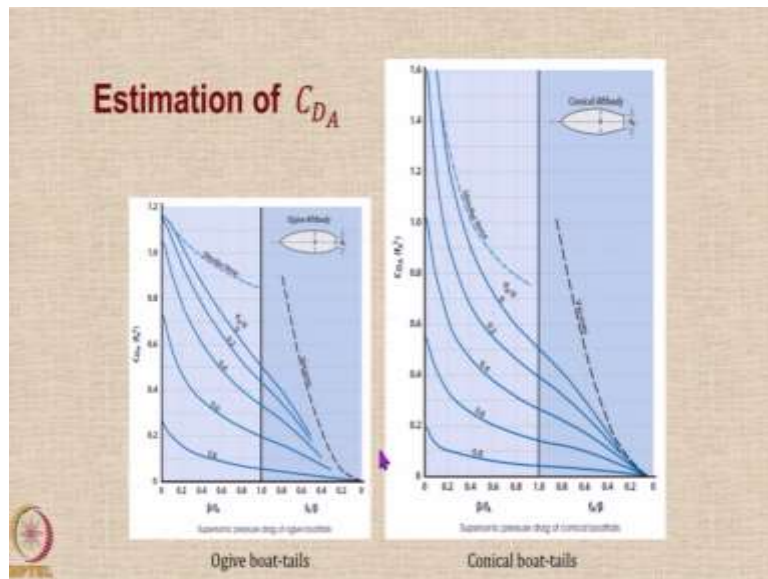
Similarly, if you have a Ogive nose for Ogive nose, you can use these parameters to calculate the value.

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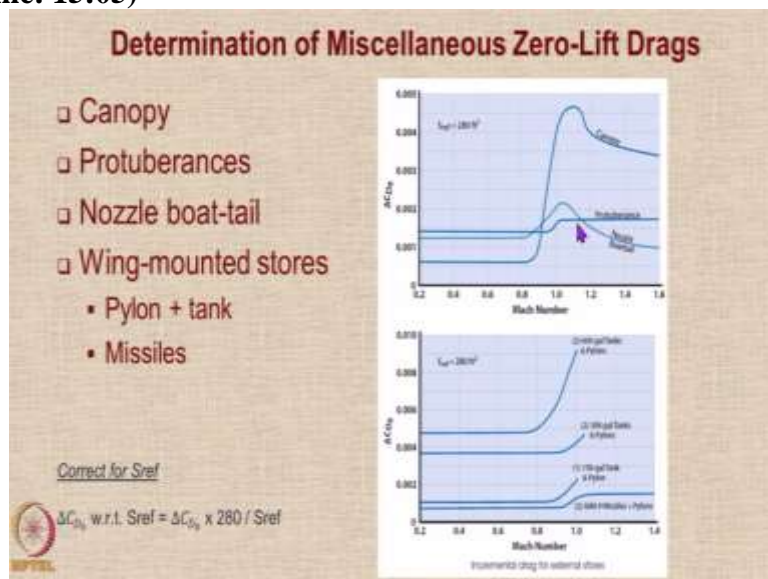
And if you have conical nose, then the graph is this 1 and if you have an ogive knows the graph is this 1.

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Finally, you have C_{DA} , which again depends upon the shape so either you have a conical after body or you have an ogive after body depending on that, depending on the boat tail shapes you can get. You can choose the correct graph and read the value on the Y axis after given the input from the X axis. So now we come to delta C_{D_0} .

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So, delta C_{D_0} is due to various miscellaneous components like canopy, protuberances, nozzle, boat tail, wing mounted doors, pylon tank etc. So, there are some recommendations given as the, what will be the drag area for various types. So, incremental drag for external stores and this is the again the incremental drag for canopy, protuberances and nozzle boat tails.


So, when you have stores, you can use this graph when you have canopy protuberances or novel boat tail you can use the one on the top and you have to correct for S_{ref} remember. Finally, we come to C_{DL} .

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Determination of Induced Drag - Subsonic

$$C_{D_L} = \frac{1}{\pi e AR} C_L^2$$

where span efficiency factor e is given by

$$e = \frac{2}{2 - AR \sqrt{4 + AR^2}}$$


C_{D_L} now there this is a very simple formula for calculation of the lift dependent drag, but you can get the more accurate formula for span efficiency factor where this expression can be more detailed.


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Determination of Lift Induced Drag - Supersonic

Step 1
Estimation of wing C_{l_α} - Supersonic

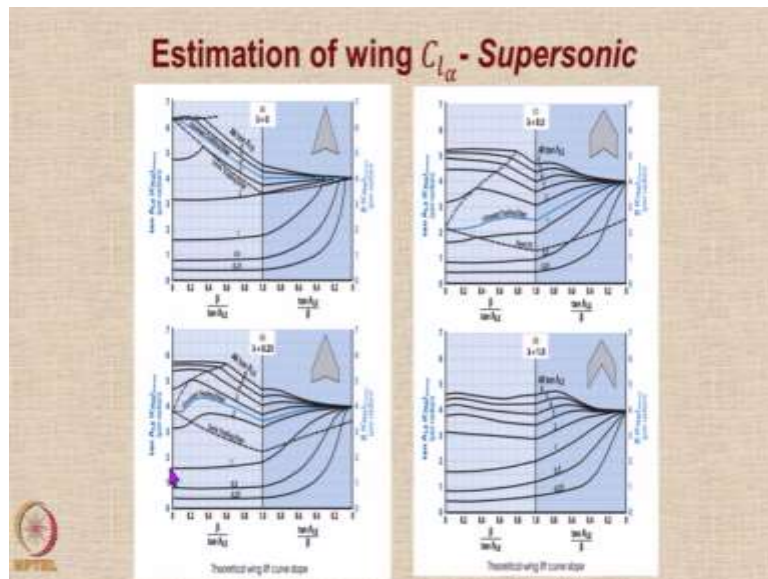
Choose C_{N_α} according to taper ratio λ (tables in next slide)

Assume $C_N = C_L$ for small to moderate angles of attack



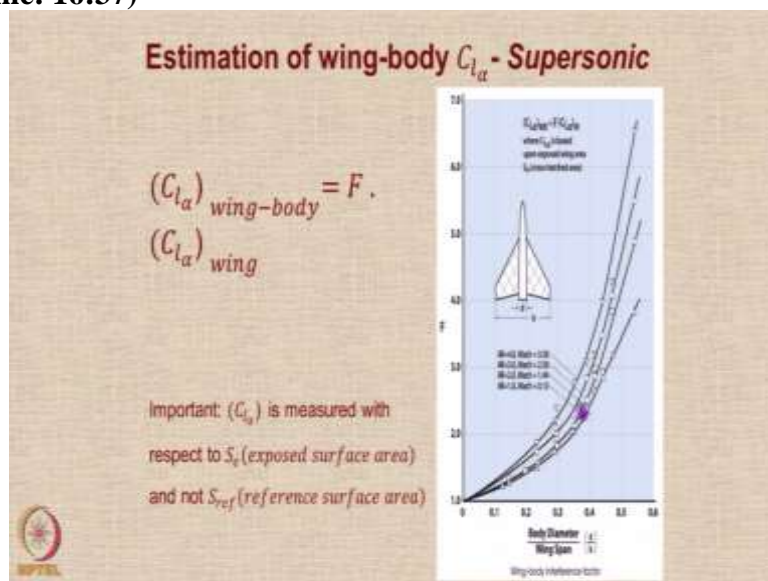
So, what is the procedure first is you calculate the wing C_{l_α} supersonic. So, choose C_{N_α} according to the taper ratio and there are tables shown in the next slide. And for small angles, you can assume that the normal force coefficient is equal to the lift force coefficient.

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So, you can see a $\lambda = 0.5, 0.25$ and 1 for various values of λ you have these graphs available. So, you have to interpolate between them depending on what kind of trailing edge you have and what kind of configuration do you have.

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Similarly, if you want to calculate the C_{l_α} for supersonic for a wing body so, you can use this particular graph now, here what you need is you need a factor F . So, C_{l_α} wing body is equal to F times C_{l_α} wing. So, C_{l_α} wing is known to us but C_{l_α} wing body for that you need F and F will come from this graph depending on the various d/b values and for various aspect ratios and Mach numbers there is some experimental data available through which we can get the value by interpreting the database.

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Determination of Lift Induced Drag - Supersonic

<p>□ Supersonic leading edge</p> <p>Lift induced drag coefficient</p> $C_{D_i} = k \cdot C_L^2$ <p>where</p> $k = \frac{1}{(C_{l_\alpha})_{wing-body}}$ <p>Note: $(C_{l_\alpha})_{wing-body}$ is with respect to S_{ref}</p>	<p>• Subsonic leading edge</p> <p>Lift induced drag coefficient</p> $C_{D_i} = k \cdot C_L^2$ <p>where</p> $k = \frac{1}{(C_{l_\alpha})_{wing-body}} - \Delta N$ <p>where $\Delta N = \Delta N_{M=1.0} \times \left(\frac{\Delta N}{\Delta N_{M=1.0}}\right)$</p> <p>where $\Delta N_{M=1.0} = \frac{1}{\gamma(\gamma+1)} - (k' + k'')$</p>
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Now, let us look at the determination of lift induced drag in supersonic condition depends on whether you are leading edge is supersonic or subsonic. So, the induced drag coefficient actually remains the same formula remains the same kC_L^2 however, the value of k will change in the case of a supersonic leading edge you have k is going to be

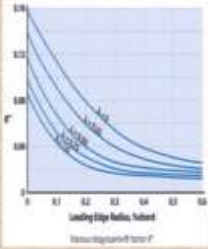
$$k = \frac{1}{C_{l_{\alpha_{wing-body}}}}$$

which we already have, but this is referred to S_{ref} .

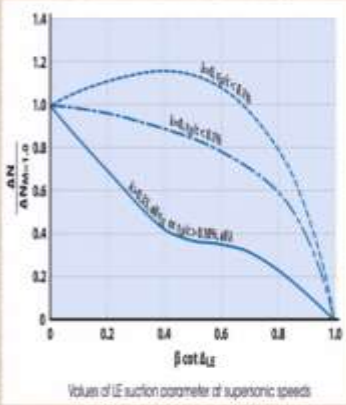
But in case of subsonic leading edge, there is this additional term minus ΔN here this can be obtained as shown in the calculations.

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Determination of k' and k'' and leading edge suction parameter $\frac{\Delta N}{\Delta N_{M=1.0}}$

$$k' = \frac{1}{\pi e AR}$$


Graph showing k' vs Leading Edge Radius/Chord. The y-axis ranges from 0.8 to 1.0, and the x-axis ranges from 0 to 0.6. Several curves are shown, all decreasing as the x-axis value increases.

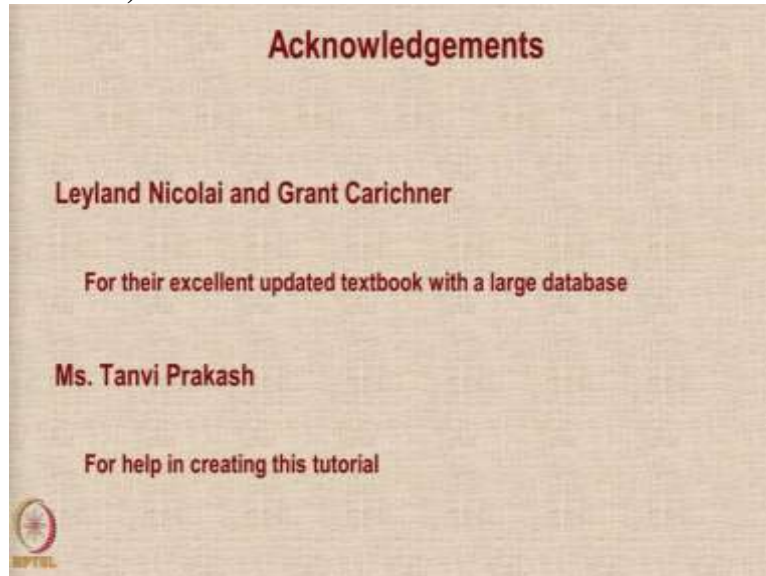


Graph showing suction parameter vs $\beta \cot \delta_{LE}$. The y-axis is $\frac{\Delta N}{\Delta N_{M=1.0}}$ ranging from 0 to 1.4. The x-axis is $\beta \cot \delta_{LE}$ ranging from 0 to 1.0. Three curves are shown: 'Indep. k'' ', 'Indep. k' ', and 'Dep. k' ($M > 1.0$)'. The 'Indep. k'' ' curve peaks at approximately 1.2. The 'Indep. k' ' curve starts at 1.0 and decreases. The 'Dep. k' ' curve starts at 1.0 and decreases more sharply.

So, for this calculation you need value of k' and k'' . So, these 2 parameters are obtained here.

So, they can be obtained here, k'' is here and $k' = \frac{1}{\pi e AR}$

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I would like to acknowledge the contribution by Leyland Nicolai and Grant Carichner in giving us the detailed formulae from their experience and their knowledge and a very large database. So, this was just a very brief overview of the procedure for doing an example and for understanding I recommend students to go and read this textbook in detail in the appendices they have given a full procedure for calculation.

I also want to acknowledge the contribution of Ms. Tanvi Prakash, my PhD student for help in creating this tutorial. She has meticulously gone through the book by Nicolai and Carichner extracted the various formulae and the procedures and put it together for your convenience. Thank you very much.