Introduction to Aircraft Design
Prof. Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology Bombay

Lecture - 48
Tutorial on Initial Sizing of Transport Aircraft

Hello, let us have a look at how we can do the initial sizing of a civil transport aircraft. And we will take Boeing 787 - 8 Dreamliner as our test case.

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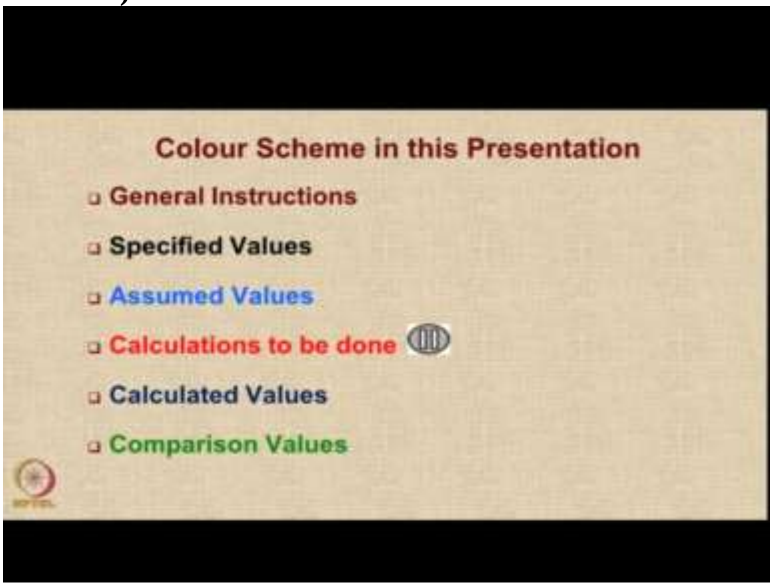

In this presentation, we will follow a color scheme, please understand this scheme so that you know what the various colors mean. General instructions like this will be in brown color, if there are any specified values from the database of Boeing 787, then they will be shown in black color. The blue color will be the assumed values. The calculations to be carried out will be marked with red color, so that the moment you see a red color, it means some calculations need to be done.

And this pause button it will be aid to your memories, so that you can stop the video when you see this button, do the calculations and then resume. And the values which are calculated will be shown in dark blue color. Towards the end we will compare our data with some source and those comparison values will be shown in green color.

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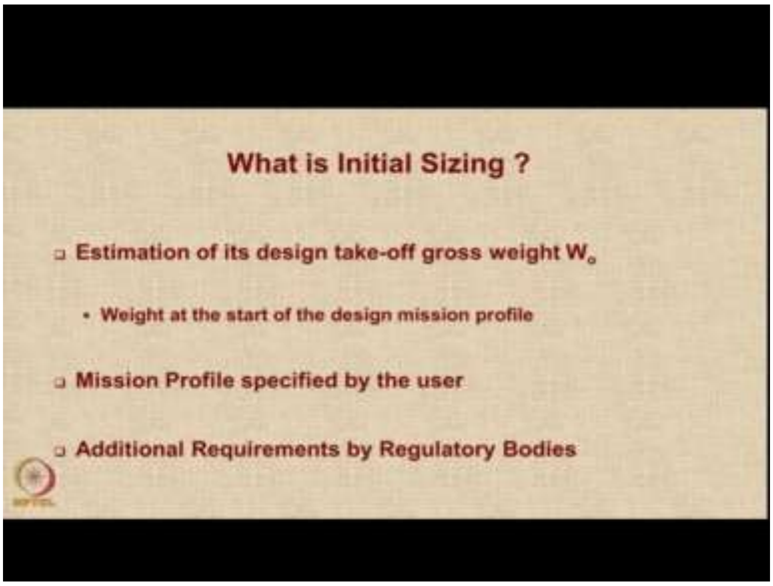

Let us understand what is meant by initial sizing. At this point I hope you have already seen the video regarding initial sizing because this tutorial can only be done if you have already seen the video and if you understand what is initial sizing. So if you have not done I would suggest you stop here go back and watch those clips. If not you can go ahead. So as we know initial sizing is the estimation of aircrafts design gross weight W_0 before it starts the design mission. The mission profile is specified by the user. But there are certain additional requirements which are given by the regulatory bodies.

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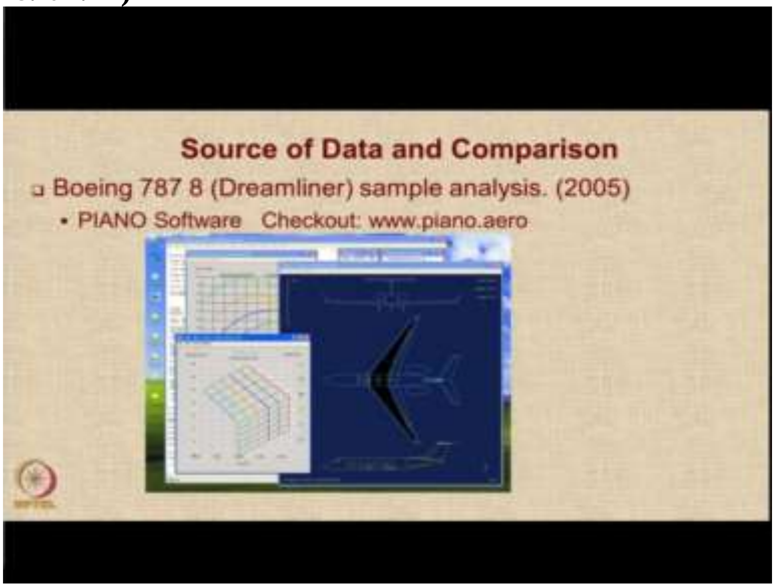

Let us look at the source of the data and comparison for this particular tutorial. We have used the output provided by the Dimitri Simos of the piano software, so check out www.piano.aero , to where the code is described. And we are going to use Boeing Dreamliner sample analysis carried out in 2005 as our baseline.

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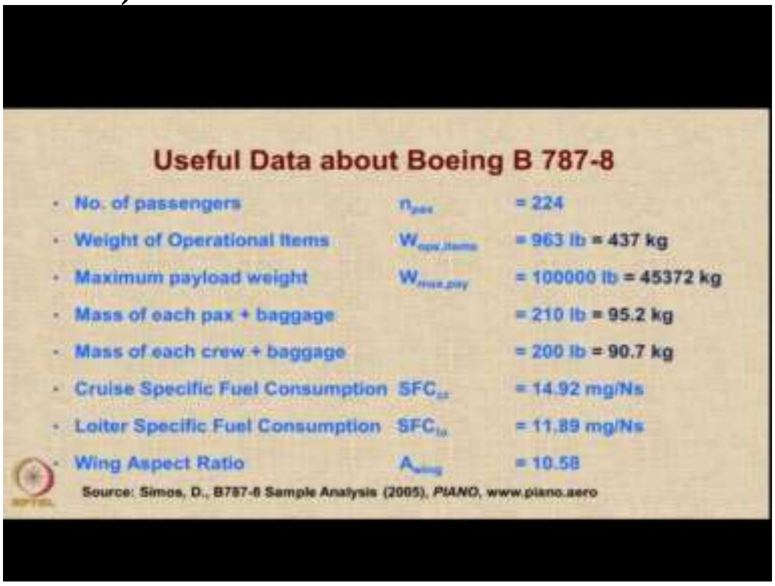

Let us look at some useful data about Boeing 787 - 8 which we will need to do this tutorial and the source of this data is also from the output on piano by Simos. We are going to look at a baseline case with 224 passengers. And the operational items which are going to be carried by the crew members to serve these passengers would be 963 pounds or 437 kg. So we will add this in the crew weight.

The maximum payload weight of the aircraft is 100000 pounds or around 45.372 tonnes. It is assumed that the mass of the baggage and the passengers would be 210 pounds and for the crew members it would be 200 pounds each. Now from the source given by the Dimitri Simos, we know that the cruise specific fuel consumption of this aircraft in loiter and cruise phases are specified as mentioned.

We have mentioned here the units in the SI system, the data is actually provided in terms of pounds per hour divided by pound force, but we have converted that number into milligrams per Newton seconds. And of course, the wing aspect ratio is 10.58.

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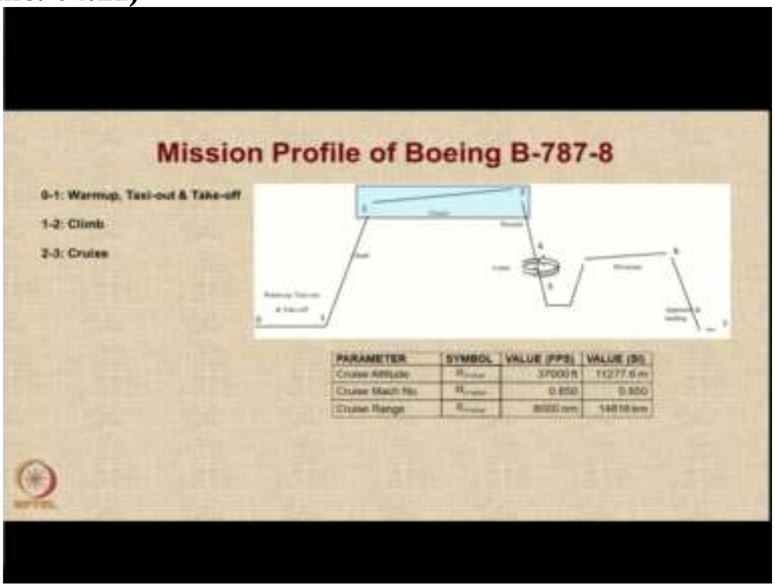

Let us look at the mission profile of Boeing 787. The mission profile that we will consider will be a simple one which will involve several segments. So the first segment will be the warm up taxi out and take off. The next would be the climb segment. After that we have a cruise segment and it is mentioned that the cruise altitude is 37000 feet, Mach number is 0.850 and the total distance to be covered in cruise is around 8000 nautical miles.

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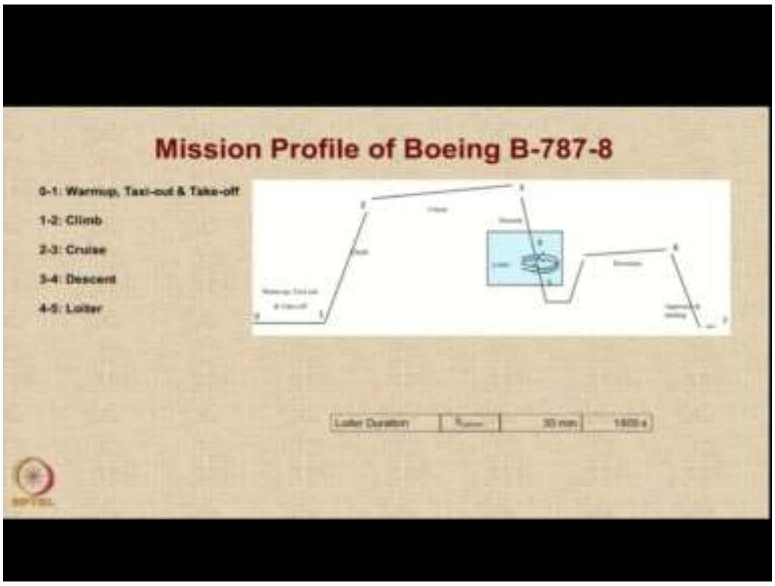

And then we have the descent segment we descend down to the loiter altitude and then there is a loiter for about 30 minutes. After which you have a missed approach.

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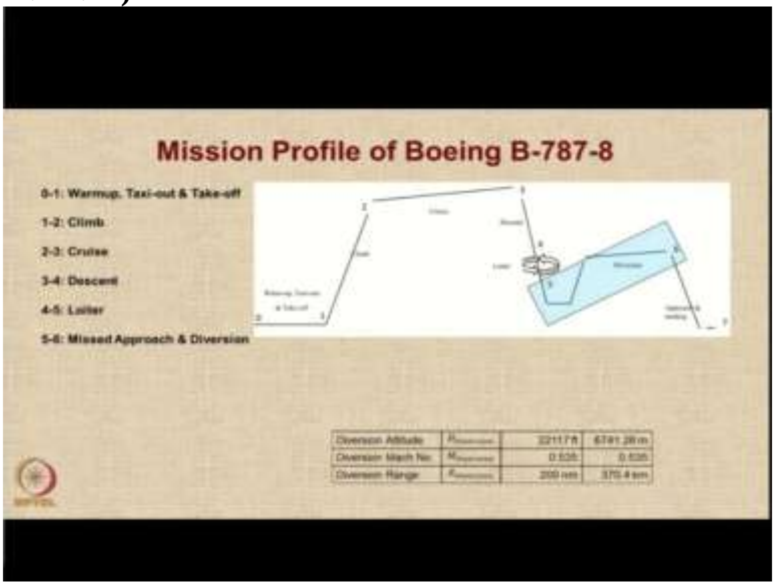

And then you divert the aircraft and travel a distance of 200 nautical miles at a Mach number of 0.535, while flying at an altitude of 22117 feet, these numbers have come from the source, which I have already mentioned to you.

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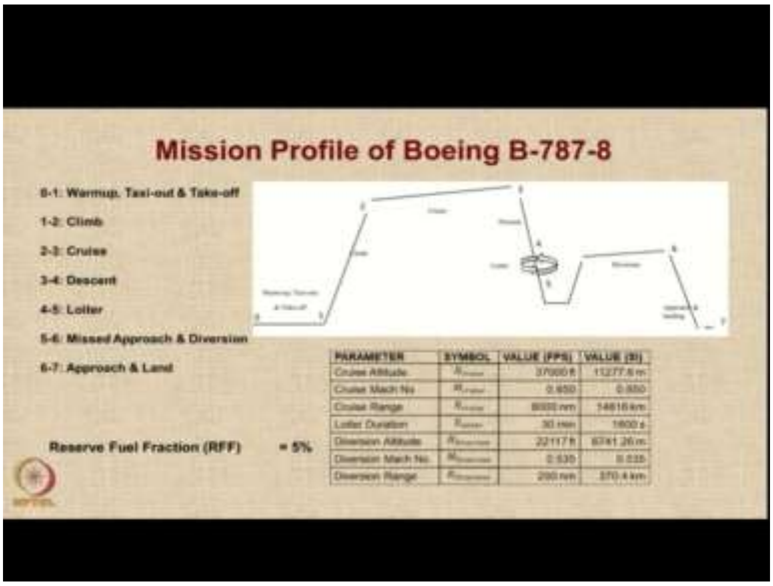

And finally, the last segment is the approach and land segment. So this table shows you the data to be used. And because it is in black color, it is understood that this information is specified by the user as a mission profile. One more point you need to know which is the reserve fuel fraction, how much is the additional fuel to be carried beyond the mission fuel that is 5%.

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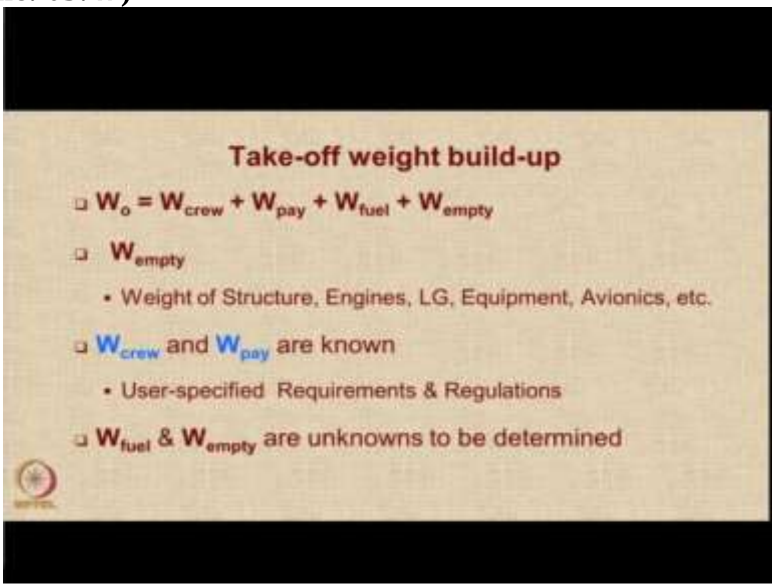

So, the first step initial sizing is the takeoff weight buildup. So the takeoff weight can be considered to be a summation of 4 items, the crew weight, which includes the items, the operational items, the payload, the fuel and the empty weight. The empty weight would be the weight of the structure, the engines, the landing gear, equipment, avionics etc. Now in this particular situation, W_{crew} is available from operational requirements as well as some regulatory requirements.

And $W_{payload}$ is a specified number. So therefore, if we can somehow estimate W_{fuel} and W_{empty}, then we can get a first estimate for W_0, which is the initial sizing. So the problem now is to determine the values of W_{crew} and W_{fuel} and W_{empty}. So these are the 2 unknowns which are to be determined.

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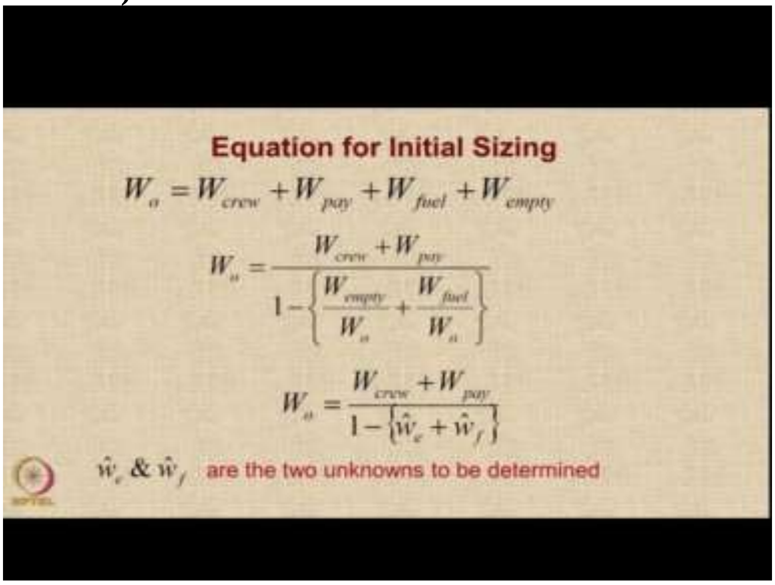

The equations for initial sizing are very straightforward. As I mentioned, you can replace W_0 by a summation of 4 terms. But since 2 of these terms are specified by the requirements of the mission, we keep them in the numerator, and we take the remaining 2 terms in the denominator, and then we divide by W_0 on both sides. So we get 2 ratios, the empty weight ratio and the fuel weight ratio. So now, since the numerator is known, therefore to calculate W_0, we only need to calculate $\overline{w}_e$ and $\overline{w}_f$, which are the remaining 2 unknowns.

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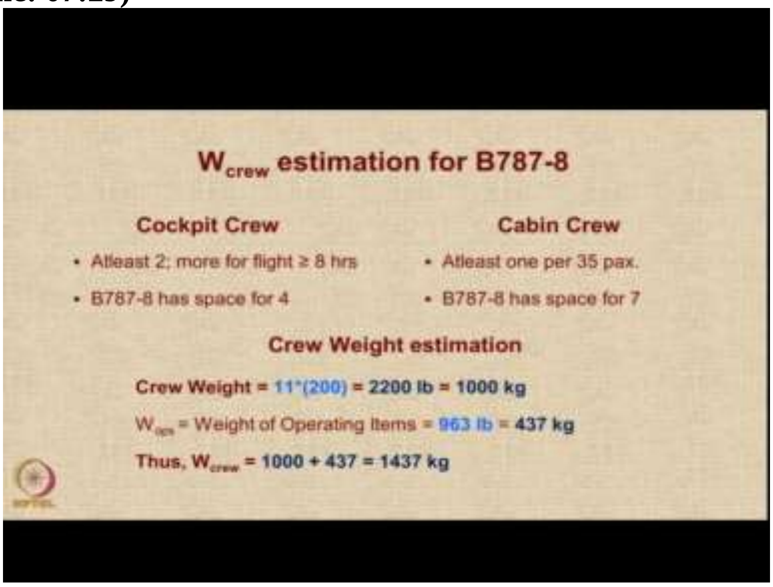

Let us see how the crew weight for Boeing 787-8 is estimated. There are 2 kinds of crew there is a cockpit crew inside the cockpit and there is a cabin crew inside the cabin. As per regulatory requirements that have to be minimum 2 pilots in the cockpit. But for durations more than 8 hours, we need at least 2 more. In our case, the flight is around 8000 nautical miles.

So surely it will travel for more than 8 hours. So we will assume that we will occupy the entire space of 4 crew members. So there will be 4 crew members in the cockpit. In the cabin, we require 1 crew member for every 35 passengers to ensure their safety during emergencies and also their comfort and to provide them services. In case of Boeing 787 there is a provision for 7 crew members. So we will assume that all 7 are presence.

Which means, there are totally 11 crew members, each of whom is 200 pounds for their weight and for their baggage. So therefore the crew weight turns out to be 1000 kilograms. But remember, we have to also carry operational items, and this has been specified as 963 pounds, so therefore the total crew weight would be 1437 kilograms.

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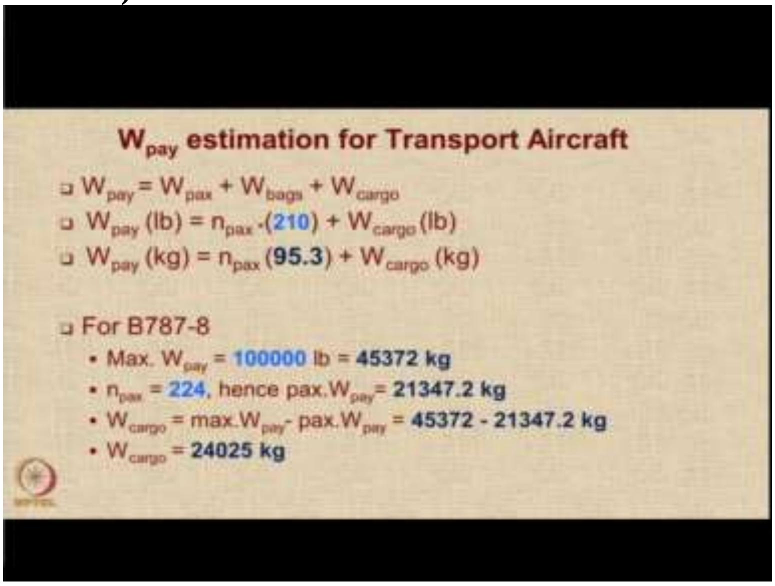

Let us look at the $W_{payload}$ estimation. In this case, of course it is already specified. But just to complete our information. The payload consists of passengers, weight of the passengers, weight of the baggage and the weight of cargo. In our case, we have been told that each passenger weighs 210 pounds. So when we calculate the payload weight in pounds, we will use this simple formula, converting it into kilograms, we can use the formula as shown.

For Boeing 787 - 8 we are told that the maximum payload is 100000 pounds. So since there are 224 passengers, therefore the payload with all the passengers and their baggage will be 21347.2. And in our design mission, we assume that we do not carry any extra cargo. However because we can carry a total of 100000 pounds, therefore if required the cargo can be carried to the tune of 24 tonnes approximately. But in our case, we are going to assume for the design mission that you are only carrying the 224 passengers and their baggage.

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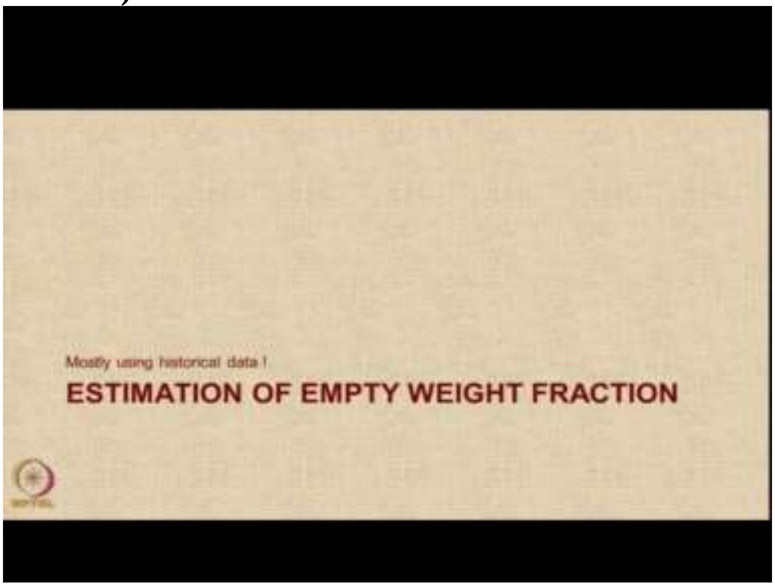

Now let us see how we estimate the empty weight fraction. As I mentioned, 2 unknowns are remaining, the empty weight fraction and the fuel weight fraction. For the empty weight fraction, we normally reach out to historical information.

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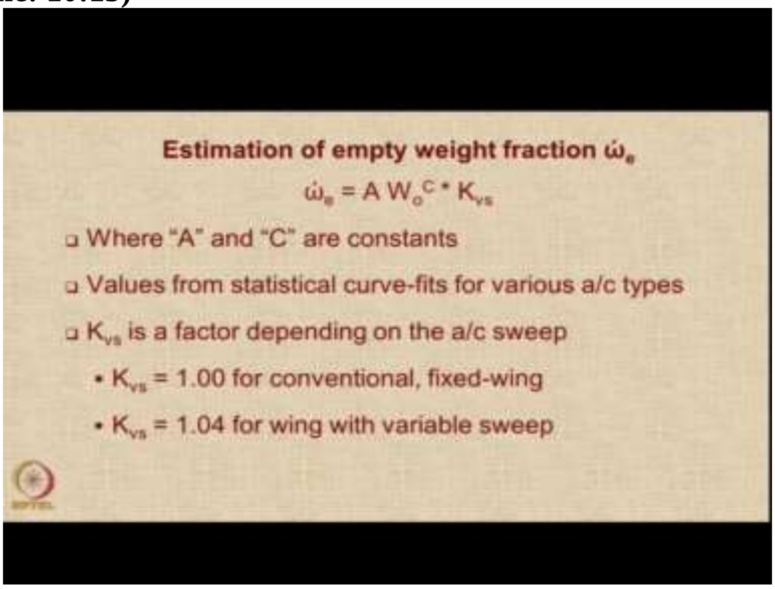

So if you refer to the standard textbook by Daniel Raymer, the formula is $AW_0^C K_{VS}$, where A and C are constants. And their values are different for different aircraft types. And we obtain these values from statistical curve fits, as I will show you very shortly. The term K_{VS} takes care of the additional weight due to variable sweep. So in our case, there is no variable sweep. It is a fixed wing aircraft, so this factor will be 1.0.

However, if we design an aircraft with variable sweep, the additional weight because of that would be approximately 4% of the empty weight fractions. So therefore, we have to add a factor of 1.04.

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So, if you look at the textbook by Daniel Raymer, there is a table given where the coefficients A and C, both in metric units, that is in kilograms, and in pounds are specified, the coefficient C remains the same, whether you use the power remains the same whether you use the value in pounds or in kilograms.

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So since we are going to work in SI system, we will not worry about the units in pounds, so we will go into metric. And since our aircraft is a jet transport aircraft, we have to use the values as 0.97 for A, when W_0 is in kilograms and -0.06 for C.

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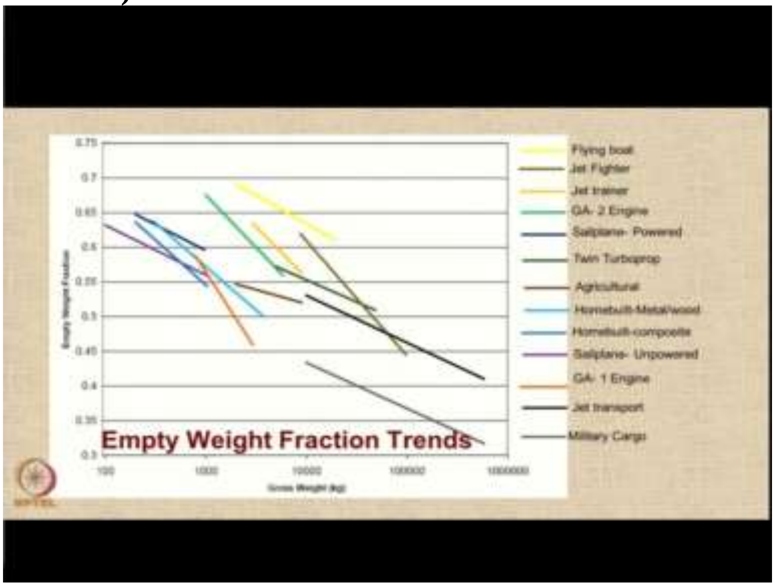

Let us have a look at how the empty weight fraction trends are there for various aircraft type. So, in this particular figure, the largest coefficients are generally for flying boats, you can see they can go as to as high as 0.7, followed by jet fighters, jet trainer, general aircraft general aviation twin engine aircraft, powered sailplane, twin turboprop, agricultural aircraft, homebuilt metal or wood aircraft, homebuilt composite aircraft, sailplane unpowered, general aviation single engine aircraft.

This is the curve for jet transport with which we are interested. Notice that the empty weight fractions can go as high as nearly 0.53 or 0.54 depending on the aircraft weight. And the lightest fractions are for the military cargo, which are seldom beyond the value of 0.44 or 0.45. So the highest empty weight fractions are seen for the flying boat and the lowest empty weight fractions are seen generally for the military cargo aircraft.

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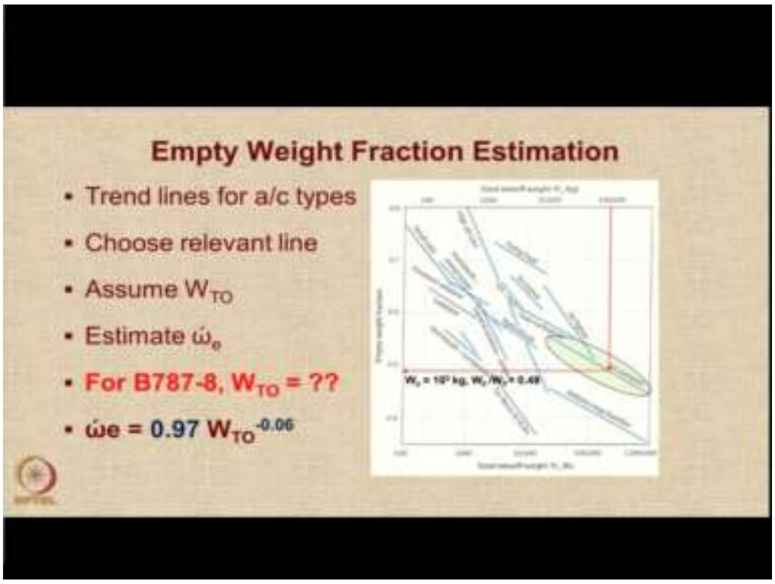

So, this entire figure is shown in one shot in this particular graph, which is taken from the textbook by Raymer. So, we have seen, so what we need to do is we need to choose a line which is relevant to our type in our case it is jet transport. We assume some value of W 0 for example, if he assumed the weight to be 100000 pounds, a 100000 kgs then we can estimate w e bar directly as a ratio. So if the extra weight is 100000 kgs then the empty weight fraction is around 0.49.

What do we do for Boeing 787 - 8. Unfortunately, we do not know its gross takeoff weight that is what we have to determine. So therefore, we cannot use this method directly we cannot get the empty weight fraction as a number. We have to use this expression because in our case W takeoff is itself a variable which has to be determined. So therefore, we will use the expression

$$\overline{w}_e = 0.97W_{TO}^{-0.06}$$

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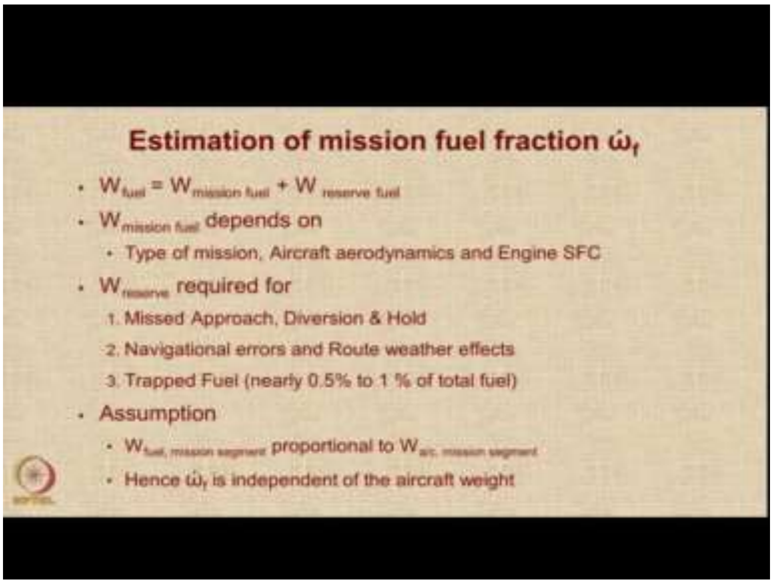

Let us look at how we estimate the next variable which is the mission fuel fraction. So, just a recap, the fuel in the aircraft is basically mission fuel and reserve fuel. The mission fuel depends on the type of the mission that you are flying, the aerodynamics of the aircraft and the engine SFCs. We need reserve fuel for 3 major contingencies either for missed approach diversion or hold.

Or some errors in navigation because of and also route weather effects and plus a small amount of fuel will be trapped in the pipelines, which will not be available to you. So in our method that we follow for initial sizing, we have a very fundamental assumption that the mission fuel segment weight is proportional to the aircraft weight in that segment. So, a heavier aircraft is going to consume more fuel and hence give a higher fraction.

And also we have to assume that the value of this view is independent of the general aircraft weight, it only depends upon the weight of the aircraft in that specific mission, it does not depend upon the weight of the aircraft in the other missions.

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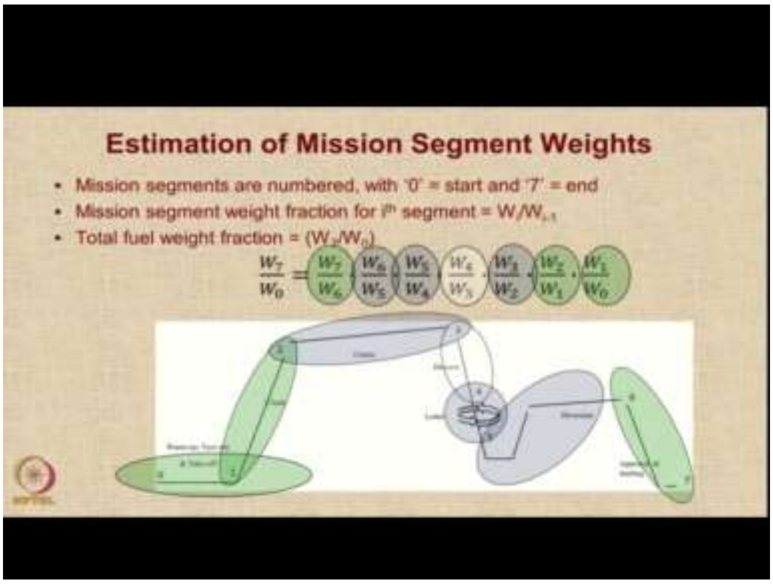

So, going ahead here is our basic simple mission profile. So we number the segments from 0 to 7 with 0 being that start and 1 being the takeoff and 2 being climb et cetera, et cetera. So for the ith segment, the mission segment weight will be $\frac{W_i}{W_{i-1}}$. And the total fraction will be $\frac{W_7}{W_0}$ in this case, which can be obtained by just a multiplication of the individual mission fuel weight segment fraction.

So, because of the assumption that we have made, this particular equation becomes very simple. So, all you need to do now is calculate using some method or by assumptions, the weight of the aircraft at the end of the mission divided by the weight of the aircraft at the beginning of that particular segment. So for every segment, we have to now calculate the ratio as weight at the end of the segment upon weight at the beginning of that segment.

So for takeoff, we are going to use some historical information, for climb also we will use some historical information, for descent we are going to ignore the weights. So this number $\frac{W_4}{W_3}$ will be equal to 1. And similarly, for the land, for the approach and landing we are going to assume from historical data. So, 3 of the mission segments are assumed from historical data, one of them is assumed to be a segment which does not consume any fuel.

So what is remaining now is 3 segments, $\frac{W_3}{W_2}$, that is the cruise segment, $\frac{W_5}{W_4}$, which is the loiter segment, and $\frac{W_6}{W_5}$ which is the diversion segment. So all we need to do now is to calculate the segment fuel fraction for these 3 segments. And then we can multiply all these terms together to get the total fuel fraction.

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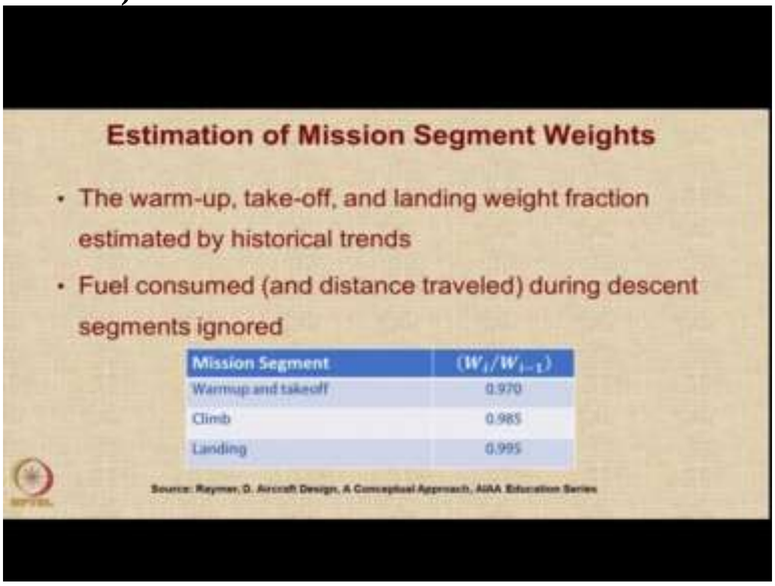

So, as I mentioned for 3 of these segments, that is the warm up takeoff and landing, and also climb, we are going to assume it from this table given by Daniel Raymer. And we will also assume that the fuel consumed as well as the distance travelled during the descent segments is going to be ignored.

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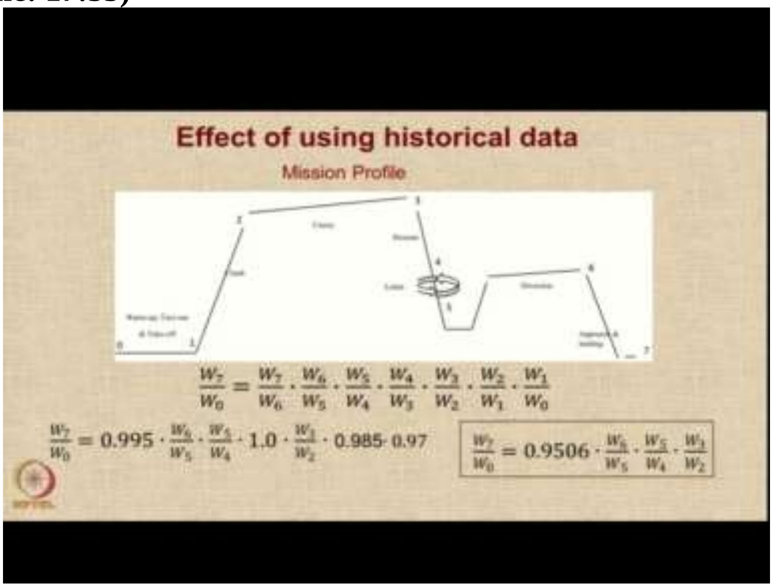

So what happens when you use historical data is that in this mission, this was the basic equation. And 3 of these ratios, or other 4 of these ratios are now replaced by, values available from historical data. So if you multiply those 3 that is 0.97 into 0.985 into 0.995, you will get 0.9506.

In other words, $\frac{W_7}{W_0}$ is simply going to be a multiplication of the fuel fraction in 3 segments the cruise segment, the descent and for the loiter segment and the diversion segment.

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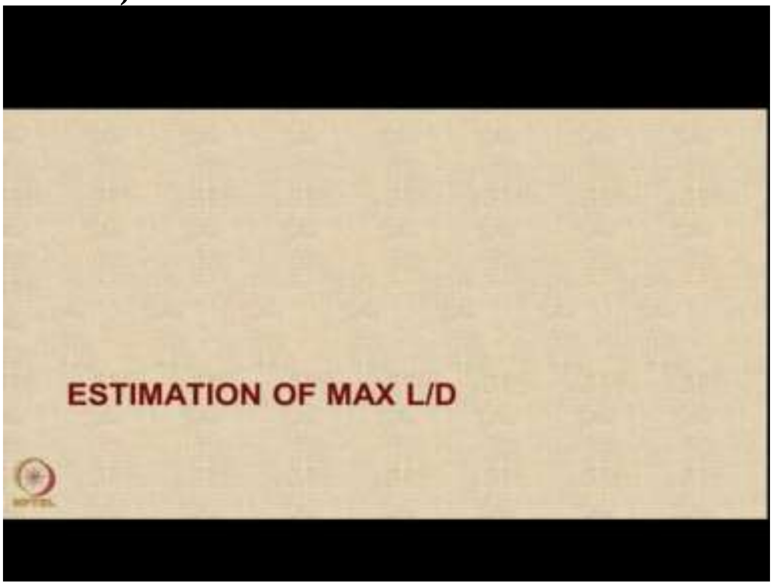

So before that, we need to estimate the $\left(\frac{L}{D}\right)_{max}$. And again, we look at historical data.

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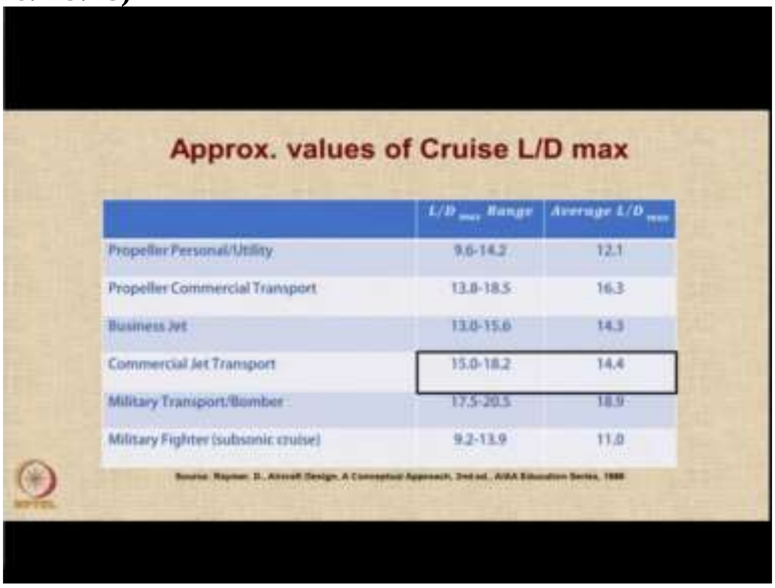

Because there is no other way you can do it at this stage of the design when you do not know much about your aircraft. So first of all, let us look at what are the approximate values of $\frac{L}{D}$, cruise $\left(\frac{L}{D}\right)_{max}$. So for a commercial jet transport is generally specified that the values of $\left(\frac{L}{D}\right)_{max}$ would be nearly about 15 to 18. And the average would be 14.4. So if you do not have any information on you do not have any data about the aircraft you are safe to assume these values. However, please note that modern day aircraft have a very high $\left(\frac{L}{D}\right)_{max}$ values typically of the order of 20.

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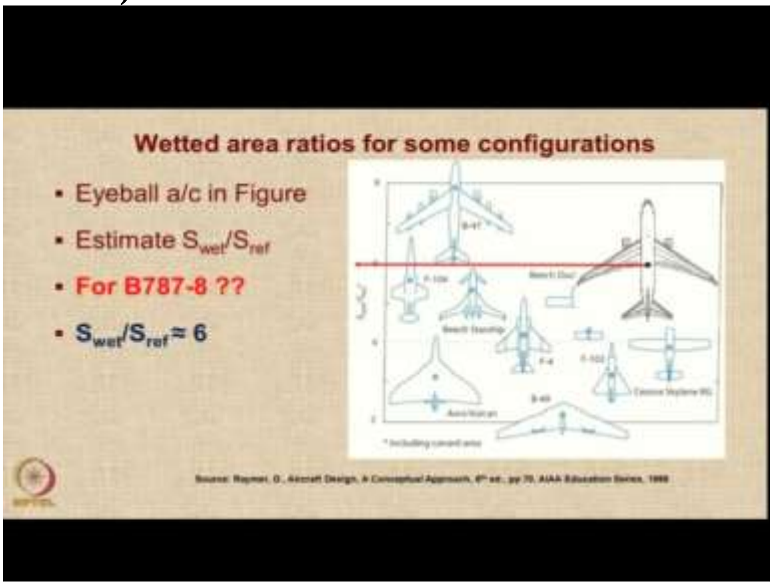

So here is a graph taken from Raymers textbook which talks about the wetted area ratio that is $\frac{S_{wet}}{S_{ref}}$ for various aircraft types on the basis of their shapes. So what we do is we eyeball our aircraft in this particular figure, we have a rough idea right now about how our aircraft will look like. So you just locate where your aircraft will roughly fit. And then just read the value of $\frac{S_{wet}}{S_{ref}}$.

So for Boeing 787 - 8 where do you think it will fit. So I think the aircraft figure is very similar to Boeing 747 except that Boeing 787 has got a much more slender wing compared to Boeing 787. So that is why it is positioned slightly below and slightly to the left of Boeing 747. So it sure turns out that the value of $\frac{S_{wet}}{S_{ref}}$ at that particular point turns out to be 6. So roughly it is a whole number.

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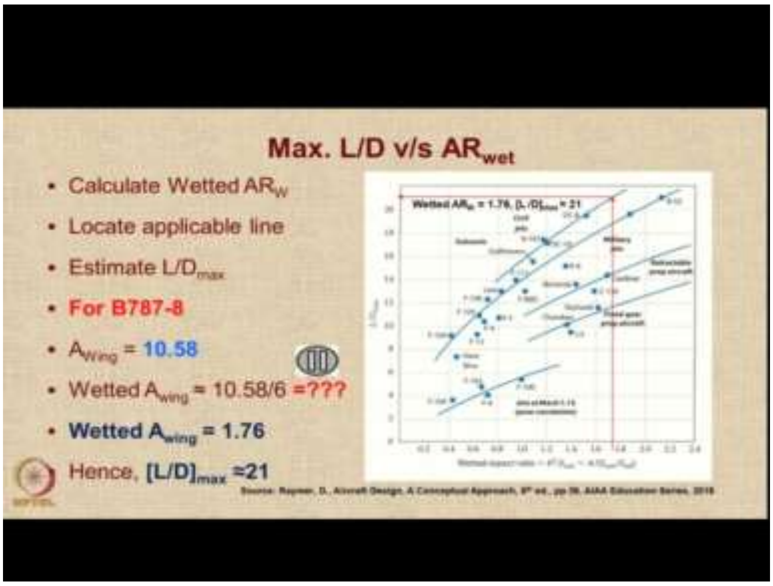

So, that means you calculate the reference area of this aircraft multiply by 6 you will get the wetted area. Now, once you know the wetted area, you can calculate the wetted aspect ratio. And then there is this figure which shows the typical $\left(\frac{L}{D}\right)_{max}$ values for various types of aircraft as a function of the wetted aspect ratio. So first you calculate the wetted aspect ratio, which is aspect ratio divide by the ratio that we got in the previous slide $\frac{S_{wet}}{S_{ref}}$

So, we locate the applicable line for us and then we estimate the $\left(\frac{L}{D}\right)_{max}$. For Boeing 787 - 8 the line would be the top most line which is for civil jets. We know that the wing aspect ratio is 10.58. And just now, we have seen that the $\frac{S_{wet}}{S_{ref}}$ ratio is going to be approximately 6. So, please pause the video here and calculate the value of the weighted aspect ratio, it will be 10.58 divided by 6, the number comes out to be 1.76.

So then in this graph, what we do is, we start from 1.76 go to the appropriate line and then take a left turn and hit the vertical axis we see that the value of $\frac{L}{D}$ is estimated to be approximately 21. So this is how you can estimate the max $\frac{L}{D}$ of the aircraft.

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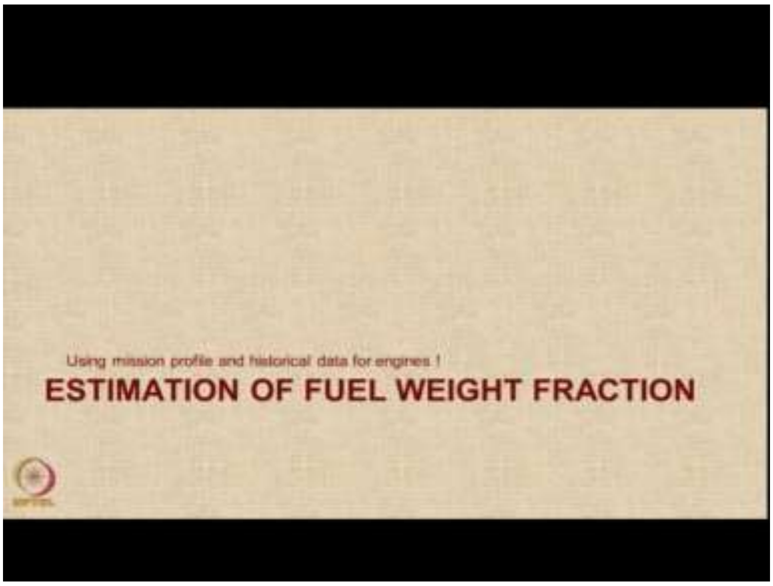

Now, fuel weight fraction is estimated using mission profile and the historical data for engines.

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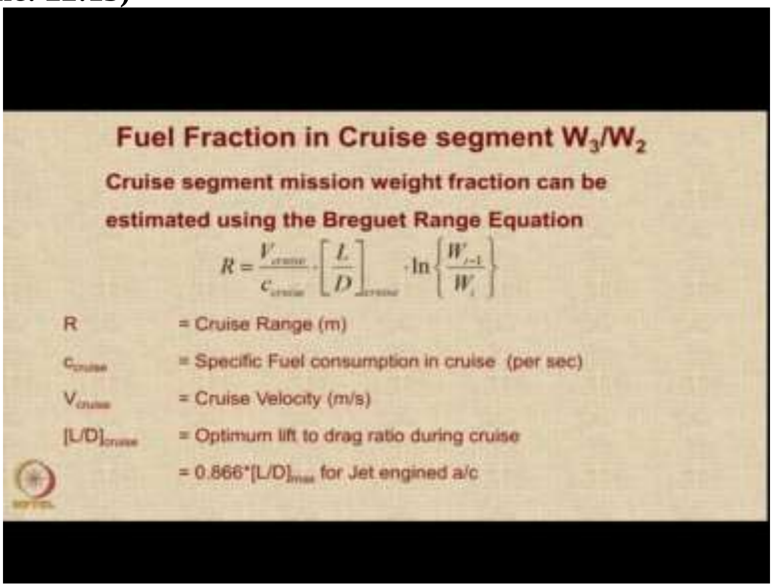

So, first let us look at there are 3 segments for which we need the fuel fraction in first is cruise, the other is the loiter and third is the diversion which is also a cruise. So, for $\frac{W_3}{W_2}$ ratio we assume that we can follow the Breguet range equation, which is as shown in the screen. So what you need is V_{cruise}, SFC_{cruise}, $\left(\frac{L}{D}\right)_{cruise}$ and if you know the value of R, then you can invert this expression and you can easily get the value of $\frac{W_{i-1}}{W_i}$

Where R is the cruise range, c cruise to this specific fuel consumption per second. Remember the units are very important. Notice that the units of velocity are meters per second, and the units of range are meters, so therefore, to balance the units on both sides, since L/D is dimensionless and log of weight ratio is dimensionless the units of c_{cruise} have to be per second. Then in that case it will balance the units on both sides.

So, the value of c cruise given in the requirements or in specifications is not in the units of per second but we will convert it. And since we are looking at optimization of the fuel fraction for a range of a transport aircraft, we know that the

$$\left(\frac{L}{D}\right)_{cruise} = 0.866 * \left(\frac{L}{D}\right)_{max}$$

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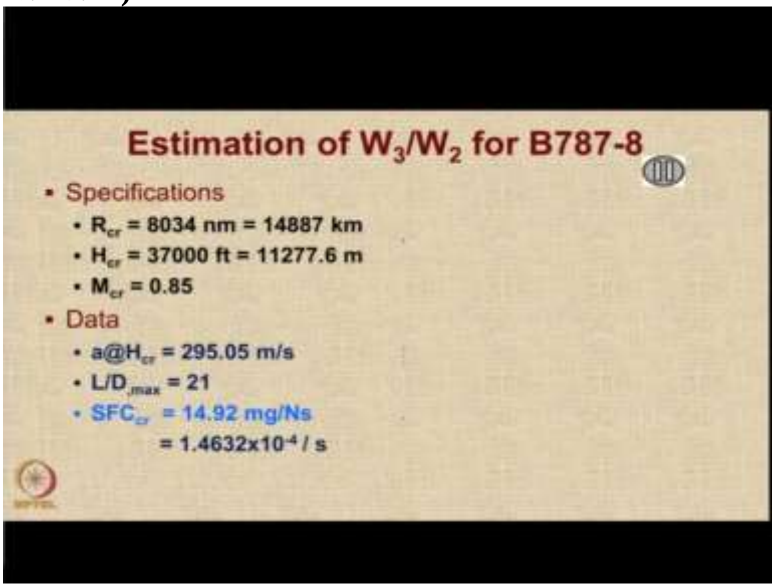

So let us see how to estimate the $\frac{W_3}{W_2}$. So what are the specifications, it is specified that the R cruise is 8034 nautical miles, which is 14887 kilometers. We also know the cruising altitude and you also know the cruising mach number. So first thing we do is we calculate the value of sonic speed at that particular altitude. This you can get from the atmospheric tables or you can calculate based on the value of temperature. $\left(\frac{L}{D}\right)_{max}$ was just previously calculated as 21. SFC cruise is given as 14.92 milligrams per Newtons second.

This is the value inputted from the Piano data, it has to be converted into per second by multiplying by 9.807 that is g and dividing by the 10 power 6 to convert the milligrams into kilograms.

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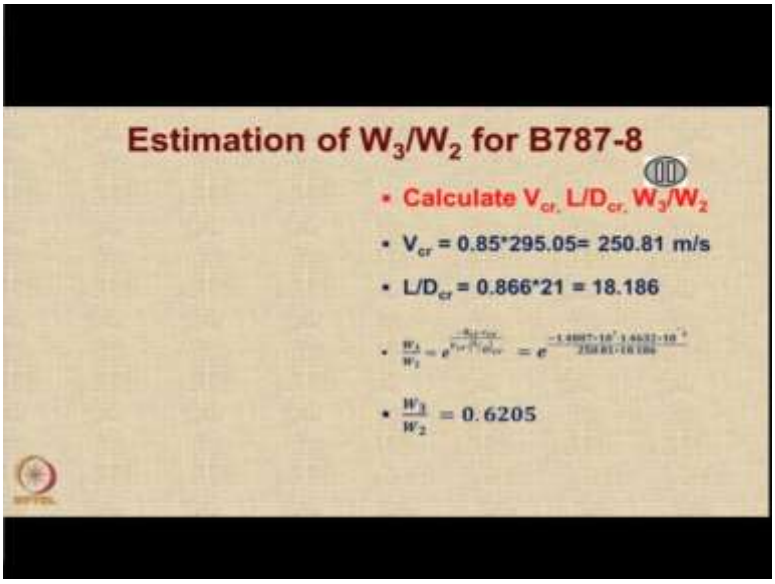

So let us calculate the values of V, L/D and $\frac{W_3}{W_2}$. So V simply mach number into the sonic speed which is 250.81. And $\left(\frac{L}{D}\right)_{cruise}$ is 0.866 times 21, which is already known. So by inverting the equation, you can simply get

$$\frac{W_3}{W_2} = e^{-\frac{R*c}{V*\left(\frac{L}{D}\right)_{cruise}}}$$

And we get the value of

$$\frac{W_3}{W_2} = 0.6205$$

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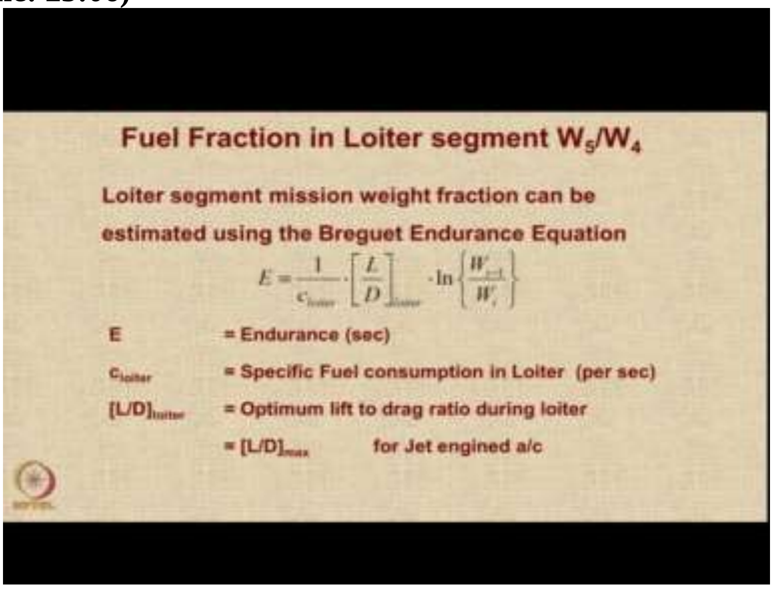

Moving ahead, let us look at the fuel fraction in loiter segment $\frac{W_5}{W_4}$. Again for this we use a Breguet endurance equation

$$E = \frac{1}{c}\frac{L}{D}\ln\frac{W_{i-1}}{W_i}$$

Here endurance ease in seconds and hence the SFCC loiter also has to be in per second, L/D and log weight ratio are dimensionless. Now, since we are looking at turbo jet engine aircraft, the L/D in loiter should be equal to L/D max.

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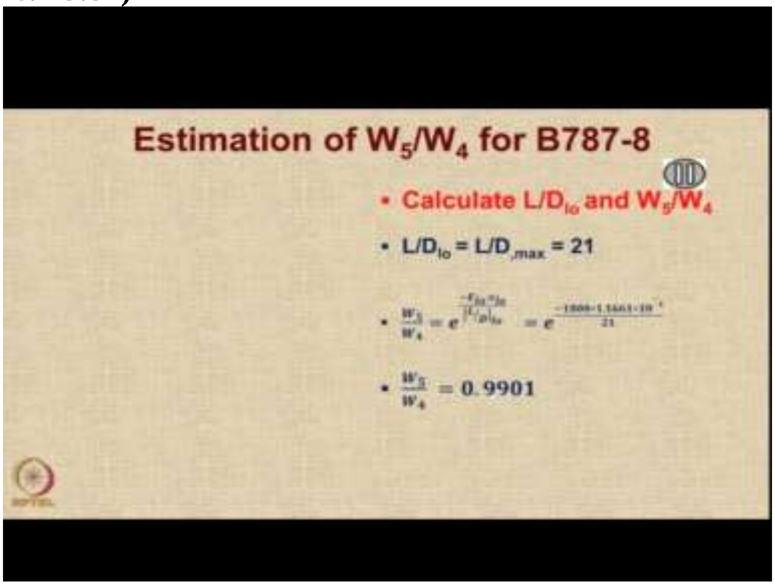

So let us see how these numbers pan out for Boeing 787 - 8. The endurance time is half an hour or 1800 seconds and L/D is 21, SFC already given in, during the loiter segment. So, just calculate the value of $\frac{W_5}{W_4}$ in the same fashion. So first you calculate L/D in during the loiter which is equal to L/D max which is 21 and then you put in e power minus endurance into SFC divided by L/D, you get the value of 0.9901. So there is a very small loss in the fuel fraction during a half an hour cruise

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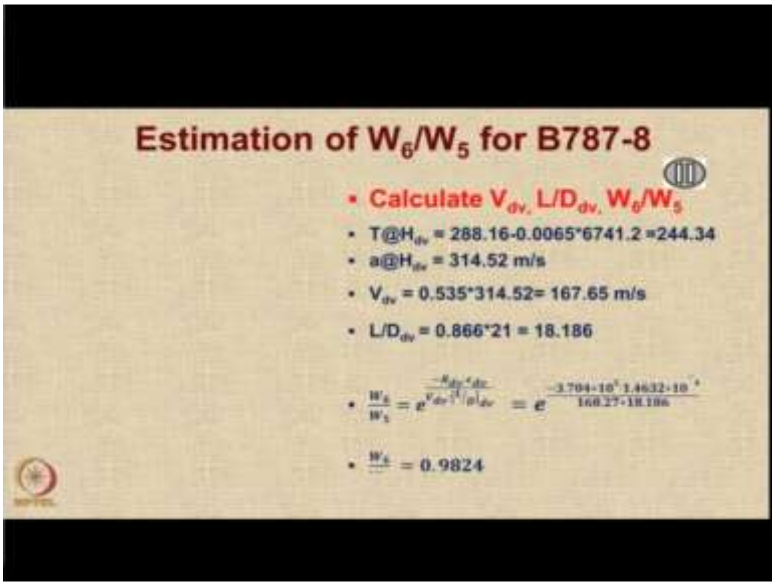

Then you go to diversion segment. Diversion segment is like a cruise segment. So therefore, the same formula applies, the same condition of 0.866 times L/D max also applies only thing is that the $V_{cruise}, SFC_{cruise}, \left(\frac{L}{D}\right)_{cruise}$ will change because it is at a different operating condition. So, let us see how it is calculated for diversion we are given that the distance to be travelled is 200 nautical miles at a height of 22117 feet at a mach number of 0.535.

So, $\left(\frac{L}{D}\right)_{max}$ was 21 as obtained earlier, SFC during diversion is given. So you can convert it to per seconds, and then you just calculate the values of V, L/D and weight ratio. So, V is obtained by calculating T

$$T = 288.16 - 0.0065 * 6741.2 = 244.34\ K$$

then when you know the value of T

$$a = \sqrt{\gamma RT}$$

and since, you know the mach number you can get the value of V and L/D is also available to you as a input. So, with that you can get $\frac{W_6}{W_5}$ = 0.9824.

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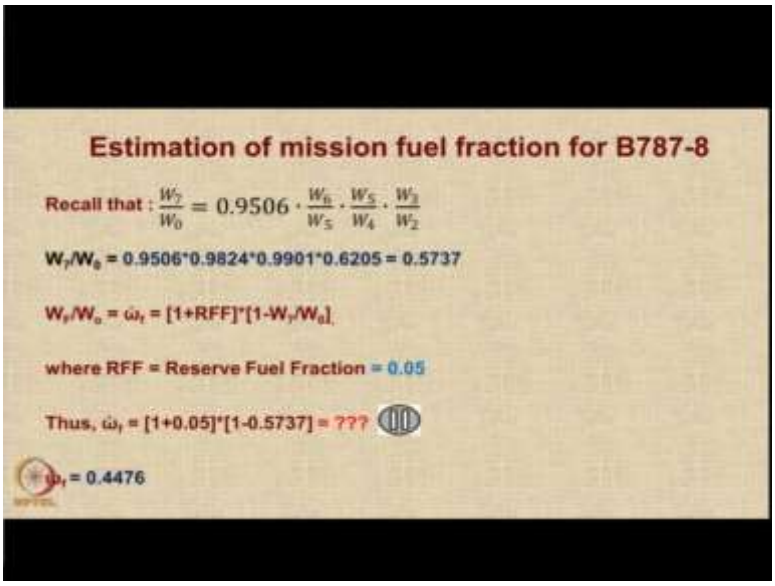

Let us estimate the mission fuel fraction for Boeing 787. This is our equation which had putting together of the various constant terms. So at this point I think you should pause and have a look at the numbers and calculate the values. So we get it as 0.5737. And to bring in the fact that we need to carry 5% more mission fuel, we are saying that $\frac{W_f}{W_0} = \overline{w}_f$ which you obtained earlier, to be changed slightly.

And the index of 0.8 has to be changed the reserve fuel fraction is 5%. So therefore,

$$\overline{w}_f = (1 + 0.05) * (1 - 0.5737)$$

Please take a pause and calculate the value. The value comes out to be 0.4476.

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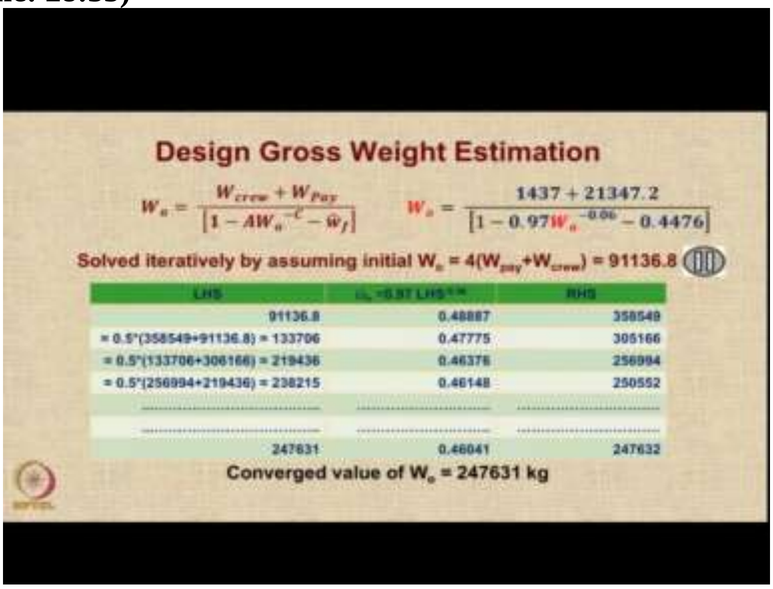

So, this is our equation the master equation for the initial sizing in which we have replaced

$$\overline{w}_e = AW_0^{-c}$$

$\overline{w}_f$ remains and we have just reproduced here the values from the previous slides. So, W_{crew} is the weight of the crew members plus the operating items which is 1437 kg, W pay is the design payload. This is the payload of these 224 passengers at the rate of 293.2 kgs each and the values of A and C from Raymers table are taken directly here for a jet transport aircraft.

And we have just calculated the value of fuel fraction, mission fuel fraction as 0.4476. So what we do now is we now solve iteratively we assume the first value of W 0 as 4 times the payload plus crew. So being while doing that, you can have LHS as that value, then you calculate the value of $\overline{w}_e = 0.97 W_0^{-0.06}$ which is the w e bar and with that you can calculate the RHS you can see there is a huge difference.

So, in the next iteration, what you do is you can take the average of the new RHS and the old LHS and repeat the calculations and you keep on doing this iteration a few times, few more times. And finally is going to converge to a value of 247631 kgs.

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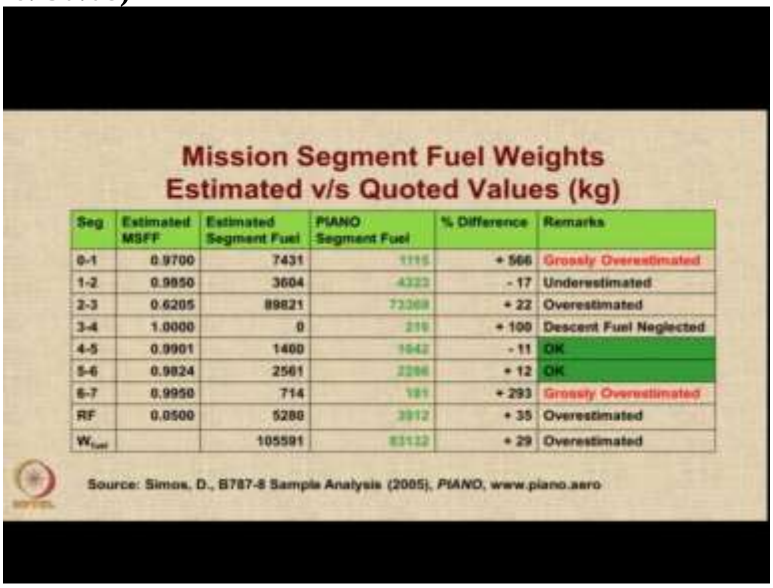

Mission Segment Fuel Weights
Estimated v/s Quoted Values (kg)

Seg	Estimated MSFF	Estimated Segment Fuel	PIANO Segment Fuel	% Difference	Remarks
0-1	0.9700	7431	1115	+ 566	Grossly Overestimated
1-2	0.9850	3604	4323	- 17	Underestimated
2-3	0.6205	89821	73368	+ 22	Overestimated
3-4	1.0000	0	216	+ 100	Descent Fuel Neglected
4-5	0.9901	1400	1642	- 11	OK
5-6	0.9824	2561	2286	+ 12	OK
6-7	0.9950	714	181	+ 293	Grossly Overestimated
RF	0.0500	5288	3912	+ 35	Overestimated
W_{fuel}		105591	83132	+ 29	Overestimated

Source: Simos, D., B787-8 Sample Analysis (2005), *PIANO*, www.piano.aero

Now let us see the comparison between what we got and what has been quoted by Piano. So for the first segment that is a warm up taxi out we have estimated the weight to be 7431 kgs, whereas piano it is 1115, there is a gross overestimation. For the climb segment, we have obtained 3604 whereas, the value quoted is 4323, so that is a under estimation by us. During the cruise, the value obtained by us is overestimated by 22%.

During the descent, we have ignored the fuel consumed, whereas in piano there is a fuel of 216 kgs. So of course, there is a very large mismatch. For the loiter segment, there is a reasonably good match of around 11% we have slightly under estimated, but for the loiter segment, we have overestimated by the same amount approximately 12%. So this is also okay

And again if you go for the landing, we have done a gross overestimation because we have used the Raymers formula, with that you, get a number of 714, whereas the actual value given by piano is only 181 kgs. Finally, the reserve fuel fraction will be overestimated because there is a gross mistake in the estimation of the fuel and hence there is also an overestimation in the mission fuel, in the reserve fuel. So the total fuel estimation is around 30% beyond.

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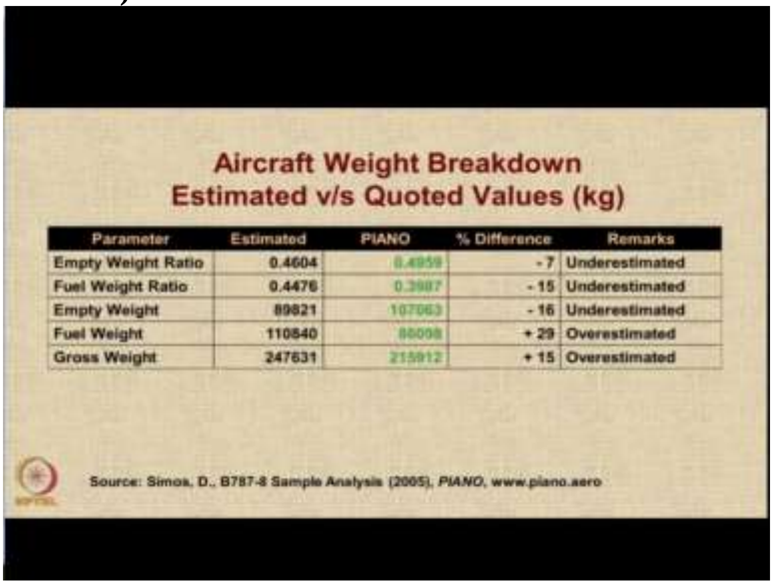

Now we have to worry, we have to also look at the ratios. So empty weight ratio has been underestimated the actual value is 0.4959, but we have got 0.4604. Fuel weight ratio as I mentioned there is a 15% excess. Empty weight has also been underestimated and fuel weight has been overestimated. In short there is a 15% overestimate in the gross weight.

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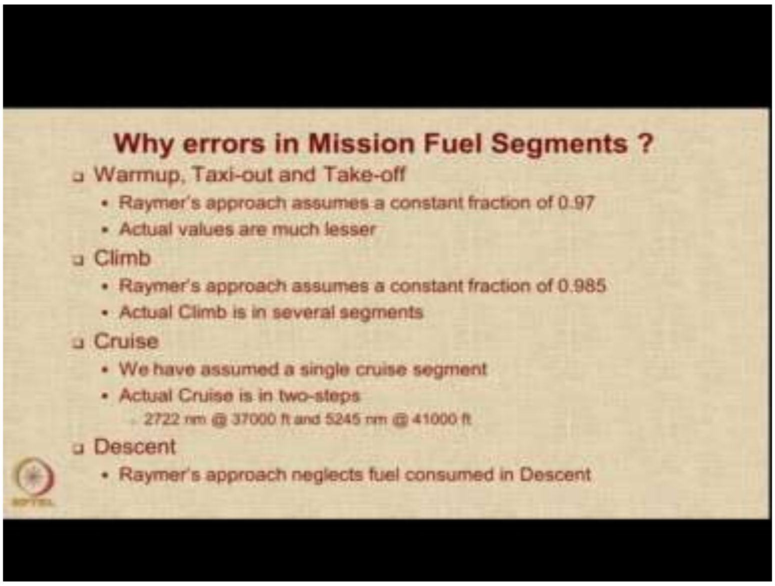

Now, why are there so much errors in the mission fuel segments as compared to Piano. The reasons are very clear. First of all in the warm up taxi out and take off Raymer assumes a constant number of 0.97, whereas actual values are much lesser. During climb in Raymers approach we have again a constant fraction of 0.985, actual value is different because climb happens in many segments.

And in cruise we have assumed a single cruise segment whereas if you look very closely at the data, we find that the actual aircraft profile involves a step. So there is a first cruise at 2722 nautical miles at 37000 feet, then you climb up by around 4000 feet to 41000 feet and then you cruise for the remaining portion. And in descent Raymer approach is the fuel in descent whereas that is actually consumed.

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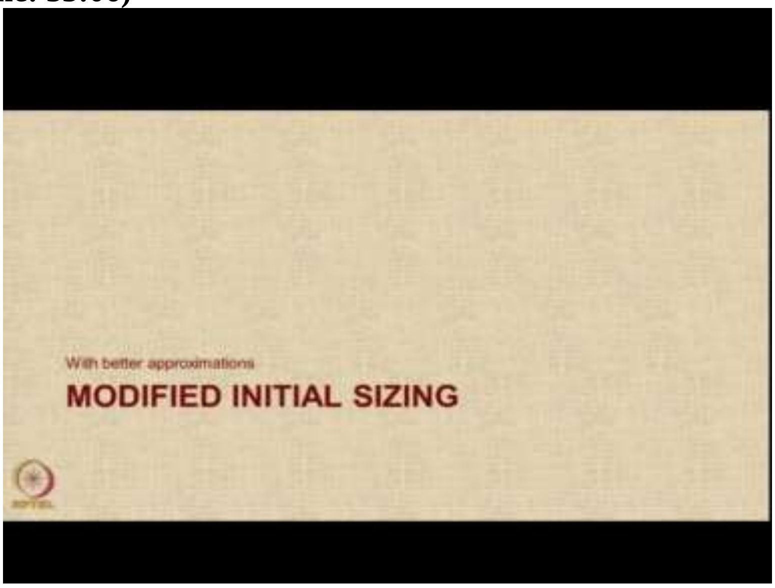

So we have to now look at how to modify our calculations. And for that we will take the values for the, we will take the values from the table given by Piano and try to modify the values according to that.

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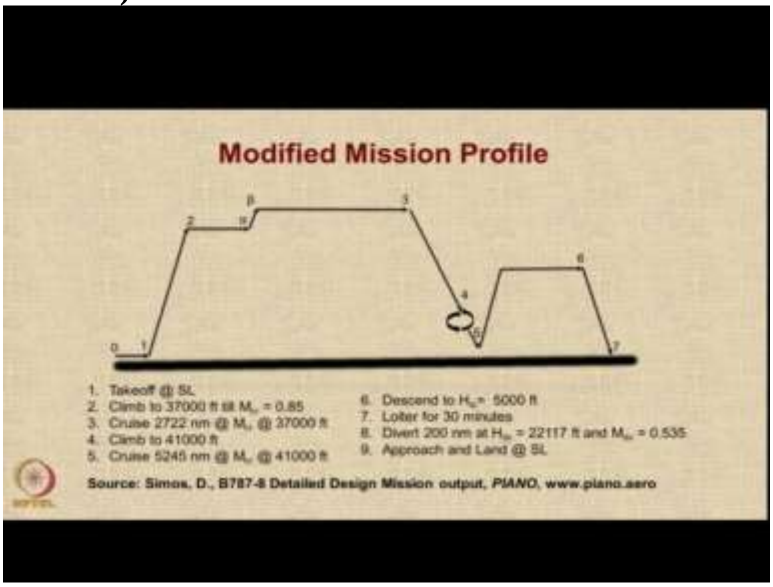

So, here is the modified mission profile as suggested by Simos in his output. So, the first one is the takeoff segment. The second one is the climb segment. The third one is a short cruise at 37000 feet at a constant mach number, then there is a small gain in altitude at the 4000 feet and then there is a bigger cruise of 5245 nautical miles at 41000 feet, but at the same cruise mach number and also the same speed. And then there is a descent to the loiter altitude of 5000 feet then there is a loiter for half an hour.

After that there is a missed approach and then a diversion. Finally we have landing. So we are going to now calculate the mission, modified mission numbers for this particular profile.

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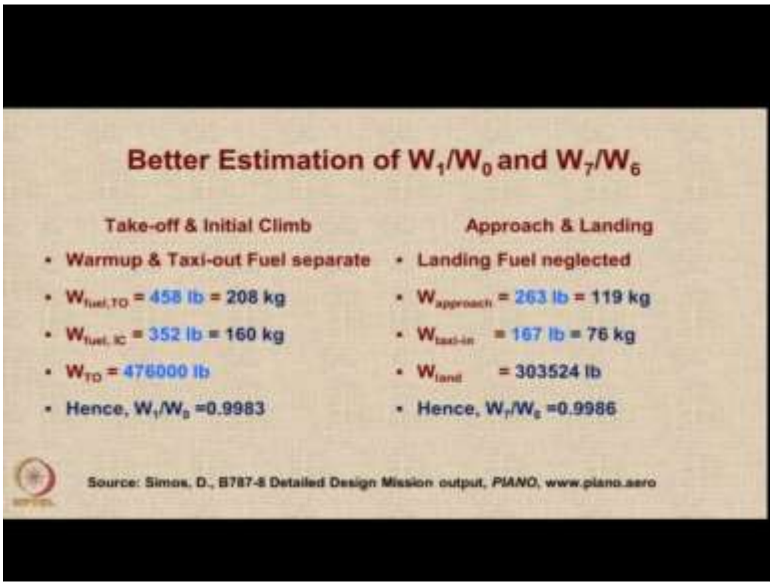

So for better estimation of W_1 and W_0 the weights in the initial first segment and the last segment, first let us look at take off and initial climb. In this the first one is that the warm up and taxi out fuel is not included in the numbers actually it is a separate value. Simos has provided the fuel in takeoff as 458 pounds and fuel in the initial climb 352 pounds, so with this we get some numbers and the takeoff weight is 476000 pounds.

Hence we can estimate that the weight in the initial segment ratio is only 0.9983 as compared to 0.97 given by Raymer. Similarly in approach and landing, the landing fuel is neglected actually and the weight of the fuel consumed in approach is 263 pounds and that in taxi in is 167 pounds. The landing weight at the end of the mission is given as 303524 pounds. Hence the weight ratio during approach and landing is only 0.9986.

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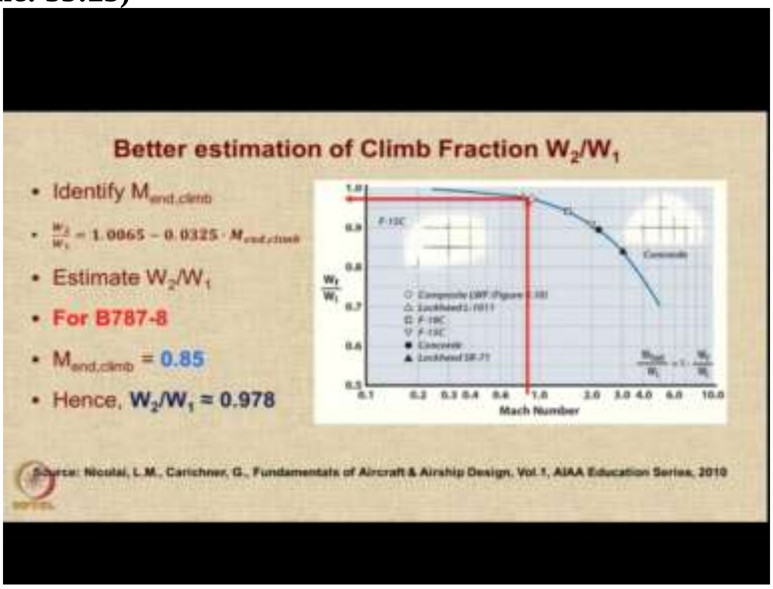

Now, let us look at a better estimation for climb fuel fraction that is $\frac{W_2}{W_1}$. Now, if you look at Raymers textbook there is a improved formula available where if the mach number of the aircraft at the end of the climb is known that is called as M end climb, then you can calculate $\frac{W_2}{W_1}$by this particular expression. So, what you can do is for Boeing 787 - 8 what we will do is we know that the mach number at the end of the climb is 0.85.

And here is a graph from the textbook by Nicolai and Carichner, where they have shown this particular fuel weight ratio for the accelerated climb or climb segment. So in our case, the mach number at the end of climb is 0.85. So with that if you proceed on this line, you can get the weight fraction as 0.978, whereas Raymer has taken a weight fraction of 0.985.

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For better estimation of the cruise fuel fraction, there are 2 segments in the first segment we go from 2 to alpha where the range is 2722, the velocity is 488 knots True AS or true air speed, SFC is given and L/D is given. So with that you can get the $\frac{W_\alpha}{W_2}$ as 0.8682. Similarly, for the second cruise segment, which is from beta to 3, we have been given the value of range as 5245, the cruise velocity is the same, SFC is slightly different and L/D also is slightly different because the aircraft is lighter.

So putting in the equation, we can get the weight fraction as 0.7580. So if you want to get a better estimate of the weight ratio during the cruise, you just multiply these 2 numbers and whatever number you get would be a better approximation than the one that we got earlier.

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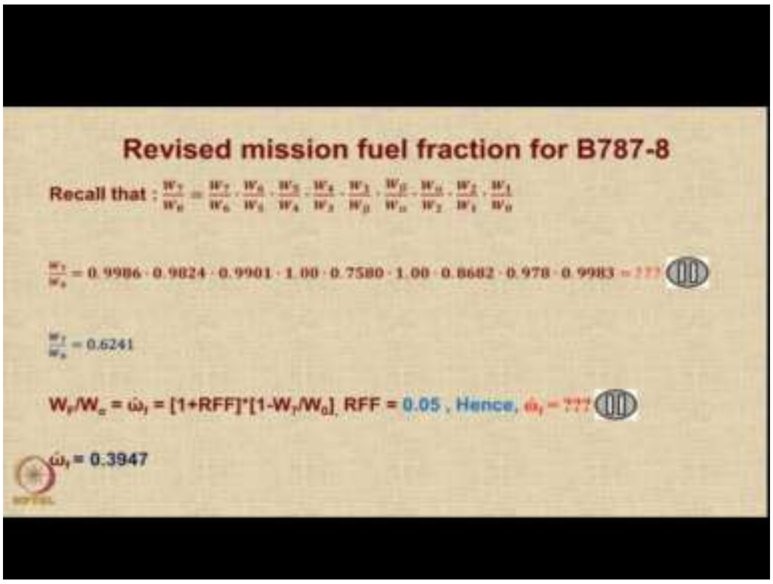

Similarly, if you look at the revised mission profile, now we have

$$\frac{W_7}{W_0} = \frac{W_7}{W_6} * \frac{W_6}{W_5} * \frac{W_5}{W_4} * \frac{W_4}{W_3} * \frac{W_3}{W_\beta} * \frac{W_\beta}{W_\alpha} * \frac{W_\alpha}{W_2} * \frac{W_2}{W_1} * \frac{W_1}{W_0}$$

Now what we can do is looking at the number that we obtained earlier, we can just insert these numbers one by one, $\frac{W_1}{W_0}$, the fresh one was 0.9983, $\frac{W_2}{W_1}$, the fresh value was 0.978, then $\frac{W_\alpha}{W_2}$ was 0.8682, beta / alpha we ignore because it is only 4000 feet.

There is of course some fuel consumed, but in this big picture, we can ignore that. And we have no other way of estimating it. The second cruise segment we get a fuel fraction of 0.7580, descent segment is again 4 / 3 we again ignore the value, 5 / 4 is the same as before, 6 / 5 is the same as before, 7 / 6 is a modified one 0.9986. So with this, the new value comes to 0.6241, which I think you should calculate by multiplying all these numbers.

So therefore, we can get the $\frac{W_f}{W_0}$, which would be

$$\frac{W_f}{W_0} = (1 + RFF)\left(1 - \frac{W_7}{W_0}\right)$$

during the mission, the RFF remains same. So pause and calculate the value of $\overline{w}_f$ the number comes out to be 0.3947.

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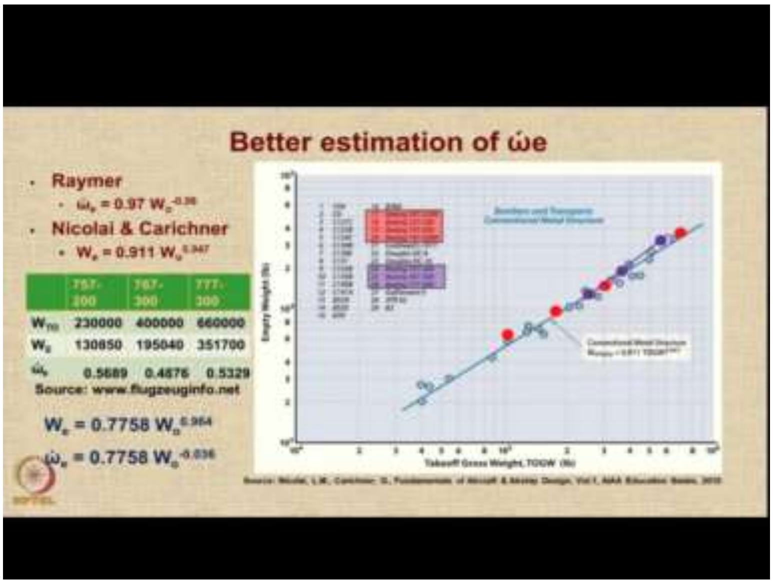

Now let us also look at how we can get a better estimate for the empty weight fraction. Empty weight fraction is very important because it is a very large fraction. Raymer gives a simple formula in terms of empty weight fraction in terms of the max takeoff weight. And here is a graph taken from the textbook by Nicolai and Carichner which plots the empty weight fraction. Empty weight versus takeoff gross weight in pounds for various transport aircraft.

These are all conventional metallic structure. Now let us look at some aircraft from the Boeing family in this. So these 4 dots correspond to 4 aircraft in from the Boeing family, which were designed and used a little bit few years ago. And these 3 dots represent the diagram the values for the aircraft which are a little bit recent. So we notice that the 3 aircraft that have been designed recently by Boeing, they follow a trend line which is slightly different from the general trend line.

So, that number the general trend line is $0.911W_0^{0.947}$. That is what Nicolai and Carichner talks in general. But if we look at the data of 3, these 3 contemporary aircraft and if you actually try to obtain the equation of this line, that turns out that the empty weight is equal to $0.7758W_0^{0.964}$ And the empty weight fraction is as shown on the screen. So if we use this equation, we will get a better estimate.

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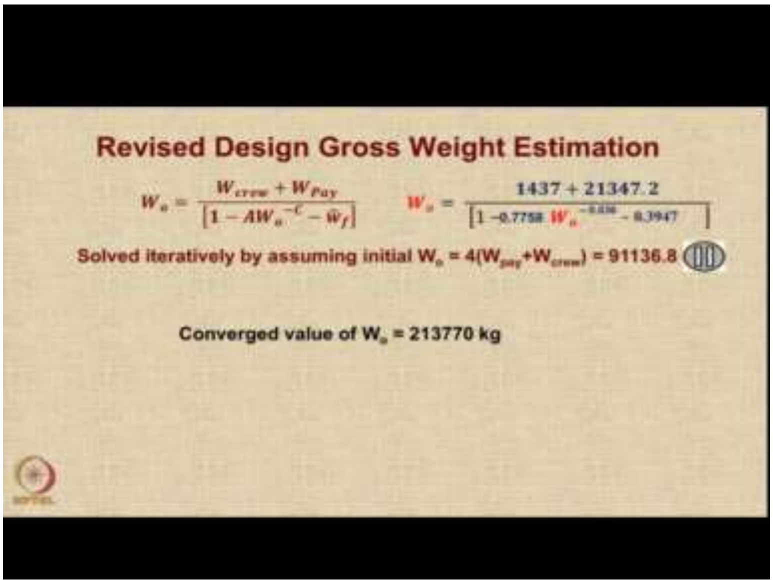

So the revised gross estimate can be calculated by using the same formula. This is the original number. Originally we had $0.99W_0^{-006}$as 0.4476. But now we know that the new fuel weight fraction is 0.3947. And the new coefficients for W_0 are 0.7758 and -0.036. So with this if you solve iteratively with the same initial assumption, finally the value would converge to 213770, which is a much better estimate for the gross weight as compared to obtained earlier.

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Hope your calculations were matching with these. Thanks for your attention.