Introduction to Aircraft Design
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Lecture - 43
Estimation of Fuel Weight Fractions

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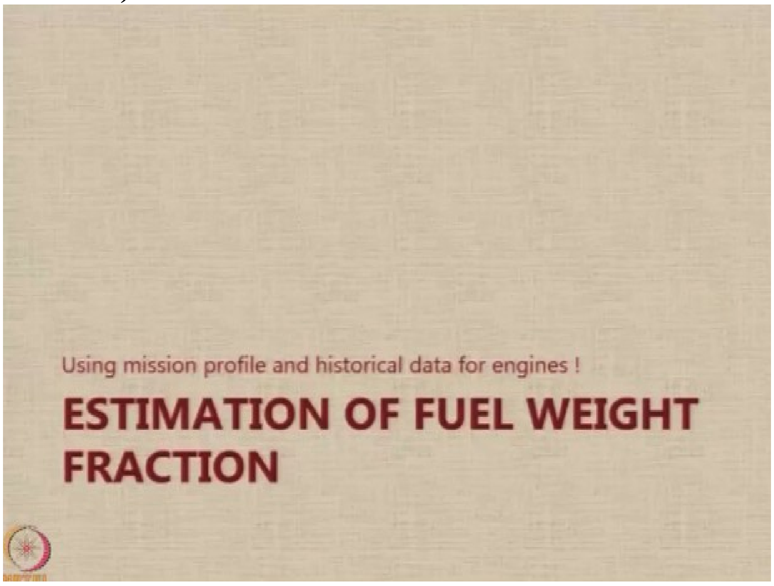

So, let us see how the fuel fraction is determined here what we do is we use mission profile information and also we used the historical data for the engines.

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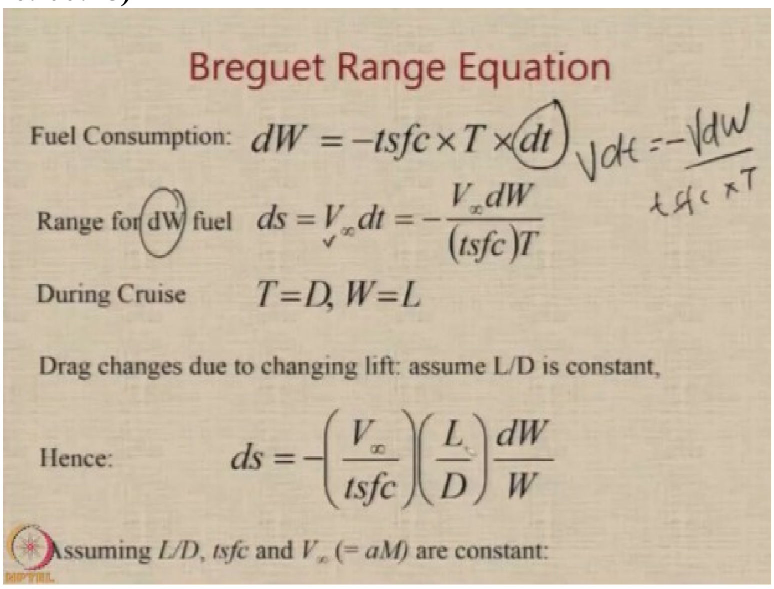

Now, to be able to determine the fuel fraction of the aircraft during either the cruise or during the endurance we can take recourse to the Breguet range and endurance equations. Let us have a look at the Breguet range equation basically, this equation helps us determine the relationship between the fuel consumed in a particular segment and certain important attributes related to the aerodynamics of the aircraft and also to the propulsion and also to its structure.

So, if the fuel consumption basically by definition of tsfc now, tsfc has the thrust especially fuel consumption is defined actually as how much fuel is consumed per unit thrust per unit time and this negative sign indicates that there is a reduction in the fuel with time. So, from there you can get a quick idea that dW or the change in the weight of the aircraft which is assumed to be only because of fuel consumption is basically going to be

$$dW = -tsfc * T * dt$$

And the distance travelled by the aircraft is actually going to be for a small amount of fuel dW the distance travelled will be ds which will be its speed assumed to be constant in that small segment into the time. So, therefore, if I just leave dt here and if I take these 2 parameters that side I get

$$dt = \frac{-dW}{tsfc * T}$$

So, if I have to multiply by V I have to put V here. So, you get this expression for the elemental distance covered in 1 particular small segment.

Now, during cruise we are going to assume that thrust is equal to drag and lift is equal to weight because we are looking at steady level cruise. And we also assume that as the aircraft fuel is consumed, while we proceed further we take care of that by assuming $\frac{L}{D}$ equal to constant. So, the lift is not going to be the same because the aircraft is going to lose fuel. So, if we assume $\frac{L}{D}$ is constant, then you can actually multiply by L and divide by D without making any great change.

So, therefore, you can get this expression that you know since $L = W$ since, so, therefore, this L is equal to this W. So, in other words, you can get the expression

$$ds = -\left(\frac{V_\infty}{tsfc}\right)\left(\frac{L}{D}\right)\frac{dW}{W}$$

and now, if you actually integrate this expression to get the value of total distance assuming now, here the integration has to be done very carefully because depending on what you assume as constant there are actually 3 different types of range and the Breguet range equation can actually be used.

So, you can assume for example, that $\frac{L}{D}$ is constant tsfc is constant and V is also constant. So, if that is assumed constant then V_∞ tsfc $\frac{L}{D}$ are all going to come outside the integration sign.

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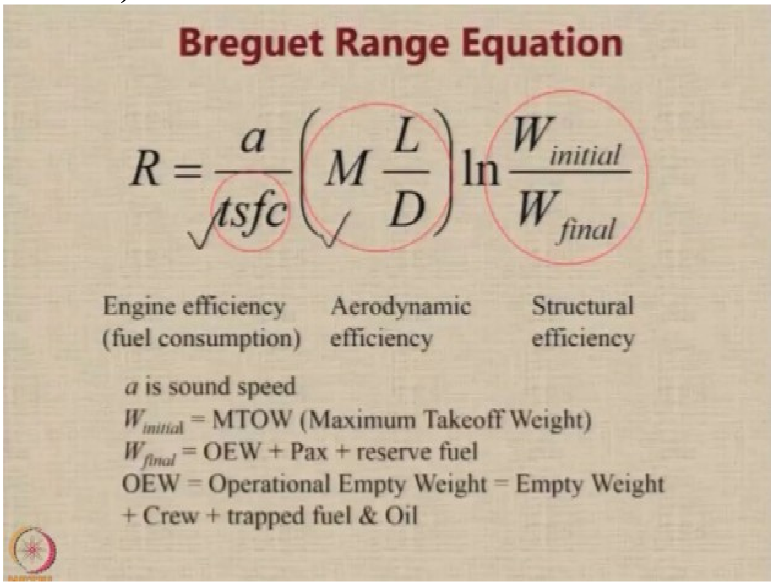

And in that case you will get the value of range as

$$R = \frac{a}{tsfc}\left(M\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

So, here what we have done is we have replaced

$$\frac{V}{a} = M$$

So, otherwise it will be

$$R = \frac{V}{c}\left(\frac{L}{D}\right)\ln\frac{W_{initial}}{W_{final}}$$

but I have replaced the value of $\frac{V}{a} = M$. Now, this is a very interesting equation. Here it captures a lot of important components of aircraft design.

So, in the denominator we have the tsfc which is a measure of the engine efficiency. We have our factor $M\frac{L}{D}$, which is an indication of the aerodynamic efficiency and we have a ratio

$\frac{W_{initial}}{W_{final}}$ which is indication of the structural efficiency. So, the range is going to be a function of how small the value of tsfc is how large the value of $M\frac{L}{D}$ is and also on the weight ratio. So, the important point is that 1 equation is capturing the 3 important elements.

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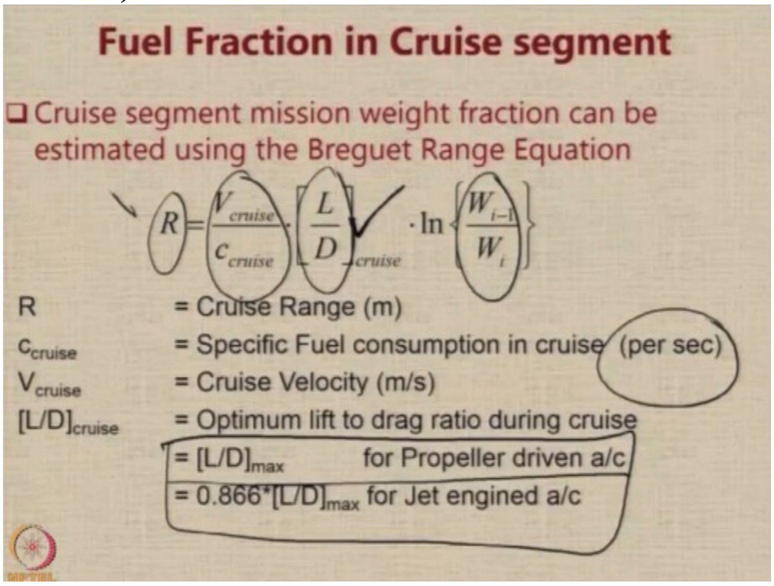

So, what we can do is we can say that the range of the aircraft will be available as a multiplication of the ratio of V cruise / c cruise where c is the SFC. Now, this is 1 place where a lot of students make a mistake in the calculation and I think it is very important for me to point out that particular mistake if you look at this equation, the units on the LHS R range which in SI system are going to be in meters. So, therefore, the units of the RHS also have to be meters.

This is a ratio of 2 weights initial and final that is dimensionless this is the ratio of lift over drag which is also dimensionless. And here you have velocity over SFC and we want the velocities in meters per second and we want this to be in meters. Therefore, the units of SFC have to be in per second this is very important because, the value of SFC which is quoted by the various agencies or the engine manufacturers is going to be in Newton’s per Newton's second or pounds per kg or pounds per pound hour etc.

We have to be very careful to use those units carefully and to correctly use now L/D in cruise should be the one that gives you the optimal condition of cruise. And it can be shown that if you have a propeller driven aircraft, turboprop piston prop, then you should use $\left(\frac{L}{D}\right)_{max}$ when you go for the Breguet range equation and you should use $0.866*\left(\frac{L}{D}\right)_{max}$ when you use a jet engine aircraft.

This is a simple exercise in propulsion in aircraft performance, which can be actually done by the students themselves it is very easy to show that the optimum range of a propeller driven aircraft occurs when it is flying at a condition when $\left(\frac{L}{D}\right)_{max}$. And for a jet engine aircraft, actually the endurance is maximum and $\left(\frac{L}{D}\right)_{max}$ condition is used and the range is maximum when L / D = $0.866*\left(\frac{L}{D}\right)_{max}$ value.

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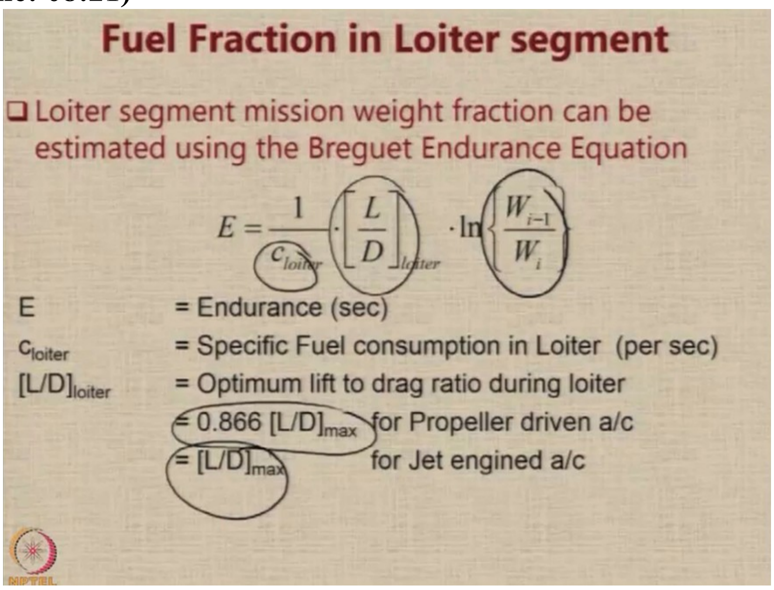

The next segment that we need is the loiter segment and again if you use a Breguet range equation, it can be easily shown that the value of endurance can be

$$E=\frac{1}{c_{loiter}}\left(\frac{L}{D}\right)_{loiter}\ln\left\{\frac{W_{i-1}}{W_i}\right\}$$

this is nothing but dividing the Breguet range equation by the velocity. But again please notice that this is dimensionless, this is dimensionless. So, therefore, if the endurance has to be in time unit of seconds, the loiter has to be in per second.

And once again this is a place where many students make mistake they will put the value blindly and they will get some very inappropriate answers. So, it is very important to caution them and here the L/D in loiter is exactly the opposite of that what you have in cruise for a propeller driven aircraft and endurance is maximized when you are flying the aircraft at an L/D corresponding to 0.866 ¿ $\left(\frac{L}{D}\right)_{max}$.

And the endurance is maximized for jet engine aircraft if you fly at $\frac{L}{D}=\left(\frac{L}{D}\right)_{max}$ So, in other words, we have to now find out the value of $\left(\frac{L}{D}\right)_{max}$ because that becomes a very useful parameter. This is something we do not know in fact, this is our target we are going to use this c loiter and $\left(\frac{L}{D}\right)_{max}$ and $\left(\frac{L}{D}\right)_{loiter}$. So, the values of SFC in loiter and in range a cruise as well as $\frac{L}{D}$ in loiter and in cruise.

We have to calculate the value $\left(\frac{L}{D}\right)_{max}$. So we will take a short break here. And in the next section when we are back we are going to see how to estimate the value of $\left(\frac{L}{D}\right)_{max}$ for an aircraft, because that is needed in the calculations. Thanks for your attention we will now move to the next section.