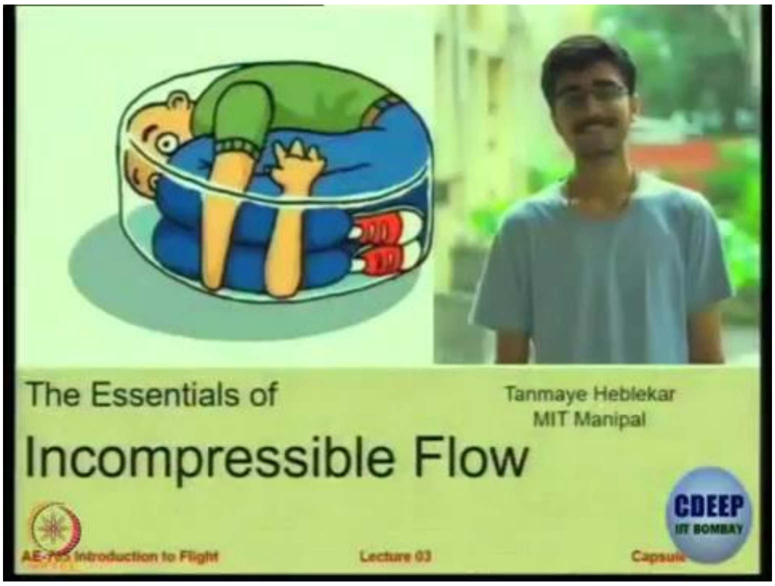
Introduction to Flight
Professor Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Lecture 03.1 - Essentials of Incompressible Flow: Part I

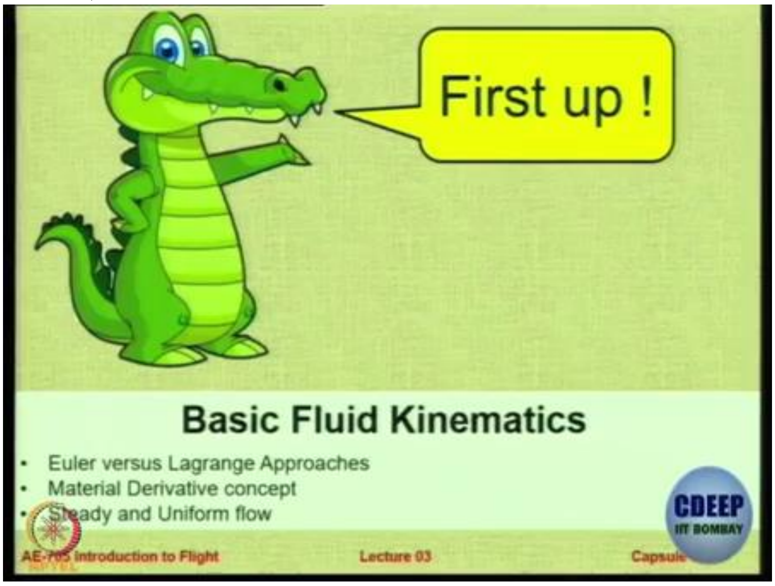
(Refer Slide Time: 0:16)



Today we start Capsule number 2 of this course which will have two presentations, two lectures. The first one today will be on essentials of incompressible flow, and the next one would be on three important concepts; the Bernoulli's and Coanda effect and Mach number, so this constitutes part 1 of the basic dynamics.

This particular presentation was prepared by Tanmay Heblekar from MIT, Manipal. As I mentioned he is the one who spent 2 months this summer working for us.

(Refer Slide Time: 1:01)



Let us have a look; I just want to first give you basic heads-up on what all we will cover. We are going to cover only very basic things and it would be limited to fluids and also we will look only at kinematics of fluids. So we cover this topic in 3 bits, the first is we look at the difference between the two different approaches which are named after two scientists, Euler and Lagrange. We also look at a concept of material derivative or substantial derivative and we see how it can be applied to basic fluid kinematics and finally we will define and understand the concept of steady and uniform flow by demonstrating unsteady flow and its features also, so these three are what I plan to cover today.

(Refer Slide Time: 2:03)

So what do you see? In this figure what do you see? Yes.

Student: Randomness.

Professor: Right, the question I want to ask is what do you see in this, so one gentleman said he sees randomness. Okay, I think there is some order also; many of these vehicles are in the same line. Anybody else can add and explain what do you see, yes?

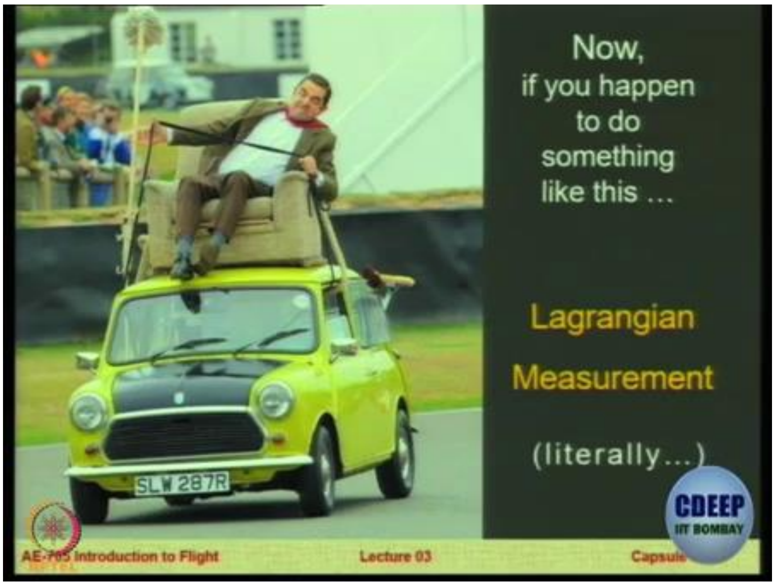
Student: Heavy traffic.

Professor: You see heavy traffic okay, but is the traffic moving? No, ok. So let us assume that these vehicles, so they are not moving because the red lights are on, that is the brake lights. But if we assume that this is a multitude of moving cars, let us imagine that each vehicle becomes a fluid particle, as you can see there are different sizes, similar but different shapes, ok.

(Refer Slide Time: 2:58)

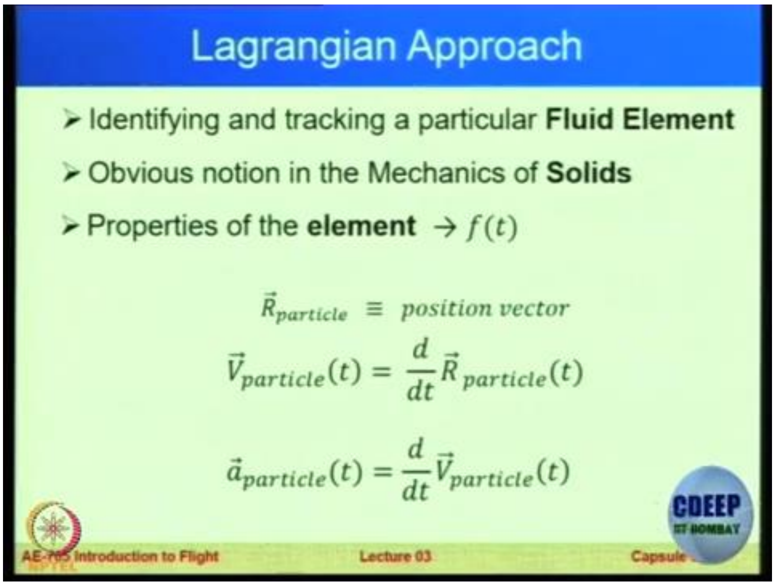
And let us focus our attention on only one of these vehicles; just on one of these mass vehicles we focus attention.

(Refer Slide Time: 3:05)



We zoom close into it and what do we see? We see there is a single car ok, but here fortunately or unfortunately there are no neighbours here but we have removed the neighbours and we are focusing on this particular car. So if you focus on one such vehicle in a moving traffic this approach is called as a Lagrangian approach, when your attention is focused on a particle that is moving that you call as a Lagrangian approach. Ok, how do you, ok, I will talk about it later.

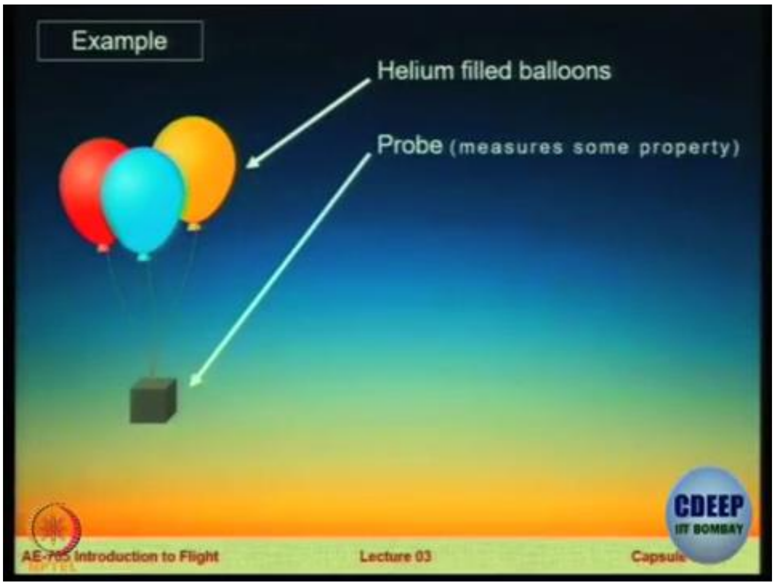
(Refer Slide Time: 3:41)

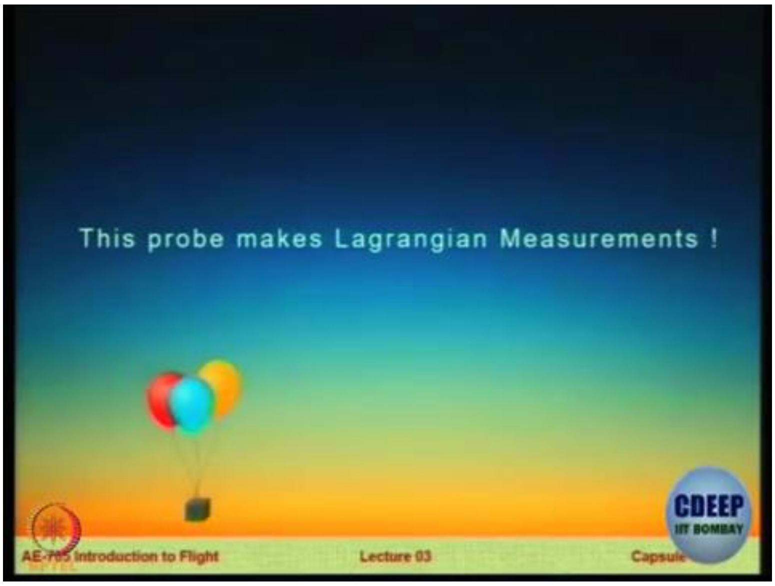


So literally identifying and tracking a specific fluid element ok, now this is an obvious notion in the mechanics of solids because in your basic classes whether when you were undergraduate or whether in the previous courses you must have seen that this is the kind of notion which is very obvious in solid mechanics. The properties of this fluid element they are a function of time they are not constant, they need not be constant, we assume that they are a function of time. You cannot say function of space because it is moving, so the space is moving along with it, the space around it is not constant. So therefore the properties are only a function of time.

So the position of the particle is a vector because it has a magnitude and direction, magnitude in the sense of the coordinate frame and direction so it is a position vector, then we have the velocity of the particle as a function of time T that would be the derivative of this position over time. Remember it will not remain the same depending on the signal depending on the traffic, depending on what is possible velocity will change as a function of time and the acceleration also will be in general a variable which is a derivative of its velocity over time, so this is a Lagrangian approach.

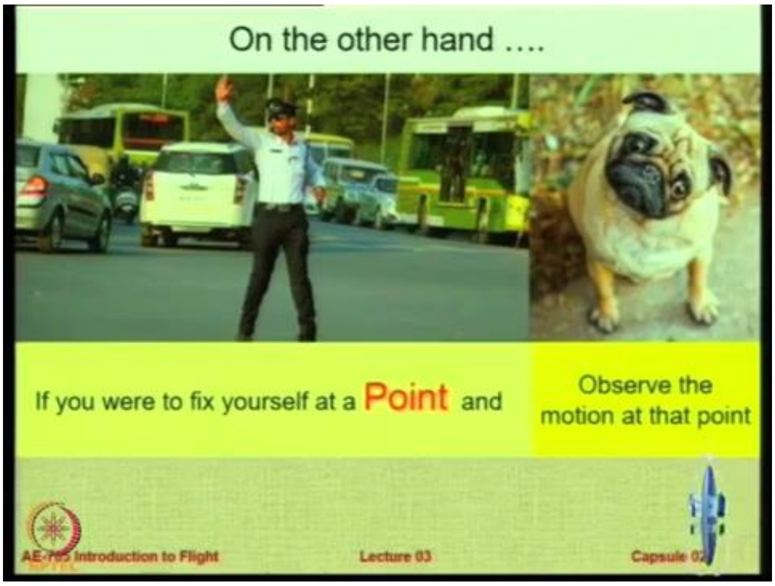
(Refer Slide Time: 5:18)





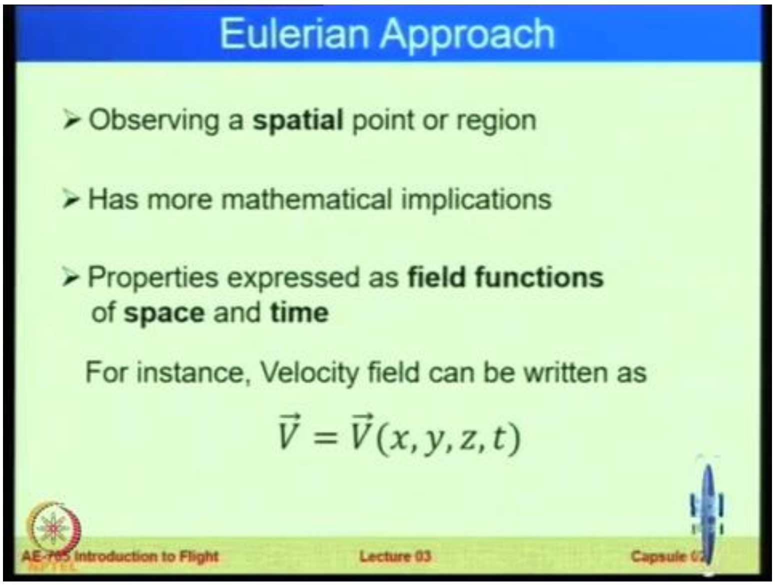
Now let us see another example of a Lagrangian approach. Assume that we have these helium filled balloons which are carrying a small payload, the payload could be a probe or a sensor which measures some property, it could be for example humidity or temperature or pressure, any parameter. So now what happens to this particle is that as a function of time it is moving ok, so where is our attention focused? The attention is focused in this example on the specific object that is moving in space, so therefore this probe which measures any property, it is making Lagrangian measurements, it is measuring a property over a function of time.

(Refer Slide Time: 6:20)



So now let us look at on the other hand, let us say you are a traffic policeman and now you are standing at one point and you only observe what is happening at that specific point over a period of time, such an approach would be called as the Eulerian description or Eulerian approach.

(Refer Slide Time: 6:49)



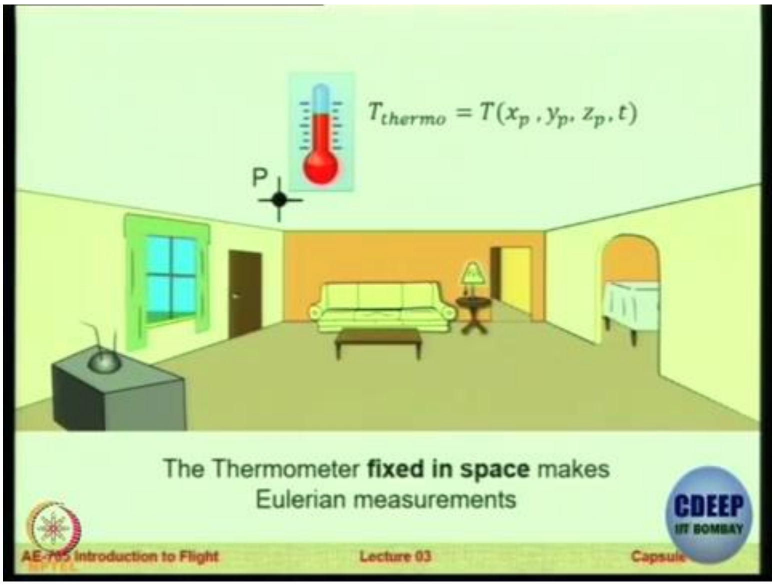
So let us understand, in an Eulerian approach our attention is focused on a particular place for example, the main gate just outside the main gate you are focusing only there and then you see a stream of cars, vehicles passing by. Now this particular approach has more mathematical implications, the previous one had more physical implications because that is what we studied in school, etc, we say two cars are moving etc etc, they were all Lagrangian approaches. This one is mathematical because the properties in this approach are expressed as field functions of space and time.

Space because it is a fixed location, time because over time the properties over that particle are observed, they may be fixed they may be changing. With this I want to ask one basic fundamental question, when is a function defined as a field function? There is a mention here of a field function, so before you answer remember our two rules, one is to raise your hands and second is to grab a mic before you answer if you are chosen to answer. So who will help me define field function? Yes.

Professor: Perfect, very good. If there is a function and that function is defined everywhere in the space, there is no region in the field or space where it is undefined, there are no holes in the domain. Its value may be the same, it may be variable, we do not care but it is defined so if the function is determinable at any arbitrary point in the field we call it as a field function, most physical phenomena tend to be field functions. So therefore it is easy for us to say okay at any point you can define it. So then look at for example the velocity field, the velocity field can be written as a velocity V which is a vector which is the function of the location in a Cartesian coordinates x y z and time, so it can vary as x varies or y varies or z varies or t varies or any of these vary, correct that is a velocity field.

$$V = V(x, y, z, t)$$

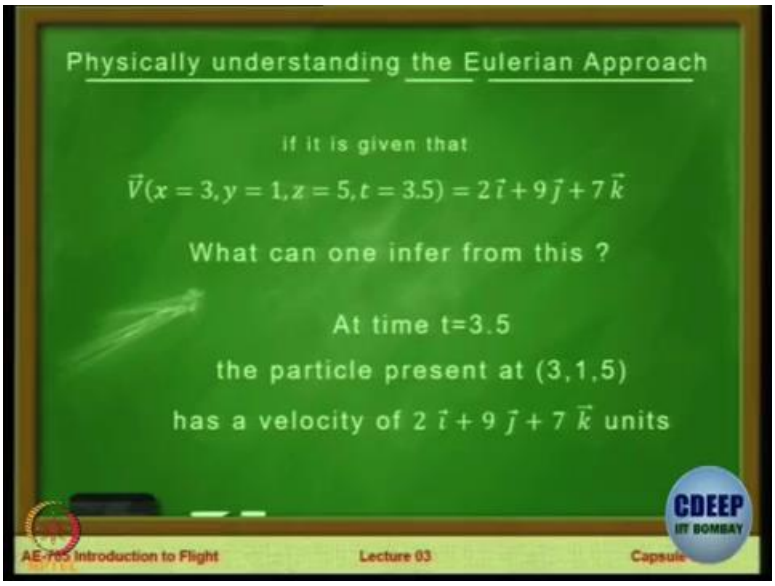
(Refer Slide Time: 9:28)



So an example, this is a typical drawing-room. You have a fixed point in space P, at that point I install a thermometer so the thermometer that is fixed in space is going to measure only Eulerian measurements. If I record the reading of this thermometer over a day I can get the value of temperature at that point xyz as a function of time, typically it will increase then become less etc, etc okay. So the T of the thermometer which is the field function everywhere it is defined along this field, is there a place where it is not defined? It is defined everywhere it is a function of x, y and z coordinates of that point P and time T, ok.

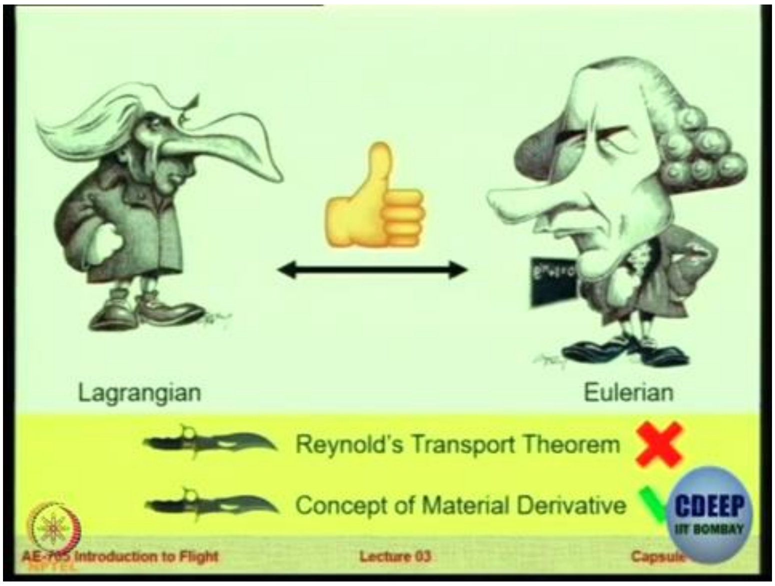
$$T_{thermometer} = T(x_p, y_p, z_p, t)$$

(Refer Slide Time: 10:28)



So we do not have a blackboard, I thought we will use blackboard as an example ok. Let us understand the Eulerian approach physically, so let us assume that for a particular scenario you define the velocity at the point 3, 1, 5 in space at time t equal to 3.5 units, it could be hours, seconds or milliseconds or nanoseconds, just define this velocity as $V = 2i + 9j + 7k$,so it is a vectorial representation in terms of i, j, k, what can we infer from this? In fact what we can infer I have already told you, so at time t equal to 3.5 the particle is present at 3, 1, 5 it is present and the velocity is, $V = 2i + 9j + 7k$ units, that is all we can infer. Can we make some other inference also? Apart from these 3 important inferences is it possible to make any other inference if we give V x, y whatever? No, I do not think anything else can be concluded.

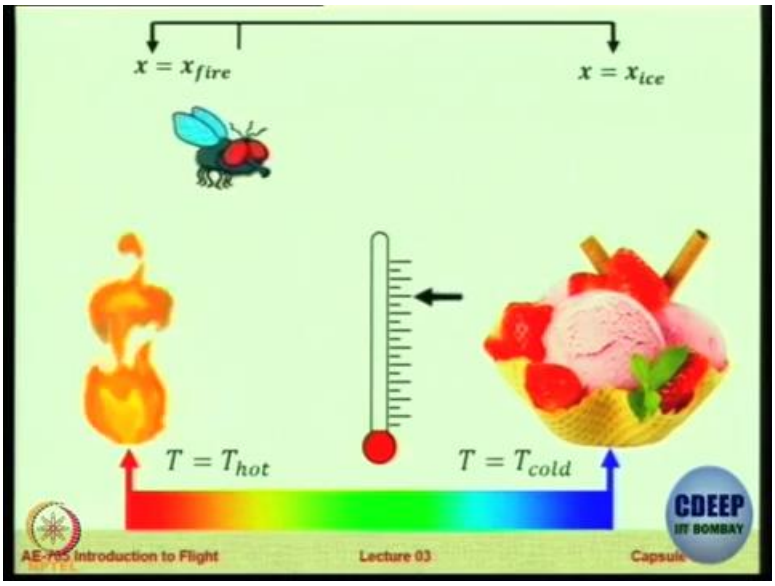
(Refer Slide Time: 11:44)



Now we have these two scientists fighting for our attention, we need to understand the difference between their approaches and why both of them are useful and important for us when we study fluid mechanics. So often people get confused and they get forgetful, so when you have your quiz for capsule two, if there is a question about Eulerian approach you will say "Kaun tha yaar Euler?" Was he the policeman or was he the car moving in the traffic? So how will you remember? To remember certain things which are very confusing and similar you need to use imagination or you need to tell yourself some interesting way of remembering so I will share with you how I remember.

I know that one of them concerns moving, the other concerns fix, so this Lagrangian has a word "range" inside and range for me brings in an aircraft the range means moving from A to B, so I remember Lagrangian as moving and hence Euler is a policeman that is how I remember it okay, you can have your own mnemonics your own way of remembering. So basically these two are connected, we can easily connect them and one way of connecting them is called the Reynold's transport theorem but we will not study that today, we will use the other one called as the Concept of material derivative ok. So once again you can understand the connect between these two scientists and their approaches using two methods there could be more, but today we will discuss only the concept of material or substantial derivative to see how these two are connected.

(Refer Slide Time: 13:34)



So let us see, if I ask once again what do you see? There is a place which is having some fire, so when you see fire which property do you get along with it? Wherever there is fire there is…

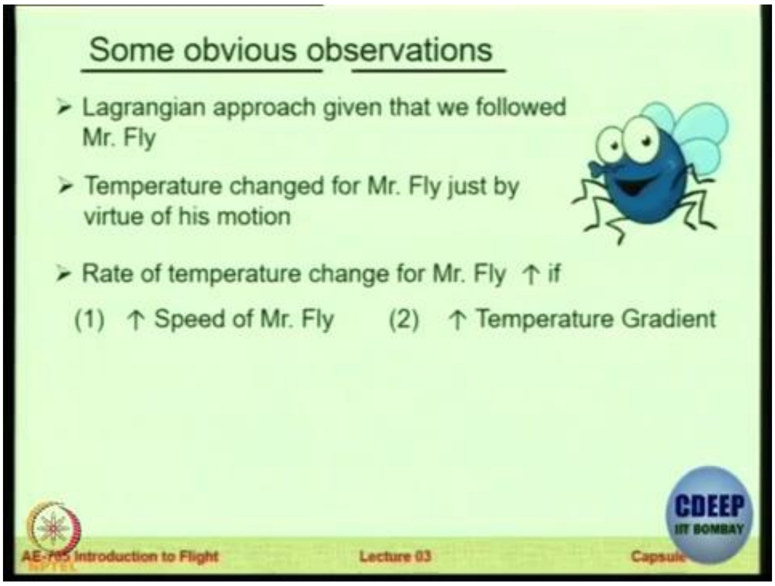
Student: Temperature.

Professor: Temperature is everywhere, even in this room there is a temperature but we are not on fire. You can say high-temperature, heat, which causes temperature ok, and the other thing is an ice cream. Which property? Do not say that Mango delight or Tutti-frutti or Peppermint when you say property, I am not talking about that property, what property or what thermodynamic property do you immediately get in your mind when you look at an ice cream? Again temperature but low-temperature, so from hot to cold it is the temperature field. So let us see this is the field and we have used this colour scheme to indicate the temperature, so blue being cold, red being hot so there is a temperature gradient. The temperature is reducing as per some particular law which need not be linear but there is some relative reducing. So there is one place at which the temperature is T is equal to T hot.

T hot is a number, it could be 512 degree Kelvin and there is another temperature T equal to T cold. So in this particular field from a hot place to a cold place in this field temperature varies. Now let us look at one bumblebee, now why I have chosen bumblebee as an example? Because as per the basic laws of Aerodynamics a bumblebee cannot fly, but a bumblebee flies. So that is why it is interesting that if you look at the basic description of aerodynamics forces then a bumblebee should not be able to fly but bumblebee does not believe in Lagrange or Euler, it flies ok. So there is a bumblebee which flies, it flies from this particular place x which is called as x fire to x ice, so two locations in space in which a bumblebee is flying.

And what we do is on the bumblebee we put a small temperature sensor and because it is difficult to read that particular scale, bumblebee is small, the scale has to be also very very small and lightweight so I have just taken a projection in the centre. So as the bumblebee moves you will see the arrow moving in that particular temperature scale. So it as it moves from there as a function of time the temperature recorded by the sensor mounted on the bumblebee is slowly recording a reduction. Here we assume that the sensor is having instantaneous probability that within few nanoseconds it is able to measure and display the temperature that is actually prevalent. In real life it will take time, typically if you put a thermometer on bumblebee by the time it reaches it will still show high-temperature because there is some soaking time needed but let us ignore those practicalities here. So the bumblebee experiences a temperature from T hot to T cold varying at a particular rate when it goes from a place x fire to x ice.

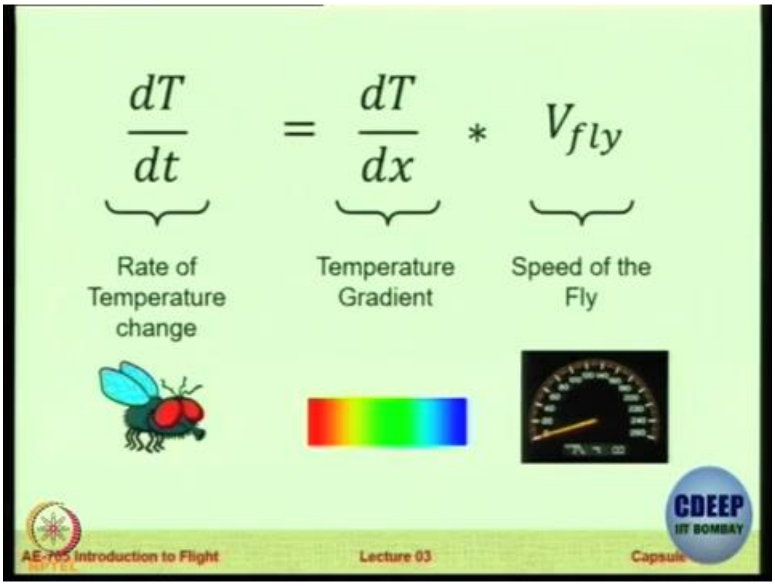
(Refer Slide Time: 17:23)



So let us now look at some basic observations. Since we are now focusing along a line ok, remember the sensor is mounted on the bumblebee but the bumblebee is moving, therefore the recordings or the approach for Mister Fly is basically a Lagrangian approach. Then the temperature changed simply because of its motion and change in location, so it is a property which is a function of its location. Obviously, the rate at which the temperature changed was not necessarily a constant. In our example, probably you must have seen it as a linear change but it need not be so, we do not know. And also the numerical value of that rate can also be altered. If the fly moves at a very fast speed the rate will be different from when it moves very slowly.

Secondly, the field may have certain elements which are sucking or expanding heat therefore the temperature gradient if it changes, the rate of temperature change also will change. So there are certain properties of the field which affect the rate of change of temperature, one of them is the speed the other one is the gradient. So now we have seen an example and we have to now put this down mathematically because something like this happens to fluid particle also when it moves in a field. So let us see how to do it mathematically okay, so what we say is that the time rate of change of this property called temperature and here temperature is a scalar.

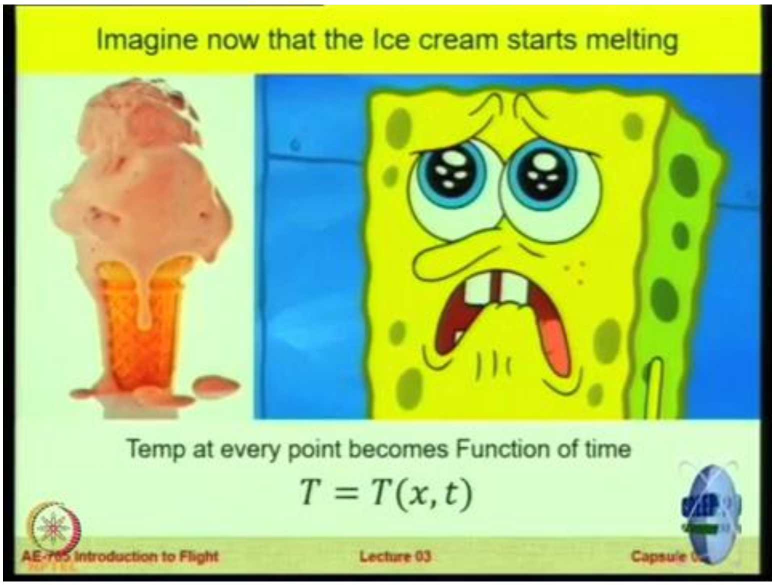
(Refer Slide Time: 19:28)



It is just one particular parameter that is equal to the rate of change of the temperature gradient as you go from this place to this place plus the velocity. So the temperature change as a function of time will depend on both the things, the velocity of the fly, so suppose the velocity of the fly is 0 and the fly remains at that particular place, so do you think if you are standing over fire and there is an ice cream somewhere nearby and that ice cream is now melting as a function of time, the temperature will not remain same okay. So there will be a change in the temperature by virtue of time also, there will be a change in temperature by virtue of speed also, so both have to be considered if you want to get the complete and total change of temperature as a function of time.

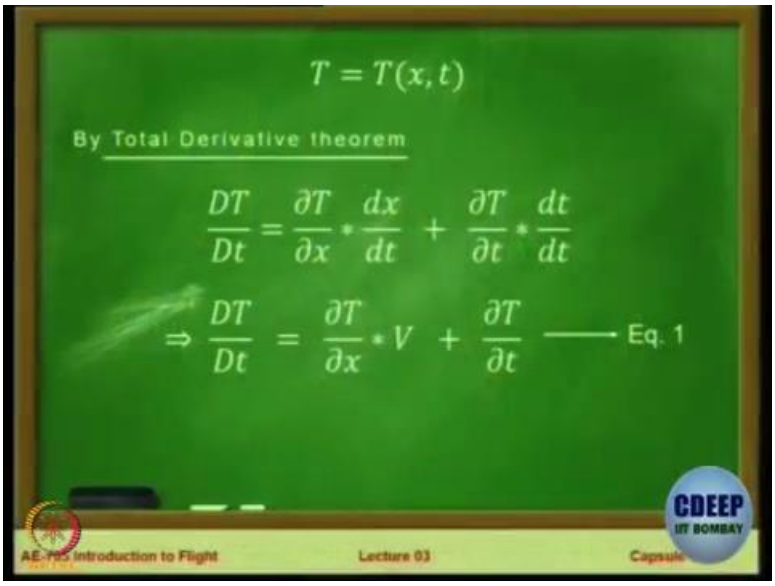
$$\frac{dT}{dt} = \frac{dT}{dx} * V_{fly}$$

(Refer Slide Time: 20:25)



So now yeah, so after some time the ice cream starts melting that means there is some heat transfer happening. So the temperature at every point, the ice cream melting will not only locally change the temperature but may be less or more but everywhere there will be a change in the temperature, it could be very less very far away but there will be a change. So at every point in the space now the temperature becomes a function of time, so T is the function now of x the location of the fly and t the time at which the recordings are being made.

(Refer Slide Time: 21:05)



Back to the green board, so temperature T is the function of location and time. So there is something called as the Total Derivative Theorem which says that if a parameter such as T depends on two parameters then if you want to find out the total derivative of that particular property or that particular item which is capital T, you can take it as the partial derivative of that with the first parameter that is x the distance and then x with t. And then you have to add the partial derivative of that probably with time into DT by Dt, so this is called as the Total derivative theorem, this is true for any property such as temperature which depends on more than one parameter. So now ultimately what will happen is so total derivative of temperature with time would become partial derivative of temperature with x into velocity because V is dx by dt change in velocity plus partial derivative of T upon time, with time sorry, and that is all. So the total derivative will be a sum of two partial derivatives ok.

$$T = T(x, t)$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial x} * \frac{dx}{dt} + \frac{\partial T}{\partial t} * \frac{dt}{dt}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial x} * V + \frac{\partial T}{\partial t}$$

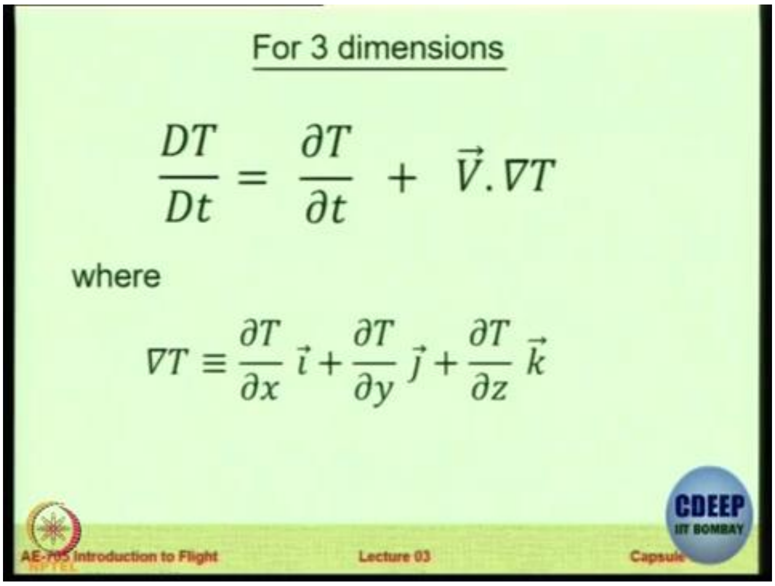
Any questions anybody has you can interrupt in between. It is actually fairly straight but if this is true then you must tell me under what conditions is this exactly true. If you have understood this derivation then you should also tell me, this is not universally true, the way it has been written is not universally true. So for which condition is this true? Yeah.

Student: Sir, it must be an isolated system that is no heat exchanges with the surroundings.

Professor: Okay fine, so it is a system which does not exchange any heat from outside to inside, yes that is acceptable because otherwise there will be an effect of the outside temperature on the temperature variation inside. Other than that any other constraint? The way it is expressed is true only in a particular condition, it is….yes.

Professor: Which means one dimensional flow correct, the question one is valid only for a one-dimensional situation when there is no variation with y and z ,ok. So now let us open it up, from a flat blackboard make it a 3D room and allow variation with x, y, z and t which is what we started with.

(Refer Slide Time: 23:57)

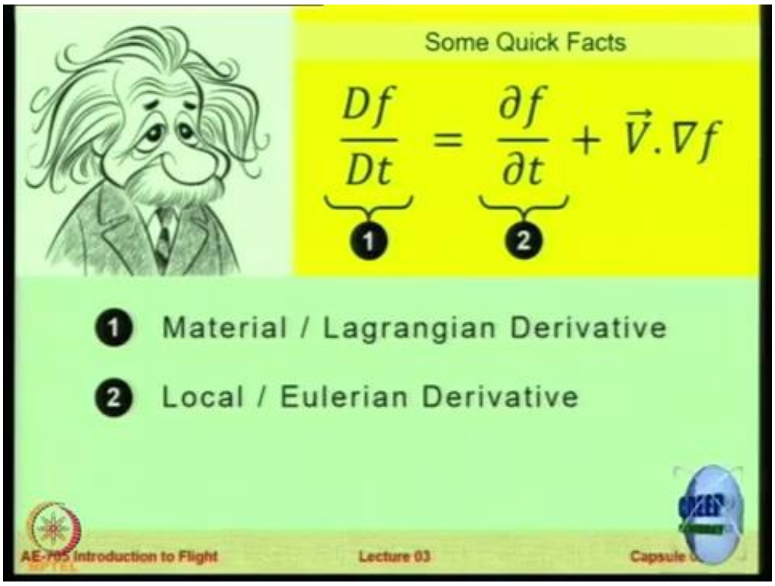


So for three-dimensional cases it will become partial derivative will become, partial derivative of T with time which does not change but now V will become the V vector dot Delta T Del T ok. So where Del T will be partial derivative of T with x into i plus T with y into j plus T with z into k, so extending the dimensions it becomes partial derivative of temperature with time plus V dot Del T ok.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + V.\nabla T$$

$$\nabla T \equiv \frac{\partial T}{\partial x} i + \frac{\partial T}{\partial y} j + \frac{\partial T}{\partial z} k$$

(Refer Slide Time: 24:28)



So now I replace the word capital T with some small function any function f. For any field function the total derivative of that function with time would be its partial derivative with time plus V dot Del f, so what are some interesting observations here? What is this? What will you call this point number one? What would you name it? Yeah.

Student: Total derivative.

Professor: This is the Material or Lagrangian derivative or Total derivative; there are many names for it. So because it is called as material derivative or Lagrangian derivative it is a name okay, what would be this?

Student: Local derivative.

Professor: Local derivative or the Eulerian derivative. So local means policeman that is Eulerian, total or complete is Lagrangian. So we are now saying so Lagrange is more than Euler always, not always, in some cases when V dot Del f is 0 it will be the same. So now the last one is very clear, the middle row what would you think is the number 3? What do you call this term? Yes.

Student: Space derivative.

Professor: Space derivative, where is the space coming there? No, it is a transport derivative or convective because it is because of moving. So the total derivative is equal to a local derivative plus a transport derivative or a convective derivative. This is how we will define basic fluid flow, so yes.

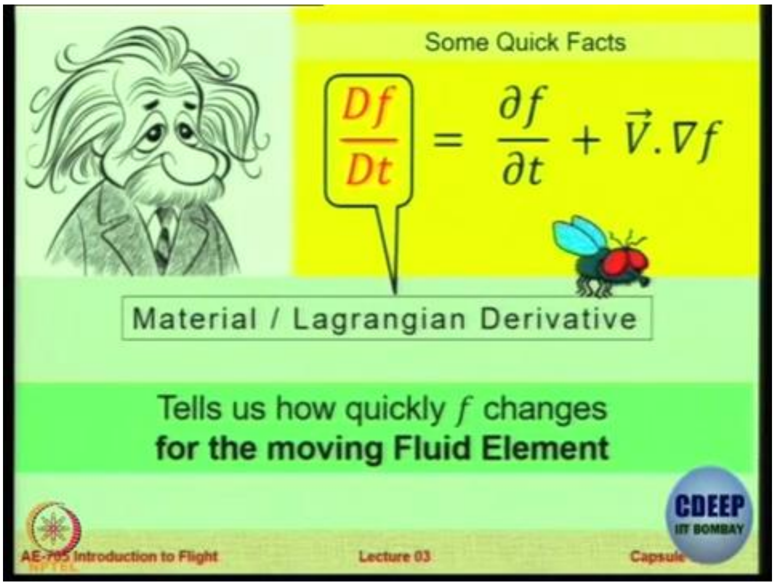
Student: Sir, if we take f as any function any field function then what is V?

Professor: V is a property which is associated with that field function of the fluid particle. It could be viscosity if you are looking at viscosity changes, in this case we have looked at changes in the distance or location. So in general the third term depends upon the convection or the transport of some phenomena, some parameter in that particular field ok. Remember it could be 0 also, so that is where how we will define steady flow, etc, ok.

Student: It is not necessary that V should be must a velocity.

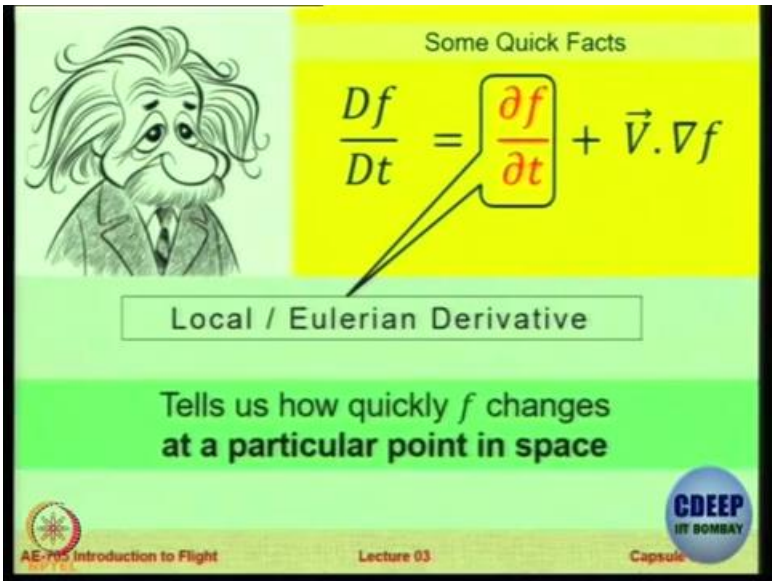
Professor: No no, it is not necessary to be only velocity it is just example okay, but in most with particles it is only velocity. But if you look at magnetic hydrodynamic modelling of flow, it may be some other particle. But when you have something moving some particle moving in a flow field then it is velocity only. So in general you can say it is some convective property, some property because of which there are changes happening over a function of time which is velocity in most cases for fluid particles. Good, I really like these interruptions; these interruptions are helpful in clarifying even to me how I can add more information in this presentation for future batches, alright.

(Refer Slide Time: 28:16)



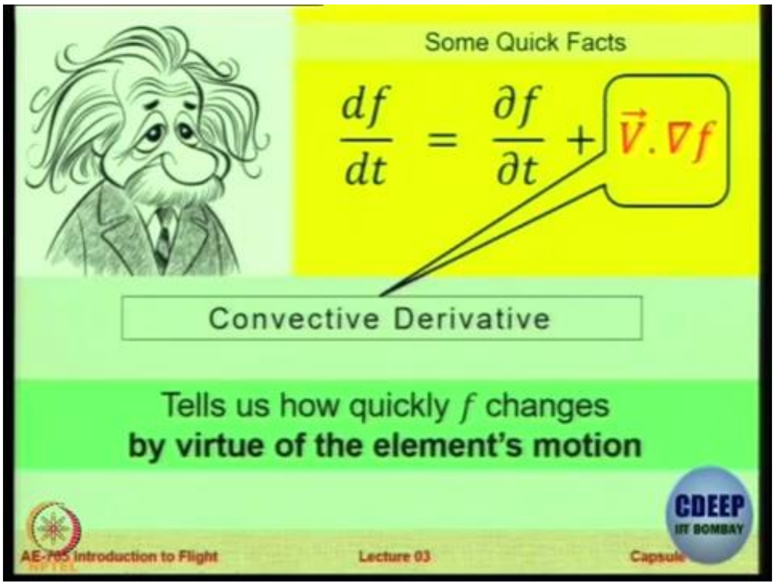
So this is the material derivative, what it tells us is how quickly this field function f changes for the moving element. So this is the total story.

(Refer Slide Time: 28:32)



The partial derivative or the local derivative or Eulerian derivative it tells you at a particular place what is happening because it is a function of only time at a particular space.

(Refer Slide Time: 28:46)



And the convective derivative tells you by virtue of the elements motion in this case by virtue of motion how f is changing, there could be some other class in which we are talking about some other not fluid mechanics but some other mechanics and there the same equation might represent not motion but something else ok. So whenever there is motion related changes, velocity will come into picture so the definition is very general.

(Refer Slide Time: 29:20)

So now we are fully armed, we have all the required mathematical support to proceed ahead with the definition of, remember what is the whole purpose of this exercise? To define what is meant by incompressible flow that is our chapter, we are here to look at incompressible flow and to define what it is. Ok now when we come to this point I am reminded this is a very interesting aircraft and it is fully armed as you can see, so just a small deviation. Can someone tell me what is this aircraft? Name the aircraft first, yes.

Student: A10 Warthog.

Professor: Yes, it is the A10 Warthog. So do you know any special features about this aircraft, ok?

Professor: Right in the front there is a gun ok.

Student: It is used for ground attack in US Air Force.

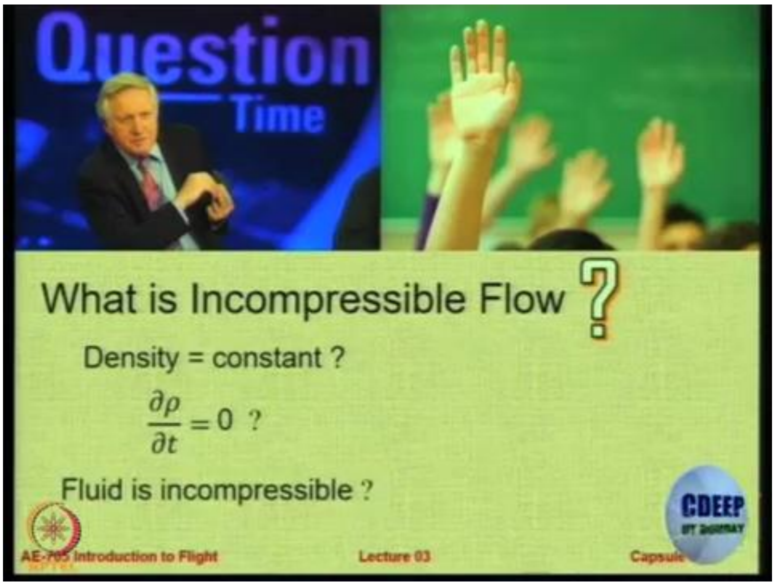
Professor: Yes, not only US Air Force yes ground attack, so it is basically an anti-tank aircraft, an aircraft designed for attacking the ground forces right, what else?

Professor: Right.

Student: Basically it is the aircraft built around it.

Professor: Exactly, I am going to say that very right, it is essentially an aircraft built around this canon air to ground canon right in the nose, yes and there are many many other interesting features ok. So now I am trying to now sell myself, if you ever attend a course in aircraft design that I would teach in the future, I will talk about this aircraft in great details because this is one of the very good examples of how you can design an aircraft for combat survivability. So we are fully armed, we have A10 with us, let us go ahead and ask some questions to understand. This will tell you, this will tell me rather and also you yourself how much do you know.

(Refer Slide Time: 31:20)



So the question is, what is incompressible flow? Is it a flow in which density remains constant? Is it the flow in which the rate of change of density with time is 0? Or is it when the fluid involved is incompressible? In which situations can we term the flow as incompressible flow? So now just like a quiz all three maybe wrong, all three may be right, two may be wrong, etc, etc, so let us see if you can answer this question. When do we define a flow as incompressible flow? So how many of you feel that the flow is incompressible when density is constant? Yeah, we have one hand raised there.

Student: My name is Aruno. When some pressure is applied on the fluid, if the change in density is insignificant I mean less than 10 percent or something so we can call that fluid incompressible.

Professor: That means using partial derivative of Rho by partial derivative of time is equal to 0, that is the answer you are giving? That is what you said?

Student: No, it will not be 0; it will be close to 0.

Professor: Okay, almost 0 okay.

Student: Almost.

Professor: When it is almost 0, fine that is his point of view. Yes, we have another person here, name please?

Student: My name is Rahul. We call the flow as incompressible when the density does not change with respect to the space as well as with time.

Professor: So density is equal to constant with respect to space and time, then it is incompressible, okay that means the first choice density is constant, any other? Anybody else would like to add to this?

Student: I am Kavita, I think the density here, incompressible flow is the one in which there is a change in density but it is less than 5 percent.

Professor: So you are again collaborating what he said that there is a change in density but very small.

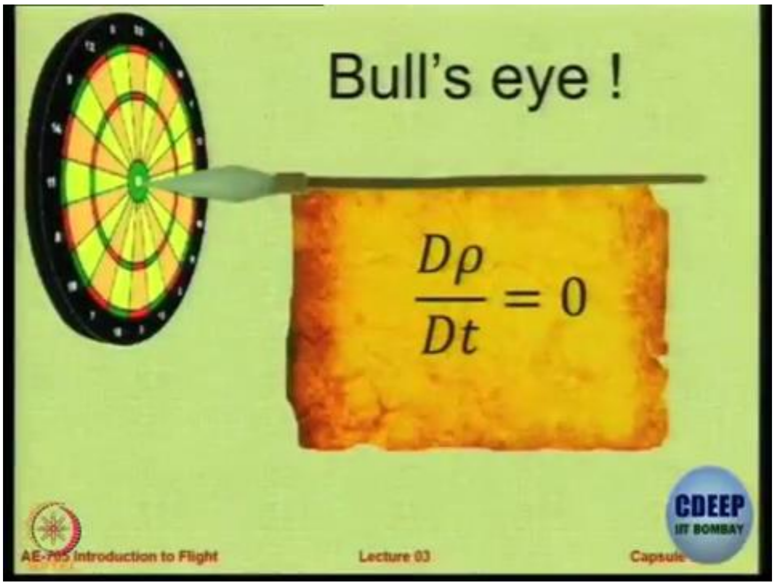
Student: Very small.

Professor: That is when you call the flow as incompressible flow.

Student: Yes, sir.

Professor: Ok, this is what we have all come up to believe so far, so I want to trash that myth. Let us see what is the, that is not exactly the truth statement. Let us see.

(Refer Slide Time: 33:41)



Basically it should be the total derivative to be 0, so that is where you stand correctly. It is not just density remains within 5 percent, all these answers are true only for 1 dimensional flow or for steady flow or for, yeah so basically if you say in a steady flow then I understand your points are correct. But the correct definition is that the total derivative, the substantial derivative, the material derivative whatever you like of density with time should be 0 only then the flow is actually incompressible flow.