Introduction To Flight
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Lecture 11.8
Tutorial on Range and Endurance

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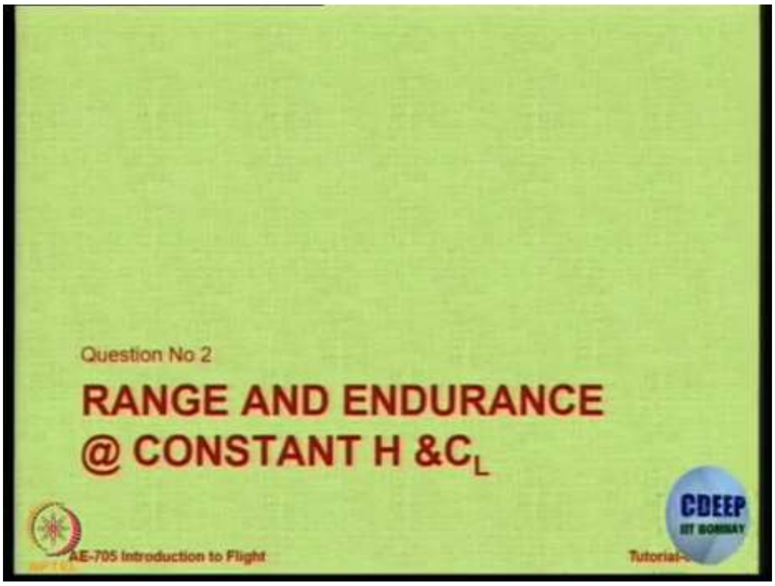

Let us go to question number 2, this is on range and endurance. As I mentioned to you, if you look at the standard equation for range. Okay It is $\frac{V}{C}\frac{L}{D}\int_0^1 \frac{dW}{W}$ or sorry it will be integral $\frac{V}{C}\frac{L}{D}\int_0^1 ln\frac{dW}{W}$, and now depending on them which of them are constant you take them out. So we will look at an example where the height remains constant and CL remains constant and you are cruising okay. So this is also a live example of a turbojet engine aircraft. First there is some data about the aircraft, so wing loading is given the thrust SFC is given, cruise fuel weight fraction is given, aspect ratio, Oswald efficiency, C_{D_0} parasite drag coefficient and the cruising speed is all given to you. So this number you can note down because you will need them now for your calculation.

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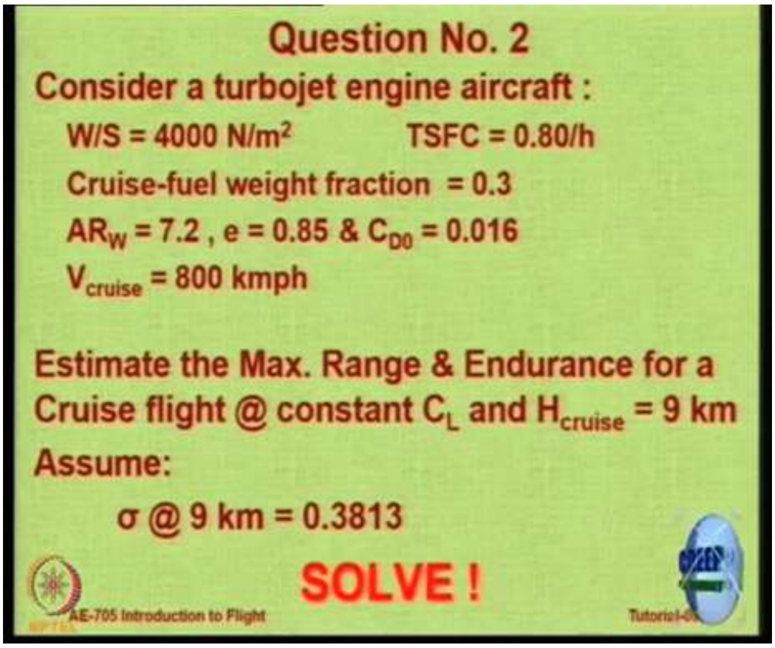

And remember the aircraft turbojet engine so normally the TSFC is given in terms of amount of fuel consumed per thrust produced per time, but if the amount of the fuel consumed and thrust are in the same units if let say both are in Kgs, you can cancel them so the units become per time and in this case they have given the units in per hour. Okay?

So this is the question, if flying at 9 kms at 800kmph and the pilot now decides to go for the range with CL constant and H cruise constant, so now as the aircraft travels, the aircraft weight will go down because fuel is consumed right. So what will happen if I want to maintain the same altitude? What will happen? What does the pilot have to do? Decrease the? when you decrease the speed will the CL remain the same? It depends on angle of attack. Okay? So does the pilot has to change the angle of attack during this flight? Remember, we want CL to be same and H to be same, the moment you change the angle of attack, CL will change, because for every given angle of attack there is a fixed CL value, as per the alpha CL curve so alpha cannot change, right and H cannot change.

So interestingly you will change the velocity to maintain level flight but you will continue to fly at the same angle, because the aircraft weight requirements are reduced. Alright. So for your convenience the value of sigma is given at 9 kilometers so that the density at 9 kilometers will be 0.3813 into 1.2256 kg per meters cube assuming ISA conditions which we assume unless it is mention, so now can you solve do you remember the equation for constant height, constant CL?

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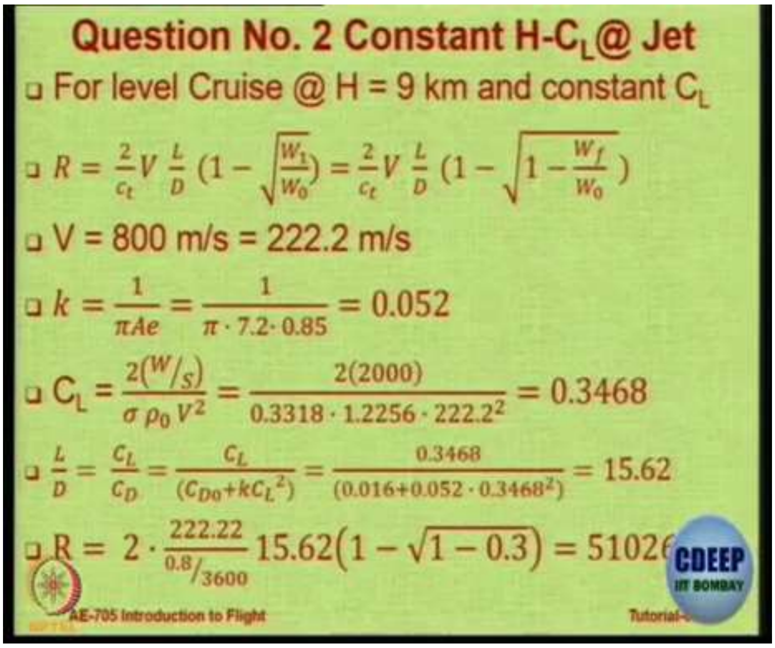

For your information this is the equation, so the range will be

$$R = \frac{2}{C_t} V \frac{L}{D}\left(1 - \sqrt{\frac{W_1}{W_0}}\right)$$

but W0 is actually the maximum takeoff weight which is W1+Wf or W1=W0-Wf, and it is given that the cruise fuel fraction is 0.3 that means Wf/W0 is 0.3, so that is why you can rearrange the equation in the terms of the known information. So V is given to you, it is given as 800 meters per seconds, kilometers per hours so convert it in toW meters per second by dividing by 3.6.

K is the induced drag coefficient $\frac{1}{\pi Ae}$, so A is given, e is given you get the value. $C_L = \frac{2W}{S\sigma\rho_0 V^2}$, sigma Rho Zero is the density by definition and L/D will be C_L/C_D which will be $C_L/C_{D_0} + KC_L^2$ you know C_{D_0} is given by 0.016, you know K now is calculated, so L/D is 15.62. Notice this is not L/D max, in fact it will be 0.866 time L/D max then R is equal to 2 into V upon C, now to convert it into the units this is where most people goof up to convert into per seconds because if you notice the units have to be maintained on both sides. So range is in meters, so you have velocity in meters per seconds therefore SFC must be per second not per hour, everything is dimension less, L/D dimension less, weight ratio dimension less.

So therefore by doing this you get the range as 5102, I made your job easy you do not have to calculate anything, I have done the calculations for you. Okay? So the range is 5102.7 kilometers, but now I wanted to calculate the endurance that I will not tell you. So what is the formula for endurance for a jet engine aircraft under this condition, actually any condition? There is a same formula for $\frac{1}{C}\frac{L}{D}$ log of so do it. All information is available in this slide, so $\frac{1}{C}\frac{L}{D}$ log of weight ratio natural log of the weight ratio. So C is the same 0.8 by 3600, $\frac{L}{D}$ is a same 15.62, it will become log of 1 upon 1 minus fuel weight ratio, so the number to calculate is $\frac{1}{C}\frac{L}{D}$, natural log of 1 upon 1 minus 0.3 yes

Student: Sir confused about how the equation for the range came up like not like the ln.

Professor: Correct, this was given to you as a home work, which many people has not bothered to do. In the class I said that the range equation is integral of $\int\frac{V}{C}\frac{L}{D}ln\frac{dW}{W}$, so now what you do is you calculate this for various conditions because there are 3 variables, there is velocity there is CL and there is L by D. okay?

So there are three possible ranges in one the height will remain constant and CL will remain constant. The other one is that we will see that in the next slide, the velocity will remains constant and CL will remains constant but height will change that is cruise climb. And here is a third one in which the height remains constant and V remains constant the angle changes or the CL changes that is more complicated because L by D also keeps changing, as CL changes L by D will change, so if you are really interested in deriving this formula there is a very very good book on performance by professor S K Ojha. So he has actually derived the expressions for all three ranges. Okay? And there you will get so I have just put the equation here for your information because this is something which I wanted you to derive yourself you can still do it. Okay any more doubts? Yes.

Student: We should not keep it constant?

Professor: You have not derived the formula so you do not know, this V is V at the beginning of the mission. Yes V will change but when you derive this formula you derive it in terms of the initial velocity ok. So you are right the value of V will change but when you derive this formula I should have put it as V 0 or V initial. You are right V will change Alright. So what is the value of endurance? $\frac{1}{C}\frac{L}{D}log\frac{1}{1-0.3}$.

Student: Seven or so

Professor: Seven or so it is 6.9 something hours, Correct. Yeah that is right. That is right Okay?

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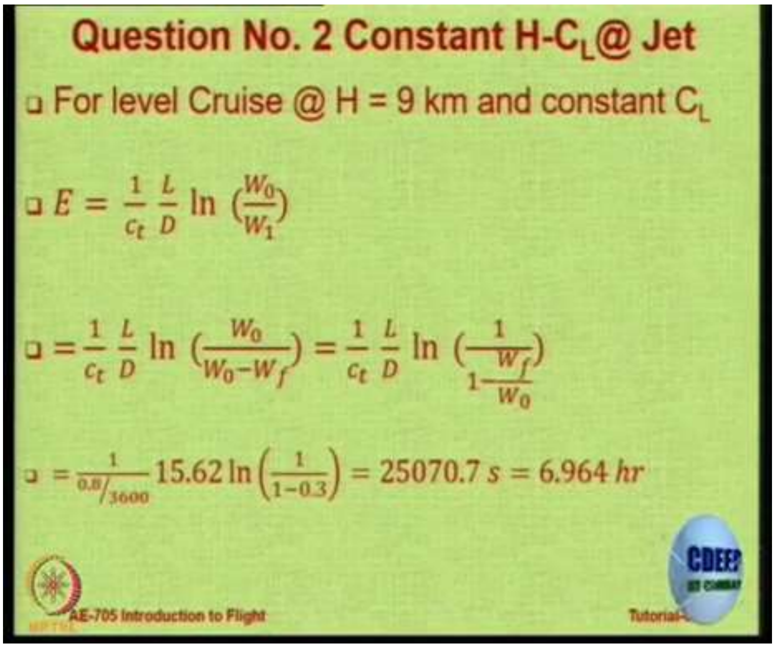

So let's see 1 by C, L by D log of weight ratio which is $\frac{1}{C}\frac{L}{D}log\frac{1}{1-\frac{W_f}{W_0}}$, 6.964 hours. That is right okay so now let us look at question number 3.

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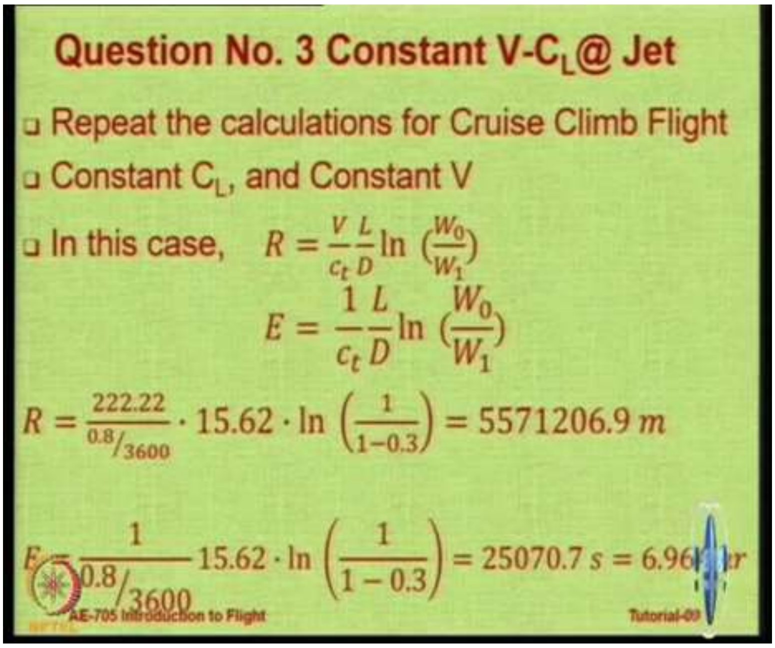

This is now height is changing but V is constant ok and CL is constant so this is what normally the pilot fly, this is called as the cruise climb if they are permitted to fly. Actually, this is what they are the one that I showed you is they are forced to fly because ATC says sorry maintain the altitude. Okay? But in the scenario where the ATC permits changes in altitude the pilots will love to fly this one, we will see the reason why? So we repeat the calculations for the cruise climb flight, constant CL constant V, in this case the formulae are very simple, this is what we are normally used to. So it is just by V by C L by D log of weight ratio and range upon V will be endurance, so 1 by C L by D so E will be the same as last time because everything is same. Just calculate this value now, once again W0 upon W1 will become 1 upon 1 minus Wf by W0, this is what I got, 5571 kilometers, Endurance is same as last time.

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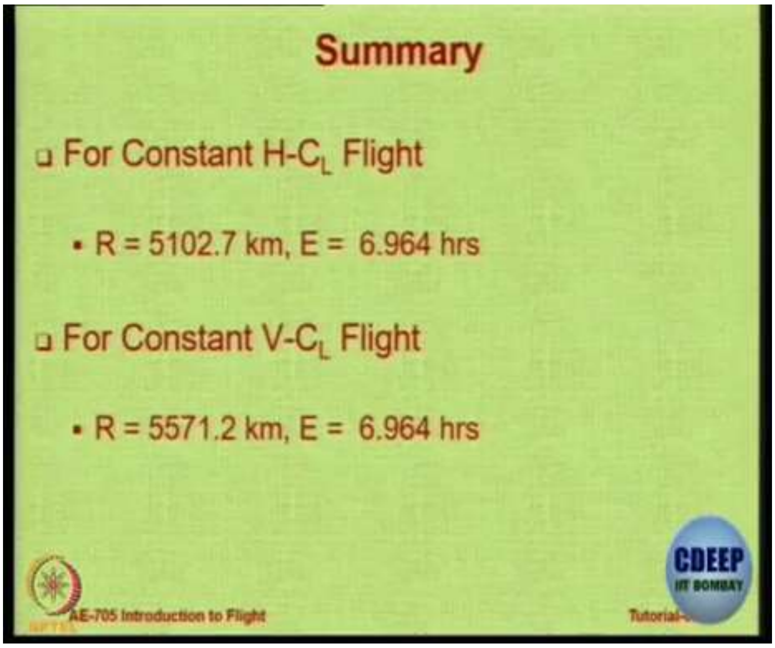

So if we sum up we find that if you have a constant H and constant CL flight then you get 5102 kilometers and you can fly in the air for around 7 hours, whereas if you go for constant V constant CL flight you can actually travel approximately 470 kilometers more on the same amount of fuel, this is the reason why pilots beg for this particular flight profile allow us to climb, and the ATC permits if there is less traffic and if there is operating conditions are such that it can be permitted. But in a very busy traffic where the ATCO's have to constantly monitor they will say sorry you may be little bit inefficient by following constant H constant CL okay. The third option is constant H, constant V, CL changing but that pilot do not prefer because it gives you more inferior value. Okay? And it does not help anybody so that one it is there as one possible scenario but it is not normally used, so pilots are forced to fly the first one, they love to fly the second one, Okay.