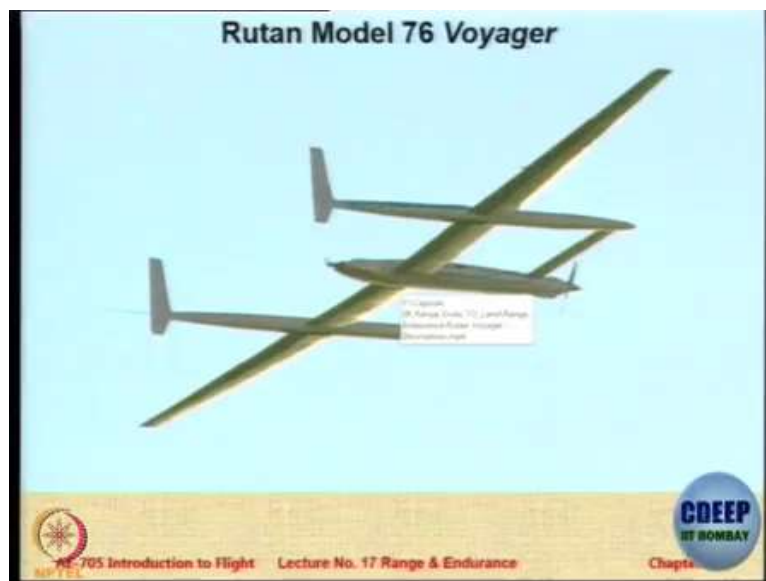
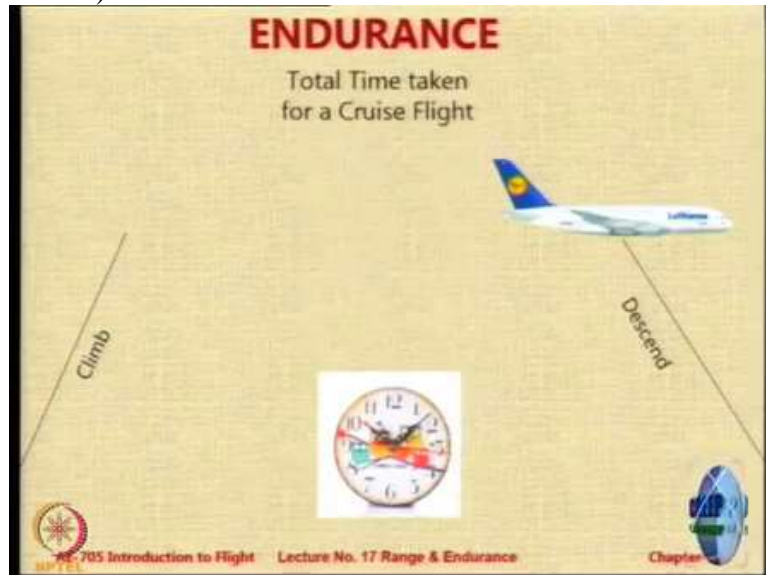


Introduction to Flight
Professor Rajkumar S. Pant
Department of Aerospace Engineering
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Lecture 11.3 – Endurance

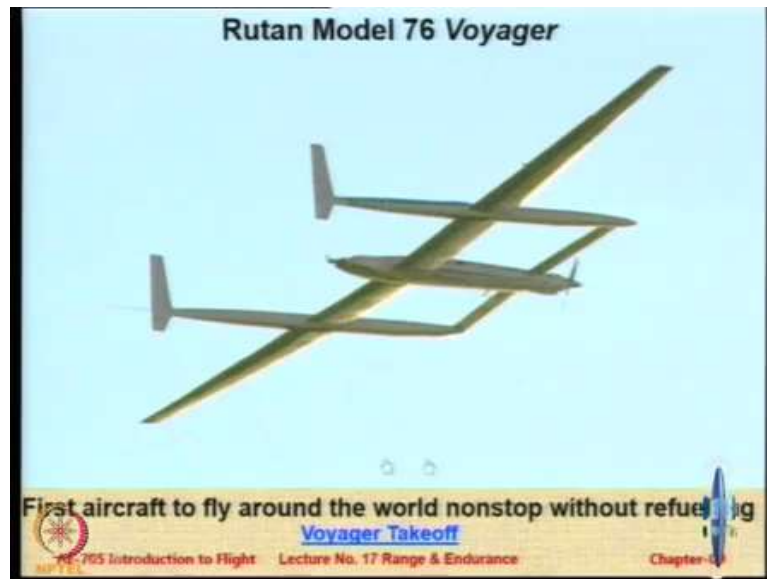
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Okay, now we come to Endurance. Basically endurance is how much time you spend in the air, so this is again a small graphic which shows some climb at the end of climb till before the sense whatever time you spent that is called as the.....

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So let us see a short film about an aircraft that....

Video representation:

You notice, they flew non-stop for 9 days. So they were so particular regarding reducing weight that the toothbrush that they took, they broke it off and said that we do not need the rear portion of the toothbrush, we can hold it with our finger. Okay. My question is, why toothbrush? So that is a different situation. Okay, so many, so many IIT Bombay students are champions in reducing weight.

Now one very interesting thing happened when they were trying to take off and that is the wing actually hit the ground because 76 percent of the aircraft was fuel. In the typical aircraft approximately 20 percent of the aircraft is fuel and the remaining is W_1 , and voyager is the opposite. 76 percent of the aircraft is fuel. So the wings are like this during takeoff just a few inches above the ground and only during the, so I will show you a very short video. We do not have time. This is a very long video, but it is an amazing video.

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You can see, so the designer is Burt Rutan, he is sitting in an aircraft and shooting this film and he is going to take off and follow them for the first four hours. Okay and you just listened to the conversation. It is amazing; Burt Rutan knows that this aircraft is safe after 61 knots. That is the lift off speed. So he tells the pilot, I just want to hear 61 knots because that means you are going to be air borne. You can see the tips. So he says I need to hear 61 knots. There are 13 fuel tanks. You can see the lift on the wings, he is now lifting the wing literally. See the flexing of the wings.

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ENDURANCE RECORD

Duration (minutes:seconds)	Date	Location	Pilot	Aircraft
278:02:44	December 14-22, 1986	Colorado Air Force Base, Colorado/Alaska	Dick Rutan and Jeana Yeager	Rutan Voyager
117:51:00	June 26 to July 2, 2015	Nagasaki, Japan - Kalaheon Atoll, Hawaii, United States (4282 kilometers)	André Borschberg	Solar Impulse 2
84:32:00	May 25-26, 1931	Jacksonville, Florida	Walter Edwin Lees and Frederick Boring	Bellanca J-2
75:23:07	February 26 to March 1, 1931	La Sienne, Algeria	Louise Berthelot and Maurice Pégibet	Bleriot XIb
67:13:55	May 30 to June 2, 1930	Marmarath, Italy	Umberto Nobile and Fausto Cecconi	Savoia-Marchetti S.64
66:25:00	July 5-7, 1928	Dresden, Germany	Johann Richter and Wilhelm Zimmmerman	Junkers W 33
62:22:21.8	August 3-5, 1927	Dresden, Germany	Comtesse Edouard and Johann Richter	Junkers W 33
61:17:25	April 12-14, 1927	Long Island, New York	Clarence Doolittle Chamberlain and Bertrand Blanchard Acosta	W3-2 "Columbus"
45:11:59	August 7-8, 1925	Chartres, France	Maurice Drouhin and Jules Grahb	Farman F-40
37:58:10	July 16-17, 1924	Chartres, France	Etienne Guynet and Maurice Drouhin	Farman F-40
36:04:34	April 16-17, 1923	Wright Field, Dayton, Ohio	Colley George Kelly and John Arthur Macready	Fokker T-2
14:07	October 14-15, 1922	Le Bourget, France	Louise Berthelot and Robert Drouhin	Farman F-40

Alright. So that is again, they use a record on Voyager. But then of course it has been broken by so many other people.

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Now let us look at another very recent record on endurance. This is on UAV now. This is Research UAV from ETH Zurich. So again in the interest of time I am just going to show you.

[Video Presentation]

Look at the this is the hours. This was in 2015. So the aircraft has been flying non-stop for 12 hours 40 minutes. So the battery is 89 percent. 18 hours, battery 54 percent. And now they enter the night flight to save time. See, 3 days, 9 minutes. It is coming into land autonomously. That is a record. So this is the world record for an aircraft less than 50 kilograms of weight flew for more than 81 hours non-stop on autonomous flight. So that is the current world record for endurance.

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GENERALIZED ENDURANCE EQUATION

Definition of TSFC :

$$c_T = \frac{w_f}{T} = -\frac{dW_f/dt}{T} \quad \rightarrow \quad dt = -\frac{dW_f}{c_T T}$$

In steady straight level flight: $L = W, T = D$

$$\rightarrow dt = -\frac{1}{c_T} \frac{L}{D} \frac{dW}{W}$$

Integrating from Full Fuel to Empty,

$$E = \int_{W_0}^{W_1} \frac{1}{c_T} \frac{L}{D} \frac{dW}{W}$$

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ENDURANCE FOR PROPELLER-DRIVEN AIRCRAFT

Using SFC and steady straight level flight condition:

$$c_t = \frac{cV_\infty}{\eta_{pr}} \quad \left. \begin{array}{l} V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}} \end{array} \right\} \rightarrow E = \int_{W_1}^{W_2} \frac{\eta_{pr}}{c} \sqrt{\frac{\rho_\infty S}{2}} \frac{C_L^{3/2}}{C_D} \frac{dW}{W^{3/2}} \quad E = \int_{W_1}^{W_2} \frac{1}{c} \frac{L}{D} \frac{dW}{W}$$

If η_{pr} , c , ρ_∞ , and $C_L^{3/2}/C_D$ are constant:

$$E = \frac{\eta_{pr}}{c} \sqrt{\frac{\rho_\infty S}{2}} \frac{C_L^{3/2}}{C_D} \int_{W_1}^{W_2} \frac{dW}{W^{3/2}}$$

$$\rightarrow E = \frac{\eta_{pr}}{c} \sqrt{2\rho_\infty S} \frac{C_L^{3/2}}{C_D} (W_1^{-1/2} - W_2^{-1/2})$$

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So just like we have the range equation, we have the endurance equation, which is also very similar. Again $dt = \frac{dW_f}{c_t T}$, but now we do not bring in V infinity into that, we just go to this T integrate it. Okay, then there are some conditions which I leave you for self-study that talk about this is not the interesting part. This is the tedious calculation part. But look at the endurance equation. It has $\eta_p C, \frac{C_L^{3/2}}{C_D}$. We have always been saying that the propeller aircraft has a maximum endurance when $\frac{C_L^{3/2}}{C_D}$ is maximum. This is a proof of that in the calculations.

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ENDURANCE FOR PROPELLER-DRIVEN AIRCRAFT

$$E = \frac{\eta_{pr}}{c} \sqrt{2\rho_\infty S} \frac{C_L^{3/2}}{C_D} (W_1^{-1/2} - W_2^{-1/2})$$

To maximize endurance:

- Fly at maximum $C_L^{3/2}/C_D$
- Fly at sea level (maximum ρ_∞)
- Maximize propeller efficiency (η_{pr})
- Minimize SFC (c)
- Maximize fuel capacity (maximize $(W_1^{-1/2} - W_2^{-1/2})$)

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MAX. ENDURANCE: PROPELLER-DRIVEN AIRCRAFT

For a specific aircraft, η_{pr} , c , and $(W_1^{-1/2} - W_0^{-1/2})$ are fixed

At a given altitude (ρ_{∞} is constant), Endurance is maximized by

Flying at maximum $C_L^{3/2}/C_D \rightarrow 3C_{D0} = KC_L^2$

For parabolic drag polar, this condition yields:

$$V_{E_{max}} = V_{(C_L^{3/2}/C_D)_{max}} = \left(\frac{2W}{\rho_{\infty} S} \sqrt{\frac{K}{3C_{D0}}} \right)^{1/2} \rightarrow 3C_{D0} = KC_L^2$$

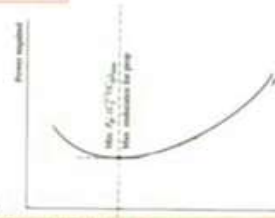
$$\left(\frac{C_L^{3/2}}{C_D} \right)_{max} = \frac{1}{4} \left(\frac{3}{KC_{D0}^2} \right)^{3/4}$$

$$3C_{D0} = KC_L^2$$

$$C_L = \sqrt{3C_{D0}/K}$$

$$C_D = C_{D0} + KC_L^2 = 4C_{D0}$$

$$C_L^{3/2}/C_D = \{3C_{D0}/K\}^{3/4} / \{4C_{D0}\}$$



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Chapter

ENDURANCE FOR JET-PROPELLED AIRCRAFT

• If c_t and L/D are constant:

If c_t and L/D are constant:

$$E = \frac{1}{c_t} \frac{L}{D} \int_{W_1}^{W_0} \frac{dW}{W} \rightarrow E = \frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$$

To maximize Endurance:

- > Fly at maximum L/D
- > Minimize TSFC (c_t)
- > Maximize fuel capacity, Reduce W_0 (maximize W_0/W_1)



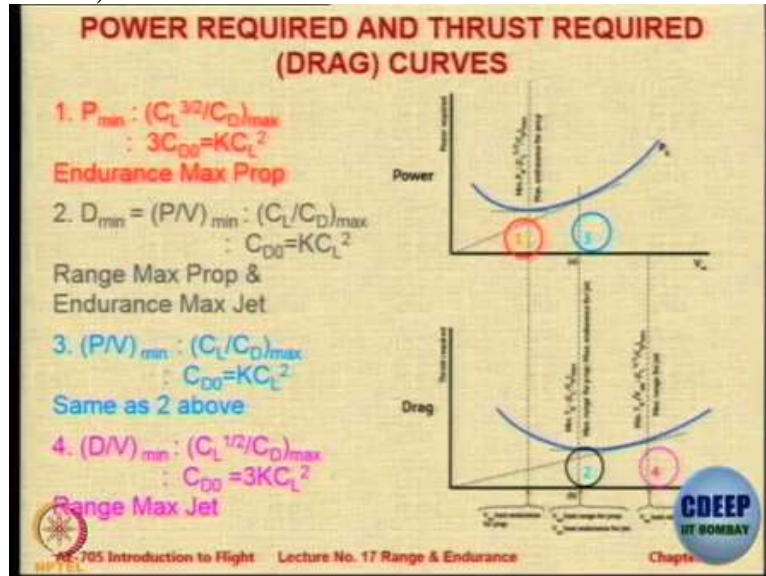
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Chapter

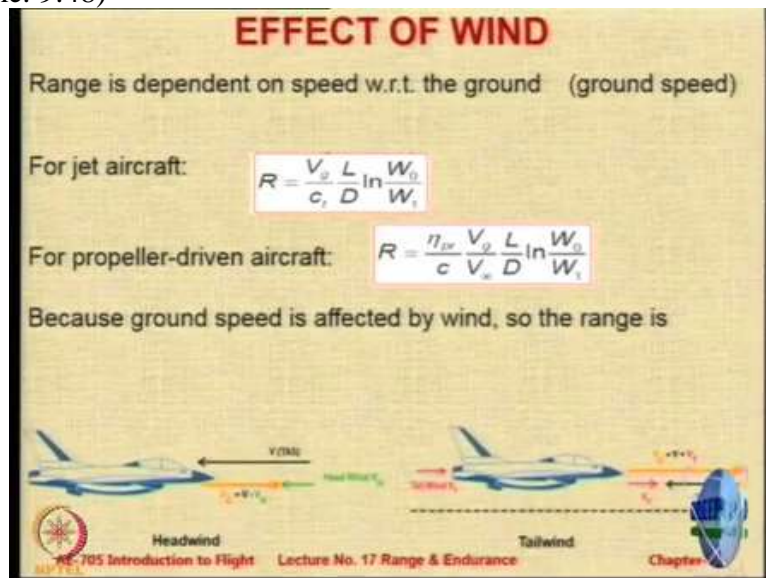
Okay, so to maximize of course just maximize these things and minimize the thing on the bottom. Okay, condition comes for propeller-drive aircraft. I am going to rush through. This also you know, because you have already derived it. For jet engine aircraft it is a simple expression, $\frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$, to the same. So the expression for maximum range for turbo, for turboprops piston props is similar to the equation for maximum endurance for turbojet system, so same conditions are applicable, okay?

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And finally, if you look at the comparison, there are these four speeds. That is one speed, number one at which power is minimum. Therefore endurance is maximum for a prop, there is another speed at which drag is minimum. That will give you the maximum range for props and maximum endurance for jets. Point number three is the same. Point number four will be the range for jets to be maximized.

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EFFECT OF TAIL WIND & HEAD WIND ON GROUND SPEED

Ground Speed (V_G) = True Air Speed (V) + Tail Wind (V_T)

Ground speed (V_G) = True Air Speed (V) - Head Wind (V_H)

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Now wind is going to make a big difference. So let us see effect of wind. So the conditions are the same, same equations are there, but now you have to replace the value of V with ground speed. So if there is a headwind, okay then the ground speed will be the actual speed plus the tail wind. If there is a tailwind, sorry. And if there is a headwind then it will be the opposite. So the ground speed is going to be true air speed plus tailwind or true air speed minus headwind. Minus because it is opposite direction, so that is it. You just replace in the equations and you will get the expressions.

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EFFECT OF TAIL & HEAD WIND ON RANGE JET AIRCRAFT

(Increase in Range in Tail Wind)

$R = \frac{V_G}{c} \frac{L}{D} \ln \frac{W_0}{W_1}$

(Decrease in Range in Head Wind)

(L/D) value as applicable for constant altitude cruise or constant speed cruise

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EFFECT OF TAIL & HEAD WIND ON RANGE PROPELLER A/C

(increase in Range in Tail Wind)

$$R = \frac{\eta_{prop} V_g L}{c V D} \ln \frac{W_0}{W_1}$$

(decrease in Range in Head Wind)

Note: (L/D) value as applicable for constant altitude cruise or constant speed cruise

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So this will be the same range expression, but V_g will now be V plus V_t . Similarly for turboprops and piston props. So tailwind is going to always give you an increase in the range. That is what we saw in the picture also earlier, there you can get much more range because of tailwind.

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RANGE AND ENDURANCE – SUMMARY

Range for Jet Aircraft

Constant speed $R = \frac{V}{c} \left(\frac{L}{D} \right) \ln \frac{W_0}{W_1}$ (h increases - Cruise climb) $C_{D0} = KC^2$

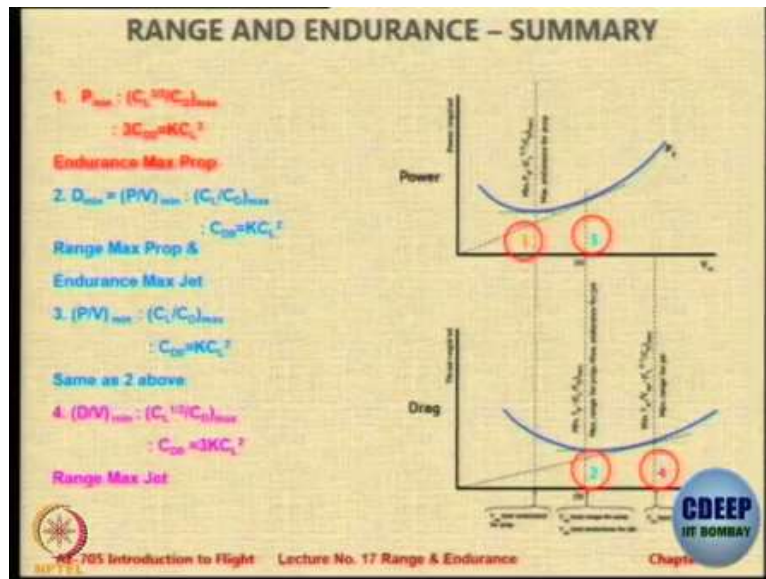
Constant h $R = \frac{2}{c} \left(\frac{2}{\rho_\infty S} \frac{C_L^{3/2}}{C_D} W_0^{1/2} - W_1^{1/2} \right)$ (V decreases) $C_{D0} = 2KC^2$

Endurance for Jet Aircraft $E = \frac{1}{c} \left(\frac{L}{D} \right) \ln \frac{W_0}{W_1}$ $C_{D0} = KC^2$

Range for Propeller Aircraft $R = \frac{\eta_{prop}}{c} \left(\frac{L}{D} \right) \ln \frac{W_0}{W_1}$ $C_{D0} = KC^2$

Endurance for Propeller Aircraft $E = \frac{\eta_{prop}}{c} \sqrt{2 \rho_\infty S} \left(\frac{C_L^{3/2}}{C_D} W_0^{1/2} - W_1^{1/2} \right)$ $2C_{D0} = KC^2$

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So this is a summary. So in summary $(L/D)_{max}$ CL3 by, $\left(\sqrt{C_L/C_D}\right)_{max}$, L by D for maximum

for endurance, $(L/D)_{max}$ for range and C_L^2/C_D . So there are three conditions. $(L/D)_{max}$,

$\left(\sqrt{C_L/C_D}\right)_{max}$ and $\left(C_L^2/C_D\right)_{max}$. So $(L/D)_{max}$ gives you maximum endurance for jet

maximum range for props, $\left(C_L^2/C_D\right)_{max}$ gives you the value of maximum range for maximum

endurance for propeller aircraft and $\sqrt{C_L/C_D}$ gives you maximum range for the jet aircraft. So

this is a summary. Again, the same thing I have shown you. So in the next class we are going to look at takeoff and landing.