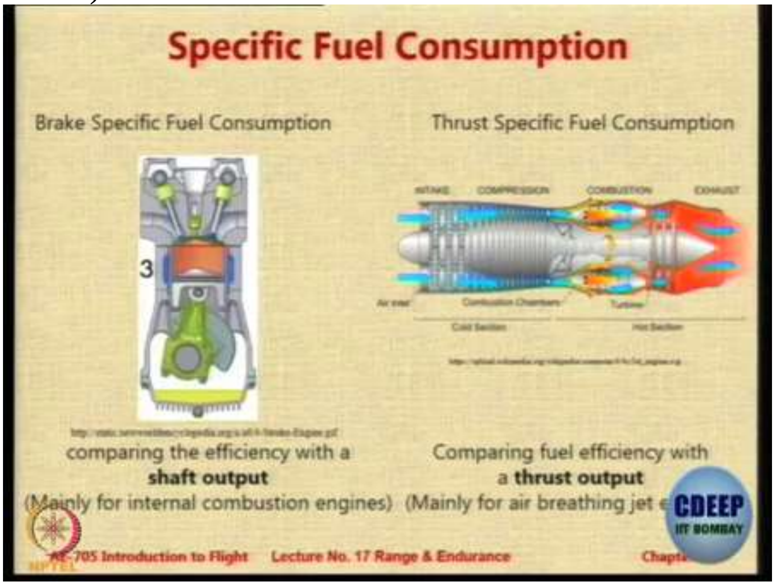
Introduction to Flight
Professor Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology, Bombay
Lecture 11.2 – Specific Fuel Consumption and Generalized Range Equation

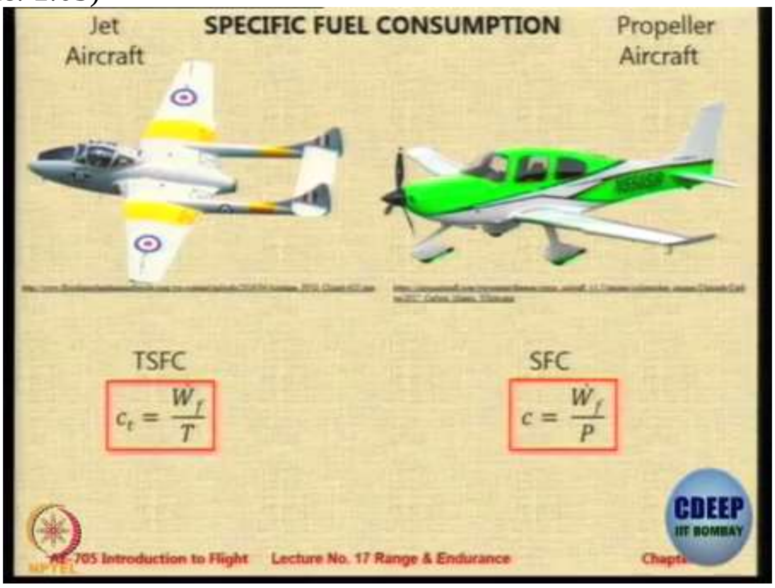
(Refer Slide Time: 0:16)



So, let us look at what exactly determines how much you can travel on a given amount of fuel and that one of the important parameter is the SFC. So, for an IC engine it is the brake specific fuel consumption and for a jet engine is TSFC, so in the brake in the BSFC or in the IC engines we are concerned about the break or the shaft horsepower. Here we are concerned about the thrust produced directly.

This shaft horsepower as you know is consumed by a propeller, so depending on whether you are concerned about power or thrust there are two different definitions for the specific fuel consumption.

(Refer Slide Time: 1:05)

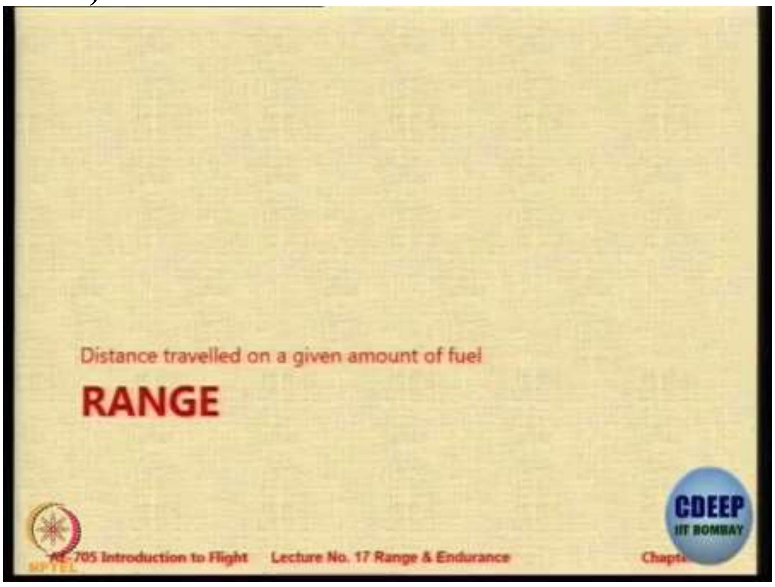


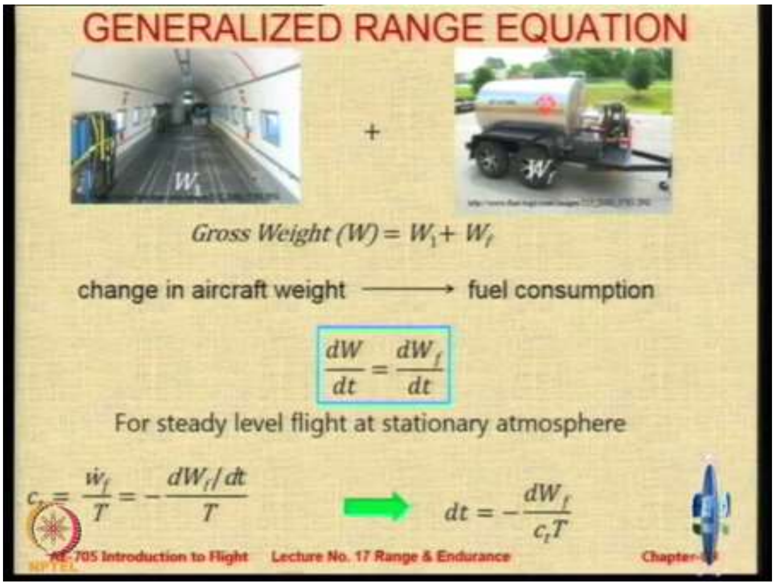
So, for jet engine aircraft we talk about TSFC or the thrust specific fuel consumption called as $C_t = \frac{\dot{W}_f}{T}$. For propeller aircraft we are concerned about power so we define SFC as the fuel flow per unit power produced.

Now is there a way to connect these two together, is there a way by which we define a single value for both the aircraft types. So, power will be thrust into velocity because at least in level flight thrust is equal to drag. So, $P = T * V$. So, you notice now, but P also has now it is not just T into V because you have efficiency also so eta P also will come. So, if you replace P with $\frac{TV}{\eta_P}$ so, then you can get an expression for Ct equivalent jet SFC for a piston prop aircraft.

Ok, so thus the equivalent jet SFC for piston prop is equal to the SFC for piston prop into V infinity by η_P. So, this is used many a times in performance calculations and I will explain to you why when I do the derivations in that. So, let us now look at some mathematical formulations for arriving at how much range an aircraft can travel.

(Refer Slide Time: 3:08)





So, just as point of memory the distance travelled on the given amount of fuel is called as a range. We assume it is steady level flight, so there is something called as Generalized Range Equation which is common to both the or all the aircraft type. But at some point some deviations will come. So for this purpose we divide the aircraft into two things, everything other than fuel, that is W_1 weight and only fuel is W_f. So, the aircraft gross weight W is equal to W_1+ W_f, simple, Right!

Other than fuel is W_1 and W_f is only the fuel. This W_f consists of reserved fuel, so all of it is not available to you for your mission planning but that distinction we will not worry right now. We will assume that W_f is basically fuel available to us. Now, we make one very fundamental assumption that during flight we do not catch a passenger and throw him out or take somebody's suitcase and throw it out or do not open payload bay and drop off something. The only reason why an aircraft in this calculation reduces its weight is because fuel is consumed. It means W_1 will remain constant, then and only then we can apply this equation.

So before we go ahead can you tell me which are the situations in which the aircraft fuel, the aircraft weight reduces not only because of fuel consumption but also because of other things? What are the scenarios in which, yes, give me a scenario in operation of an aircraft.

Student: Sir, when the military transport aircrafts airdrop vehicles bombs...

Professor: So, do not call them transport aircraft then, the word transport means nothing is dropped. But I know military cargo aircraft or military aircraft which drop things off for them we cannot calculate the range by this formula. Agreed, but I will not use the word transport. The word transport basically means transporting people without dropping them. So it is ok or cargo without dropping it but cargo aircraft or military aircraft agreed. Bombers, fighters, if they lose anything we cannot use it, so that is one reason, any other?

Any other scenario in aviation where we cannot apply this equation? Because the aircraft weight during flight changes for reasons other than fuel consumption. Think about it. Yes?

Student: Maybe in dive pull out, the weight is also changing.

Professor: Why weight is changing?

Student: We have seen that $\frac{mV^2}{R}$ that part is into the weight.

Professor: Correct, $\frac{mV^2}{R}$ in which m remains constant. In fact even V remains constant. You can have a constant velocity dive pull out. So, the aircraft weight will not change during dive pull out. Aircraft weight will remain the same. Any other scenario?

Student: In one class you told about the second world war plane, in that this landing gear and all will be blocked.

Professor: Ok, that is right, it is just at take-off so that is, ok. Technically speaking, yes, you drop the landing gear just at take-off, that is one very special aircraft, agreed. Answer is not wrong, but there is one more mission that is regularly flown in which W_1 changes, air to air refueling.

When you acquire fuel from some aircraft, or when you give fuel to somebody it is not consumed it is only transferred. So, when we do air to air refueling or when we have bombing or drop of payload we cannot use this equation. For all other applications you can use it. So, we will assume that change in the W with time is equal to change in the W_f with time. I am going to put this in the formula to derive the expression. Another assumption will be no headwind, no tailwind so stationary atmosphere and no acceleration. Yeah?

Student: Sir in some case we can see that plot used to dump the fuel.

Professor: That is at the end of flight when you want to suddenly reduce the weight.

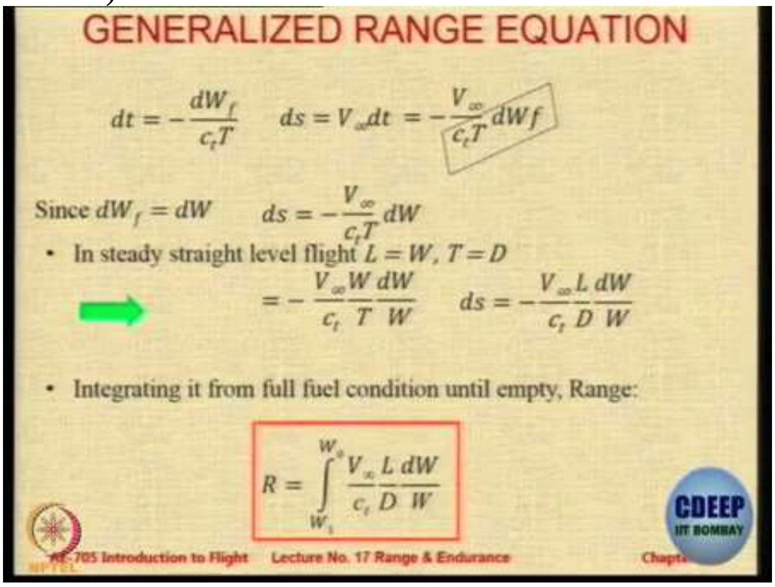
Student: In that case also W_f changes while the fuel is not being consumed.

Professor: Exactly, but you have already reached the destination. So, you are not at the state when that will affect the range. So, either when you drop landing air in the beginning or when drop fuel at the end you are doing it not to increase or decrease the range, but for some other reason. During flight during steady level flight basic reason would be only one of those two, I agree with you.

If you can also suddenly dump fuel at the end, so if you look at steady level flight, and if you look at stationary atmosphere and if you look at a situation in which the change of weight is only because of fuel consumption then only then we can apply. So, let us start with fundamental definition of the thrust specific fuel consumption.

So, SFC or C_t, I am using C_t because I want to talk about both the aircraft together, the reason is this is generalized range equation, this does not say applicable only to turbo prop, or piston prop or turbo jet or turbo pulse state. It is applicable to all aircraft types. So, the thrust specific fuel consumption basically is rate of consumption of fuel divided by the thrust and the rate of consumption of fuel basically is nothing but at any particular instant is actually $\frac{dW_f}{dt}$. Ok, so this gives us an expression that dt, in a small elementary time dt you have a change in W_f upon SFC into time and because the change is negative because we lose fuel we have put a minus sign. So, this becomes our basic building block, $dt = \frac{dW_f}{C_t T}$

(Refer Slide Time: 10:20)



So let us stick it and copy and paste here and we start looking at the manipulation. So, first manipulation that I will do will be to replace, let see one by one what do I do. You just see what I do. So first I say the distance travelled a small distance travelled in the elemental time t will be V infinity into time t. The assumption here is that V infinity is constant, steady flight, so at least in that small elemental time t, we do not assume change in V and therefore the distance travelled in time $dt\ is\ V_\infty dt$. Now, dt is already given there, so I can replace dt with $-\frac{dW_f}{C_t T}$. How have I done this? Because, $-\frac{dW_f}{C_t T} = dt$.

In other words $ds = -\frac{V_\infty dW_f}{C_t T}$. Now dWf is basically dW. Do you agree? Change in the fuel weight is the change in the aircraft weight, so we replace it. So, we get $ds = \frac{V_\infty}{C_t}\frac{W}{T}\frac{dW}{W}$. Now, what we have done here is we have multiplied and divided by W. Is it permitted? Because, W is non-zero so it is permitted. And now the last thing that we do is because it is level flight therefore T=D and L=W. So, what I am going to do is because L=W, the W in numerator I will replace with L, with your kind permission and the T on the bottom I will replace by D. So, that is it, this is the elemental equation or sorry, Generalized Range Equation.

For any aircraft the distance travelled in a small elemental time dt is given by the product of the velocity of the aircraft during that time assumed to be constant, divide by the thrust specific fuel consumption into the lift over drag into $\frac{dW}{W}$. So, now we start integrating this over a range, so the total range will be from the aircraft weight equal to W0, W1 to W0 from initial condition to final condition. Now, this minus sign I have consumed in changing the integration limits, so instead of putting minus W0 to W1, I have put W1 to W0. So, from this point onwards now there are many many paths available. So this particular equation has a very interesting name, it is called as Breguet range equation.

What is special of this equation? Number one, it is valid only for steady level flight. Number two, it is valid for any engine type provided we use the equivalent thrust SFC. If the engine is jet engine then you directly use Ct. $\frac{\dot{W}_f}{T}$. If it is a piston prop, turbo prop we use the converted value. From this point onwards you will have different paths depending on what is assumed constant. So, the most common assumption is a constant altitude flight, flight at which the value of L/D remains constant so that means a constant angle of attack flight.

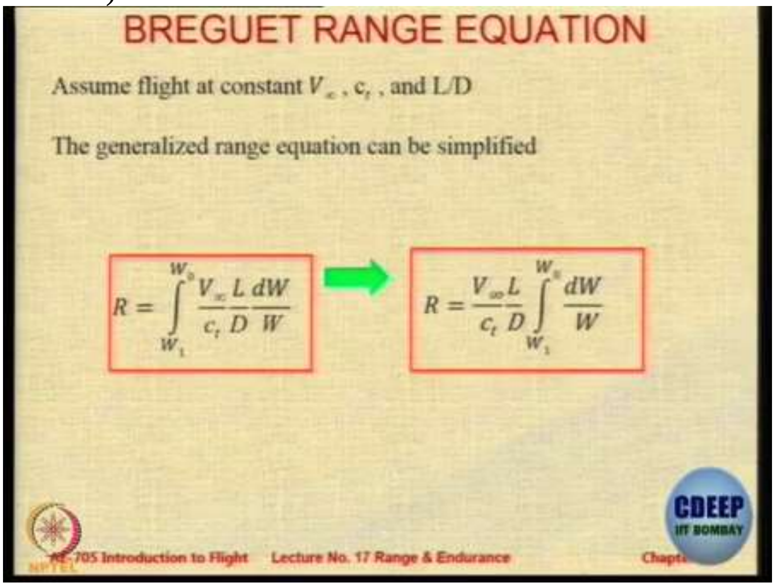
You know that corresponding to any angle of attack the aircraft has a given value of CL that is alpha CL curve for the aircraft. So, for any alpha value there is a CL and for that CL value you have $C_D = C_{D_0} + KC_L^2$. CL is constant K is constant, $\frac{1}{\pi Ae}$ that does not change. C_{D_0} is also a constant that is the function of the aircraft shape, geometry etc. So, as long as the angle of attack of a flight during flight is fixed, it may not remain fixed. But, as long as the pilot flies at a given angle of attack you have a corresponding velocity. So, level flight at a constant angle of attack one can assume that Ct does not change. Ct is a function of the engine behavior and the engine consumption of fuel does not change if you have the same density.

So, if V is constant, Ct is constant, L/D is constant then all of them can come outside the integration sign then,

$$R = \frac{V_\infty}{C_t}\frac{W}{T}\int_{W_1}^{W_0}\frac{dW}{W}$$

So, that is one very simple expression that is what it is, but remember you may not have this condition always.

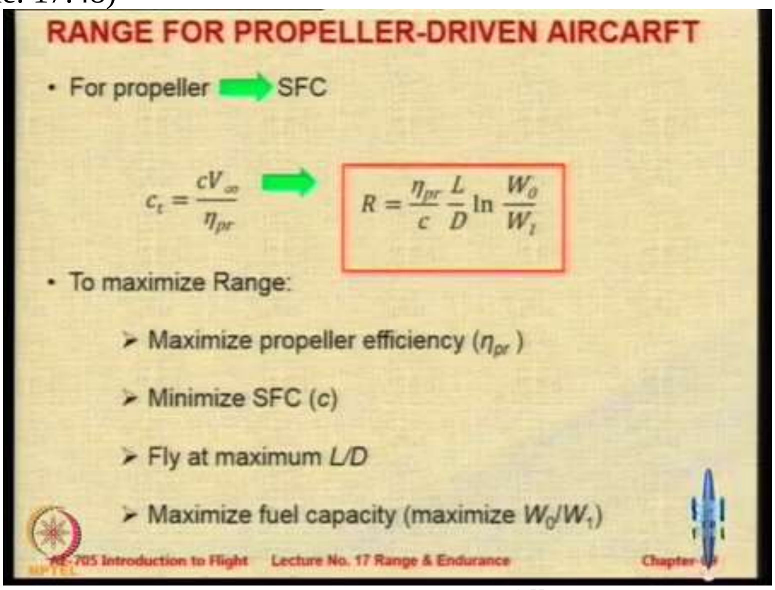
(Refer Slide Time: 16:33)



For example, there is one condition of flight in which velocity may remain constant but altitude may not remain constant. Do you remember I talked about cruise climb condition in which as the aircraft is losing the weight, you slowly increase the altitude. So, during that time this cannot be acceptable because Ct will not remain same, as density changes Ct will change.

So, you may have a constant V, constant L/D but Ct changing. So any combination of these three parameters is permitted and therefore three different types of ranges also are there, and there are three different equations for those ranges but we will not go into that direction because this is not a class in aircraft performance where you want to go in detail, so we will look at only a fundamental expression. Ok, so let see.

(Refer Slide Time: 17:48)



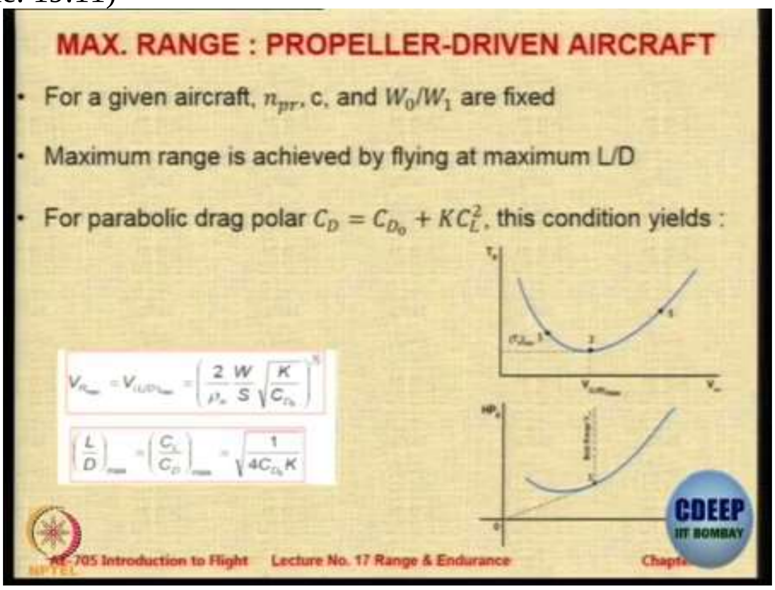
So, for a propeller aircraft we use that relationship $C_t = \frac{C \cdot V_\infty}{\eta_P}$. So,

$$R = \frac{\eta_P}{C}\frac{L}{D} ln\frac{W_0}{W_1}$$

. So now if I asked you a question, let say I would like to maximize the range, what should I do? Tell me. Yes, slide is there in front of you, first thing is fly at a condition at which eta p is maximum, Second thing is minimize the SFC, third thing is maximize the fuel capacity by increasing W0 by W1 and also fly at maximum L by D.

There is one particular angle of attack at which the aircraft acquire maximum L by D. That angle is approximately 3 to 4 degrees for most aircraft. So, the pilot will fly at that condition but when you fly at 3 or 4 degree angle of attack it is quite possible that the aircraft may be also inclined which is not convenient for passengers. So, there are many years of doing this, one is you set the incidence angle of that aircraft itself so that the aircraft is horizontal when you have the optimum L by D.

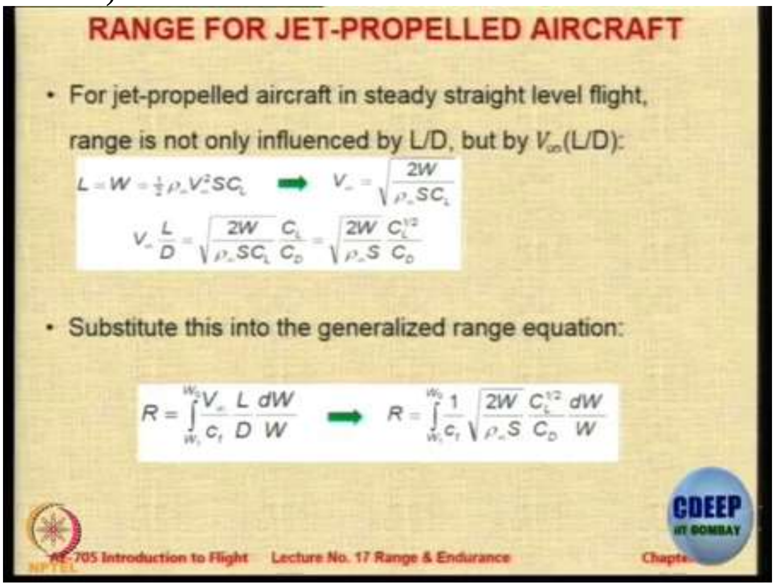
(Refer Slide Time: 19:11)



Ok, so let us look at propeller driven aircraft, so we can derive a condition. Now, this is something that I would like you to derive yourself. Ok! If you just look at the board and say yes, then you will be in real soup. So, it is a very simple expression which I would like you to do as homework.

So, I will just show you back again. Basically, you have to derive the condition for L/D maximum. What is the CL at L/D maximum? So that can be really calculated and if you know the L/D maximum, you can put it there. So the at what speeds you fly, so that your range is maximum for turbo prop, you can estimate by this particular expression, so here K is the induced drag factor.

(Refer Slide Time: 20:03)



For a jet propelled aircraft the only influence is L/D. So, one can easily derive the expression. So, it is very messy it is not a very simple expression. So L/D alone is not important because V also is there, so you have to optimize V, L/D.