

**Introduction to Flights**  
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**Lecture No. 10.4**  
**Introduction of V-n Diagram**

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Welcome to lecture number 16, which is second lecture of capsule number 8. Before I start this lecture I want to share with you some interesting things, about the person who has created the content for this course. This is Dr. Rahul Goyal; who is currently a research assistant at University of Houston. He was one of our very distinguished undergraduate student joined in 2004. And after one semester of stay in IIT Bombay, when most students want to go home he came along with his friends and said, "I do not want to go home because all my friends have gone to various engineering colleges, medical colleges, there is nobody at home. I want to do something."

So, I said, "You are so fresh, just fresh into engineering; forget about aero." But he was highly enthusiastic, he insisted so I said, "Ok, go home for a week so that your parents can see you and your friends and your relatives can see you then come back." He came back with his friends. And the content that you are going to see today was first initiated in 2004. So, it is very old, ok. As a winter intern it was done; he did many things. One of things was to prepare this presentation.

Now, today PowerPoint is very common, it is available everywhere, we do lots of animations etc., etc. video insertion. Go back in 2004, Power Point was very new. It was the latest sensation

software in the market, and we had not explored PowerPoint; prior to that we were using transparency sheets. So, this presentation was created by Rahul and his friends using my transparency sheets for this lecture and inserting some more material. So, take it from the point of view of some historical approach also.

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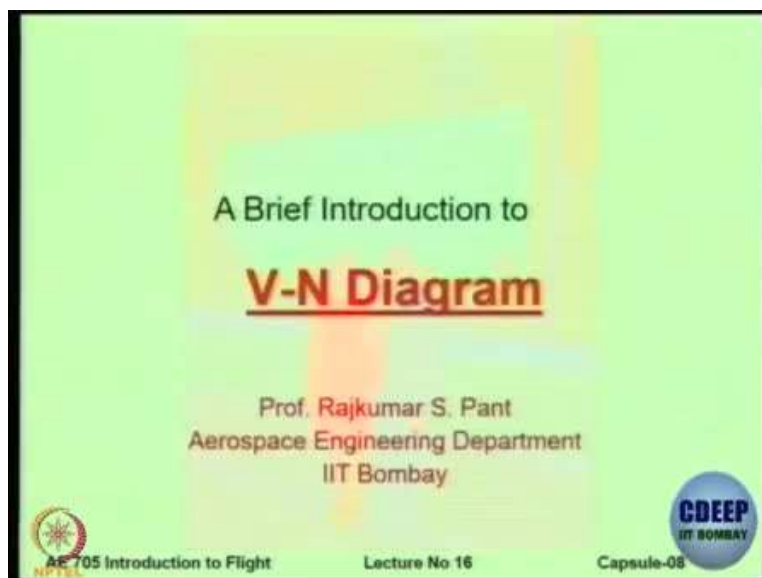
After completing his graduation in IIT Bombay, Rahul went to MIT on a fully funded Masters Program. After completing Masters he did many interesting things; he went to Germany for a space program, etc., etc.

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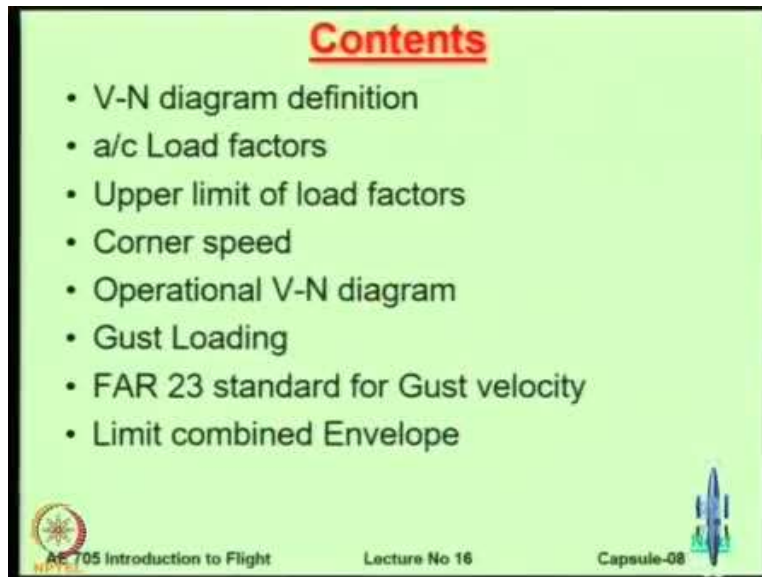
And interestingly just about, today is 13, about 7 days ago he defended his PhD at University of Houston, ok. This is Rahul and his wife Supreeti. I had the pleasure of attending his wedding last year. And he is now an expert in Space Physiology. So, the topic of his thesis is Sensory Motor Mechanisms Underlying Postural Control for the astronauts, ok.

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So, I present to you, the present, the PPT made by Rahul and his friends on V-N Diagram.

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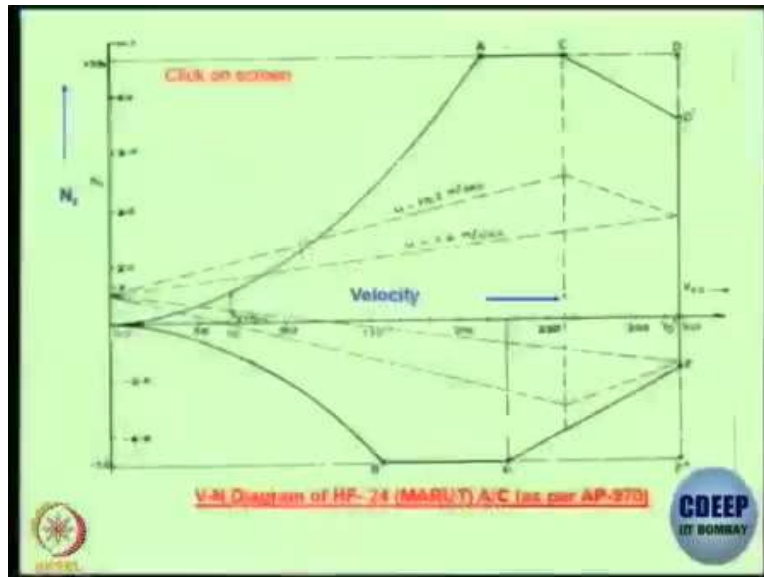
Ok. So, these are the contents of this particular presentation. And one more interesting fact the entire content of this presentation have also been recorded as a video lecture. So, back in 2001 I think or 2, we had one education technology cell in IIT Bombay; ET Cell. I can call it as the father of CDEEP basically. CDEEP has formed basically out of ET Cell. And ET Cell approached us young faculty members, at that time I was quite young, 2001. To record video lectures, so, I volunteered to record this lecture. It is available in the library; on VHS cassette and I think it is also available as a soft copy. So, the same lecture is already recorded and you will find me, a very young person with lot of hair and also darker in black presenting this lecture, ok.

If I locate that, I am not getting that DVD somehow, I will locate it I am going to share with you. So, what we are going to look at is, definition of V-N diagram. We are going to look at load factor acting on the aircraft, what are the limits on them? Remember, limits, upper limits are not necessarily just positive upper limits, that are also negative upper limits. I do not call them as lower limits, I call them as a positive and negative limits.

There is a concept called as corner speed which is very important, and in the lecture that you attended on turning flight, we have discussed some formulae and methods to calculate turning rate, tightest and fastest turn; I will come back to that. The V-N diagram is of many types, one of them is called as a operational V-N diagram, and then we move on to the effect of gusts; how disturbances are acting on the aircraft? And how they generate load on it?

We will look only at vertical gusts in this particular presentation, and then look at some regulations on how the gust velocity is specified and finally, how these impose limits on V-N Diagram and make a limit combined envelope, ok.

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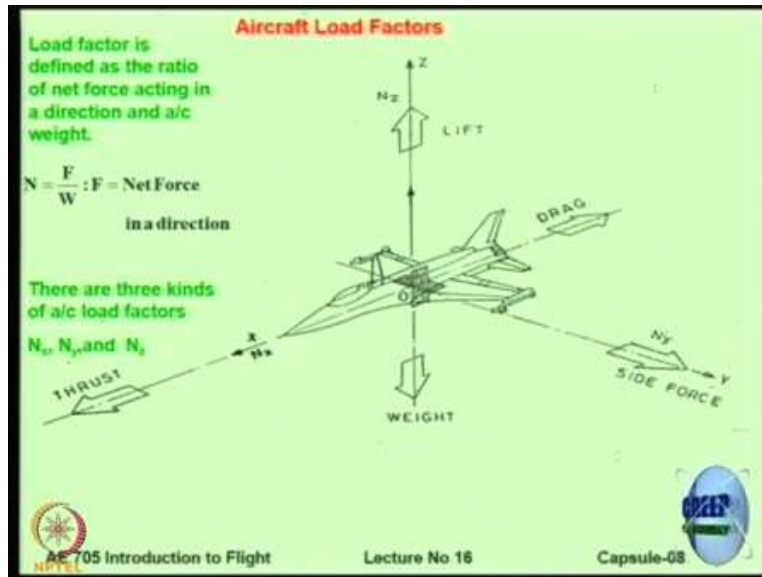
Ok. So, this is a V-N Diagram of an aircraft called HF-24 Marut which was one of the first transonic fighter aircrafts designed in the country. It was designed by a team lead by Dr. Kurt Tank from Germany Dr. V. M. Ghadge from India and his team.

At its time, during the time when it was designed, it was considered to be one of the best aircrafts in the world in this category. So, what is a V-N Diagram? Basically we see we have an x-axis which is load factor, on the y, x-axis we have velocity, on the y-axis we have the load factor, ok. Now, the velocity on the x-axis is not a normal velocity but a special velocity and also the load factor on y-axis is also not the y load factor but z load factor as we will see.

And I want to show you the diagram first and then we will see how to derive it. So, you see there are curve lines from origin OA upwards and OB downwards. These are the 2 lines; there are 2 horizontal lines on top and bottom which are the limits of maximum vertical positive and maximum vertical negative load factors. And then, we have a vertical line here which is the maximum speed; and then there are some cuts here, and AP970 stands for the British regulations, they were called as Aviation Publications, that is AP; AP970.

AP970 was a regulatory document provided by the Britishers for a military aircraft. And they gave a procedure and calculations and methodology to calculate the V-N diagram. Similarly, there are other agencies who also give, who also give the requirement, ok.

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So, let us take a typical aircraft in flight, and although we have shown level flight here, it could be at any particular angle. But, consider level flight, you have THRUST, DRAG, LIFT and FORCE; do not worry about point of action of forces, the of course the points are not co-linear.

But, in a direction along the thrust line, or in the forward direction, if we ignore the thrust mounting angle, we have the force  $T - D$ . So, that force,  $\frac{T-D}{W}$  will be  $N_x$ . Similarly, you will have lift acting on the wings, plus or minus lift acting on the tail, mostly it will be plus lift on the tail because tail has down load that divide by the aircraft weight would be called as  $N_z$  or vertical load factor.

And is there is any turn with acceleration, then there is also going to be side force. So, that side force divide by aircraft weight will be  $N_y$ . So, an aircraft typically has 3 load factors;  $N_x, N_z, N_y$ . So, this already I've covered, yes? Yeah, that force upon, correct. Net force acting on the aircraft, external force except weight. It is except weight; it is a net force acting on the aircraft, except the weight.

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**Some General Points**

**V-N diagram is applicable only for symmetrical maneuvers in the vertical planes. Why?**

Because  $N_z$  has the highest numerical value and in symmetrical maneuvers in vertical plane  $N_x$  &  $N_y$  remain constant.

**V-N diagram is drawn only for  $N_z$ . Why?**

Because the numerical values of  $N_x$ ,  $N_y$  are small and can't lead to structural damage to a/c if they are too high.

It can be seen that  $N_z \propto V^2$  and (AOA) [How?](#)

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Ok. So, let us see some general points. First point is that, V-N Diagram, when we talk about V-N Diagram we are looking at symmetrical maneuvers in the vertical plane. So, can you name or describe any symmetric maneuvers in the vertical plane? That you have heard of? What is it? Barrel roll, is it in vertical plane? It is not. What is a barrel roll? That is not the way it is. Barrel roll is this way. So, some people are moving their fingers now, it is a name for moving fingers; yes! Anybody here knows? Yeah?

Yeah dive pull up maneuver, pull up maneuver we have studied that maneuver. You have answered a question in the quiz based on that. So, why is it only for symmetrical maneuvers in vertical plane? That means we cannot use this V-N Diagram for Barrel Rolls, for Flip Rolls, or for any things that is not in a vertical plane; there is one maneuver called as a Knife Edge Pass, so, you go like this and then, you turn like this, then you go like this. Ok.

And if you turn it will create side force, apart from the load. So, we do not consider those maneuvers. Only symmetrical maneuvers that means not even this, symmetrical maneuvers in which the wings remain level and in a vertical plane. So, basically dive pullout or a loop. So, why only for these? What do you think? This is a question; I am not going to give the answer. I want the answer to come from you. What do you think?

Yes? What could be the reason, correct! So, the, the question actually can be answered in many ways. One way to say is that, you already told that V-N Diagram has  $N_z$  as the y-axis. So, that

means you should have pure Nz. Pure Nz will come only when you have a symmetrical maneuver, ok. The other reason is, someone can say, "Ok, then let us make one more V-N Diagram for Ny versus V, Nx versus V." The answer is, that, Nz has the highest numerical magnitude in a symmetrical maneuver in a vertical plane, Nx, Ny remain constant.

And the numerical value of Nx and Ny, generally are much lower than Nz. So, Nz is the real difficult thing they are large also we worry only about Nz; we do not worry that much, we do. Otherwise, if you do a turn, the engine may be thrown away by the aircraft if you do not take care of enough side force. But those numbers far lower, ok. This is the reason; numerical values are much higher for Nz, ok. Now, we have to see, now, the line in the V-N Diagram, if you notice, was parabolic, of Nz versus V.

So, we have to prove that, Nz is proportional to V square; only then the line is parabolic. And also with the angle of attack. So, let us see, let us see how we can prove it, ok.

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$N = \frac{L}{W}$

$L = \frac{1}{2} \rho v^2 S C_L$  : Lift

$L = \frac{1}{2} \rho v^2 S (AOA) a_0$

where

$\rho$  = density of air     $C_L$  = Lift Coefficient  
 $v$  = a/c speed         $S$  = wing area  
 $a_0$  = Lift curve slope

Thus  $N_z \propto \rho V^2$   
 and  $N_z \propto (AOA)$

But this would imply that we need to draw a different V-N diagram for every possible altitude.

So how do we eliminate this problem?

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So, here is a typical CL alpha curve for an airfoil, and this is the cambered airfoil, just for general generality, the same argument also applies for a symmetrical airfoil. So, the load factor N, normally shown as a small n, not capital N. That is  $L/W$ .

$L = \frac{1}{2} \rho_{\infty} V_{\infty}^2 S C_L$ , here S is the wing reference area.



But,  $Cl$  comes from lift curve slope into the angle of attack, assuming that is linear, and assuming that we don't go into the areas where the curve is non-linear, ok. We would never fly the aircraft beyond or near alpha stall. So, the operational angle of attack range will be probably 5 degrees or 4 degrees below alpha stall. So, it is a linear area. So,  $L$  is proportional to  $V$  square, because density of air is constant, if you ignore density change in one small height, ok.

$S$  is the wing reference area, it remains constant. Angle of attack is going to remain constant because you want lift equal to  $Nw$ , you cannot change that. And lift curve slope is a parameter that is married to aircraft geometry and aerodynamics so that has to be constant. So, in another words, other things are constant, and hence, angle is proportional to  $\rho$  into  $V$  square. So, now there is a problem, when I do the maneuvers that are 3 kilometers altitude, although the maneuver will be only 100 meters height.

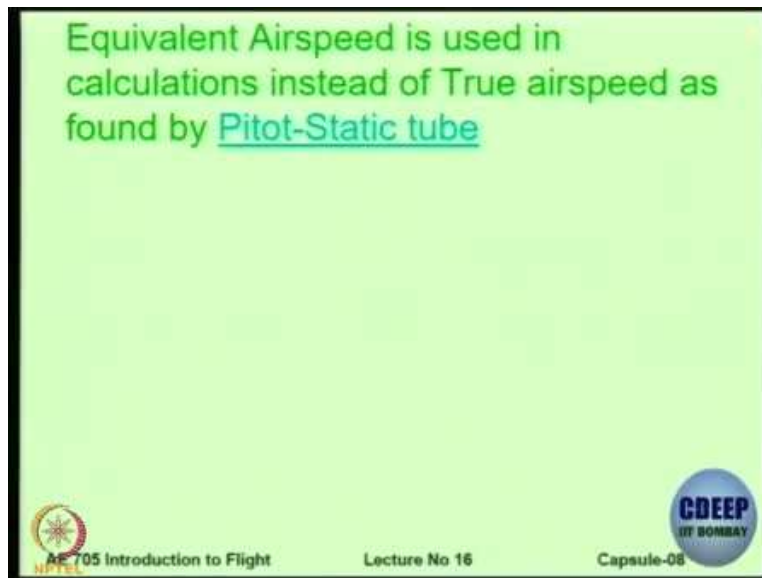
But, density at 3 kilometers is far lower than density at sea level, ok. So, the problem is that  $Nz$  is proportional to  $\rho$  and  $V$  square, and also to the angle of attack. That means, the pilot can actually be required to do a vertical maneuver at any altitude. You cannot say please do not do vertical loop at 3 kilometer, do it only at 2 kilometers or lower; we cannot say that. Correct. You might say, does not matter; take the highest density which is at sea level.

So, if the aircraft is safe at sea level, it will be safe at higher altitude, ok. But, still it is important for the pilot to know whether they are within the limits of operation or not. So, now how do you solve this problem? This will be a nightmare right? 25 different V-N Diagrams for each altitude. Remember, you had a curve which said, power variation with altitude; and we saw that the curve tilts towards the right and moves up. So, that kind of drama we have to do now for V-N Diagram also, which is not a good idea.

So, can you suggest something? How do we take care of this? So, is the question clear to you? The question is, how do we remove the dependence of  $Nz$  and  $V$  on  $\rho$  and have only one V-N Diagram applicable to all the altitudes. So, let us see your suggestions, how do we take care of it? Yes? Let's ask, let's ask, yeah. If you are going to divide by half  $\rho V$  square, then, the dependence of  $V$  is also gone, dependence of  $\rho$  also is gone, then how do you draw diagram between  $N$  and  $V$ . I want a graph between  $Nz$  and  $V$ . So, what do I divide by half  $\rho V$  square, both side are those equations.

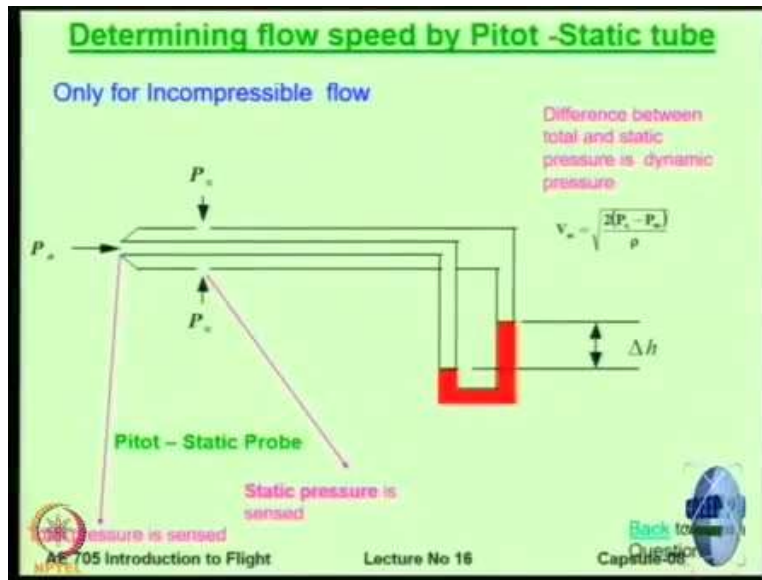
So,  $L$  is equal to half  $\rho$ , so that means you are saying divide, take half  $\rho V^2$  on the denominator of the left hand side. Correct! Dependence of  $V$  is gone. Yes. Sushil, I think Sushil has some, some point. Yes. Converting it to equivalent airspeed, so, yes that is a right answer. So, let us see, let us see how to do it, ok.

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So, let us use the speed so that, dependence of density on speed is gone; and that is gone through equivalent airspeed. So, for the same dynamic pressure half into  $\rho$  at altitude into  $V$  at altitude square, is equal to half into  $\rho$  at sea level into  $V$  equivalent square. That is how you define equivalent airspeed. So, something to what you said, something like what you said. Do not use  $V$  true on the x-axis, use  $V$  equivalent. Now, if you use  $V$  equivalent, then  $V$  equivalent is same at every altitude, because it takes care of effect of density. So, that is, what is the solution, ok. Using the pitot-static, this all you know, just wanted to quickly repeat.

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So, you take a pitot-static tube that measures the dynamic pressure, and that gives you, that gives you the value of  $V_{\infty}$ , which is the true airspeed, ok.

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**Equivalent Airspeed is used in calculations instead of True airspeed as found by Pitot-Static tube**

- The velocity (True Airspeed [TAS]) indicated by the Airspeed Indicator is proportional to dynamic pressure
- Taking into account the errors in calibrated instruments we get the calibrated airspeed [CAS].
- And after taking into considerations the compressibility effects we get Equivalent airspeed [EAS] (so it is that speed at which the a/c would be flying at sea level under same conditions of pressure and temp.)
- By using this equivalent speed the variable ' $\rho$ ' can be eliminated
- So  $N_z \propto AOA$   
 $\propto V_{eq}^2$  ONLY

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So, the true airspeed or TAS is indicated by the airspeed indicator. Now, this is not correct because the true airspeed indicator shows IAS not TAS. But, interestingly I told you some many instruments are there which also have a TAS indication in the instrument. They correct it for the errors and give you the true airspeed. So, this all of you know because we have already covered this, right. So, taking care of compressibility, taking care of position error, taking care of

instrument errors. If you have equivalent airspeed, and if you use that, then the problem is solved; by using equivalent airspeed the variable  $\rho$  can be eliminated. In that case  $N_z$  is proportional to angle of attack and  $N_z$  is proportional to  $V$  equivalent square only, ok.

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Factors that governs the upper limit of  $N_z$

- Structural strength of a/c
  - high  $N_z$  means designing the aircraft structure to bear higher loads
- Safety and Comfort of Passengers and Pilot

See this TABLE

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The other thing is about limits on the load factor, these limits as I said are specified by the regulatory bodies. So, let us look at now, 2 categories of aircrafts; the aircraft in which the people like you and me normally fly, these are the transport or commercial aircrafts. So, there are 2 important considerations in deciding the upper value of  $N_z$ . The first consideration is the strength of the aircraft. Obviously if I say that the aircraft is permitted to fly at the load factor vertical of 10, and if the aircraft weights 20 tons; it means the structure has to be designed, especially the wings to take care of 200 tons of vertical load.

Which is quite a lot. That will make the aircraft heavy, ok. So, the designer would like to design the aircraft for low load factors. The designer is interested to sell you the aircraft, the, the commercial organization are are wanting to sell you the aircraft. So, they will design it for load factor of 1.2. But, if you use an aircraft with load factor 1.2 and if the load factor increase in a slight turn, the structure will break.

So, the regulatory bodies say no, no, no, not permitted to design for 1.2; design for at least so much. So, from our safety point of view, the regulatory bodies insist on certain max value. So, they say show us the compliance of structure, and other systems at higher load factors, do not, we

will not give you certificate unless you show that the aircraft can take so much load factor, ok. So, please understand, the limits are specified by the regulatory bodies.

Left to the designers or to the aircraft manufacturers, they will design them for lower load factors because, while competing with the competitor they will have a lighter aircraft. It will meet the performance requirement but it will not meet the safety requirements. Second thing is, high load factor also means more discomfort for the passengers. So, let us look at this table.

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Aircraft Type	N(positive)	N(Negative)
General Aviation-normal	2.5 to 3.8	-1 to -1.5
General Aviation-utility	4.4	-1.8
General Aviation-aerobatics	6	-3
Homebuilt	5	-2
Transport	3 to 4	-1 to -2
Strategic Bomber	3	-1
Tactical bomber	4	-2
Fighter	6.5 to 9	-3 to -6

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Now, this table is taken from FAR23 regulations. As per these regulations, typically the aircrafts are divided into some categories. The first one is called as the general aviation category. Again there you have 3, normal, utility and aerobatic. So, I hope you know that, general aviation basically means aircraft to be used by the people who are not paying any fare. Non fare paying passengers. So, who are these people? Who do not pay any fare for the aircraft? Personal aircraft, ok. So, let us say, he want to go to his hometown after today's class.

So, he just takes a taxi, Ola, Uber to Juhu, gets into his aircraft, takes off and lands; personal use, ok. We had a very good panel discussion recently on, 'why this category is not so popular in India?' The general aviation category. Then we have utility, what are utility aircrafts? Can you give an example of a utility general aviation aircraft? What could it be used for? Well, let us say we have to drop some medicines at some place where there is a problem, or we have to rescue some people.

Now, these are aircrafts which are used by the agencies for non-profit, non-commercial reasons. But, they are going to be subjected to slightly more loads compared to our normal aircraft; that's why the load factors are higher, ok. Load factors, higher, basically means, compliance in design should be shown to that load factor. So, if you are going to make or buy a general aviation aircraft, the structure need to be only designed to maximum 3.8 .

Now, 2.5 to 3.8 depends on the mass of the aircraft, there is the formula. So, but, nothing less than 2.5. So, you can conclude that any aircraft has to be able to take 2 and a half times its max takeoff weight, as the vertical load; below that, you cannot get certification. But, it could be 2.8, 3, 3.2, 3.5 or 3.8 for this category. If it is utility, it is more, 4.4. If it is aerobatic, that means for shows etc., etc., then higher. Because they will be doing aerobatics, so, they will be doing lot of maneuvers and hence they will be subjected to higher loads; so, there it goes to 6 value, ok.

Then, there is one category called, home built aircraft. Homebuilt basically means, as the kits are available, you buy the kit, assemble it together. So, because it is expected that when you do homebuilt, you will not be able to do very good job in the structure, and also from enhanced safety, the load factor is 5, ok. Transport aircrafts, between 3 to 4, depending the max takeoff weight. So, then that means this table is not for FAR-23 only, it is for all aircraft value.

If you go to military aircrafts, regulations are given by the military agencies; like defense standards or def stand, or earlier AP-970 or some other regulatory requirements, ok. The FA and the JA requirements are not applicable for military aircrafts, they have their own requirements. In India we have something called CEMILAC, center for military aircrafts airworthiness, they specify the values. So, typically if it is a strategic bomber, that means an aircraft that is used for long distance travel, heavy bombs, for strategic applications, not just go and kill whatever you see, load factors are only 3.

If it is a tactical bomber that means, you have to go and evade some enemy aircrafts or if you expect to fight with some ground forces, just like A-10 aircraft which I showed you, then it will have 4. And if it is a fighter aircrafts which does the lots of maneuvers it can go from 6.5 to 9. So, the highest vertical load factor is for, fighter aircrafts, who undergo very, very tight and complex maneuvers which load the aircraft too much. Now, let us go on the negative side, when will you have negative load factor acting on the aircrafts?

What do you think? When will a load factor be negative? When the lift force acts downwards. So, when does it act downwards? We have seen in the examination also there was a question. In a vertical loop there are situations, when you go, inverted; and also when you come down inverted, you have lift acting downwards. So, that is when the negative load factors come. So, the negative load factors are not equal to positive one, they are nearly half.

Because it will be very difficult to expect a person to be doing maneuvers, while being negative. There will be a requirement, but not so much. So, therefore concession is given; it is a concession. If the airways agencies were cruel they would have said, "Give us negative 6 also". But, the people said, "How do you achieve negative 6. You normally don't do maneuvers when you are having negative loads, and then you do not do accelerations".

So, that is why, by, now these numbers have come from experience; they have not come from somebody's toss, or just arbitrary, they are coming on experience. People have fitted aircrafts with load factor meters and measure load factors which occurs. In fact, I have done that when I was in HAL Nasik. We have fitted accelerometer on MiG-27, done flight testing. Because there was a requirement on MiG-27 to subject the aircraft to load factor of 5.5g, with no armaments loaded; and 4.5g with drop tank loaded.

So, we used to do that, we used to measure the load factor, the flight recorder used to give us the data and we had to ensure. I have a very interesting lecture on that, which I will give some time when we have. It will be a general lecture for everybody in the department, ok. So, yes there are requirements for negative load factor but, they are typically half of the ones. Now, these numbers are generally non-negotiable, generally non-negotiable.

But, if you go to agency and say, "Look I am designing a very special aircraft, and I can prove to you that this aircraft will never experience more than  $N_z$  equal to 3.1" And if you can convince the agency by testing, by calculation or whatever they demand, they may give you a concession. Such things have happened. For example, there is an aircraft called Voyager, right. What is the famous thing about Voyager?

What does it do? Why is it so famous? Do you know what is a Voyager aircraft? So, why do not you check on Google and tell me? More importantly, from today's lecture point of view, I want to know what was the concession given to Voyager in load factor calculation or load factor estimate?

Why was it given? It's a very unique aircraft. So, these are guidelines. So, it's like unless otherwise stated, you have to follow this table. If you want to, if you want to go beyond this table, then you need to make special provisions and proofs.

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Factors that governs the upper limit of  $N_z$

- Structural strength of a/c
  - high  $N_z$  means designing the aircraft structure to bear higher loads
- Safety and Comfort of Passengers and Pilot

See this **TABLE**

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Yes. Yes, yes, because, see as a passenger if aircraft is subjected to vertical acceleration, you are also inside the aircraft although there is a seatbelt. But, if the aircraft does like this, you will not be comfortable. As a passenger you will have serious problems right? So, if aircraft is going in level flight, let us say,  $N$  equal to 1, and suddenly there is a disturbance which we will see very soon.

You will have some  $N_z$ , and if the  $N_z$  because of disturbance is 4, that will get added to the level flight. So, it will become 5. So, if  $\Delta N_z$  is 4, because of a huge jerk, so, you will get a, you will get a force on your body which is 5 times your weight. How will you feel? You will be highly disturbed, ok. So, if a passenger, from a passenger point of view; the load factor which the aircraft can be subjected to should not be very high.

But, from safety point of view, it should be sufficiently high. Because if the load factor is kam the aircraft should not break. Just to give you the comfort I cannot say, from comfort point of view the aircraft should take only 1.2 load factor, if it is 1.4 aircraft will break, I hope there is no problem. So, this you cannot say. Safety is paramount. So, from safety point of view there is a limit, ok. But,



it cannot be that, ok, to make it safe, make a transport aircraft also  $N_z$  equal to 7. Safer, but, that will cause discomfort.