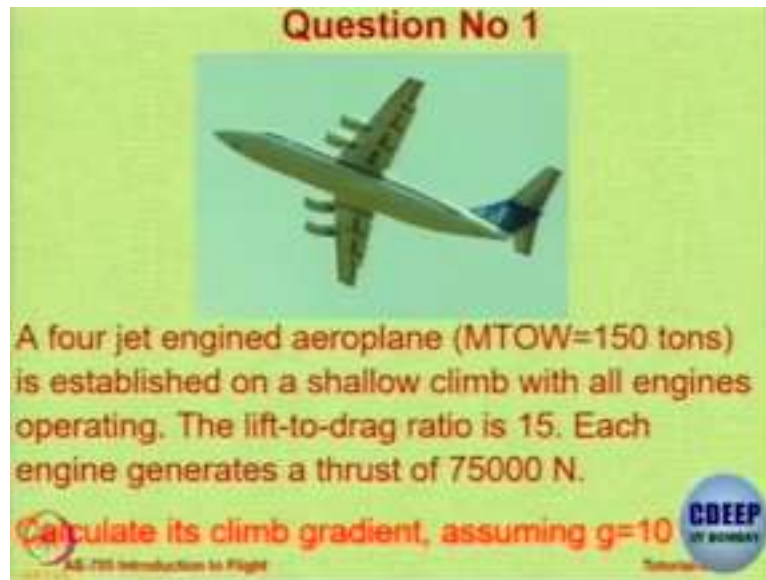



Introduction to Flight
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Lecture: 09.6- Tutorial on Climbing Flight and Turning Flight

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Question No 1



A four jet engine aeroplane (MTOW=150 tons) is established on a shallow climb with all engines operating. The lift-to-drag ratio is 15. Each engine generates a thrust of 75000 N.

Calculate its climb gradient, assuming $g=10$

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Okay, so question number 1 would be what is this? Can you guess the aircraft, this aircraft belongs to a flight training school. So what do you see basically? What do you observe? Is this a jet engine aircraft? There are four engines, are all four engines working? How do you know? I can not say, they are just there, so is this a dive or a climb? Is this a very very steep climb?

No, that's the thing. So here is a four engine jet engine airplane with a maximum take-off weight of 150 ton and just after takeoff the pilot establishes this aircraft in a very shallow climb thankfully with all four engine working.

So the aircraft is designed very beautifully aerodynamically as you can see so the $(L/D)_{max}$ or L/D in climb is 15, this is L/D in climb with flaps deflected etc, each engine generates 75,000 newton, so what, this is just a story, ok. The question is can you calculate its climb gradient and to make things easy for you g will be taken as 10 meter per second square, so do it please. So what is climb gradient? What is the formula for climb gradient?

Yes? What is climb gradient? How do you calculate it? Let's say? But that is the dh by dt. What is the rate of climb? That is the rate of climb. Yes? What is climb gradient? How do you calculate in this case? Is there a formula available for climb gradient calculation? Yes or no? Yeah? How do you calculate tan theta in this example? Ok, so with tan theta, get me the value. 1.3 ok, anybody else?

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Solution to Question No. 1

- For shallow angle of climb, $L = W$
- Climb Gradient $\gamma = (T - D) / W$
- $T = 4 \times 75000 = 300000 \text{ N} = 3 \times 10^5 \text{ N}$
- $D = W / (L/D) = (150000 \times 10) / 15 = 1 \times 10^5 \text{ N}$
- Thus, $\gamma = (T - D) / W$
 $= (3 - 1) \times 10^5 / 15 \times 10^5 \text{ N} = 2 / 15$

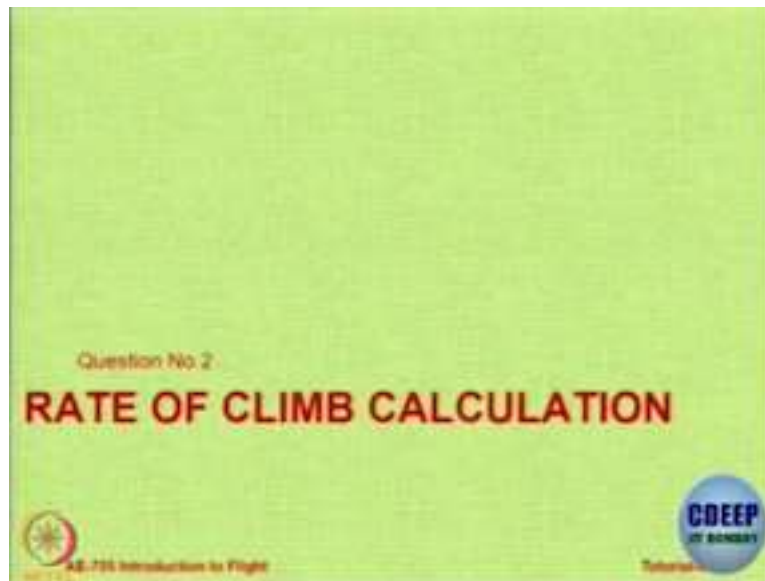
$\gamma = 0.1333$ or **13.33 %**

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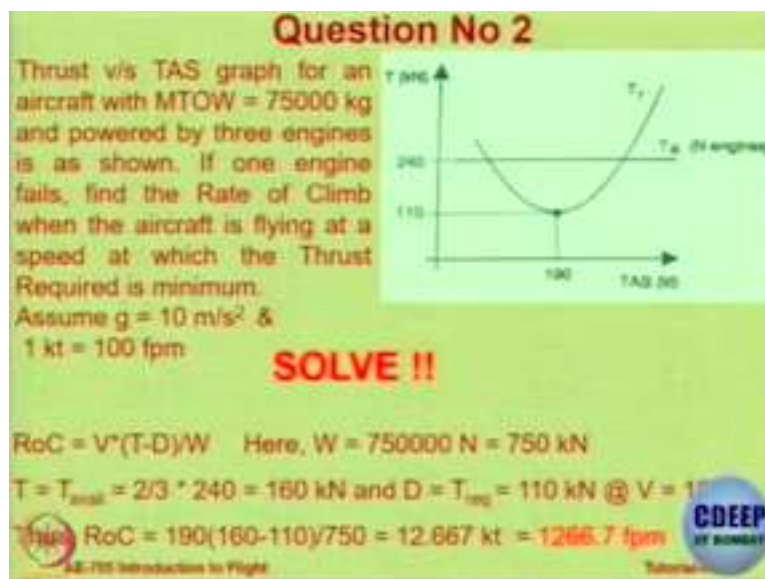
So for a shallow angle of climb $L = W \cos\theta$, $\cos\theta = 1$ so L equals to W, so the climb gradient is $\frac{T-D}{W}$, excess thrust by weight, that is climb gradient. So thrust is 4 into 75,000 newton, drag will come from L/D because $L = W$, now at least do it and give me the answer. If you know the answer, please raise your hand if you know the answer. Yeah? 12 P. what is it 2 by? 2 by 50 cannot be calculated? 0.1333 and express in percentage 13.33 percent, yeah, so that's it. Ok, now is it clear.

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Now the next one is the next step after getting gradient the next thing is $\frac{dh}{dt}$.

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So for that we have picked up an aircraft and we got its thrust available and thrust required curve, so notice their X axis is two airspeed in knots and the Y axis is thrust in kilo newton, ok. The first question is what kind of engine does this aircraft have? It's a jet engine, why is it a jet engine? Thrust available is a constant line, this is not the actual story, actually there is a slight change in thrust available, ok, it goes up and comes down slightly but in the classroom we take it straight problem.

So here is an engine, this aircraft has three engines and its takeoff weight is 75 tons, 75000 kilograms, now one engine out of three has failed and the aircraft goes into a climb. So the pilot immediately brings the aircraft to a speed at which the power required is minimum, Okay. And we want to know what will be the rate of the climb.

So question number 1, at what speed will be the power required minimum. Yes? Ok, at what speed will be the thrust required minimum? Both are equal? So are you sure at the speed at which the power required and thrust required minimum are same? They are not same, okay.

So at what condition, how will you get the speed? So I will make the question easy for you, what I will do is just to make it easy for you because... okay, how about making it fly at a speed at which thrust required is minimum. So first of all how do you calculate rate of climb? Anyone can speak out please raise your hand, yes, what is the rate of climb? Excess power by weight, how do you calculate? Excess power by weight that means $\frac{T-D}{W} * V$.

So please do it now, you have the data with you and because the rate of climb is normally expressed in feet per minute, feet per second rather than knots, there is a conversion given that one knot is equal to 100 feet per minute. So therefore the speed at which the thrust required is minimum you can convert that into feet per minute to get the answer in feet per minute because $\frac{T-D}{W}$ will cancel, units will be cancelling, okay.

So now I want the answer of this question in feet per minute, let us see how soon you can do it. By the way please note the line that you are seeing horizontal line of thrust available it is for all engines working, it is n engine here, this is not thrust available per engine this is thrust available from all the engine. Yeah? What is the answer you got? 4 feet per minute, does it sound reasonable to you? An aircraft with 75,000 kg weight, its too low, 4 feet per minute is not acceptable, that is near the ceiling, 4 feet per minute. 3300, 1200? No, slightly more than 1200, that sounds reasonable, ok.

Let's see, the RoC is basically $\frac{T-D}{W} * V$, so the thrust available will be two thirds of the total and the drag will be equal to thrust T required because, you are flying at a speed at which thrust required is minimum at which and the velocity is given to you. So the

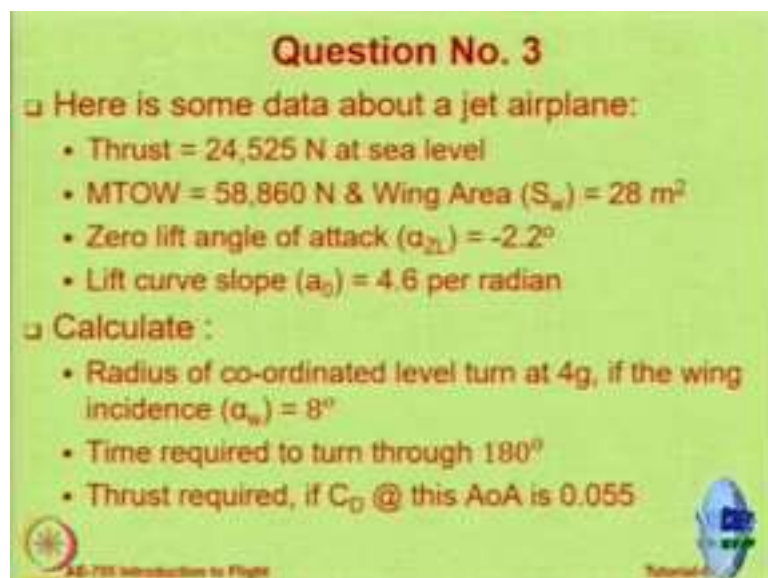
RoC will be 12.667 knots or 1266.7 feet per minute. So this number is reasonable 1200, okay.

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Next question, question number three, this is regarding turning flight, ok.

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So here is a question, now I would request you to note down in your notebooks the first part, which contains the data, because you will need it in the calculation. So sea level thrust is 24,525 newtons, max takeoff weight is given, wing area is given, its zero lift angle of attack is negative 2.2 and the lift curve slope is 4.6 degrees per radian. What we need to calculate is the radius of a coordinated level turn or steady level turn at 4g

and at that condition the wing incidence is 8 degrees then what is the time required for it to turn through 180 degrees.

This is because the aircraft has to now come back, it has taken off and gone and then it say you forgot something please come back, so it has to take a 180 degree turn and come back, so how much time will it need, this time is helpful in planning the operation at the airport and what would be the thrust required? If the drag coefficient at this angle of attack is given as 0.055.

So this is a slightly involved question, you will take time to do it, ok. So what will be nice if someone can first, do not solve the question, just figure out how you will do. So for that you first draw the horizontal circle in top view and write down the equations that you need, for radius of turn

What is the equation for radius of a turn? $\frac{V^2}{g\sqrt{n^2-1}}$, ok. So now what all things are available to you? n is available to you, though it is not known to you, okay, n is known to you. Now you need V , you need V , so how will you get V ? Anyone? Please raise your hand and tell me how do you calculate V ?

The aircraft is in a coordinated turn at n equal to 4, how do you get V ? Yes? Ok? So let me ask you how do you calculate C_l of the aircraft at the data given? Right. Correct. From the real lift line so that means the net or the effective angle of attack, or the absolute angle of attack as it is called will be 8 minus, minus 2.2 or 10.2 degree, okay then, so how do you convert to degrees? 0.3 correct. That's right.

So you convert the lift thrust slope into degrees per second, per degree multiply by 10.2, what will you get? C_l , okay, then? How much will it be? Correct. So the lift will be 4 into W and assuming sea level, you see in any question if nothing is given then you assume sea level and it is given thrust at sea level so I don't want to mention it everything there so assume sea level.

So what is ρ at sea level? Correct. So you know ρ at sea level. S is given, C_l is calculated, lift is 4 times W so you can calculate V . When you calculate V you have V square by root n square minus 1, you are on right track. With that you will get V , okay. Hold on, hold on, let's say we got V , now how do you get time required to turn through

180 degrees? So this question should be answered by this particular group now. They are always answering so you should answer now, how do you get? Please?

How do you calculate omega? Very good, so we know everything, so we get omega, 180 so, we can get from there, how much time it will take, okay. Then the last one is thrust required, now that is for this particular group. How do you get thrust required? It is not a level flight, it is a flight in the coordinated turn, in level flight, lift is equal to weight, here lift is equal to 4 times weight. It is a steady level coordinated turn, yeah, but the question is about drag. How do you calculate the thrust required?

So because it is a leveled turn you cannot allow thrust to fall below drag. It has to match the drag otherwise you will start becoming slow, you cannot maintain the V, okay. So what will be the drag? How much? No no no, that is the thrust, that it produces, the maximum thrust at sea level that is not the thrust required. Yes? That's it.

C_D is given to you, you know V, so $\frac{1}{2}\rho V^2 S C_D$ is D and that is equal to T. So now you know the method, so now let's get the numbers, okay. So again we will follow the same procedure but we will reverse it. So the radius will be given by this particular group, somebody from here will give the radius of turn. You are going to give the time required for turn and you are going to give us the thrust required.

So for all of that you have to calculate V. So before you calculate these numbers the first thing we need is the velocity. If you get that right then everything will be very straight forward. So let's see if anybody can tell me the velocity. Root of n times W by half rho, half rho S C_L . So we need C_L before that, ok.

So how much is the C_L ? 0.818, C_L is reasonable, in level flight C_L is approximately 0.3 to 0.4, in turning flight C_L tends to higher because n equal to n times W, C_L tends to be bit higher, so acceptable. 0.818, so if that is the value of C_L , then what is the value of V?

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Solution to Question No. 3

- $a_0 = 4.6 / \text{radian} = (4.6 / 57.3) / \text{deg} = 0.0803 / \text{deg}$
- $C_L = a_0 (\alpha - \alpha_{0L}) = 4.6(8 - -2.2) = 4.6(10.2) = 0.819$
- $V = [nW / (\frac{1}{2}\rho S_w C_L)]^{1/2}$
 $= [4 * 58860 / (\frac{1}{2} * 1.2256 * 28 * 0.819)]^{1/2} = 129.45 \text{ m/s}$
- Bank angle $\phi = \cos^{-1}(1/4) = 75.52 \text{ deg}$, $\tan \phi = 3.867$
- $R = V^2 / (g \tan \phi) = (129.45)^2 / (10 * 3.867) = 433.34 \text{ m}$
- $\omega = V/R = 129.45/433.34 = 0.2987 \text{ rad/s} = 17.1 \text{ }^\circ/\text{s}$
- Time for 180° turn = $180/17.1 = 10.52 \text{ s}$

$T_{\text{req}} = \frac{1}{2} * 1.2256 * 129.45^2 * 28 * 0.055 = 15814 \text{ N}$

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So this is the first step, second step which most of you have got so far, okay. Now the next step, next step is just V. So V will be basically square root of n times W divided by half rho Sw Cl. So please confirm this number, tell me if you get the same number. Something wrong? What is wrong? Yeah, sorry, that, correct, correct, correct, you are right, it is a mistake in copy paste, correct, the number is correct but, the numerical value is correct, this is a mistake in typing out, I have to correct it.

So much is V, do not look at me. I also got that, so I am very happy, Okay. That is fine, 4845 that is ok, some people take density as 1.225, 1.2256, 1.226 that can create a problem. Cl could be 0.818 or 0.819, okay.

Next step, next step is to get the value of phi. You can also do it as V square by, some other formula, we have taken this particular formula. So is that what you got? Radius of turn, okay. So the next thing was time, so for that we need omega. So V by R and then 180 degrees will take how much time? Almost around 10 second, right, assuming that the turn is constant and it will be because the steady level turn. So the turn rate 10.52 seconds, almost 11 seconds.

So finally last question was drag, okay. If there is a mistake please tell me. So notice the thrust required is far below the thrust available.