Introduction to Flight

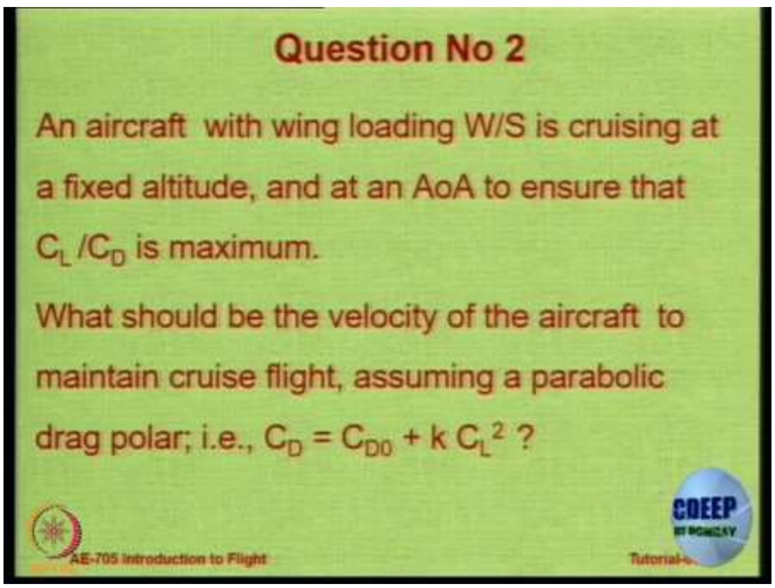
Professor Raj Kumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology, Bombay

Lecture 08.4

Tutorial on Steady Level Flight

So, we will have a look at the two chapters or the two lectures that we covered the first one was basically on introduction to Propulsion System. So, it is an introduction on that I do not want to do any numerical, we will do numericals based on level flight. So, the first question is an analytical question, you have to derive something and that question is about estimation of the velocity that is to be used for level flight. So here is the first question, ok.

(Refer Slide Time: 0:56)



There is an aircraft with wing loading $W/_S$ that means aircraft weight is W and the wing area is S. It is cruising at a fixed altitude so that means ρ is constant and also the pilot flies it at an angle of attack at that velocity to ensure that its $C_L/_{C_D}$ is maximum. Given this condition, what should be the velocity of the aircraft to maintain the steady level flight or the cruise flight assuming a parabolic drag polar; that is $C_D = C_{D_0} + KC_L^2$?

Remember, that the parabolic drag polar is only an assumption. In real life the drag polar or the drag variation of the aircraft is not parabolic, typically we see $C_{D_0} + K_1 C_L^2 + K_2 C_L$ that is a k into C_L term also but we ignore it because by ignoring it you get a simple expression which can be used to derive some formulae easily so, now the question is open to you. You have to derive the condition for V the velocity for cruise such that ${C_L}/{C_D}$ is maximum and what is that velocity in terms of ${W}/{S}$, in terms of C_{D_0} and in k. So your answer should contain a formula V for minimum ${C_L}/{C_D}$ is equal to a function of ${W}/{S}$, k and C_{D_0}. So, can you please derive the expression? So the way to do this first to get the condition for ${C_L}/{C_D}$ maximum. What is the value of C_L or V at which ${C_L}/{C_D}$ is maximum? Does anyone know the answer already? What is the condition for ${C_L}/{C_D}$ to be maximum? Yes please.

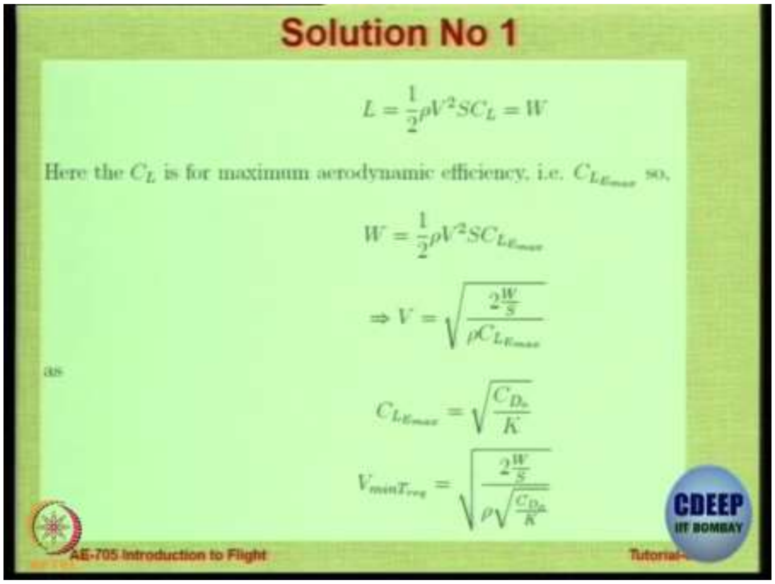
Student: Ok that is great, so when the two drag terms are going to be equal that means when C_{D_0} is equal to k that is a condition, but can you derive this condition? So to derive the condition what you will do is you will say that $C_D = C_{D_0} + KC_L^2$, take a partial derivative with respect to C_L such that ${C_L}/{C_D}$ is maximum because you can easily derive it ok, so ultimately as we heard the condition you will get will be C_{D_0} is equal to k, ok. Let us just save some time, let us assume that the optimum condition for ${C_L}/{C_D}$ to be maximum is C_{D_0} is equal to k. If that is a case, now it is very simple to derive the expression for velocity. So please do that, ultimately I should get the expression which says V is equal to a function of ${W}/{S}$, C_{D_0} and k. So whenever somebody gets the answer, you just raise your hand. What is your answer, V is equal to?

Professor: You have to take k. I told you I want the answer in terms of only C_{D_0}, k, ${W}/{S}$ and may be ρ that is all, I do not want to see anything else in the expression, ${W}/{S}$, ρ, C_{D_0} and k.

Professor: All of it is to the power 1 by 4, then you do it if you have something like square square and power 4, 1 by 4 you do it make little bit more elegant. Your answer may not be wrong but you can reform it slightly. It is actually very straight forward, you have to just say lift equal to weight therefore $W = \frac{1}{2}\rho V^2 S C_L$, ok. So take S on the denominator on the left hand side, you get

${W}/{S} = \frac{1}{2}\rho V^2 C_L$, and C_L is nothing but $\sqrt{\frac{C_{D_0}}{K}}$. So that is it, it is a very straight forward expression, ok.

(Refer Slide Time: 6:11)



So that means if you know the drag polar of the aircraft, that means if you know C_{D_0} and k and if you know its wing loading $W/_S$, for a given altitude you can calculate what is the true airspeed required such that the $C_L/_{C_D}$ is maximum and if $C_L/_{C_D}$ is maximum then that is a condition corresponding to thrust required is minimum because we got it from drag required is equal to minimum, ok. So it is a nice elegant expression which can be very handy in calculating at what speed should you fly an aircraft so that the thrust required is minimum, just look at the $W/_S$ which is easily available for every aircraft C_{D_0} , k and Rho, alright.

(Refer Slide Time: 7:13)

Moving on, the next question is to try and get a variation of thrust available and thrust required at sea level for an actual aircraft, and to do this exercise I have chosen this very beautiful aircraft called as the Fairchild Republic A-10 or also called as a Warthog. I say it is a beautiful aircraft but many people find it very ugly, ok. But as I always say in war in combat we do not have a beauty contest, we have a contest of capability. So let me present you my friends Fairchild republic in a war scenario.

(Refer Slide Time: 7:48)

See how this aircraft can be used to create havoc. I have a question as this aircraft is drifting away or pulling away it is throwing out some things. Can you see that? There are some white things coming out from the aircraft. Have you observed, what are these? What are these white things which the aircraft is shedding as it goes into a turn? Yes, smoke flare. What is a smoke flare? Is the aircraft a smoker? Before going for a fight let me have a smoke. What is it for? It is not a smoke flare. There is a better word for it, you are near the point but not exactly. Anybody else? May be this side somebody? Yes

Student: After burner.

Professor: After burner is after burner behind the engine, this is ahead of the engine. See once again, see where is it coming from and why will it come in pulses?

(Refer Slide Time: 9:12)

After burner will be a continuous you see it is coming out in pulses, but it is not a pulse jet, but it is coming in pulses, ok. Anybody else?

Professor: Correct.

Professor: Not any kind of missiles, only the heat seeking missiles correct.

Student: Yes.

Professor: So an aircraft that flies low and in the enemy territory for attack is always going to be under the radar of heat seeking missiles so these are called as magnesium flares. These are a kind of a safety or a counter measure. So, to confuse the infrared heat seeking missiles you throw out magnesium flares which is also called chaff dispenses. So you create very high heat source so the missile can be misguided, ok so it is a defense or a safety mechanism. So the aircraft is has peeled off and now is going to combat, so therefore it is sending out these to confuse any of the missiles, ok.

(Refer Slide Time: 10:33)

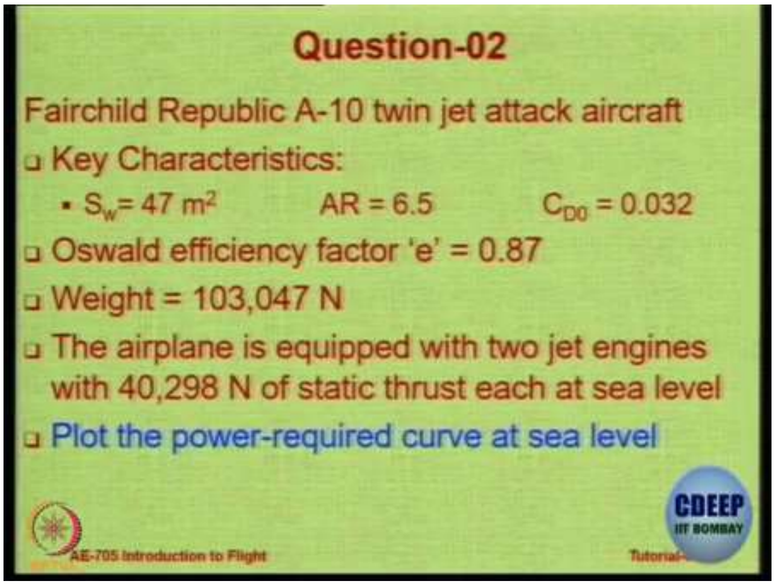
This is a machine gun mounted on the aircraft.

(Refer Slide Time: 12:31)

Video: A-10 saving the day again, baby. It is an awesome testament to the aircraft. I think that the same gun that we used to kill main battle tanks in 1991 is the same gun where we can shoot a single insurgent that is fleeing on a motorcycle or shooting at our guys from a tree line. You will not pursue the elimination of the finest close air supports weapons system in the world. Senator, I will give you the facts of how many targets have been struck by which kind of platforms in Iraq and Syria over the last year. Yes, a significant number of them have been done by the A-10. Is that true or false? No, it is true. It is true? Then why would you want to retire the least expensive, most accurate close air support system. Enough said, general, Ok?

Professor: So this is politics in air force. The air force wants to buy the new aircraft F-35 which is very expensive as you saw to buy fifty F-35 they have to cancel or retire 300 of these aircrafts and this is very effective, it is doing a lot of good work. So, therefore John McCain says I will not support this activity of encouraging F-35 and downgrading or retiring A-10. So, they have got a new leads of life, they have been refurbished and they are still being used. So there are several interesting design features of this aircraft but our question is today on performance, ok. So we will look at a very-very boring thing that is steady level flight of Warthog, no combat, no bombs, no missiles, ok.

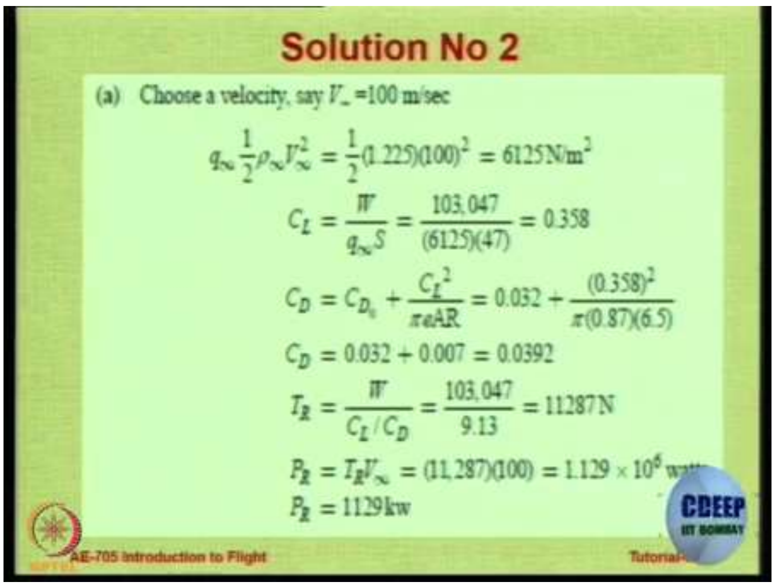
(Refer Slide Time: 13:39)



So, we are going to look at this particular aircraft and these are some of the key characteristics the wing reference area, the aspect ratio and the zero lift drag coefficient. Notice that the value of C_{D_0} is very high it is 0.032, whereas the value is normally of the order of 0.02 or even 0.01815 for very smooth aircraft. So this is not a very smooth looking aircraft, it has got lots of appendages which come out because it carries armaments. It is meant to carry a lot of armaments, so it is not a very pretty aircraft, not aerodynamically very smooth but does not matter, ok. It has extremely powerful engines and not one but two of them from reliability point of view. So two jet engines each of them giving you approximately 40 kilo Newtons of static thrust at sea level, so 80 kilo Newtons of thrust available at sea level from two engines. The weight of the aircraft is 103,000 Newtons.

Our job is to fold, we were to plot the power required curve at sea level and also we have to plot the power available curve at sea level and hence, get the value of the maximum velocity at sea level, ok that is our job today. Ok, so to do this first thing you do is please note down these four or five important things somewhere in your notebook, S_w =47 square meters, AR of wing 6.5, C_{D_0} 0.032, the Oswald efficiency factor or e as it is called 0.87, W 103047 Newtons and T will be 40,298 Newton, these numbers please note down, ok.

(Refer Slide Time: 15:44)



So how do we do this? We look at some velocity say 100 meters per second. Is this a realistic number for an aircraft of this type, 100 meters per second? Some people are saying no. What is the problem? Is it too high? Yes, but may be it can fly at this speed, ok it is not too high it is not beyond the expected V max. We will know very soon, whether it can fly or not depends on whether the thrust available is equal to thrust required or power available equal to power required or more, ok. Let us choose a number 100 meters per second, ok so dynamic pressure at sea level for an aircraft which has got a speed of 100 meter per second is $\frac{1}{2}\rho V^2$.

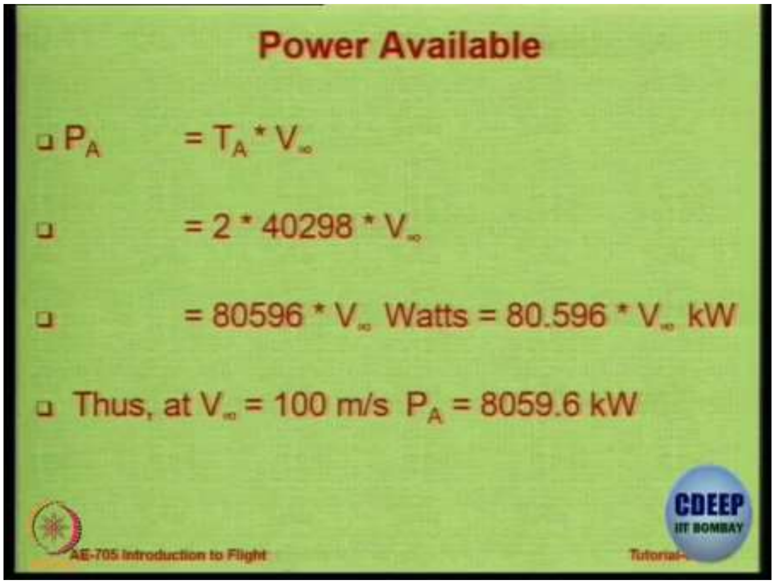
Now this a first place where many students make a mistake. Observe and notice carefully you are putting density in kg per meter cube which is 1.225 at sea level, you are putting velocity in meters per second. When you put these numbers, although the density is in kg per meter cube the answer q infinity will not come in kg per meter square, it will come in Newtons per meter square. The reason for that is units have to be balanced on both sides, ok. So q is dynamic pressure, it could be kg per meter square, it could be N per meter square, but you check density is kg per meter cube, velocity is meter per second so this is kg per meter cube into meter square per second square where you cancel the various units you will find that g or meter per second square remains. So therefore the answer that you will get will be in Newtons per meter square.

Many people make this mistake because they have taken kg per meter cube they think the answer in kg per meter square, but be careful about this thing, ok. So 6125 N by N square is the dynamic pressure at sea level at this velocity. Now the lift coefficient C_L for level flight is $\frac{W}{qS}$ because lift has to be equal to weight. W is in Newtons 103047, q is 6.25, S is 47, these numbers are not going to change, W and S are going to remain the same so therefore C_L will be 0.358 this is very a appropriate number. C_L in level flight normally tends to be approximately between 0.3 to 0.4 so that is a right a right ball park between 0.3 to 0.4 is the expected value of C_L for level flight.

Once you know the C_L then you know the value of e which is 0.87, AR with 6.5 so therefore you can get the value of C_D. Once you know the value of C_D you can get the value of D drag and thrust equal to drag so the thrust required will be $\frac{W}{C_L/C_D}$, ok. Remember there was a mistake in my notes, in my notes I had shown this thing little bit differently I think. So $T_R = \frac{W}{C_L/C_D}$ so that is 103047 by C_D, C_L/C_D which is 9.13 which is 11287 Newtons. The thrust required is only 11287 Newtons, the thrust available is actually 40,000 into 2, quite a lot. So we are comfortable at this speed we have much more thrust than required.

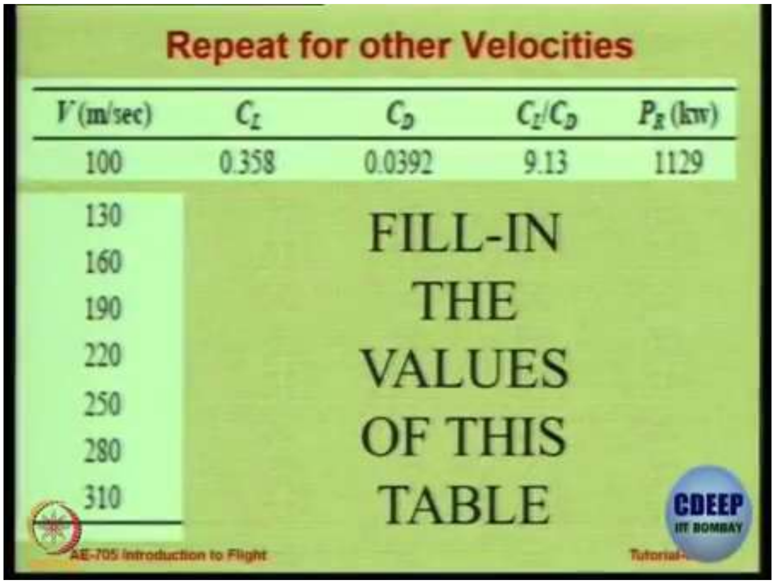
Power required will be thrust into velocity, velocity is 100 meter per second, thrust required is 11287 so the power required will be 112900 so it will be in watts 1.129 into the 10 to the power 6 watts or 1129 kilo watts, ok. So now in a graph paper or in a sketch against 100 meter per second on X axis you to have to mark a point 1129 kilo watts on the Y axis. Simple, this is how you get the power required at a given speed for steady level flight. So we have used we have used two conditions here lift equal to weight, thrust equal to drag.

(Refer Slide Time: 20:15)



Power available is basically the thrust available into the velocity, thrust available is going to be given to you two engines each of 40298, so therefore it will be 80596 into V infinity in watts which will be 80.596 kilo watts I have just divided by 1000. So total power available is 80 kilo watts so at 100 meter per second the velocity of 100 will give you 8059 kilo watts, required 1127, available 8059, required is only one eighth of available so absolutely no problem, ok.

(Refer Slide Time: 20:59)



So now we need to repeat this for other velocities, so I just show you what we have already obtained for one particular speed, correct. So now we have to take a guess, at this speed, power available is eight times power required roughly and the power required is proportional to V cube so we need to take a guess up to what number we can go but we do not know apriori, our job is to find out where it is maximum. So what we can do is take some rough number. So, I have just taken numbers 130, 160, 190. So now each one of you, you have a choice of any two velocities, just write down on your notebook the number. Choose any number between 130, 160, 190, 220, 250, 280, 310. You can choose some other number also but then you cannot verify because for these I have the values so you can verify, ok.

So pick up any two numbers, any two velocities of your choice and what you do is fill up this table for your aircraft sorry, for your operating condition. Also there is one more column to be put which is the power available. At sea level, we assume that the power available is not going to change much with the velocity. Power available will change sorry, it will be proportional to velocity. So it is a sea level so the maximum velocity remains the same just multiply by the velocities you have chosen. So put one more column where you have P available in kilo watts, ok. So this is an exercise for individuals but you have a choice of two velocities whatever numbers you like. The first thing you need to calculate is dynamic pressure for which density is the same but velocities are different, so Sushil what velocities have you chosen?

Student: Sir, 310.

Professor: 310 ok, that is it one velocity? no I said choose two velocities, so at 310 do you get power required more than power available?

Student: Yes.

Professor: It means that you have crossed the maximum speed. So we have one hint from Sushil, if his calculations are right which I assume will be then the maximum velocity is below 310, ok. So Sushil take 280 maybe because 310 is more than what your aircraft can handle. 310 meter per second would be almost, almost near mach one, mach one is 330 or 340, ok. So 330 and 310 means you are almost near Mach number one. In another words, it seems that this aircraft cannot fly supersonic at sea level, ok. But please note this is not an exact calculation, this is a small example, small example for the classroom purposes so do not take it literally that these numbers are perfectly correct.

Also remember one more thing that $C_D = C_{D_0} + KC_L^2$ is actually a parabolic approximation. If you reach very high Mach numbers near 1 for example there will be other drag terms will come into play. You will have probably a critical Mach number much before 0.9. So, therefore this particular simplistic calculations may not be applicable but nevertheless we want to use it for calculations purposes.

Ok, anybody else have you finished two velocities?

Student: Yes.

Professor: You have finished, which one have you chosen?

Student: 220.

Professor: 220 and?

Student: 310.

Professor: And 310, ok.

Professor: Ok, so you have three velocities now. So, are you confirming to me that the aircraft can fly at 280?

Professor: Power

Professor: Approx, so which is more?

Professor: PR is more so which means the maximum velocity is slightly more than 280.

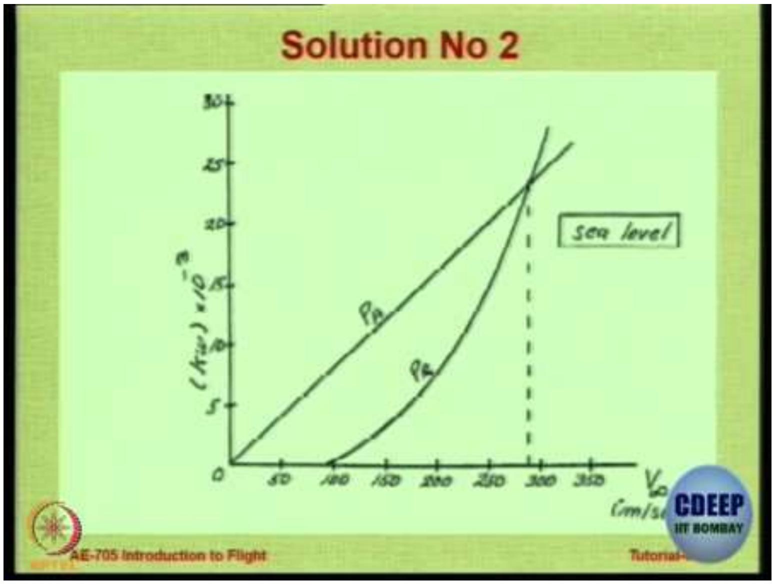
Professor: No, no more than 280, more than 280, Does anybody know what is the maximum velocity? You cannot. It will be more than 280 less than 310 unless somebody does for 300, ok. So let us see the table now, ok. Look at the numbers, I have the numbers only for power required, power available I do not have the numbers here. So check for the numbers against the velocity which you have chosen, if the numbers are not matching then please look at your calculations. So, is there anybody who has a difference of opinion? So are these numbers ok, at least for your velocity? They seems to be ok.

So we notice that the C_L/C_D is actually maximum at 100 itself amongst the numbers given, as the velocity increases beyond 100, C_L/C_D is reducing. So it will be worthwhile to try 80 meter per second as the velocity maybe at that velocities we may have a C_L/C_D higher than 9.13, we do not know, it is the matter of just calculating, ok. So this is something you can try out yourself, in fact what you can do write a small code and using that code you can calculate the value of power available and power required and then plot the graph. What will be nice if somebody can upload on Moodle the value of P required, P available for velocities from say 10 meter per second, that will too less I think, something like say 20 to 25 meter per second to the maximum value at sea level, ok. So whoever wants to do it just to brush up their knowledge can do it and upload. So this is a, yes, yes there is the question.

Student: Can we assume that maximum thrust we can provide and then do it from behind?

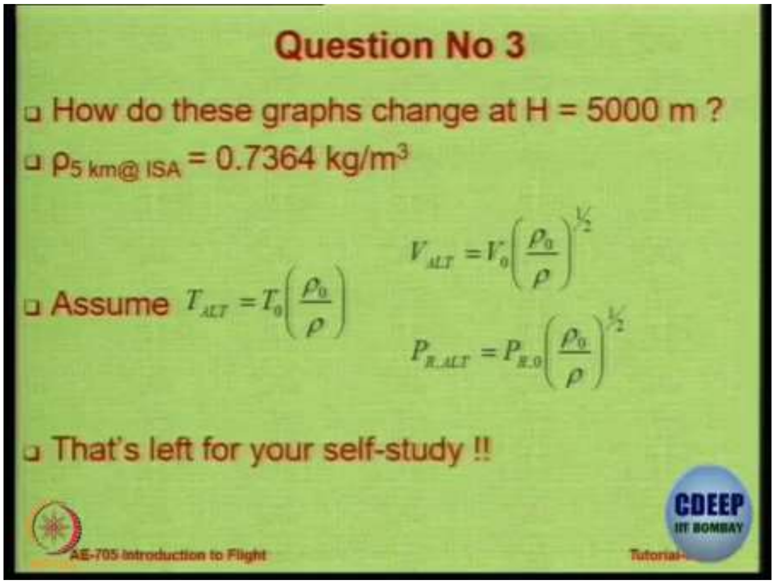
Professor: So you can do that also, but how do you know? Yeah you can do that to get the value but then you will get two answers you will get a lower value and the higher value so you can take the higher of the two. Yes, you can do that if your question is just to find the velocity at which you can fly max, you do not have to do all this, you can do it directly by a single equation, but I wanted to see the plot because I want to look at many more things. For example, I also want to know what is the velocity to fly at which thrust required is minimum, what is the velocity required at which the power required is minimum. I want to confirm that C_L/C_D is actually minimum and I am more curious, I also want to plot $C_L^{\frac{3}{2}}/C_D$ and I want to confirm that $C_L^{\frac{3}{2}}/C_D$ is also maximum or minimum at a particular value, ok. so that is why I want you to do this.

(Refer Slide Time: 28:56)



So, here is the graph for power available versus power required for this example. For speeds below 100 we have not drawn it because we have not taken it up, but you could do it and with that we get the intersection at around 295 meter per second. So by this very simplistic calculation the maximum speed of the aircraft at sea level will be 299 meter per second, actual value will be less than this because, the drag will increase very rapidly. May be this velocity is more than the critical Mach number. So, therefore the drag will rise rapidly so roughly this value can be considered, ok.

(Refer Slide Time: 29:39)



Let's look at what happens to the same thing at a higher altitude. So question number three is everything else remaining same, just change the altitude of operation as 5000 meters. So earlier you had a thrust required and thrust available at sea level. Now we want thrust required and available at a height of 5000 meters. So to do this we need to know two things; one thing we need to know is what is the density of air at 5000 meters? Now, this is a number which you can actually calculate because if you assume ISA conditions which we have to assume unless it is stated explicitly in the question you can assume the conditions to be ISA, ok whenever there is a question if not mentioned you have to assume ISA. So under ISA conditions, how will you find density of air at 5000 meters? What is the procedure you will follow?

Student: Sir under ISA conditions the temperature drops to 6.5 degrees per kilometer.

Professor: Ok, under ISA conditions I agree that temperature drops at the rate of 6.5 degrees per kilometer, fine.

Professor: So this whole formula just to memorize it, under ISA conditions it becomes T by T0 to the power 5.258.

Professor: Yeah what is T1? What is T0?

Professor: How much T0 at sea level?

Student: 288 degree Kelvin.

Professor: 288 degree Kelvin and density?

Student: 1.22.

Professor: 1.225 or 1.2256 kg per meter cube correct, very good. So you know T0, you know Rho 0, you can get T1 by subtracting 6.5 degrees per kilometer and this subtraction is acceptable till 11 kilometers. From 11 to 25 it remains constant above that do not bother it is not aeronautical it is space, ok. So it is very simple up to 11 kilometers reduce 6.5 degree per kilometer and T by T0 to the power 5.256 will give you the value of density so that number comes too, you can just cross check this phase quickly. 288.16 minus 6.5 into 5 upon 288.16 to the power 5.258 should be 0.7634 no, that will give you the ratio and that ratio into 1.2256 should give you 1.73, just check it. Right, so density of air at sea level is known so 5 kilometer is known.

Now we have to assume, we have to assume something now. We have to assume some variation of power required, velocity and temperature. So temperature we do not have to assume you know it is a variation like this. Now you may not, this you can say I do not want to worry, I will take the value of, I will take the value of V true at some altitude and calculate the value of CL, then K, then CD, then T and then P, ok.