Introduction to Flight
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Lecture 08.2
Power Required for the Steady Level Flight

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So let us see, now we are going to do some derivations, we are going to derive some conditions. The first important derivation is for, how much power is required by an aircraft when you are in the level steady flight or unaccelerated level flight.

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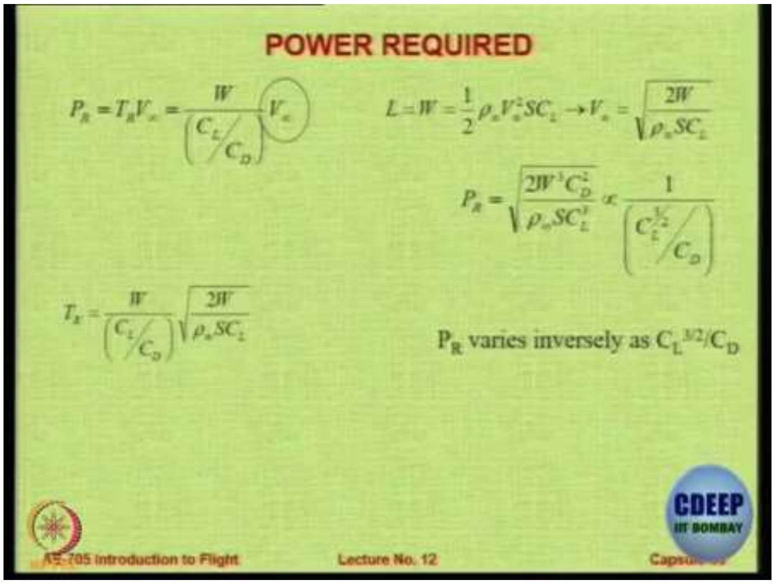

So $P_R = T_R * V$, the thrust required is correctly shown here now as $T_R = \frac{W}{C_L/C_D}$. Okay. Now can you derive an expression for V infinity in terms of W for level flight? How can you, can you do that? Now do not look at the screens please, now you have to derive some expressions. So please copy down this expression first. $P_R = T_R * V = \frac{W}{C_L/C_D} * V$. And V infinity now has to be replaced by an expression that incorporates W and CL for level flight.

So here, we have used the simple expression that we derived last time. So can you replace V infinity with something? Who will tell me what is the replacement? For steady level flight, what is the V infinity equal to?

Student: Yes, $\sqrt{\frac{2W}{\rho S C_L}}$.

Professor: Okay so you have used the condition that in level flight lift is equal to weight, so therefore, $L = W = \frac{1}{2}\rho V^2 S C_L$. So she is right. $V_\infty = \sqrt{\frac{2W}{\rho S C_L}}$. So now you replace this quantity V infinity inside the circle by $\sqrt{\frac{2W}{\rho S C_L}}$. What do you get? You will get an expression for

$P_R = T_R * V = \frac{W}{C_L/C_D} * V_\infty$ will now convert. So it is very straight forward this $V_\infty = \sqrt{\frac{2W}{\rho S C_L}}$ will go in that particular place. And the W outside will come inside the square root sign as W square, so it become W^3.

The C_L/C_D outside to come inside the square root will become $\left(C_L/C_D\right)^2$. So the C_L^2 will multiply with the C_L already in the denominator to become C_L^3. And the C_D^2 which came inside to make it go up, it will become C_D^2. So in other words, the expression will be $P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$. Now for a moment, let us not worry about aircraft weight because that is equal to lift and it is a constant number. ρ_∞ and S are also fixed values for a given altitude and for the given aircraft. So in other words, P required will be directly proportional to $\sqrt{C_D^2/C_L^3}$ or $C_L^{\frac{3}{2}}/C_D$. So if the value of $C_L^{\frac{3}{2}}/C_D$, so$\frac{C_L\sqrt{C_L}}{C_D}$. If this ratio is large, then the power required is going to be less. Now please tell me, is this expression valid for only a piston prop aircraft or jet engine aircraft or both of them?

What do you think? It cannot be none of the above, it has to be either only for piston prop. So when I say piston prop, I mean piston prop and turbo prop. When I say turbo jets, I mean turbo jets and turbo fans. So jets and props, these are the two words we will use so is it applicable for only jets, only props or both? What do you think? Yeah.

Student: Sir valid for prop.

Professor: Why only for prop?

Student: Sir because as you said earlier that thrust will be constant for jet.

Professor: It does not matter, whether the thrust is constant or variable, does that affect the expression? The expression come from basic physics yeah, maybe thrust is constant. Maybe it is constant. Okay so this argument is not acceptable to knock off turbo jets just because T produced is constant. This is about T required not T produced okay. So, what do you think? Is there some other compelling argument because of which this expression is not applicable for turbo jets. Anyone? Yes, there is no compelling arguments because when we derive the expression, did we say, turbo jet hence this, turbo jet, prop, no we did not. So this is true for any aircraft. For any aircraft the power required for steady level flight is minimum when $\frac{C_L\sqrt{C_L}}{C_D}$ is maximum, it is independent of the aircraft type.

The interesting thing is that the velocity at which this velocity occurs is maybe different but the expression is the same okay. So, now comes the interesting part, interesting part is, let us now go to thrust required. Power required you have already seen. Thrust required is actually much more straightforward because thrust required will be

$$T_R = \frac{W}{C_L/C_D}$$

Okay.

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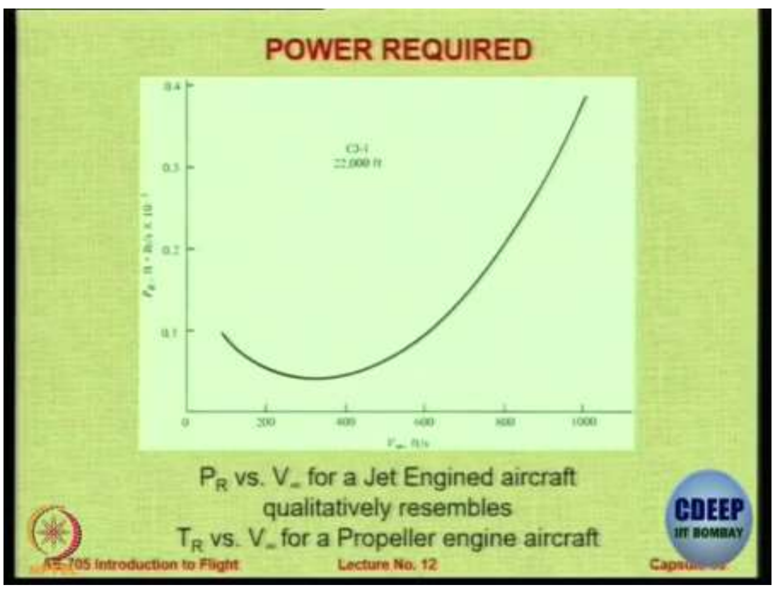

So, let us go ahead and see this thing graphically. Graphically what do we see?

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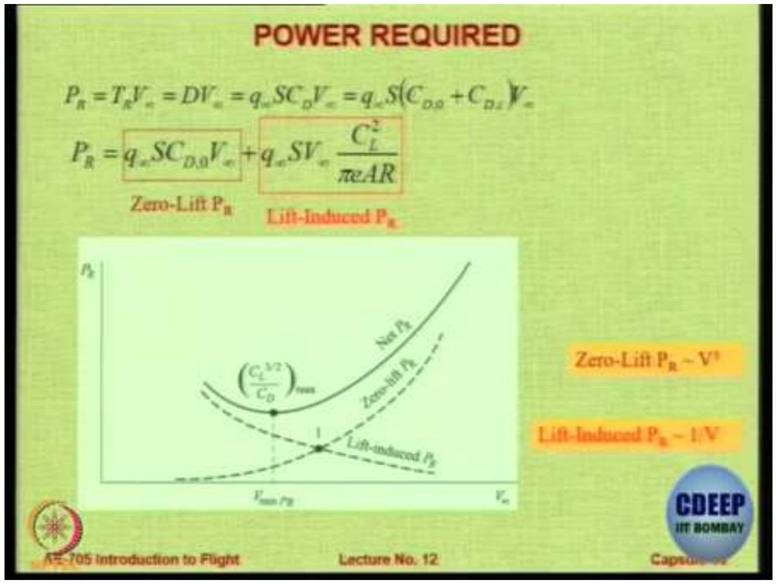

Graphically we see that okay one more interesting expression. Now it is not enough to say that $C_L^{\frac{3}{2}}/C_D$ should be maximum. It is also important to find out the actual operating condition under which it happens. So what do we do? We say that $P_R = T_R * V_\infty$ for any aircraft. And because thrust required is equal to drag, you can also call it as drag into velocity. $D = q\ S\ C_D$, this I have covered in the capsule on drag estimation where I have defined the drag coefficient. So, D will be $q\ S\ C_D$. And C_D itself has two components, $C_{D_0}$and lift dependent component C_{D_i} or induced drag component. In other words, the power required will be

$$P_R = q_\infty S C_{D_0} V_\infty + q_\infty S C_{D_i} V_\infty$$

$$C_{D_i} = \frac{C_L^2}{\pi\ AR\ e}$$

So now we have an interesting task to do and that task is, we have to now find the velocity to fly at which the power required is minimum.

So how will you get that? You have an expression that says power required is something plus something two terms: the first term has V infinity, the second also has V infinity but the second term also has C L square right. Our task is to find out what would be the condition at which, or what should be the V infinity at which the Pr is minimum. So how will you do that? Anyone can help me to proceed? Yes, Susheel, what do you want to suggest?

Student: Differentiate Pr with V infinity.

Professor: But what differentiation? Total differentiation or partial differentiation?

Student: Sir, total.

Professor: Why total? Does this velocity have a time component? So partial is sufficient? If there was a, remember our discussion about compression flow where there were two components, changing with time, here it is steady so it does not change with time. So if we take a partial derivative of Pr with V infinity and put it equal to 0, you get a condition for either minima or maxima, then you have to go for second, third, etc you have to find the first non negative. Okay. So the first component of this term is $q_\infty S C_{D_0} V_\infty$ which is the power required to zero lift drag. So we call it as zero lift power required. The second term will be the lift induced power required so there are two terms. Let us see how they are varying, so the first one is a function of $q_\infty S C_{D_0}$. So q infinity itself contains V infinity square.

So basically, this term is proportional to V_∞^3, it is a cubic term. So therefore it is going to increase like this. You can see, there is a dotted line there that dotted line shows you how it is increasing. The second term again contains q_∞, which is V infinity square but it contains CL square. Now CL, as you have already seen, in level flight, L equal to W. So as she rightly said $V_\infty = \sqrt{\frac{2W}{\rho S C_L}}$. So therefore CL will be… what will be Cl, proportional to V square.

Student: $1/V$

Professor: $1/V$ correct, so now what will happen is, this expression, it will be proportional to 1 by, tell me how is it correlated with V infinity? 1 by V or 1 by V infinity square. Okay. So therefore, this term is going to actually reduce as the V infinity increases because you are dividing it by the term, so this is the second term. It is a large value when you have a low value of velocity and it comes down Right. There will be some point which is marked as point number 1 in this figure where these two will powers are going to be equal okay. So that is a very interesting intersecting point about which we will talk later. Right now, when you add these two terms up, because they come from different expressions, they will be having a minimum value at a velocity at which $C_L^{\frac{3}{2}}/C_D$ is going to be the maximum.

So our job now is to get that value of velocity. We are going to derive an expression for that value of velocity at which $C_L^{\frac{3}{2}}/C_D$ is maximized. So to do that, yeah so this is what I was telling you that the first term is proportional to V Cube, second term is proportional to V square upon V power 3 that will be V.

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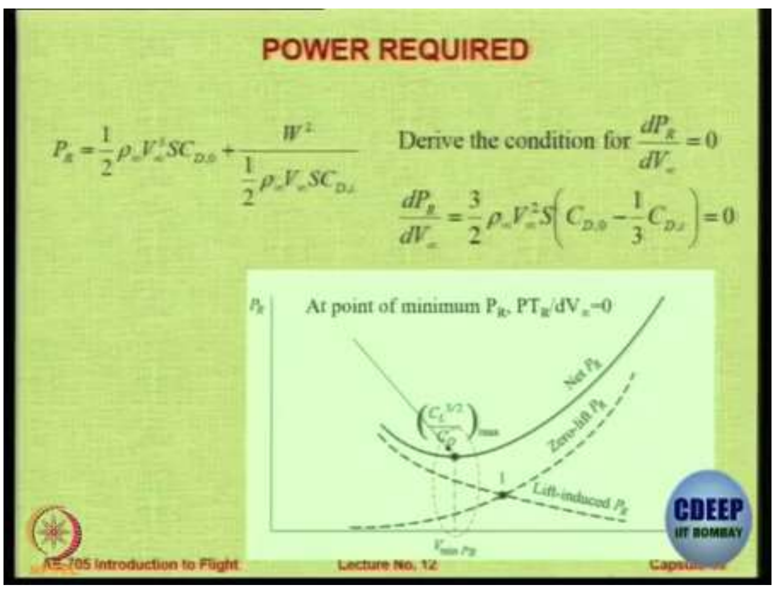

So this is our job, finding out the value at which this value is 0. So for your convenience, I have repeated the expressions now. So now you put pen to paper and now try and derive for me the condition for which the partial derivative of Pr with respect to velocity is 0. So you must tell me, what is the condition, what is the condition at which this happens. So when I say condition, I must also tell you what condition it is. So what can you say about point number 1? At point number 1, what is the same? What is the same? Yeah, at point number 1 the two components of power required, that is the induced power and the power due to zero lift are same they are equal. Okay. But now we are looking at this point where we have this value minimum. So can you derive the expression now?

So for this what you have to do is. You have to take the partial derivative or take the derivative of this expression with respect to velocity and remember that this term W square already has inside it velocity because $L = W = \frac{1}{2}\rho V^2 S C_L$, okay. So you have to now derive the expression this will need some time. So I am basically interested in is, see if you have some difficulty, then you please ask me, do not just look at the screen. You should be now doing your calculations. I do not want people to stare at me. I do not want people to stare at the board I want you to do the calculation. What I am interested in actually is the relationship between the C_{D_0} and C_{D_i}, where C_{D_0} is the parasite drag coefficient and C_{D_i} is the induced drag coefficient. C_{D_i} is basically

$$C_{D_i} = \frac{C_L^2}{\pi\ AR\ e}$$

So I want the link between C_{D_0} and $\frac{C_L^2}{\pi\ AR\ e}$ at which this particular condition is met.

So if you have got the answer then you have to raise your hands. Yes, what is the answer you have got?

Student: C_{D_0} equals to one third $K\ C_L^2$.

Professor: Okay so what is $K\ C_L^2$?

Student: K is $\frac{1}{\pi\ AR\ e}$.

Professor: Correct so $\frac{C_L^2}{\pi\ AR\ e}$ is C_{D_i}. So you were saying that C_{D_0} be one third of C_{D_i} right okay. Anybody else? Yes. CL square is equal to? It is the same thing actually. Now you have to go further because $\frac{C_L^2}{\pi\ AR\ e} = C_{D_i}$.So you will get the same condition if you probe further, you have to probe little bit further. So essentially, what I got is this. If I differentiate it and if I take all the terms common then I get an expression which says C_{D_0} equal to 1 by 3 C_{D_i}.

So the parasite power or the parasite drag coefficient C_{D_0} should be much lower than the induced drag coefficient that is the condition at which you have the minimum power required. Once again, this is aircraft independent or engine independent okay let us see.

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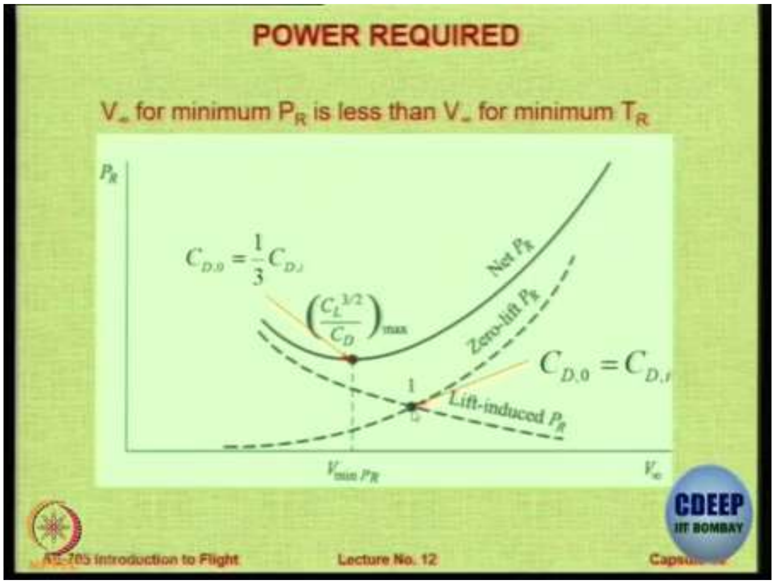

So the interesting point is that the velocity at which this particular thing occurs is in general lower than the velocity at which the two powers are equal. okay. Now when you look at this point at which the two powers are equal. That means $q_\infty S C_{D_0} V_\infty$ is equal to $q_\infty S C_{D_i} V_\infty$ which means C_{D_0} equal to C_{D_i}. So the condition at which the induced power equal to the profile power or the zero lift power equal to the net is equal to the induced power is 1 condition which will be useful little bit in the future.

The other condition is when it is one third. Now if I now look at the thrust required to be minimum not power required ok and thrust is now defined as power by V, just because power is equal to T into V. So, then interesting thing is that the value at which the minimum. Now this is something I want to leave to you for homework. This is the same thing now if you now go for T =D and if you want to derive now the condition for D into V that is power. So if you say if I want to have a condition where T required is minimum that means D is minimum. That means $q_\infty S C_{D_0} + q_\infty S C_{D_i}$ is minimum. You can take a partial derivative of again with V and you will get C_{D_0} equal to C_{D_i}.

So interestingly, the operating condition at which the power required is minimum is different from the condition at which the thrust required is minimum.