**Introduction to Flight**
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**Lecture: 06.7**
**Tutorial on Critical Mach number and Wave Drag**

Two chapters that we have studied, mostly it will be on the first chapter Critical Mach number, wave drag and we use the formulae to get some numbers.

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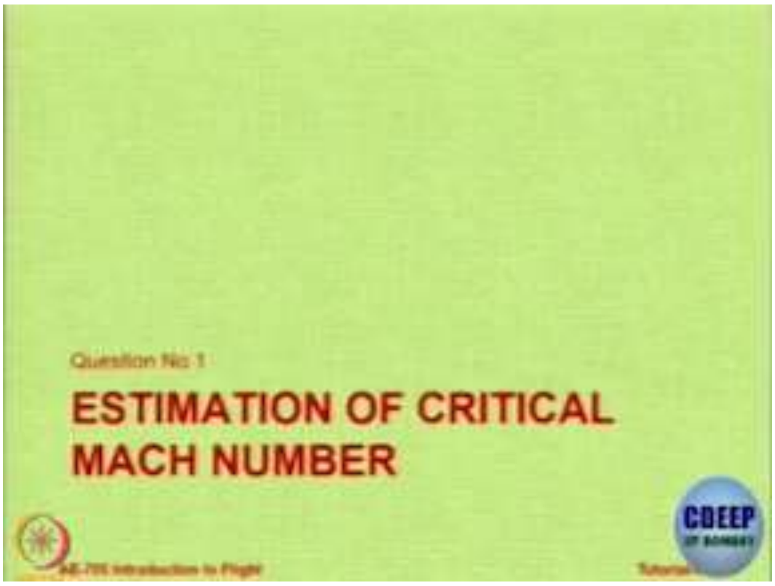


Let us first start with the estimation of critical Mach number. So this is a real life example so we take NACA 0012 airfoil at 0 angle of attack and the CP distribution is as shown, notice that the minimum CP value is minus 0.43 which is…

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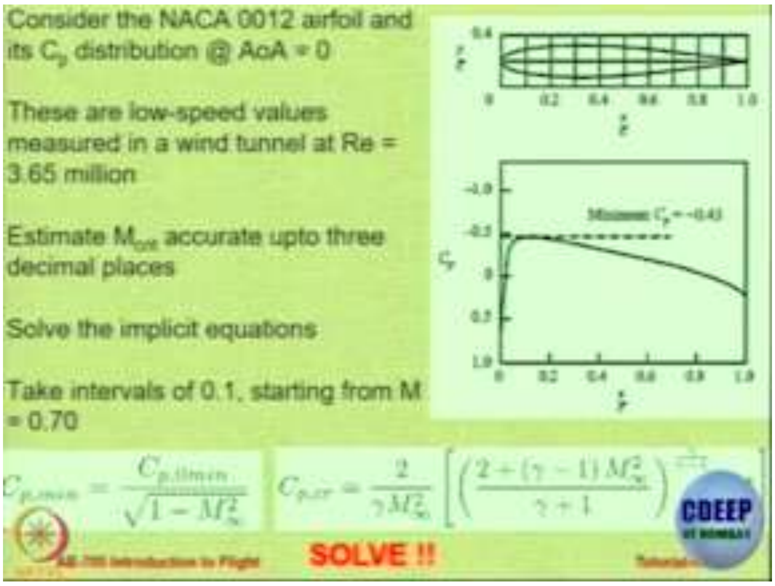


Now this information or this particular result was obtained using wind tunnel test at a load 3.65 million is a quite low Reynolds number. So, this is a low speed Cp minimum value, so called $C_{Po} = -0.43$. Our job is to estimate the critical Mach number for this airfoil accurate up to three decimal places, so how would you do that?

Student: Sir, my name is shiv. We will try to plot Cp with the Mach number and wherever we can intersect with the universal line, so that we will consider or analytically also we can say.

Professor: That is right, so there are these two equations, one that says that

$$C_{P,min} = \frac{C_{Po,min}}{\sqrt{1 - M_\infty^2}}$$

on the right hand side we have the formula for Cp critical. So just solve it, that is your task now, ok, take out your calculators, take out your note book, I am giving you a hint, start with M equal to 0.7 because Mach number below 0.7 normally we do not expect, you have a critical Mach number, ok.

So please start. So what you do is take M equal to 0.7 put it in the left equation, get the value of Cp min, put the same Mach number in the right equation, get the value of Cpcr, if they do not match then you change the value of Mach number and like that there will be a Mach number at which both these equations will give the same value. So, the first person to solve this will raise the hand and we want it accurate to three decimal places which means you have to go first 0.7.

Then may be 0.71, 0.72, 0.73, 0.74 and very soon you will come to know at what Mach number both these equations give the identical solutions. If you have a calculator that is programmable then you can set a small program, that will be easy, just put $\frac{C_{Po,min}}{\sqrt{1-M_\infty^2}}$ is equal to the RHS and solve for M infinity. Yes.

Student: Sir, the answer coming 0.737.

Professor: Right, that is the correct answer, M equals to 0.737 is the right answer, ok, in the interest of time I think I ask you to leave it, do it yourself offline, you can see the solution that we got.

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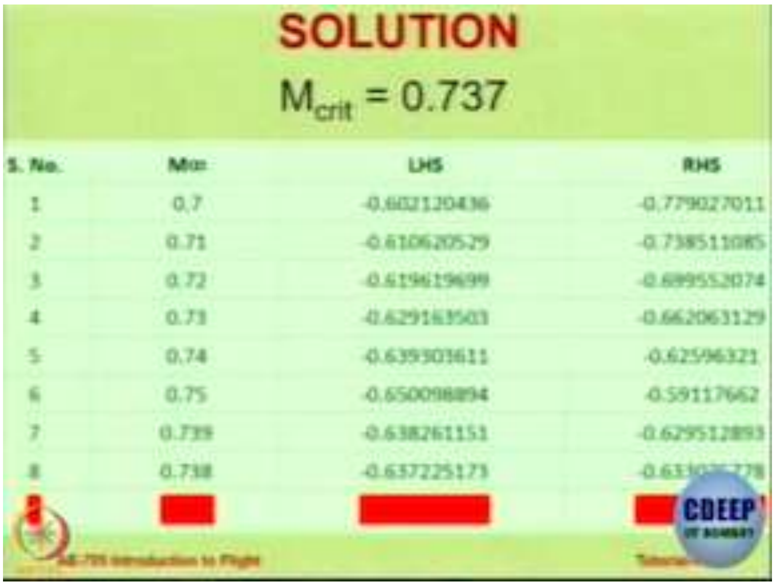


So we began with 0.7, LHS, RHS and then we kept on increasing at M equal to 0.737 LHS and RHS are matching with the three decimal places, ok. So this I think one question in the quiz you can already solved the giveaway. Yes.

Student: What did we take the value of the gamma?

Professor: What do you think?

Student: 1.4?

Professor: Why 1.4? What are we dealing with? Which field are we dealing with? Air? At Mach number 0.7 or so what is the value of gamma, 1.4, that it. So it dissociates at Mach numbers which are very high then you will take a lower value, so you are actually trying, now solve the quiz question, ok, any more doubts? Straight forward, it is just a matter of matching left and right, ok.

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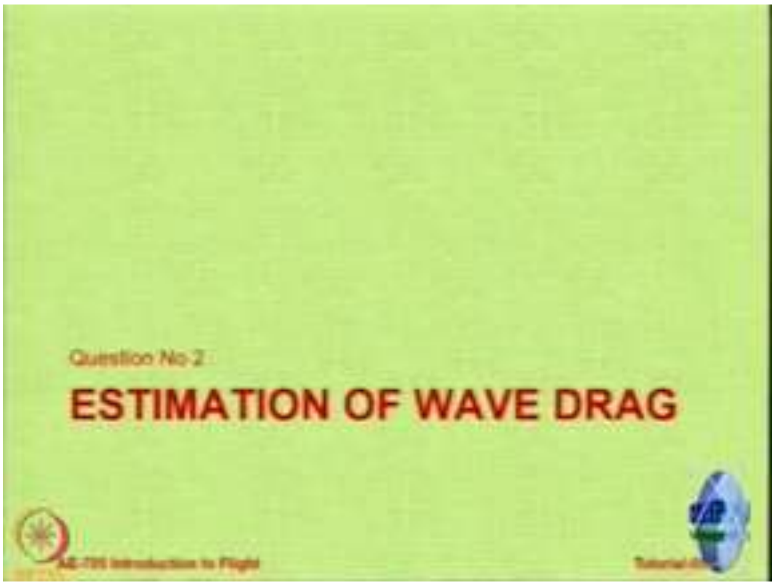


Next question is slightly more difficult, now our job is to estimate the wave drag of an actual aircraft, ok.

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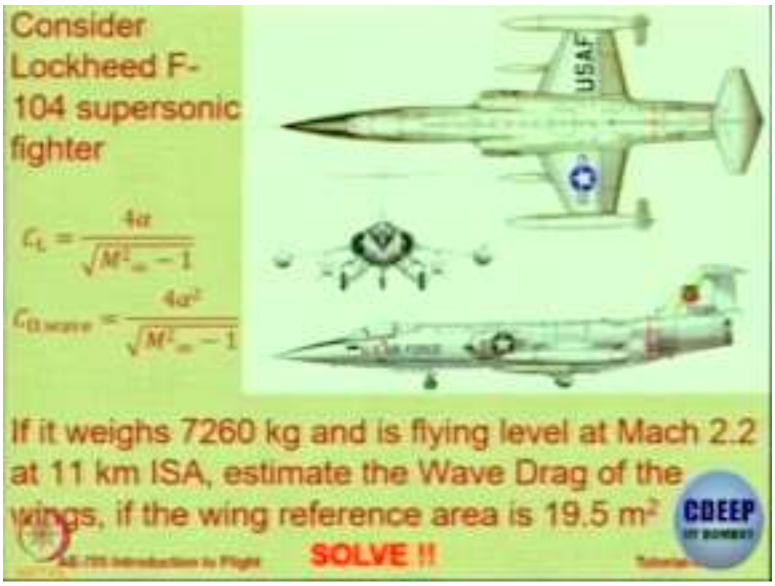

So, I have chosen this aircraft, Lockheed F 104 supersonic fighter aircraft, I will talk about this aircraft slightly as you solve your tutorial. First of all by the look of the aircraft what can you say? What are the features that you see? As far as wave drag is concerned what kind of features do you see? Can you call this as a sweptback aircraft? Yes or no? No, the answer is no.

How do you measure sweep? Not just the leading edge, not at the trailing edge but at the quarter cord, ok. So the quarter cord sweep is not there, so this is the straight wing with taper, it has taper but it doesn't have sweep so now we have spend so much time looking at sweep, variable sweep, forward sweep, but this is a supersonic aircraft and there is no sweep.

So in a supersonic aircraft, how do you allow aircraft to fly without providing sweep back. What needs to be done?

Student: The thin wings.

Professor: The thin, thin wings, that is right. Do you remember which is the first aircraft to fly supersonic? Bell X1, Bell X1, did it had sweep back? No. no, it had very sharp leading edge and thin wings, so that is the right answer. If you want to fly at a high speed and still do not want to give sweep back one of the many option available is thin wing so that means this aircraft is a thin wing aircraft, can you see in the front, wing it is very clear.

It is a very thin wing aircraft, is not it, t by c is very small, you agree? Yes or no? So the aircraft weighs 7260 kilograms and it is flying level at Mach 2.2 at 11 km under ISA conditions. If the wing area is 19.5 square metre and this is the reference wing area, which means the area of the wing as seen in the top view including the part inside the fuselage but nothing to worry that is reference area and that is what is in the calculations for lift and drag, ok.

So when we say $L = \frac{1}{2}\rho V^2 S c_L$ that S is the wing reference area which is given to you 19.5 square metres, Mach number is known, altitude is known, weight is known and there are these two formulae for a thin airfoil, for a thin wing. Now we are going from airfoil to wing, actually speaking there are slight variation but for this purpose we are going to just neglect it, ok.

So assume that these formulae are applicable so for thin wings, so

$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$ and $C_{d,wave} = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$ ,because wave drag is be predominant only when you have number more than one. So using this information please solve, get me the value of wave drag.

If you have any queries and questions you can ask me. First of all I want to know the procedure? How will you solve this question? What do you need to estimate the wave drag? Yes, so tell me how do you get wave drag? Because you said you know lift, ya, so lift is equal to weight and level flight so because W is known, lift is known, ok, then?

Student: So lift is known after that we need alpha from Cl, so we know $C_L = \frac{L}{q_\infty S}$

Professor: Correct.

Student: So dynamic pressure can be estimated using the velocity of the aircraft.

Professor: Right.

Student: So velocity of the aircraft can be estimated from the like Mach number into the velocity of the sound.

Professor: Right and that altitude. How do you get the velocity of that altitude?

Student: Velocity of that altitude, we know velocity of sound is the root of gamma P by Rho, ok, so pressure at that altitude.

Professor: You have to calculate pressure of the altitude, ok. How do you calculate that?

Student: That I do not know.

Professor: Ok, so let us go to somebody else now, please take a mic. So let us start from the place where he says velocity of sound has to be estimated at 11 kilometers.

Student: Velocity of sound can be estimated by equation velocity of sound is equal to $\sqrt{\gamma RT}$ and for the T relation we know that from 0 to 11 kilometer in the standard atmosphere the decrease in the temperature is minus 6.5.

Professor: Decrease is not minus 6.5, decrease is 6.5 that will become increase, ok got it, 6.5 degree per kilometer altitude that is the drop in the temperature from what value?

Student: From sea level.

Professor: How much is that?

Student: Sea level is 298 kelvin.

Professor: No, you have already made it 10 degree hot, ok. So, now I will take you step by step in the solution. Now, what I have done is there is a color coding here, so there are some red lines means question and there are some blue ones, blue ones are the answers, straightforward, what is the value?

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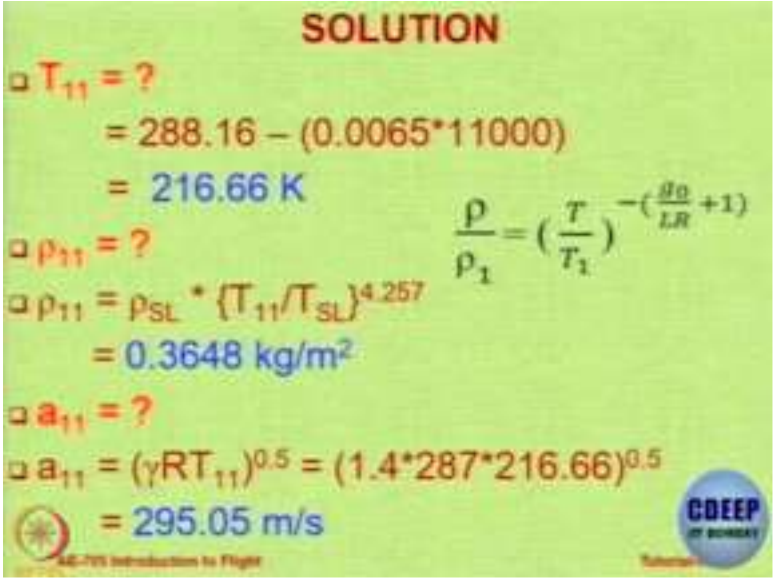


So, T11 stand for the temperature of the ambient air at 11 kilometers under ISA conditions, we are assuming ISA conditions here because it says at a standard height of, at a standard height of 11 km, ok. Please calculate and just announce the number, no need to now raise the hand because that is too cumbersome, just speak out.

216.66 degree kelvin is the temperature. So, if that is known to you how do you get the value of density rho because you need half rho V square, so you need density, does anybody remember how to get density ratio. Yes, what it is, it is T by T knot to the power, I will help you, here is the formula, ok, this is the formula.

Now remember there is a negative sign in the exponent plus this L is the lapse rate which is also minus 0.0065. So, actually the exponent is positive, if you take it as, what is R? Universal gas constant, 287 joules per kg degree kelvin, so R is 287, L is 0.0065, G0 is 9.807, so rho by rho1 is equal to T by T1 times that ratio, if you take rho as rho11 then rho 1 will be, so here is the solution.

So this is the formula, I have simplified it for you, and this formula I would request you to memorize, you will use it many times, density ratio upto 11 kilometer, chances are when you do aeronautical engineering you will not go beyond 11 kilometer, when you do aerospace engineering you will go beyond 11 but for most cases you will be within 11, within 11 it is very simple, temperature ratio to the power 4.257, so please do it now.

T1 or T11 is 216.66, Tsl is 288.16, find the ratio and multiply by sea level density which is 1.2256 kg per metre cube. Just speak out the value of density at 11 kilometers, if you calculated, 0.3648 kg per metre square, actually metre cube, that is a mistake, I will correct the mistake, thank you, ok, everybody got it, right, so that is the density.

Now, here I want to ask you one more small question, suppose the conditions are not ISA but lets say ISA plus 20, once again I am giving you a hint for the quiz. Suppose the temperature is not ISA but ISA plus 20 then how do you calculate density? Can somebody tell me the procedure?

Student: By adding 20 degrees to the sea level temperature.

Professor: Ok. If I take sea level temperature as 288.16 plus 20 will it be correct? No it will not be enough that is one more thing to be done. Remember the value of density at sea level 1.2256 is under ISA conditions, so even that you have to change to ISA plus 20. The density of air is a function of temperature also, correct, so now how do you get density of air at temperature T equal to 20, more than ISA.

So, then now you invoke $P = \rho RT$, ok, R is the same constant. Now, here we make an assumption that the pressure is going to be the same. So, ISA plus 10 or 20 or 30 does not change the pressure, it changes only the temperature, so assuming P equal to P0, 101325 newton per metre square rho P is equal to rho RT, so P is known R is known, T is known which is 288 plus 20. Get the new density, it will be 1.6 approximately then use 1.6 and 288 plus 20 you will get density at 11 kilometer under ISA condition. Yes?

Student: Why ca not we directly use ISA condition?

Professor: Because you may be flying this aircraft in the desert in Rajasthan where we do not have ISA condition.

Student: But if we say ISA plus 20 we calculate the values at 11 kilometers taking this as reference. So instead even if it takes ISA as a reference we should be getting same values as well. How can you get the same values because it on the same, did you attend the first lecture on atmosphere, I clarified to you that forces under load acting are a function of temperature.

So if the density changes then the dynamic pressure will be different so then the whole calculation will be different. So if you are told that operations are happening at a condition of ISA plus 20 you can not use ISA numbers, you will get wrong values. See the atmosphere at a place where we are flying is not equal to ISA. In the question, in the classroom, textbook, we assume ISA for ease in calculations, ok, alright.

How do you get A? Root of gamma Rt, so what is the value? How much? 295, yes, that is correct. So we have now the value of A that is the speed of sound at 11 km, under the ISA condition that is 295.

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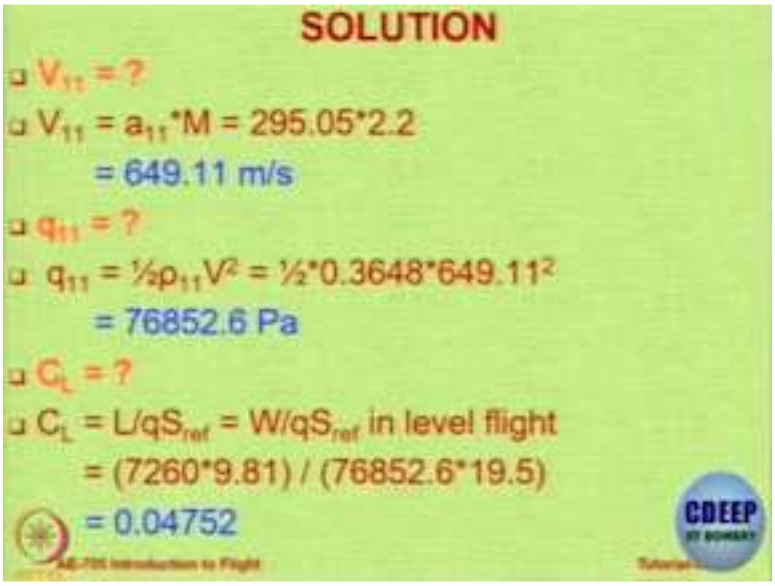


Now let us go ahead, how do you calculate now the V, that is simple 2.2 times this particular value, ok, so keep going, give me the answers. How much? 649.11 metres per second that is the speed of the aircraft. So, if that is the case what do you need next? You need the value of half rho V square, dynamic pressure. So rho is the value that you got 0.3848, V is 649.11, get the value of dynamic pressure.

Now, let me warn you this is one place where many people will make a mistake, in the numerical value, so tell me the answer, what is the dynamic pressure? 7685 what? What units? 7685 newton per metre square, ok. Does anybody dispute this? Yes? 75.8 kilo pascals. So, who is right now? That is 75000 newton per metre square. So this is called as a calculator mentality.

So density, punch the value, velocity, punch the value equal to something dynamic pressure, units are in newton per metre square or units are in kg per metre square because density was in kg per metre cube. You do not balance the units, ok. The correct answer is what she said is right, it will be 76852 newton per metre square.

So what you have done is? You divided by G and to divide by G then you have to put kg per metre cube. So, what I want to tell you is if you put the units of density as kg per metre cube and if you put velocity in metres per second you will get the answer directly in unit per metre square, ok.

Let us go ahead now. How do you get the value of Cl, someone said lift is equal to weight, so weight in kgs divide by, no then you have to divide it, be very careful when you use, now the calculation the unit should be balanced, so that is why every time you have to check the units. Cl is dimension less. You have to put W upon QS, Q is known to you, S is 19.5, that is why I multiplied it by 9.8 to get it in newton, ok, so that is the value of lift.

Notice it is a very small lift coefficient, why is it so small? High speed flight, so what will be the induced drag Cl square by pi AE, this 0.04 will be squared, so it will be negligible, induced drag coefficient will also be very small and induced drag will be small because Cl is very small, ok.

So we are assuming that no lift is created by the fuselage etc, the whole lift is produced by this wing which is simplistic assumption but very much acceptable, the wings in this case would produce perhaps 95 percent of the lift. So, since we know the value of Cl 0.0425, we know the Mach number 2.2, it is very straight forward, you can get the value of alpha please.

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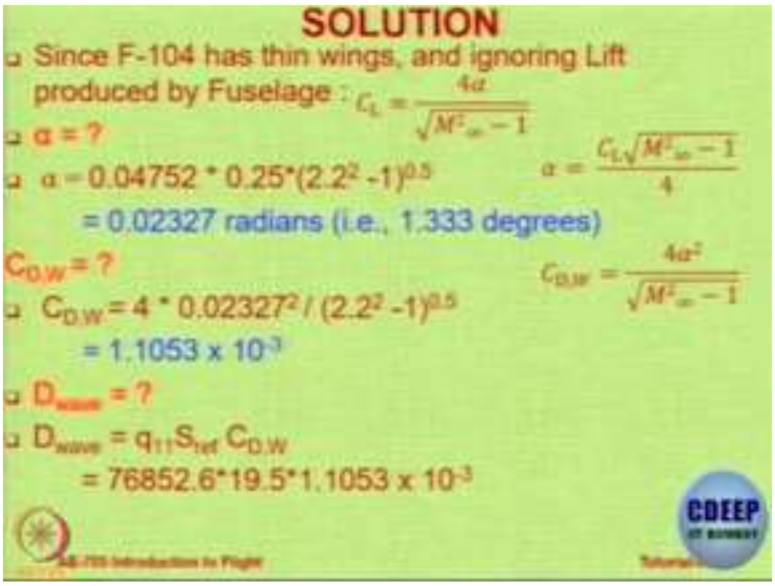


Student: 0.0233 radians.

Professor: Yes, its radians 1.33 degree. So it is a very small angle so obviously when you fly at Mach number 2.2 you can not expect aircraft to fly at a very high angle there will be huge amount of pressure drag or form drag if you fly. So, now what is the value of the wave drag coefficient, which was 4 alpha square so just this into alpha. How much is Cd wave, drag coefficient for wave.

Student: 0.0109.

Professor: Yes 0.00111. So, now you can calculate the drag. This into Q into S what is the value? So, notice here I have not used, I have not used G again because already used in newton per meter square. So, therefore, the answer I will get will be in newton, it is newton per meter square into metre square into dimension less quantity, so what is the value?

How much? 1651, approximately 165 kilograms of drag is acting on the aircraft just because of wave drag, then you have skin friction, you have form, interference all other drags, ok. So this is what the second question was. Now the last question before we start our quiz any doubts, any body has so far, ok.

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Then the aircraft, the information given on the top is just for your self study later, just to arouse your curiosity if you have regarding the aircraft. This was not a very successful aircraft, it was a failure, actually it was a failure and there are some reasons for the failure.

I can discuss it, in fact we have MOODLE to discuss that, it will be nice if somebody can reactivate the group by telling us why F-104 is a failure, what went wrong? What was the problem with it? Why such a nice aircraft or such a good-looking aircraft did not actually succeed? So our job is to plot lift coefficient versus Mach number.

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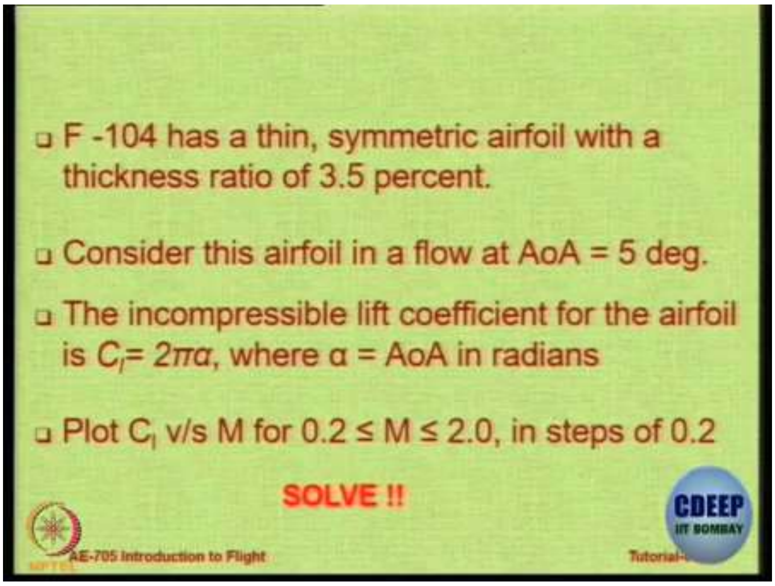


So, we have a very-very thin wing of 3.5 percent thickness, lets assume alpha at 5 degree, Cl is 2 pi alpha in incompressible flow for very high speed. So you just start from Mach number 0.2 and go to Mach number 0.5, in fact its there for all Mach number, ok. So in steps of 0.2, so you start with 0.2, 0.4.

Now this you can divide if you want, I mean groups in the class so roughly 10 people in each group, so from 1 to 10 each one will take 0.2, 0.4 , 0.6, 0.8 etc up to 2 and just calculate the value in steps. So, let us see just for confirmation let us calculate the value for M equal to 0.2, how will it change?

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Actually the question was not Cl versus M but Cd versus M, none of you have observed it and none of you has pointed out that question.

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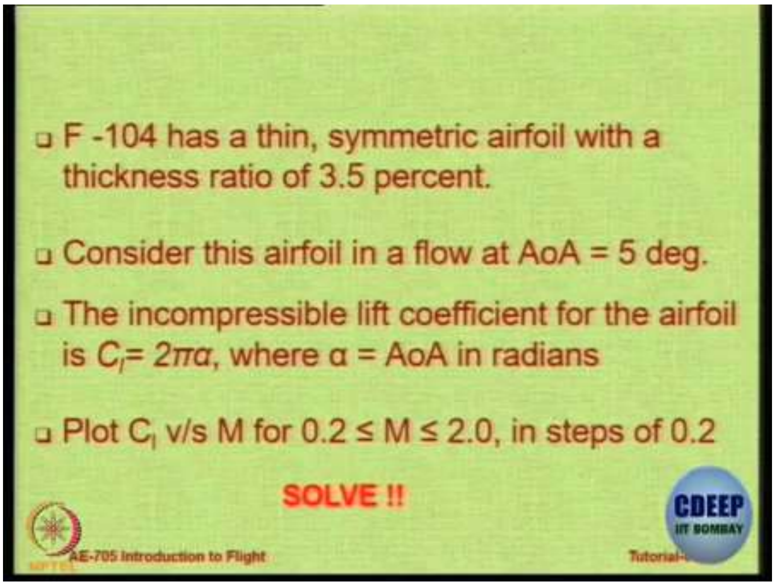


So just see, Cl is already given as 2 pi alpha, so what is there with Mach number, if alpha is 5 degree then Cl is 2 pi alpha and what happen in compressible, that is right, so you have to start with the value at low Mach number and then apply Prandtl Glauert rule and calculate the value before and after, ok.

So here it is, this is how it will come and the area between, the area between Mach number 0.9 and 1.1 we are not plotting because that is the area where you cannot use these Prandtl Glauert  rule. You can use it upto some transonic value and beyond that value. Ok.