Introduction to Flight
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Lecture Number-03.4
Mach Number

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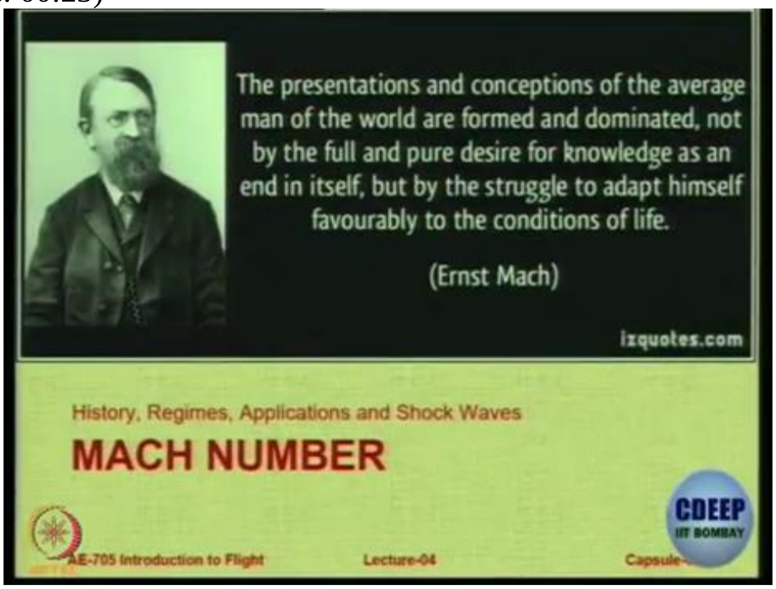

Okay, now we look at the Mach number. Mach number as all of you know is very useful when we look at high-speed flow. Ernst Mach was; so here is an example of the effect of high speed flow on the pressure acting on the surface.

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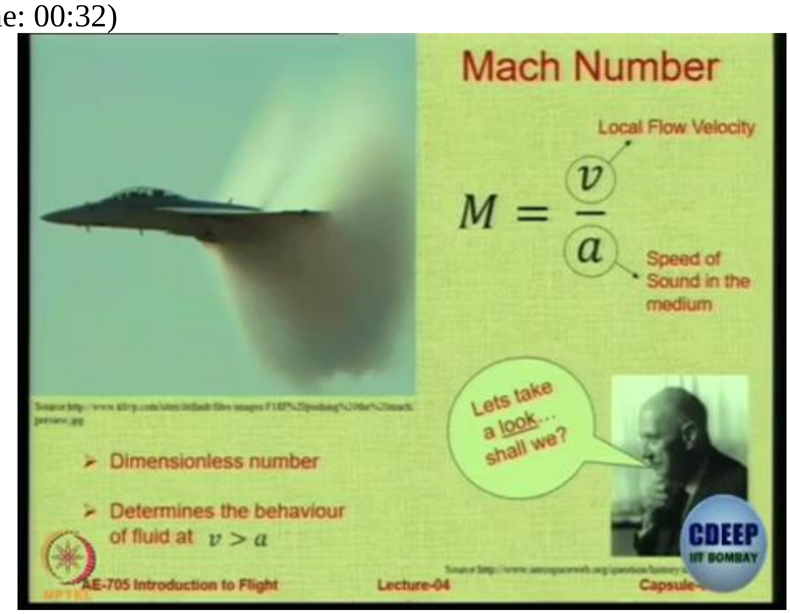

So what is happening here is there is a condensation of atmospheric water vapor due to the differences in the pressure. We will study about it little bit. So Mach number as we all know is the ratio of the flow stream velocity ambient speed of the moving vehicle and the speed of sound under those conditions. So you have a ratio of local flow velocity and the sonic speed in that medium. It is a dimensionless number because both of the parameters have the same dimension and it determines the behavior of the fluid when the velocity is more than the way. When Mach number is more than one then we have a huge effect of this Mach number. So let us have a look at very briefly.

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Video: At higher speeds approaching the speed of sound the exact relation between the speed of the aircraft and the speed of sound becomes very important. It is called the Mach number, usually shortened to M. This is how it is calculated. This aircraft has flown six tenth of a mile in the time the sound wave traveled ten tenth of a mile. It is flown at six tenth of the speed of sound in the same atmospheric conditions. That is, it is flown at Mach point six. This aircraft is flying at Mach point nine.

Professor: So if the (shock) if the sonic and This this Sound speed and the velocity of the aircraft match then by the time the sonic wave arrives the aircraft will also arrive and the Mach number will be equal to 1. Let us look at its significance.

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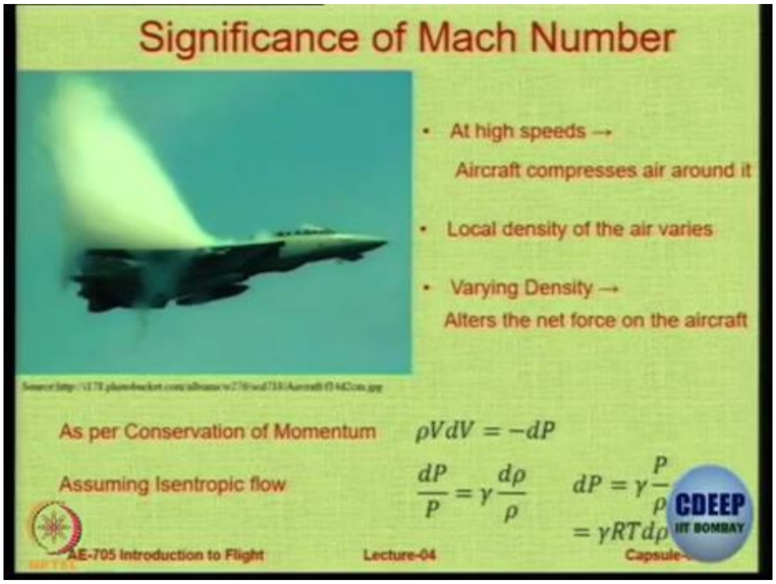

Okay, so essentially what is happening is that when Mach number increases not beyond 1 but even beyond 0.3, 0.4 when it goes beyond 0.3 you start seeing that the density of the air does not remain same and the air starts getting compressed. Ok, so the local density of the air changes and there because of that the force acting on the aircraft is not the same. So if you apply the conservation of momentum then it can be very easily shown that,

$$\rho * V * dV = -dP,$$

the minus sign showing that if this increases this decreases and vice versa. Similarly if you assume isentropic flow and I will define the isentropic flow very soon.

In that condition it can be shown that the relationship $\frac{\gamma dP}{\rho}$ is valid. So using these 2 quantities one can derive further an expression which say that dP that is change in the pressure is equal to

$dP = \gamma RT * d\rho.$

Now, if you remember gamma RT basically represents root of gamma RT is speed of sound,

$$a = \sqrt{\gamma RT}$$

$$a^2 = \gamma RT$$

$$V = M * a$$

Okay. Correct. So you can use that.

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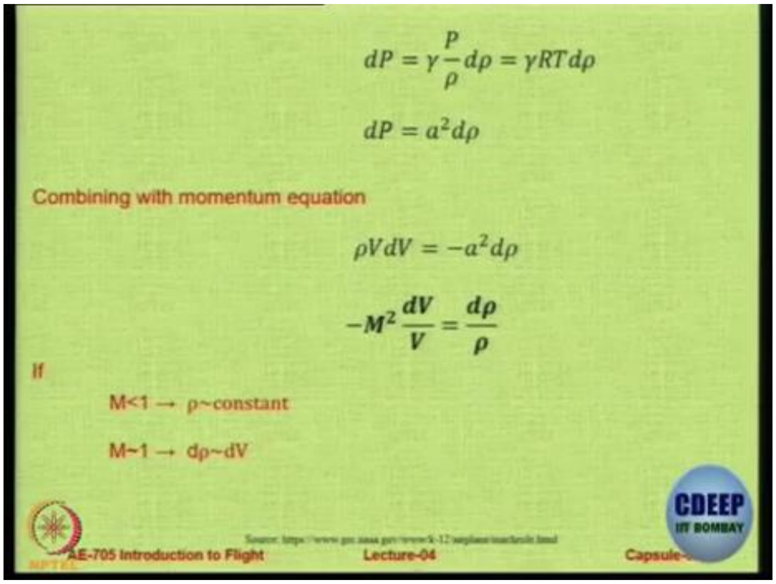

So I will repeat the same thing again so

$$dP = \gamma \frac{P}{\rho} dP$$

$$\frac{P}{\rho} = RT$$

$$dP = a^2 * d\rho$$

and therefore if you combine with the momentum equation where

$$\rho * V * dV = -dP,$$

$$\rho * V * dV = -a^2 * d\rho$$

With that you can get;

$$-M^2 \frac{dV}{V} = \frac{d\rho}{\rho}.$$

This is not to be derived or not to be really memorized or something. This is just to tell you, the link that Mach number gives us between velocity and density. That is the whole purpose of this small derivation.

Mach number is a link; for example,

if M=0 then $\rho = 0$, and $d\rho = 0$.

If $M = 1$, then $\frac{dV}{V} = \frac{d\rho}{\rho}$, therefore the change in the velocity in change in density are related. If Mach (saw) if if if Mach number is less than 1 then you know it is a very small number. Because it is a square.

So let us say Mach number is 0.1. Now it will be 0.1 square times $\frac{dV}{V}$ which can be neglected. So that means it will almost be equal to 0. In which case $\frac{d\rho}{\rho} = 0$, in which case density remains constant. It does not remain constant but it is almost constant. As the Mach number keeps on increasing, the link between the change in density with velocity becomes prominent. Okay, so when Mach number is 1 the change in the density is change in velocity. At Mach number more than 1, the factor becomes M square. So this is the significance of Mach number. We do not take Mach number just as a ratio for no reason this is the real reason because it gives you a linkup between velocity and change in the density. Okay.

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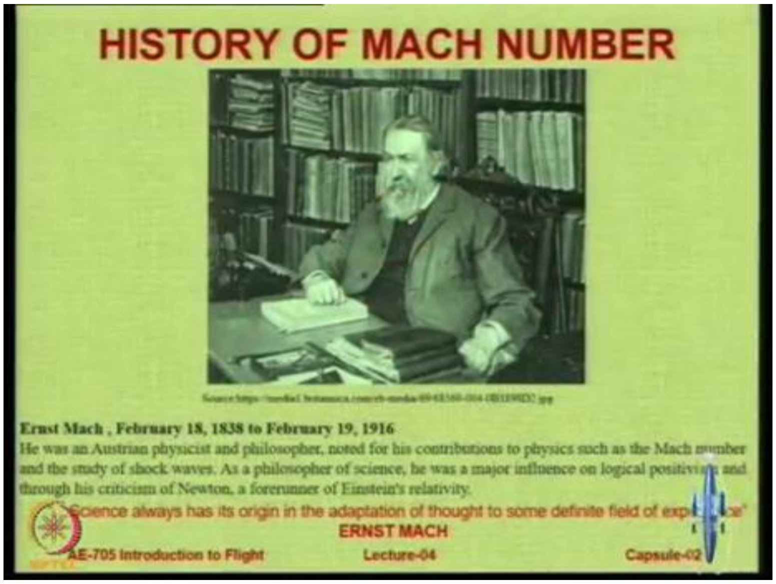

So what is the history of Mach number this gentleman; he was born in 1838 and he was a physicist and he was the first person to actually study and quantify the effects of high speed flow on a body. Before Mach number there were bodies flying at very high speeds. For example, bullets have been there for many years but he was the first person to photograph a bullet. Ok. I will show you.

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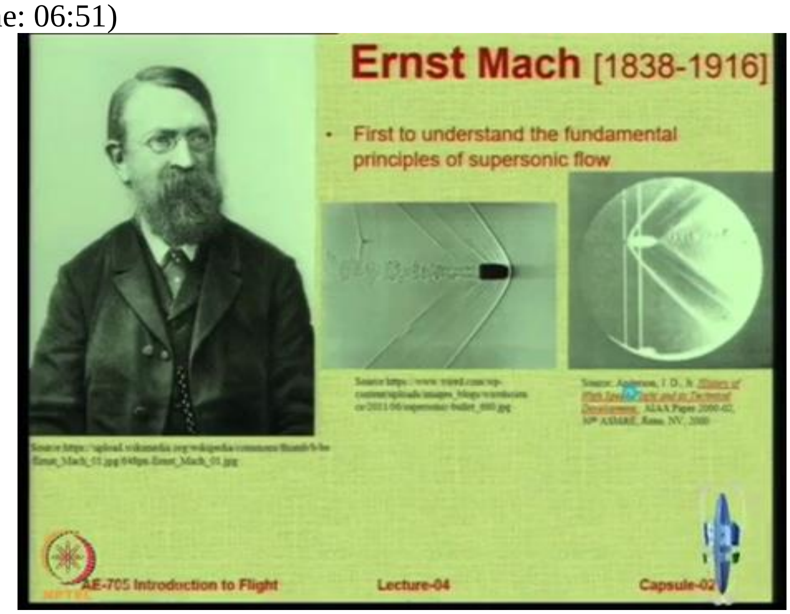

So here is Ernst Mach. Okay. He was the first to understand the principles of supersonic flow and this is the photograph taken from a very interesting paper on history of high-speed flight. So notice there is a hyperlink there, so there is a paper by the same J. D. Andersen whose book we are following. There is a Von Karman memorial lecture in the year 2000, given at the aviation sciences meet and exhibit which happens once a year every year. So in that you normally have some invited lectures. So he was an invited speaker in that and he spoke about the history of high speed flight and in that I have taken this picture from there. This is the first photograph of a bullet taken by a scientist to understand the flow around the bullet. You can clearly see that ahead of the bullet there is something behind the bullet there is something and we have those 2 vertical lines. We will talk about that in the next slide.

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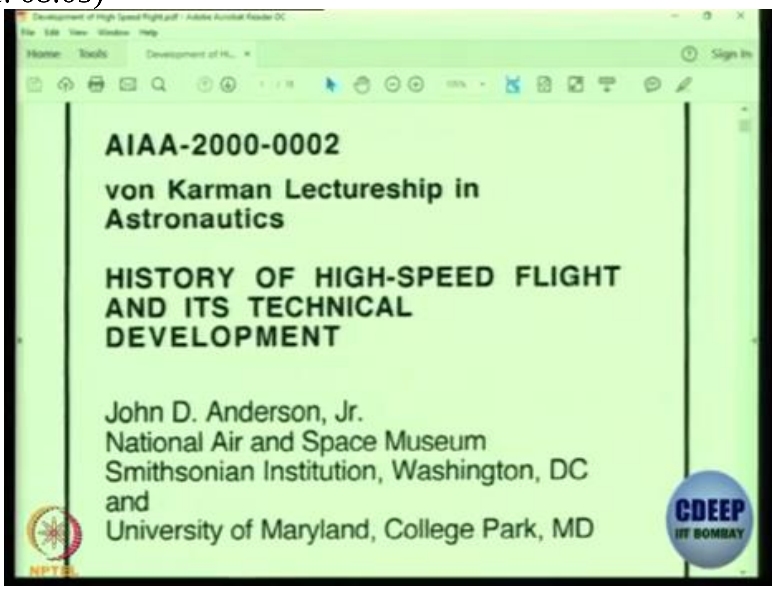

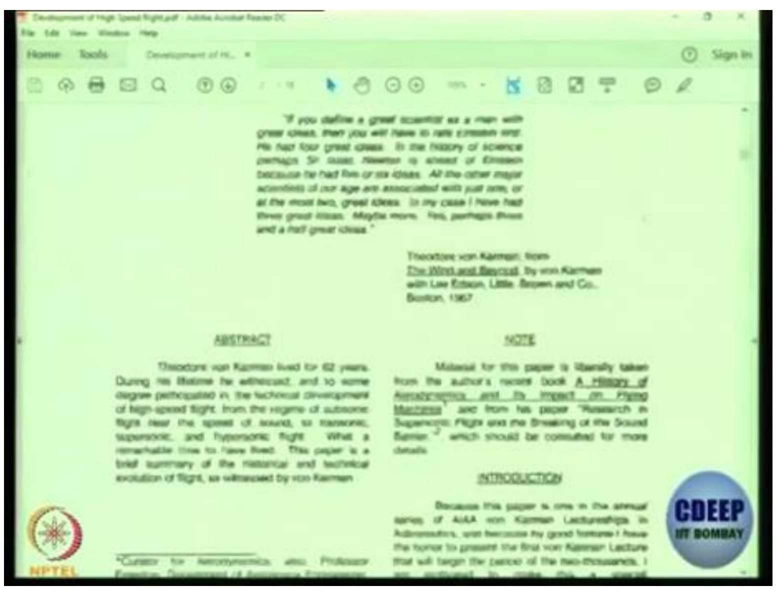

So for those who are interested in little bit of history; this is a paper and I am going to upload this paper also on Moodle. It is a fascinating paper it is historical piece. It talks about what happened?

How did it happen? Who said what? You will not find equations too much here but you will mostly find statements about what happened? What was a problem faced and how this thing all came about? So I will sharing it on Moodle, it is a very interesting read.

On the left-hand side you have a more recent photograph of a bullet. Okay and this particular photograph has been taken using the technique of schlieren photography. So here you do not see those two vertical lines and this particular photograph is you can see it is much more clearer. But fundamentally it is the same. The two photographs are doing the same thing, they are trying to capture the flow field around a bullet. This particular photograph by Mach was presented in this very outstanding paper. Look at the a you know (the) the date is very old 1887. That is when he is able to study.

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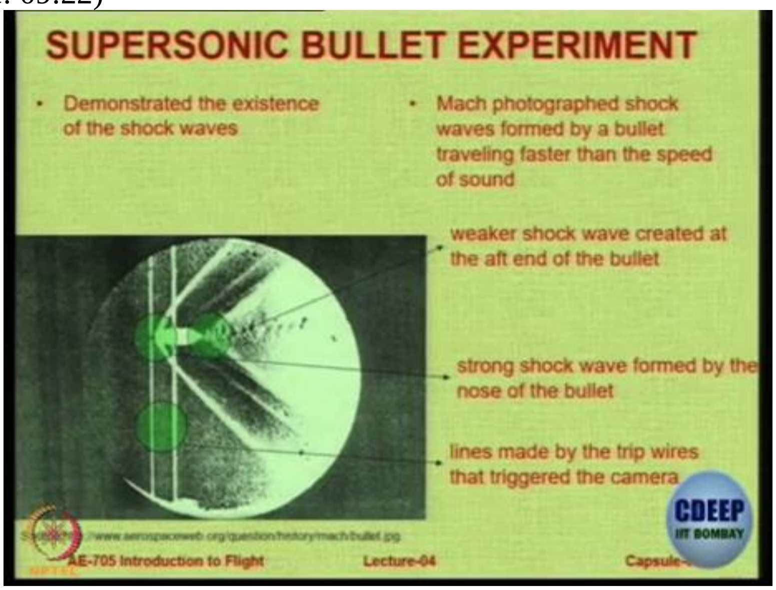

So what did he do basically he showed the existent of shockwaves and he photographed the shockwaves formed by the bullet; same photograph. So you notice in the nose there is a strong shockwave which is also curving in the nose and then it is tapering back on the back. Then there is a weaker shockwaves at the aft end of the bullet because the end of the bullet is not flat. It is having some shape, some conical kind of a shape.

So therefore there are going to be weak shockwaves created at the end, those of few we will study aerodynamics in the courses that follow; especially when you study high speed aerodynamics, you are going to study about the phenomena of strong strong shock waves, bow waves, weak waves, mach waves, etc and these two vertical lines are not anything physical; these are the lines made by the tripwires that triggered the camera which was used by Mach to record this photograph. So we should ignore those 2 vertical lines many people have a wrong idea it is a vertical shock. It is not a shock, it is not it is not a normal shock, it is just the lines made by the tripwires. Yes.

Student: what does a shock wave mean?

Professor: Good question. What does a shockwave mean? I am tempted to answer the question but I would request you to wait till the next capsule in which we discuss shockwaves. We will discuss about them. Right now, we are talking about a little bit of historical things. Today's talk is mainly about introduction to 3 scientists who are stalwarts in fluid mechanics. So we will talk about shockwaves in the next in the next capsule. Just bear with me for a few more days you will get the answer there. Okay let us go ahead.

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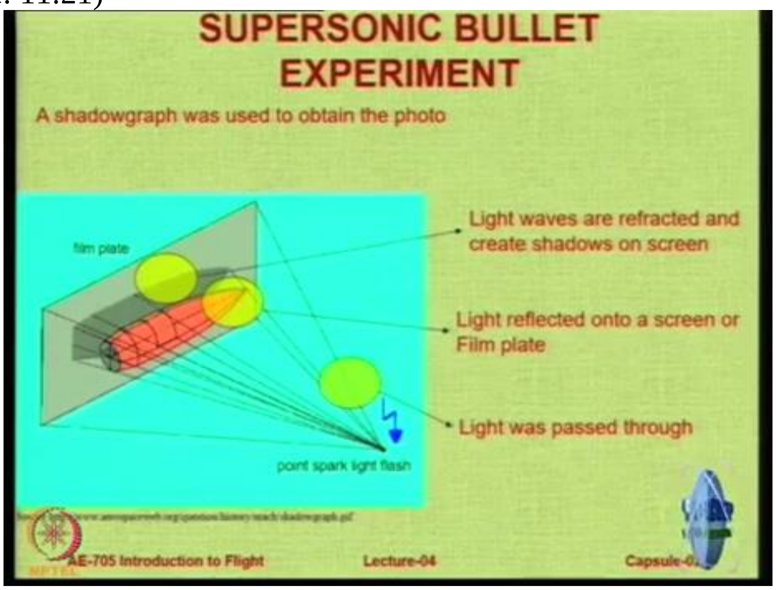

This is a technique that is used to obtain the photo called as a shadow graph, which is different from schlieren. Ok. So light was passed through this particular and there was a reflection on the film plate and then those film plates created shadows and those shadows are what you are seeing. Ok.

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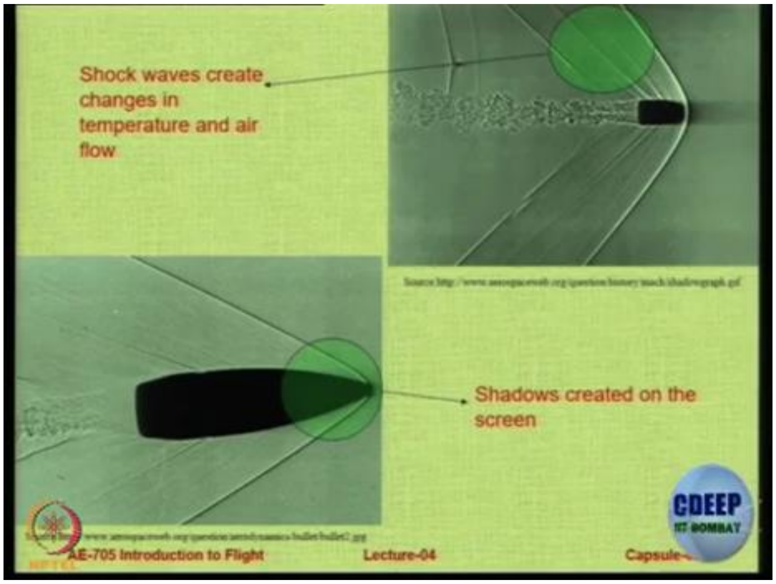

Now, the important thing is this so for the moment let me just tell you that these lines are basically called as shock waves. What they are? How they come? We will come in the next few lectures but basically the effect of the shock waves is to create changes in temperature and in the air flow. This is another beautiful picture of a bullet, this time the bullet has a base and notice that the angle of the shock wave in both the pictures are not the same. So this angle is a function of the flow speed as we will see very clearly.

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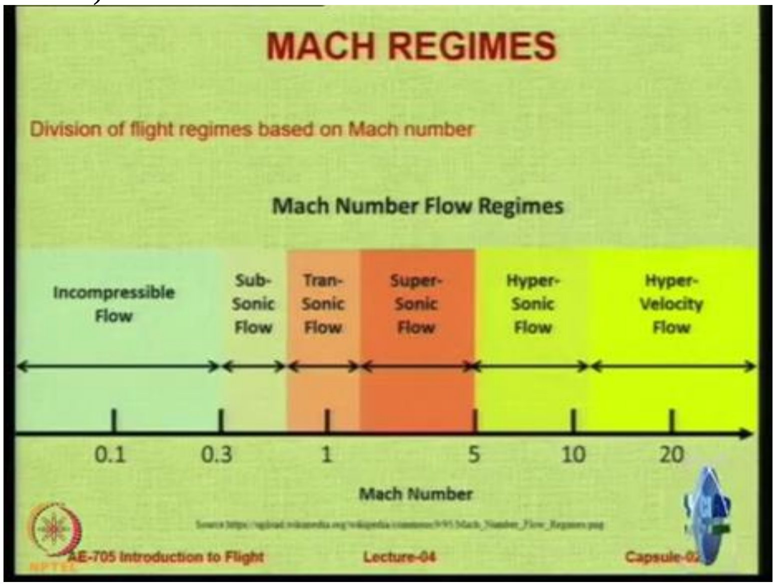

So with this we now start defining the regimes of Mach number. Okay. So there are different flight regimes based on the Mach number. Typically we consider up to 0.3 as low Mach number.

Then you have subsonic flow 0.3 to approximately 0.7 or 0.8from 0.8 to 1.2 we have transonic flow which encompasses sonic conditions but because we take a small margin before and after 1 and call it as transonic flow. Then we have supersonic flow. Technically speaking flow at Mach 1 will be sonic flow. Then we have hypersonic flow which starts normally from Mach number 4.5 or 5 and goes up to around 10 to 11 and then you can have hypervelocity flow. So very soon we will have a look at examples of vehicles that operate in various flow regimes.

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So in subsonic flow; small turboprop piston prop aircraft I would like people to identify the aircraft.

Yeah is there a question there? Can somebody identify this aircraft? Yes?

Student: Spitfire

Professor: Spitfire, very right. It is a super Super Marine Spitfire. A distinct aircraft which is distinguished because of the shape of the wing. If you notice the shape of the wing is elliptical. There is a reason for that which we will study in the next class about we talk about lift generation we will understand why this aircraft is good because it has got an elliptical wing planform. We will also study why you need not make it like this to make it good? Okay.

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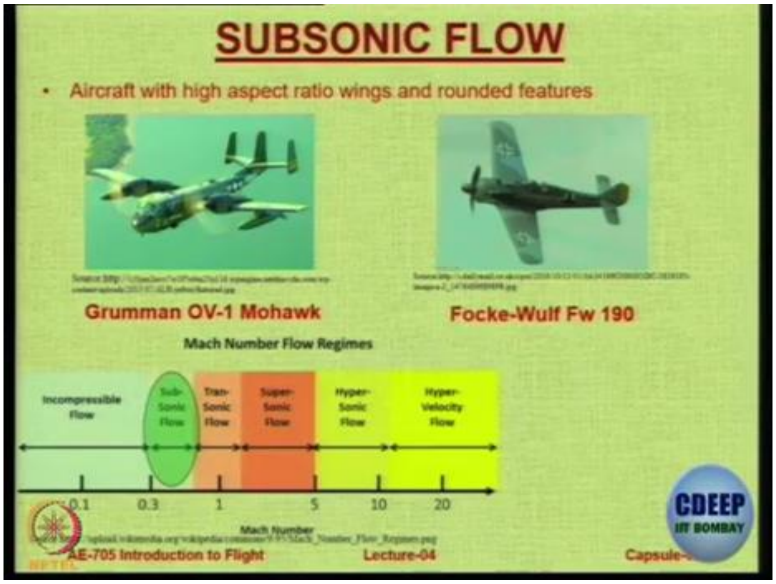

So this kind of aircraft, which flying subsonic flow are generally ones with very slender wings and rounded features. So they belong to the subsonic regime there are a few other examples of aircraft which are falling in this particular category. Mostly these are low-speed aircrafts, generally turboprop, piston prop. They normally fall in this category. I am not saying that all of them fall in this category; I am just saying that generally low-speed aircraft are the ones which are flown by propellers.

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Okay, then go to transonic flow. In transonic flow the example that I have chosen is to show you the creation of shockwaves on top of the wing. I do not know why the video is not clear. You can notice there is a small thin line here. This line is actually air, this photo was taken from the window of an actual trans transport aircraft and you can see there is a line which is a line of air which is adhering to the wing. This is the sonic line okay.

The flow behind this is at the Mach number higher than 1 or I am sorry. Not higher than 1. This is a transonic flow line. So this shows that the flow is transonic okay. So it is the region that divides the flow between subsonic and supersonic the aircraft is flying at a transonic speed Mach number point 8 but the acceleration of the flow over the wing has led Mach number to be towards the sonic Mach number.

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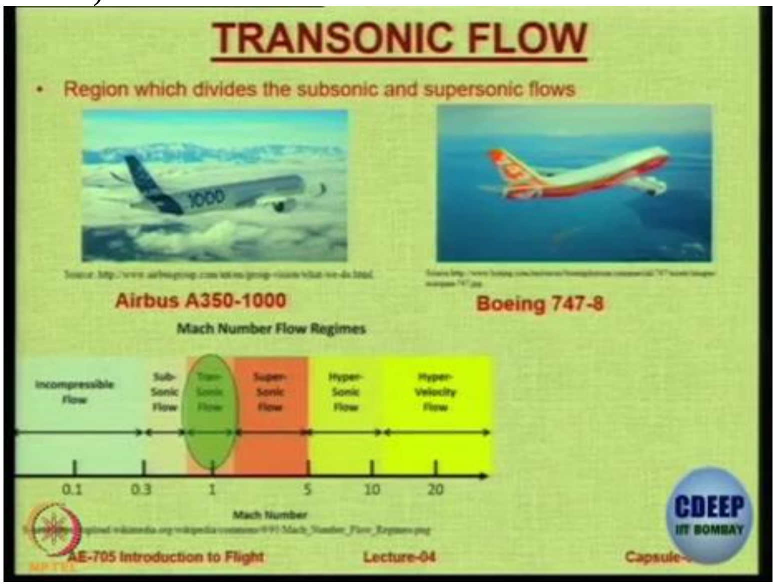

So this is transonic flow regime 0.8 and most aircraft in which we fly long distances for example, Airbus A350 or Boeing 748 they are all transonic flow. They all operate in transonic flow regimes. The typical cruise Mach number will be point 8 for these aircraft.

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All right, let us look at supersonic flow this is a very interesting video and you have to also you have to see and hear the plane is coming this is over a ship condensation of and you hear the plane after it have crossed you. Which means it is supersonic. Because the sound came to you after the plane crossed you. This is called as a supersonic flyby and the clouds that you saw on the plane were basically the condensation.

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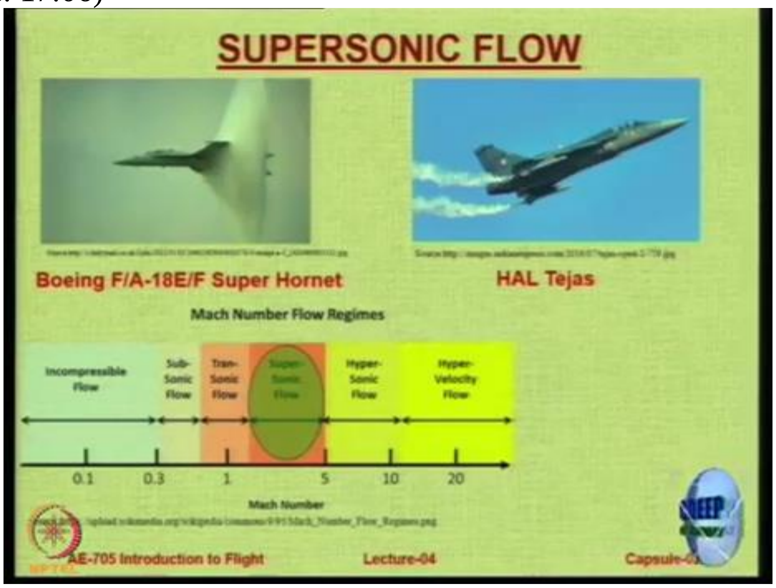

So supersonic flow is more than around 1.2 Mach number up to around 4 Mach number and the 2 examples of aircraft which can fly supersonic are our own a HAL Tejas and the Boeing F18 Super Hornet; which is the 1 that you saw in the fly past yes.

Professor: The the clouds condensation can happen even before it break the sonic barrier. Okay. Basically the condensation takes place only when the pressure drops; but if you notice here there is an angle at which the clouds are attached. So that is an indication of the fact that it has gone supersonic. Even if you look at a subsonic aircraft, you can see condensation sometimes depending on the weather conditions. So the condensation pertains is not an indication of sonic or supersonic flow. Okay. But the condensation pattern of this type which shows a special is this is actually a Mach cone as I will talk about in the next few slides, so that becomes visible because of condensation.

Professor: No no no no condensation occurs only if the atmospheric conditions are such that there are large concentration of water molecules present. Let us say, just after rain. This is a very good season to see condensation, the rainy season. So it is not that every time it will happen. This is this is you know natural flow visualization. It happens when there are conditions that allow it to happen. It is not going to happen always, so do not assume that every time an aircraft goes supersonic there will be some cloud like this. No, you can have; I will show you some videos of flight of supersonic aircraft without condensation visible.

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Let us go to hypersonic flow this is a hypersonic air-breathing vehicle launched from an aircraft. Of course, you cannot measure the velocities you do not know whether it is hypersonic
or not but you have to believe. So the rocket motor is fired after it is released from the body. So we are also developing the vehicle like this called as the HST DV. DRDL in Hyderabad is

working on their project and there is quite substantial progress in that our department is very actively involved in the project through some studies which my colleagues are doing.

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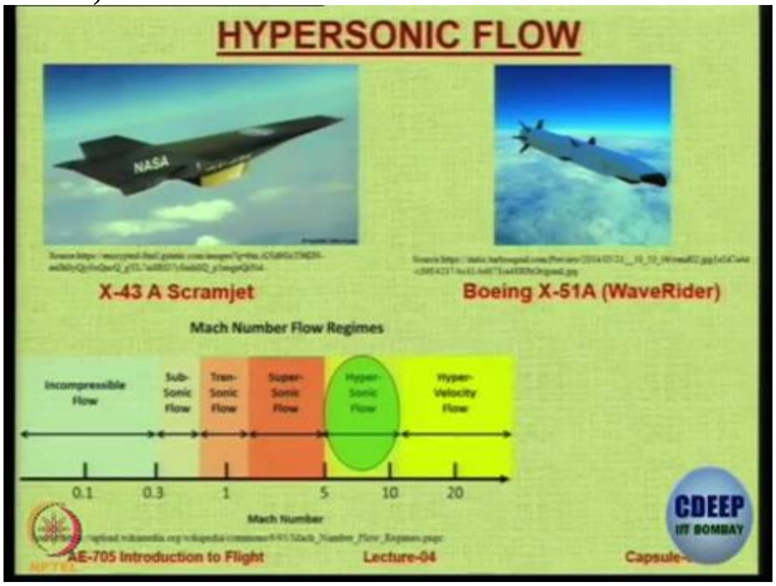

So an example is X-43 scramjet, which is a supersonic ramjet. We will have 1 lecture, especially on engines and the other example is the Boeing X-51A Wave rider which is an example of a hypersonic aircraft.

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Okay, let us go to hypervelocity flow. This kind of a flow is encountered only by missiles and spacecraft. Ok. So this is a video of a rocket which is launching a satellite in space only these kind of vehicles are able to with be subjected to hyper hyper velocity flow.

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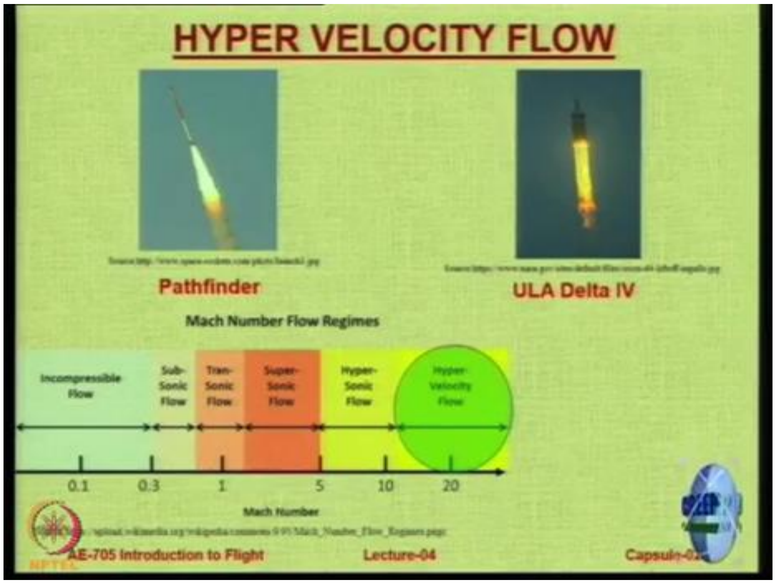

For example, the Pathfinder and the ULA Delta both of which Path finder is basically a missile and ULA Delta 4 is a rocket used to launch satellites. So that is where, now there is nothing like hyper super velocity flow beyond Mach number etcetera. There is no demarcation beyond that. Normally any Mach number beyond 11, 12 we say it is super hyper velocity flow okay. Yes?

Student: Sir, Sonic boom also exist in this?

Professor: Sonic boom will exist everywhere and anytime you exceed sonic velocity but the boom may not be heard because it may be dissipated or because of weather conditions because of the altitude. So the the sonic boom is a phenomena which is; I will come to that in the next few slides. Let me just go there then you will be able to understand it but little bit better. Okay.

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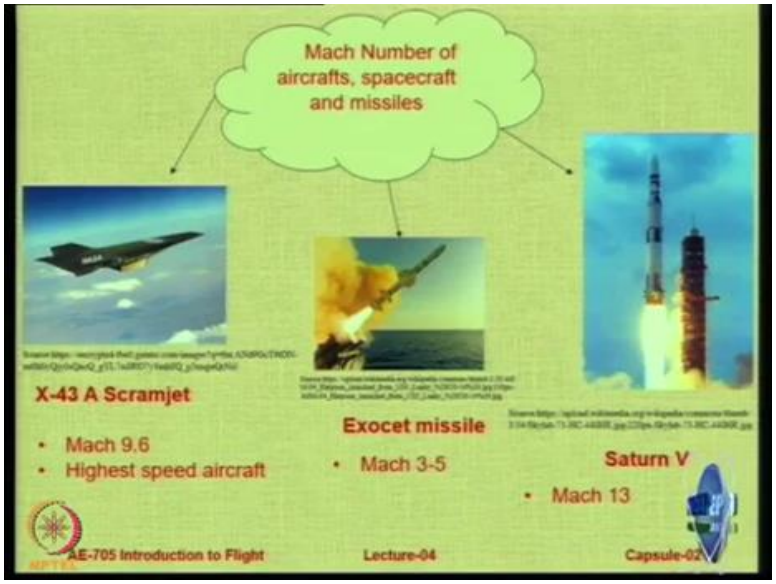

So scramjets operate at around Mach number 10. This is the highest speed aircraft known to human beings. Missiles operate between Mach number 3 to 5 normally and then you have these rockets which go to much higher Mach numbers. The Space Shuttle for example encounters Mach numbers of 20 during its flight. Okay? Ok.

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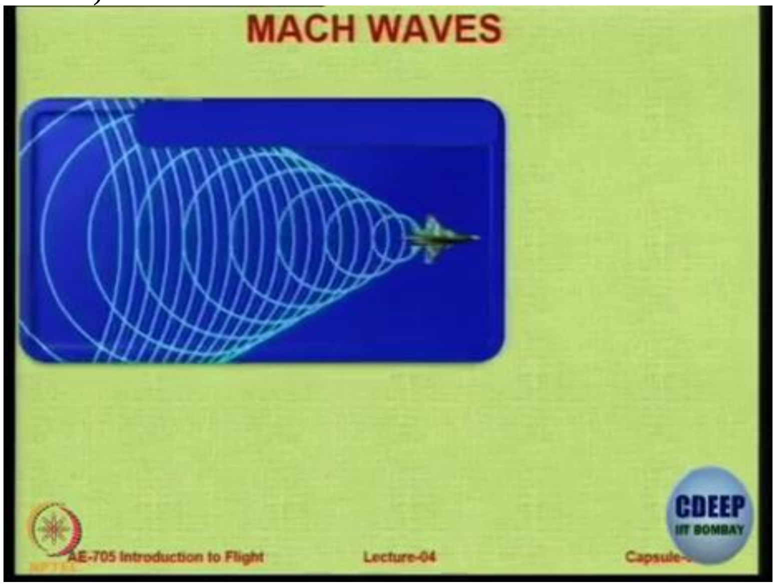

Then there is something called as Mach waves. What are Mach waves? As you can see Mach waves are essentially locus of the intersection of the sonic waves released by an aircraft as it proceeds. Now, since the aircraft is flying higher than speed of sound that is why these the sonic waves the sonic waves are behind the aircraft and they coalesce together and this particular the angle that is created by the intersection of these are is called as a Mach wave.

Okay, so this happens only when the velocity is more than the velocity of sound and the repercussion of this is they create a sudden change in the density and temperature and also the load or the stress which is acting on the body. So as the velocity of the shock wave increases this Mach waves you come at a steeper angle and their amplitude also increases.

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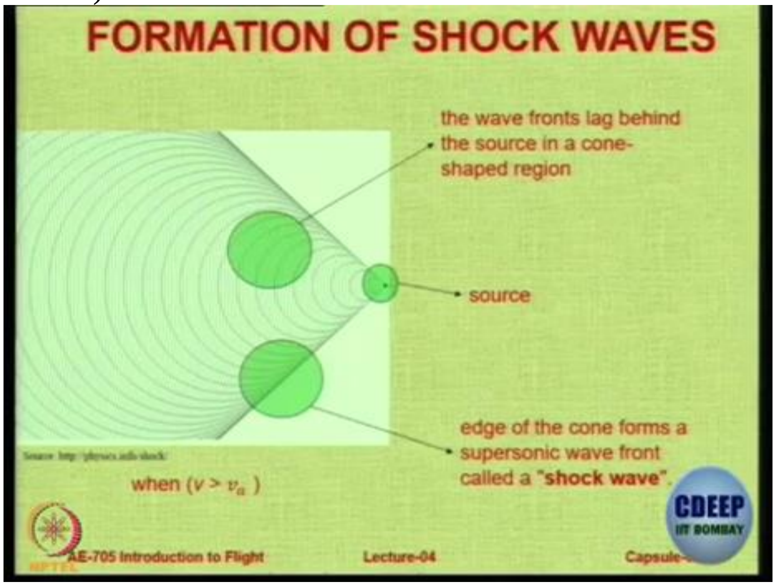

So now let us look at formation of shockwaves. This is a picture which shows you that when the velocity is more than velocity of sound, when V is more than a or in this case it is called as Va. So as I showed you in the previous video, the locus of these particular lines. So the wave fronts are lagging behind the source in a conical region and the edge of the cone is called as a shock wave. This is the answer to your question. The shock wave basically is the edge of the cone which forms the wave front and this particular this particular angle of this shock wave is directly a function of the speed of the object.

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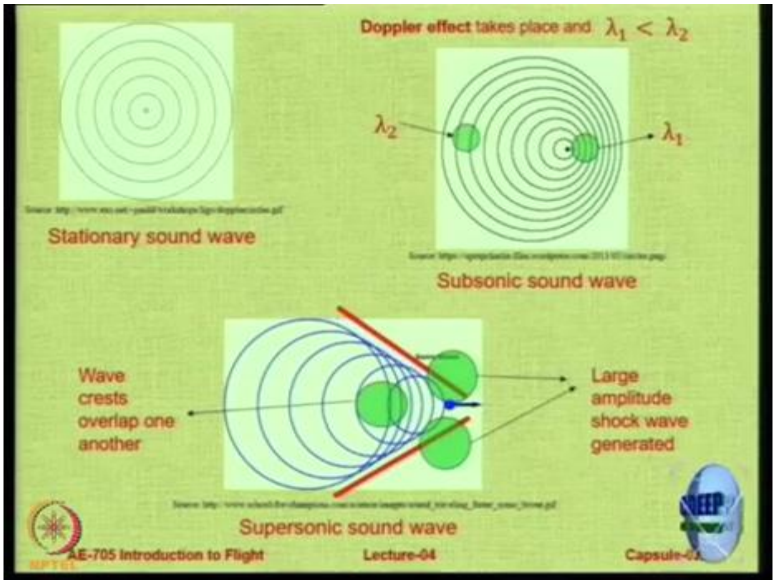

So as we will see now, so this is a stationary sound wave at any point of time there is a sound source and the center and at T equal to 1 unit, you have 1 circle, 2 units, 3 units, 4 units you have these concentric circles. If the source starts moving at a speed that is lower than the speed of sound then you will have still concentric circles but you will find that the gap between them is more on the back and less in the front because the object is also not moving.

Okay, so what will happen is, because there is a compression in the front the frequency at which sound emanates is going to change and on the back because there is a expansion the frequency this is called a Doppler effect and this particular effect is used when the body moves at a speed more than the speed of sound then this is what happens. The wave crests are going to overlap and you have a large amplitude shock wave. So the red lines are basically almost tangential, here it is parallel to a tangent for improved clarity but basically the line that is tangential to all these blue lines is the shock wave. Okay.

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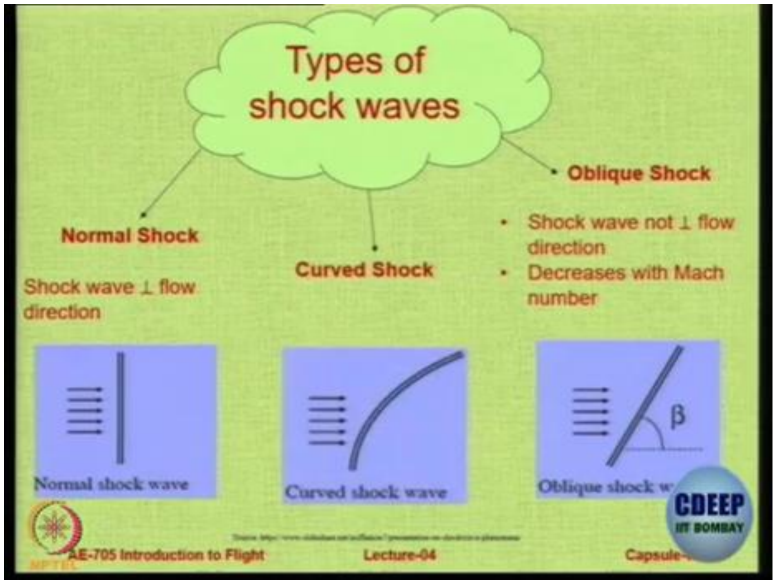

There are 3 or 4 types of shock waves. The first one is a normal shock wave. This is like a front, which moves. This particular kind of shock wave occurs in front of blunt bodies at the in the central region. The next one is an Oblique shockwave. This happens mostly on bodies which are having a very sharp point. So from the point at an angle there will be the shockwaves and this angle beta will depend upon the flow Mach number. You also have curved shock waves. Ok.

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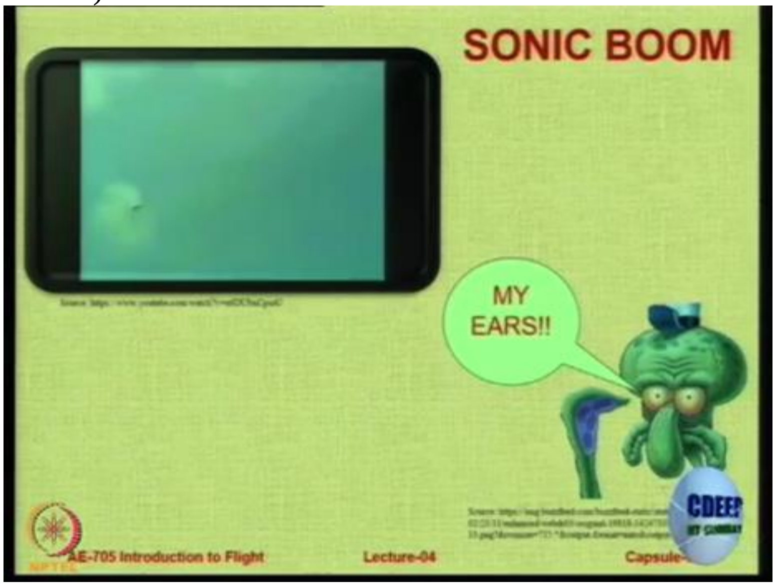

It is an example of sonic boom. The sound which we hear is the sonic boom. So why are you hearing a sound and also remember if you see the sound is coming after the plane has gone. It is not overhead the plane is ahead of you and then the sound comes to you. That means the plane is somewhere there they have crossed me and then the sound is coming to me.

Now, for you it is only a sound right now. But there are many instances in which this particular pressure wave can break the windows; window glasses. There are examples, at when military aircraft are practicing near the air bases there are many many situations in which the people nearby complained breakage of windowpane, shattering of window. That is because of the sonic boom and if this is happening above your house routinely every day then you will start getting serious complaints also.

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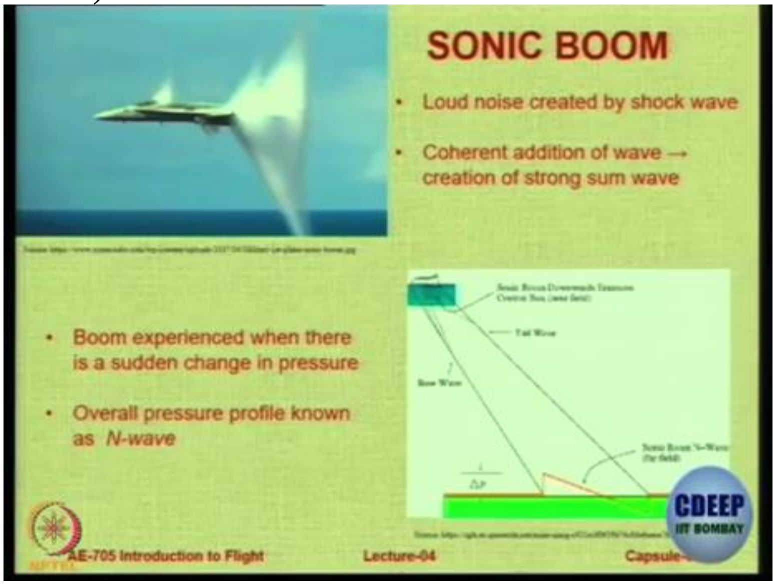

So, let u look at sonic boom. In this particular photograph, we actually see 3 different Mach cones. The one which is just behind the canopy, then we have one which is little bit which is on the tip of the wing and then one on the tail. So it is a loud noise and it happens because of the constructive interference or the cohesion of the waves will see it and essentially it is a pressure difference which comes down. So if you notice if there is a plane flying there will be 1 shock wave from the nose and 1 from the rear portion.

They both are going to hit the ground, if you are at a sufficiently low height. If you are at a very high altitude then the shock the the sonic boom may be attenuated. It may not reach or the pressure may fall because of the losses in the atmosphere. But if you fly supersonic at say 100 meters from the ground, then there is a very good chance that the bow wave and the tail wave are going to hit the ground and create this N shaped pattern which is the sonic boom.

So as the aircraft flies over a terrain at low altitude at supersonic speeds, you have this horseshoe type area of influence on the ground. Okay. So this is a plane at supersonic speeds, you have a conical pressure wave front which is the Mach cone and that conical front is hitting the ground and moving along.

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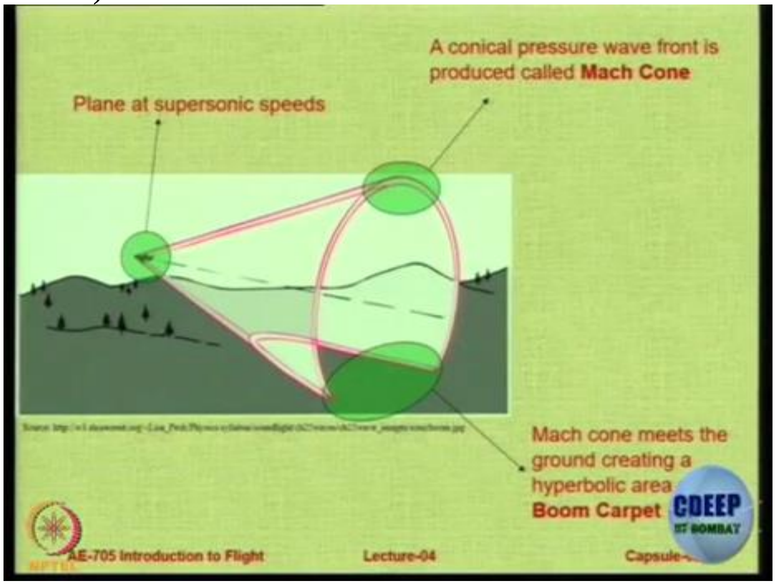

So this creates a parabolic boom carpet. So any and everything that is on that on the ground along that horseshoe type ground interference pattern they are going to be affected by the sonic boom okay.

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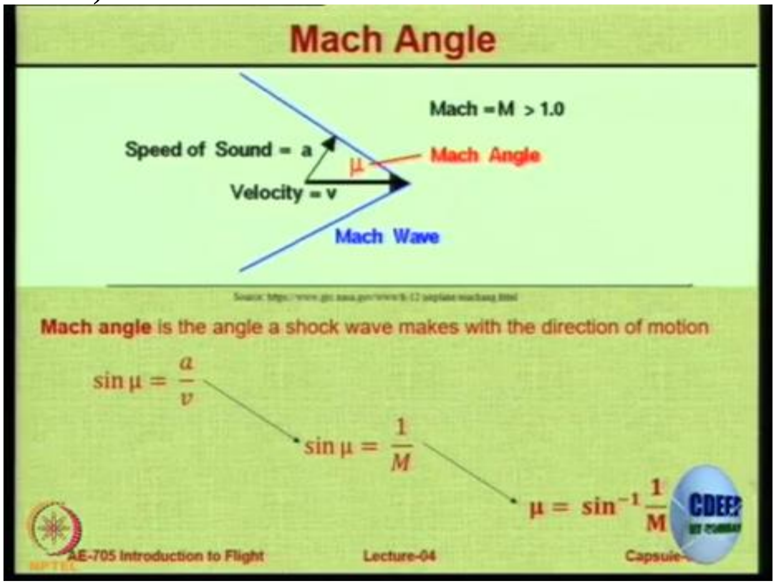

So now the Mach angle or the angle at which the shock wave is inclined at a particular speed is a function of the speed. So you can see very straightforward relationship exists between this Mach angle μ and the velocity V and sonic speed a. So basically

$$\sin\mu = \frac{a}{V}$$

$$\sin\mu = \frac{1}{M}$$

$$\mu = sin^{-1}\left(\frac{1}{M}\right)$$

So as Mach number M increases the value of μ will reduce.

So by looking at the shock by looking at the by by looking at the by looking at the Mach or by the numerical value of the Mach angle you can actually figure out at what Mach number the aircraft is flying. It gives you a kind of a visual indication of the speed. Now let us do a little bit of calculations regarding the shock.

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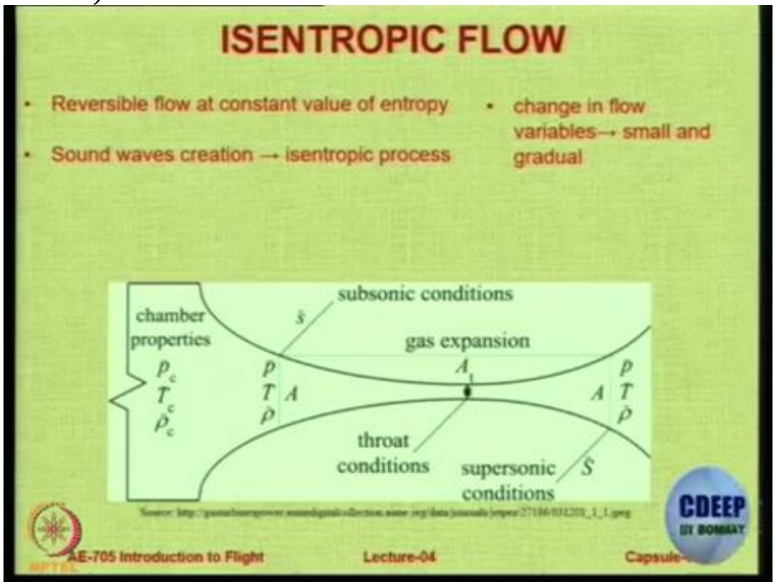

First of all we must look at and understand what is meant by an isentropic flow, it is a flow which is reversible and we assume that there is no entropy loss. It is an assumption. It is very difficult to get in real life any flow where there is no loss in entropy. Entropy the disorder which continuously increases, but for a theoretical calculation purposes or ignoring the loss in the change in the entropy, you can assume the flow to be isentropic.

So if the flow is isentropic the benefit for us mathematically is that the changes in the variables are small and gradual. They are small and gradual which means the functions are continuous and differentiable that is how it helps us. Ok. So the creation of sound waves can be assumed to be an isentropic process. It happens slowly and the changes in the variables are very small. So for example if you take if you take a simple nozzle where you have chamber which contains a fluid let us say air at pressure Pc, temperature Tc and density ρ_c.

It proceeds, it encounters there is 1 section where the condition is subsonic, you have pressure P, T and ρ and there is a area A. Area cross section A and then as it goes and comes back we have a full recovery. If we assume it, this is an assumption then the flow is isentropic. So through a nozzle the flow can be consider to be isentropic.

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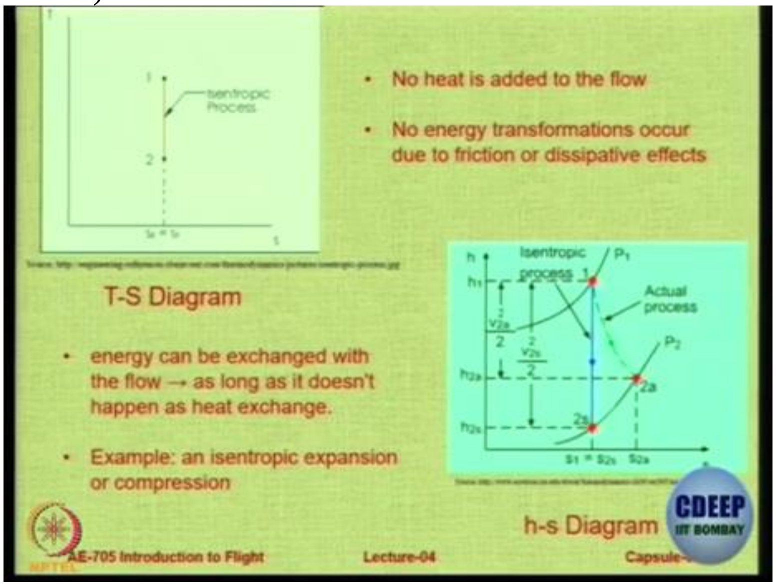

So if it is the process as isentropic then in the T- S diagram it is a straight line, there is no change in s. Ok. Because no heat is added to flow and there is also no energy tranformation because of friction. This is only an assumption, in real life very difficult to get such flows. Okay just a detail.

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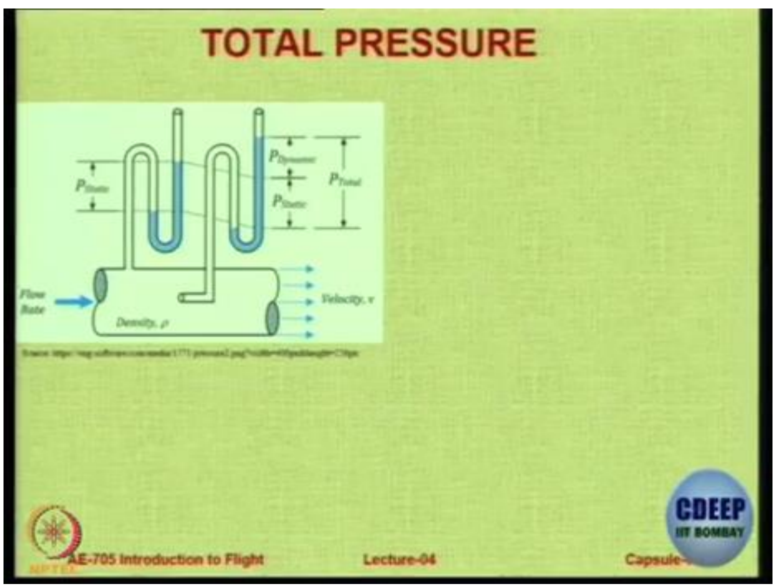

So the idea is we want to define something called as total pressure. So, in this particular example, you see a tube in which there is a fluid flowing at some flow rate. Velocity is V and when you connect the wall of this particular tube to a pressure line, there is some pressure acting because of the atmosphere. Sorry at the at the inner surface, at the border that pressure is the static pressure. So you can see it pushes the fluid column to some value. But when you insert a tube at the center or down below, then the pressure is higher.

That additional pressure is the dynamic pressure, which is there only because of the fluid flow. It is actually $\frac{1}{2}\rho V^2$. Ok. So the total pressure would be the static pressure which is there even in the because at the at the edge that means where you assume there is no flow at all plus the pressure because of the flow.

So hence, summing up the total pressure is the pressure that would occur if you bring the fluid to rest isentropically and the entire $\frac{1}{2}mV^2$ of the particle, the entire kinetic energy of the particle will be utilized just to create pressure. There will be no losses, no friction, no heating. If we assume it; I keep on repeating, if we assume it then we have total pressure. In reality the total pressure that we get is less because of losses. So, this is possible only in isentropic process. So therefore P total will be P static plus P dynamic.

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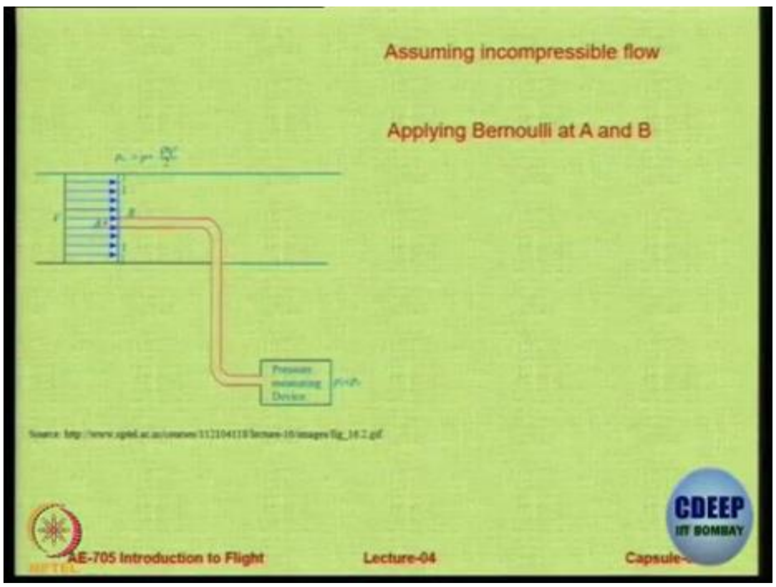

Another example, you have on top a fluid, okay and then if you apply Bernoulli's at A and B where a is just before in a cylinder in a in a container. There is some pressure A and B is just inside the pipe outside and then you put a pressure measuring device which is measuring the pressure. You will find that at B the pressure is P knot, inside the pressure is P and the velocity is V.

So the p knot is going to be the pressure P which is the static pressure plus $\frac{1}{2}\rho V^2$ depending on the velocity at which the fluid is coming out. So there P is static pressure and $\frac{1}{2}\rho V^2$ is dynamic pressure. So with this you can get the value of V. So this is a very simple way of measuring velocity; using the incompressible Bernoulli's equation by measuring the pressure difference under 2 places. This principle is used to measure the velocity.

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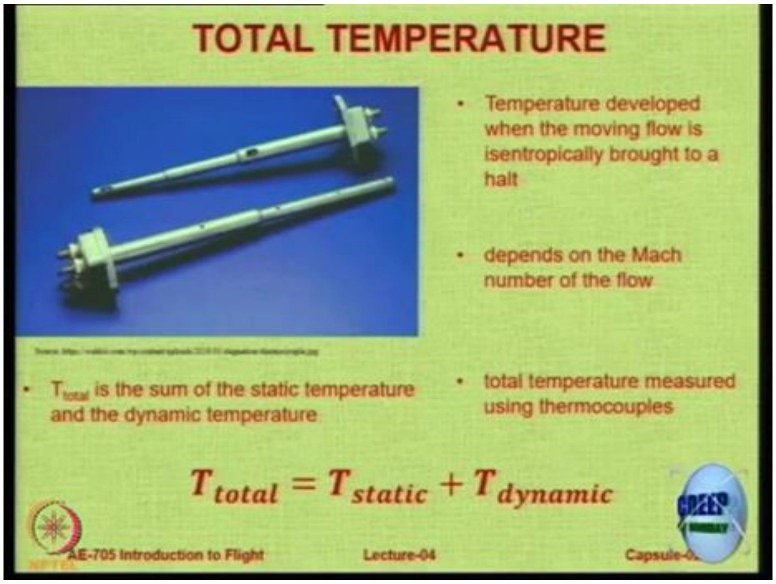

Similarly, we have a concept of total temperature. Again it will be static temperature. But here we have to bring in the effect of Mach number okay. So we use thermocouples to measure the total temperature. So it is the sum of the static temperature and the dynamic temperature again in the same manner as the pressure. Okay.

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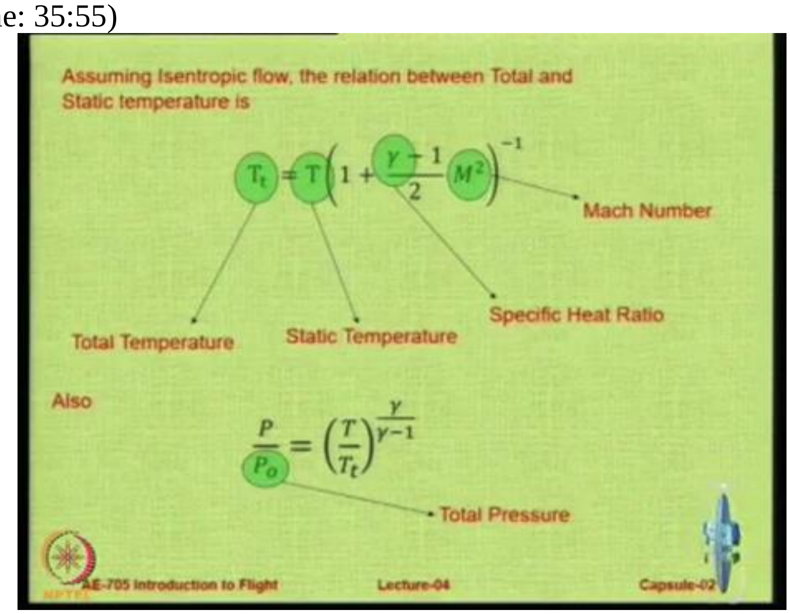

So how do you get the relationship? It can be shown, I do not want to prove it and I do not want you to also worry about it. We are not into fluid mechanic calculations. We are into introduction to flight. So our purpose is to get a general overview of how things are going to be happening.

So right now it is sufficient for you to assume that the total temperature T is going to be static temperature plus depending on the specific heat and the Mach number. So this will tell you that as the Mach number increases there is going to be a problem. Is there any issue? It should..yes, I am glad that you observed it. The minus 1 should not be there. Thanks for pointing it out. So from this you can get

$$\frac{P}{P_0} = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}}$$

Ok. So that is all, that is the total pressure we come to an end on today's class.