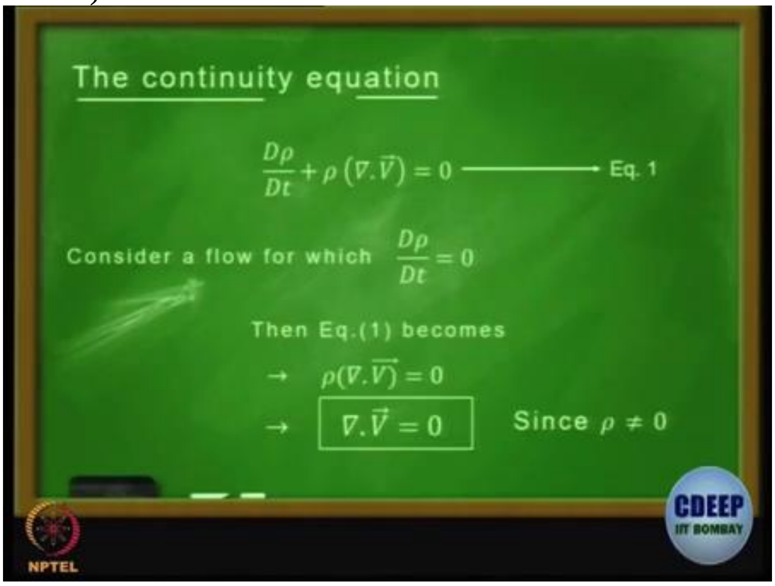
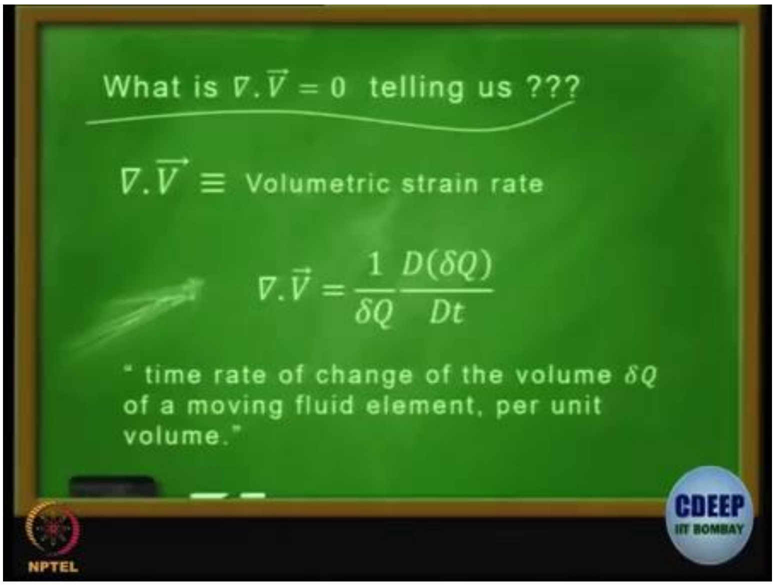
Introduction to Flight
Professor Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Lecture 03.2 - Essentials of Incompressible Flow: Part II

(Refer Slide Time: 0:27)





Okay, so let us see. Let us understand this with the help of an equation of continuity that we have all learnt. I will just state the equation. The equation says that

$$\frac{D\rho}{Dt} + \rho * (\nabla \cdot V) = 0$$

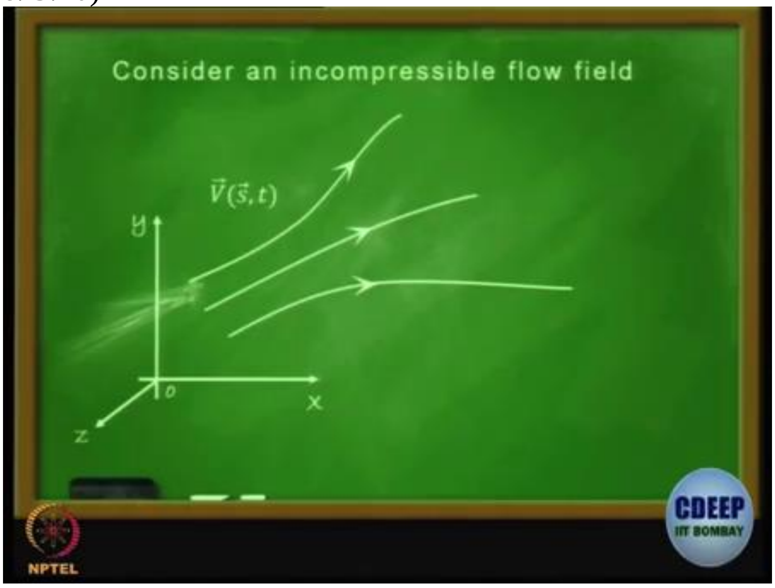
This is in the same form as we saw last time. Okay. But the difference is now I will not derive this equation. This is something which I would leave it to you for homework.

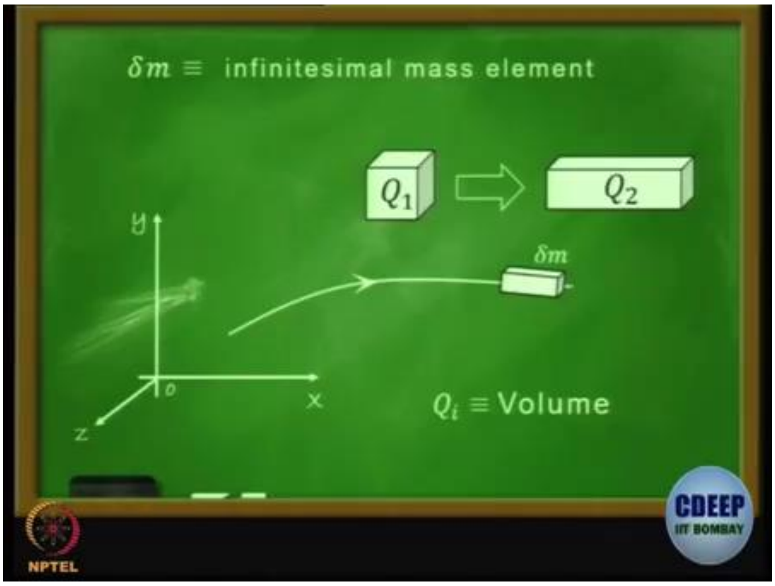
So all the Moodle champions and there are many of you in Moodle who are champions today. If you can cut and paste from some source or if you can do it yourself, for the benefit of those who do not know continuity equation, just do it and upload on Moodle. Just do not go over board and put 20 versions of that. Just 1 version is enough. I want you to just put down the continuity equation, derive it and get this particular term.

So, today we assume that this is true. If that be the case, okay, then let us look at the flow for which the density change over time; total partial derivative is 0. If that is the case, then this becomes $\rho * (\nabla \cdot V) = 0$. But rho is a property of fluid. It cannot be 0. zero rho means no fluid. So therefore the condition becomes $(\nabla \cdot V) = 0$. So, $(\nabla \cdot V) = 0$, is a condition for incompressible flow.

What it means is the volumetric strain rate, that means the change, the time rate of change of the volume of a moving fluid element per unit volume. If that remains, if that is 0, so look what we are saying is, when you say time rate of change, it means change with time. So if you take a small fluid element and now capture, capture it. So you focus your attention on that element. Now as that element moves in the fluid, if its total volume remains the same of that element, the element may transform but if its volume remain the same then the flow is incompressible. You get the point? It is not that the fluid particle which is a square remains a square. No, the square can become a rhombus. The square can become a cylinder also.

(Refer Slide Time: 3:20)





As long as the volume of that element remains invariant with time, we can call the flow as incompressible. So let us see by an example. So we assume an incompressible flow field. The moment we assume this, remember that the total derivative of, you know, of what? What will remain constant? The total derivative of density will remain constant. So here is some velocity field. So the velocity field basically means that as a function of some position vector S in xyz plane. I will show you very soon. And time. There is some velocity.

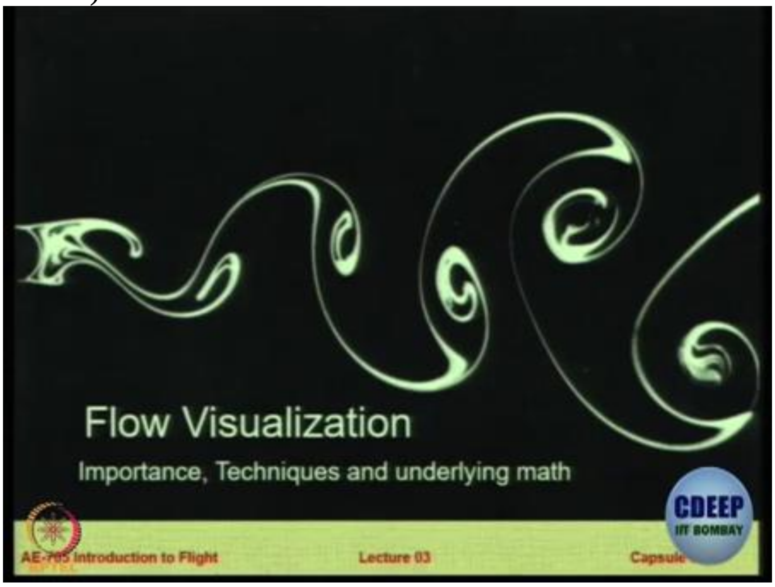
So let us focus on one small element of mass delta m. Delta m, m being very small, delta m being very small. So that means it is a fluid particle. For all practical purpose, it has some shape. This particle has a volume Q1. Now this particle moves in the fluid and its volume becomes Q2. If Q1 equal to Q2, then the flow is incompressible. Okay. The mass would remain the same because there is no loss of mass.

We know that there is a conservation of mass assumed unless we state otherwise. So if I start talking about mass not being conserved, then I will request you to attend the class after my class in which Professor Yagnik is talking about those kind of phenomena. Okay. But here we are assuming that mass is not consumed or added. Mass is conserved but the particle undergoes little bit of transformation, change in the shape.

However, if Q1 equal to Q2, only then we will call it to be incompressible. So $(\nabla \cdot V) = 0$, ensures that these 2 volumes are same. Okay. Are you clear about the difference between the two? What, there is something called as incompressible fluid. This fluid is incompressible if its density remains constant or nearly constant as a function of time and distance. But the flow is not that. So rho equal to constant is incompressible fluid not incompressible flow.

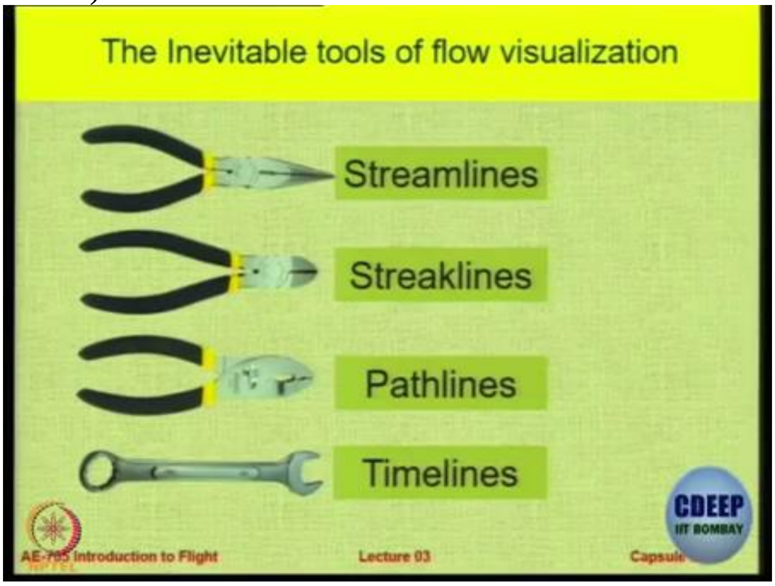
Okay. For the flow, it is the total derivative of rho with time to be constant. And therefore, there is a question which I would like to ask you that, can compressible fluids, that means where rho is changing can they undergo incompressible flow? Yes, because two different things are needed.

(Refer Slide Time: 6:20)



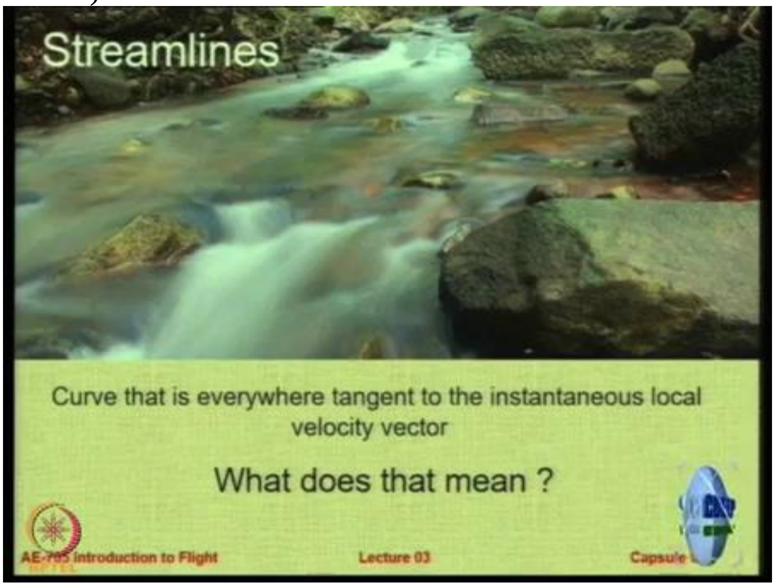
So there is a technique called flow visualization in which we look at the flow and to understand the flow visualization we move to the next aspect of this particular presentation today which is on how do we define and explain the various….what are the tools available to us , the aerodynamicist. Now this is a very interesting figure which shows the fluid patterns behind a cylindrical body.

(Refer Slide Time: 6:50)



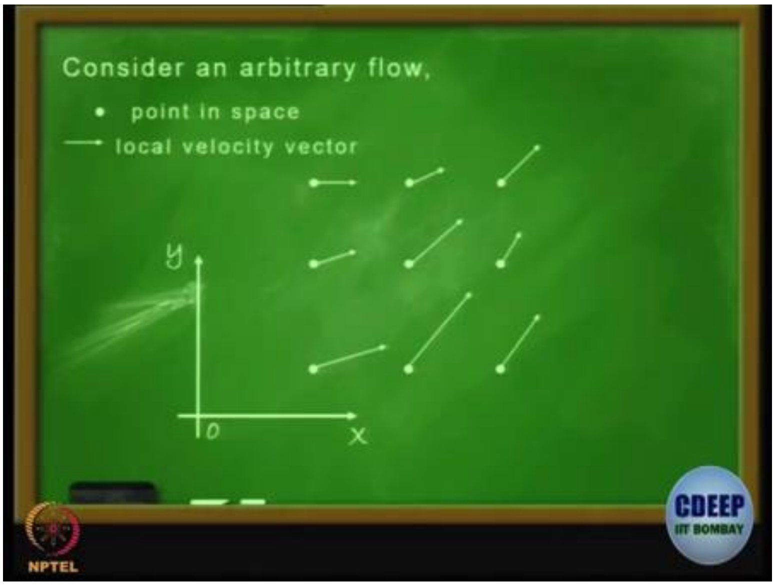
Okay. So let us look at the tools of flow visualization. The first tool is streamlines. The second tool is streaklines. The next tool is pathlines. And the next tool is timelines. They are not the same. And today, in the remaining part of this class we are going to look at these differences.

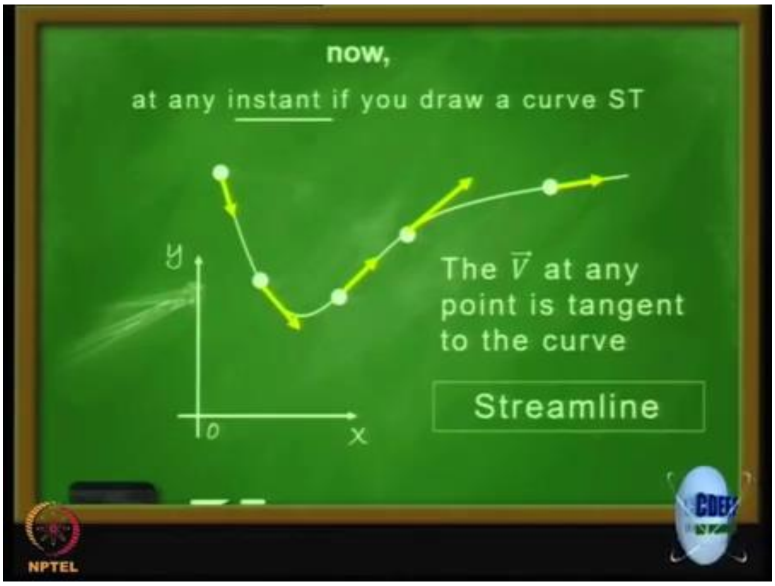
(Refer Slide Time: 7:12)



So what do we see here? We see here a river or a small brook flowing across, there is a flow. So if you draw a curve that is tangent to the instantaneous local velocity vector at every point, that particular curve is basically a streamline. Okay. So I want to ask you, is streamline a property which is Lagrangian or Euler? The definition itself will tell you. The moment I say instantaneous and local, it cannot be Lagrangian. Because Lagrangian means range. So, it is Eulerian.

(Refer Slide Time: 8:02)





So let us see. We just consider an arbitrary flow. In xyz plane, we just assume that these are points in space and you have a flow. And this gives you the instantaneous velocity vector. So I just take a picture and in the picture I have recorded the instantaneous velocity field. So what we do now is you draw a curve such that through this flow field, I draw a curve which is tangent such that the velocity at any point is tangent to the curve. That is a streamline. Okay.

(Refer Slide Time: 8:58)

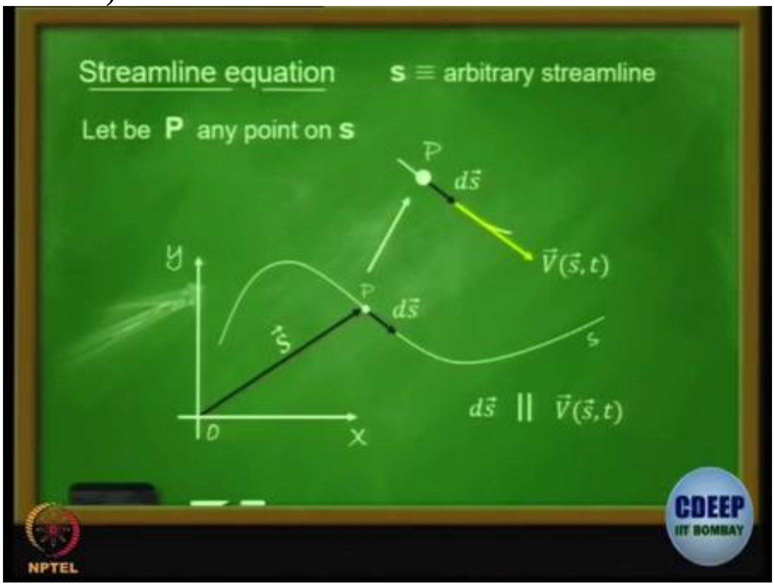
So we should keep in mind a few things about the streamline. First of all, they are instantaneous. So they change with time. Or I should say, they can change with time. Streamlines need not remain constant as a function of time because they are Eulerian. They never intersect. Why is it so? Why cannot you have two intersecting streamlines? Rahul, yes.

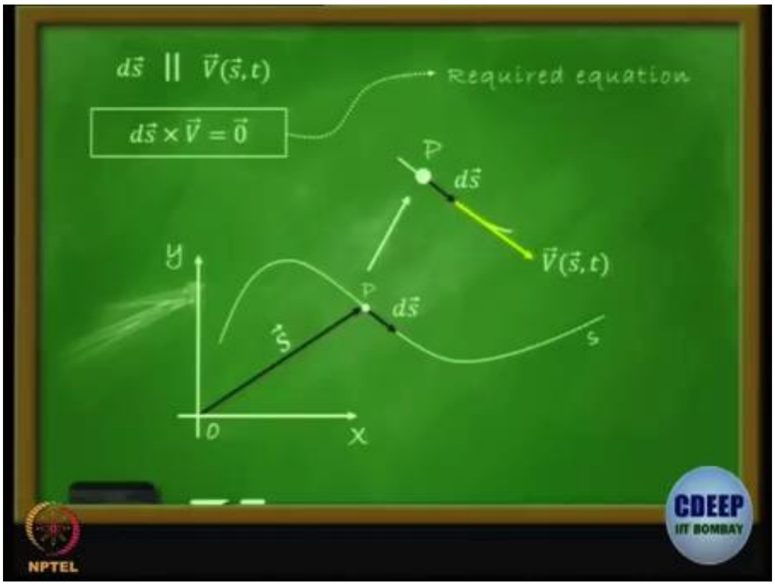
Student: They will not have unique direction.

Professor: Yeah, because at the intersection where will it go? This way or that way? There are two directions possible on an intersection. So by definition the direction should be tangential to the flow direction. If there are intersections there could be two, that will cause confusion. Okay. Not always the path, it can be along the path but you could have a different path but a different streamline.

If the velocity is changing over an instant, it will not be the same. The other thing is, it is only a mathematical notion. The physical significance of a streamline is not very apparent. It is just a mathematical notion and we have already discussed, it is the policeman's concept. It is the Eulerian concept. Okay. So let us do little bit of maths. Just a little bit of maths to model the streamline. Very simple.

(Refer Slide Time: 10:27)

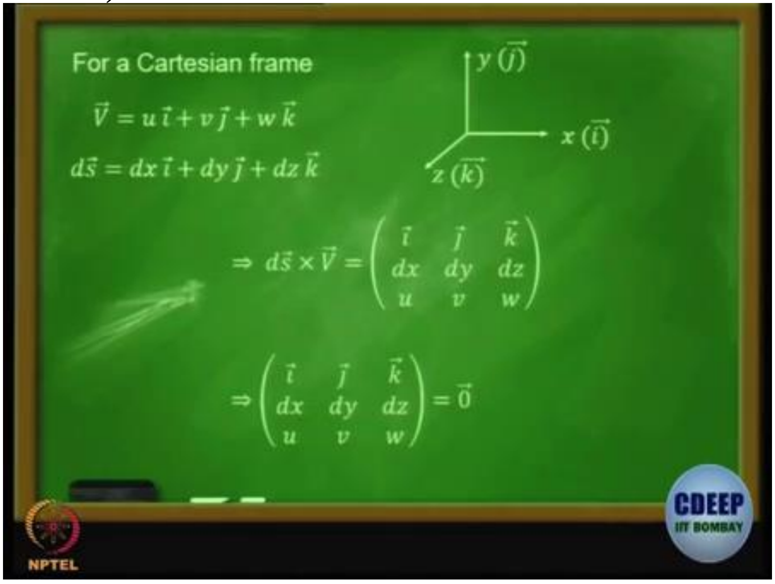


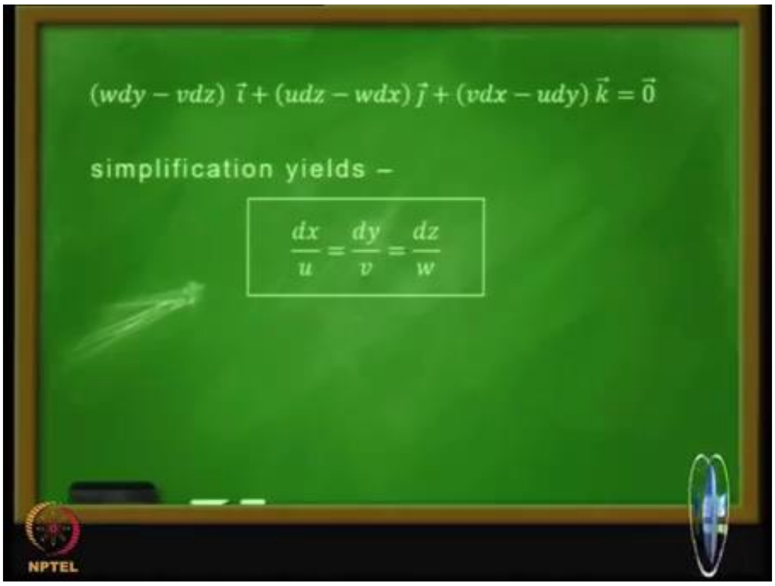


Back to the blackboard, or sorry, green board. Look at the equation of a streamline. Again you have the flow field. And we have any arbitrary streamline S which passes through a point P. So this point P is located in space at the position vector of $\bar{S}$. Okay. And we know by definition, if we have a small distance dS along the streamline and if you focus there, the velocity has to be tangential to that particular point P, by definition of streamline, where V is the function of s and t. Okay. So therefore, dS has to be parallel to velocity. So the local elemental direction has to be parallel to the velocity at that particular time. Correct. So, which means the cross product will be 0. By definition, if two vectors are parallel, their cross product is 0. So we invoke that property and we say that dS ×V = 0. But it is not 0. It is 0 vector.

What is mean by 0 vector? This is $0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$. It is not a, it is not a scalar. It is a cross product of two vector which will be a vector and that vector has 3 dimensions; x, y, z and it has got $0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$. So therefore, for a Cartesian frame this is what we are looking at. So dS or the elemental direction will $dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$. in any 3 dimension, arbitrary direction if you look at that.

(Refer Slide Time: 12:17)





So therefore dS×V which is the cross product between the local direction and the local velocity, will be a simple determinant and that has to be equal to 0. So technically you should write equal to bracket 000. But we are just trying to be lazy here. So we say 0 bar. When I say bar, it means it is a vector. Therefore, let us go ahead. If you expand this, so how do you expand it? If you remember, what I remember is,

$$(wdy - vdz)\hat{\imath} + (udz - wdx)\hat{\jmath} + (vdx - udy)\hat{k} = \vec{0}$$

That is what you get and that has to be equal to 0. Okay. So if you simplify this, you get

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

This is what you get if you simplify it. So we will use this property in our tutorial to try some very interesting experiments with streamlines. At that time, please remember I will come back and address this particular equation. So, $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, is a simple equation of a property of a streamline.

If it is a 2D flow then z goes away, because dz is 0. You get $\frac{dy}{dx} = \frac{v}{u}$ So very simple the rate of change of y with x would be the ratio of the v and u velocities in a 2D flow along the blackboard. Clear? Yes. We have a question there?

Student: What do you mean by tangent in 3D? And my second question is, what would happen if we write $\bar{S} \cdot \bar{V} = 0$?

Professor: No, it will not be a dot because these are two vectors. Okay. So we know that a cross product of two vectors is 0.

Student: No, no. I am not talking about dS bar, I am talking about $\bar{S} \cdot \bar{V} = 0$, then we would be getting a plane equation.

Professor: $\bar{S} \cdot \bar{V}$

Student: Because the position vector and velocity vector will be perpendicular. So the position vector of the point and the velocity vector would be….

Professor: Position vector and they will be perpendicular because….yeah, okay.

Student: So if we take a dot product and…..

Professor: You can do that also. You can try that way also. You can try that way also.

Student: Then we would get a plane equation. What would that plane represent?

Professor: I think you should derive it and put it on Moodle. Tell us. I have not done it. I have not done that but it is a good observation that you have a position vector and you have a tangential vector. Now the second question is about, what do you mean by tangent in 3D? So the tangent in 3D is basically a tangent with respect to the x, y and z directions in uniformity. In the sense using the i, j and k properties, you should get…So I have already mentioned what it is. Yeah. Yes.

Student: Sir, to his point, they are not perpendicular; the position vector and the tangent they are only perpendicular. They will be perpendicular only in the case of a circle or something like that.

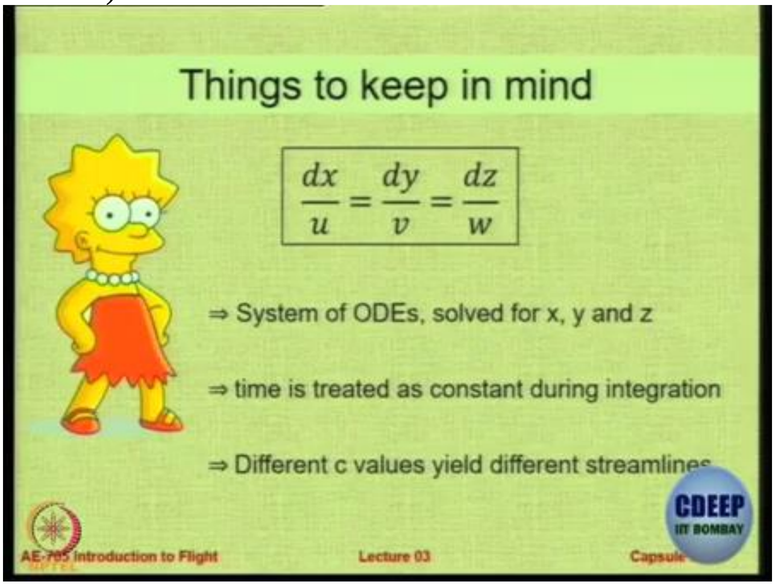
Professor: Only in the case of a curve.

Student: Yeah, in any random curve they do not have to be perpendicular.

Professor: So I think this is a very good discussion. I think you should take it up in Moodle. It is a good point. You bring out an example where they may not be perpendicular and hence it may not be valid and let him also argue. And other people can also join him. Good point. We would like to encourage such discussions. We would like to clarify and attend such discussions.

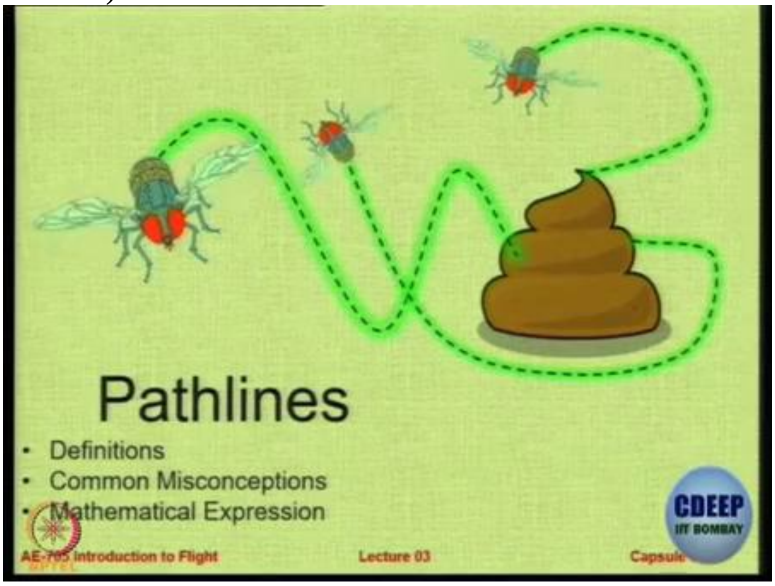
So it is a good idea. If you feel that they are perpendicular always, then you can argue out and derive an expression. Somebody else can counter. No problem. We are welcome for such discussions. Okay.

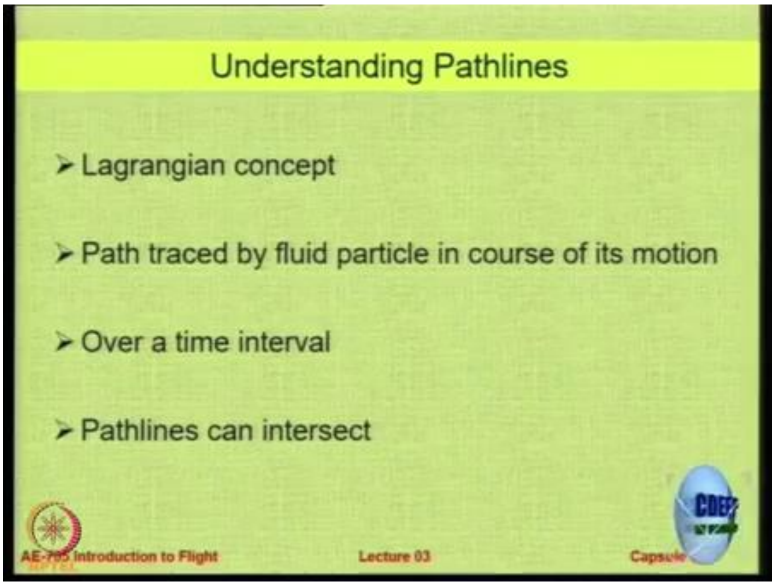
(Refer Slide Time: 16:30)



Let us keep a few things in mind. This is the prime equation which we have to consider. And it is a system of ordinary differential equations which we can solve for x, y and z. How do you solve an ODE like this? It is a simple ODE. Okay. How do you solve it? We have many ways of doing it. You could do it numerically. You could do it analytically. So hang on a minute. In the tutorial that we will take up. We will take up some examples and then we will solve it be substitution or some other methods. So during this particular integration, you can treat time as constant. And different values of the constant will yield the different values. So it will not be that it is equal to the same constant. It will change.

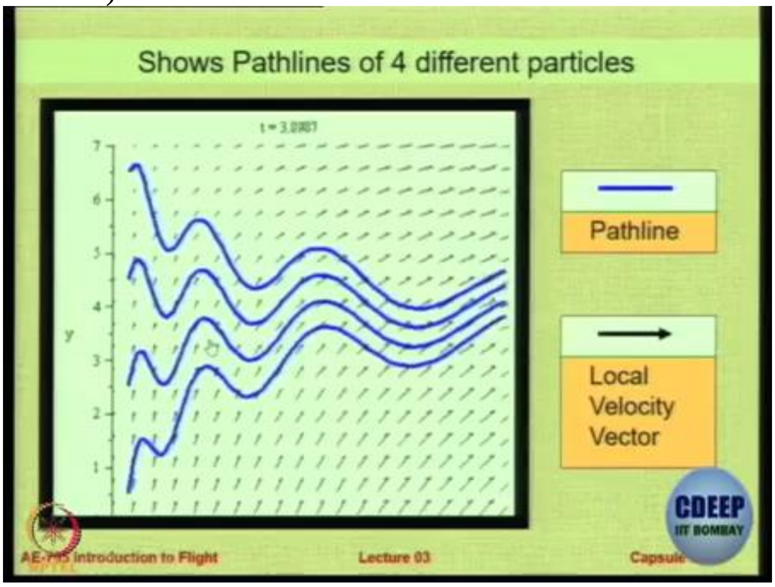
(Refer Slide Time: 17:15)





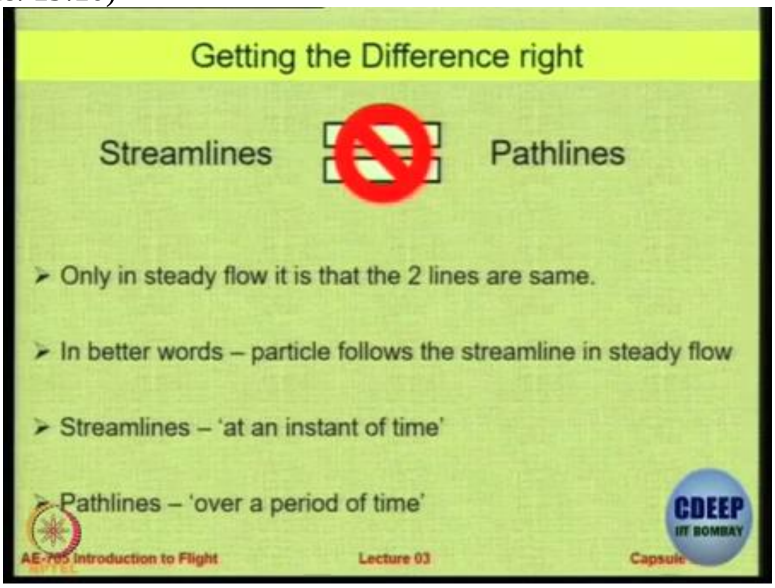
Now we go to the pathlines, another important tool available to the fluid mechanics experts. Let us look at the definitions, common misconceptions and finally a mathematical expression for pathlines. So pathline is a Lagrangian concept because there we look at the path followed by a particle over a period of time. So during its motion, what is a path and that will happen over a time interval t equal to t1 to t equal to t2. These can intersect. There is no binding on them to be parallel or never intersecting because two particles can have the same point at different times. What about same time? Can two particles be…possible! They will collide, but they can be at the same time.

(Refer Slide Time: 18:16)



So here is a small animation and on the top we have a t. So we ran a small animation for a typical case where the blue colored lines are the pathlines of these four particles and the local velocity vector is shown by the red arrows, blue arrow, black arrows. So let us see, as a function of time, so a simulation was run by one of our students, the guy who made this presentation, he ran a small simulation and he captured the value of the pathlines and the streamlines at various time intervals. So as you can see, there are 4 particles starting from 4 different locations and in this case they do not intersect. That is fine. But it need not be.

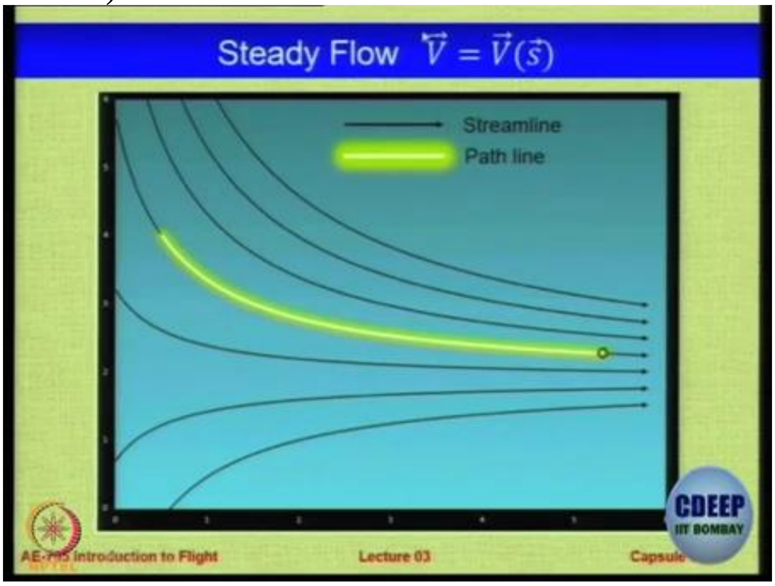
(Refer Slide Time: 19:10)

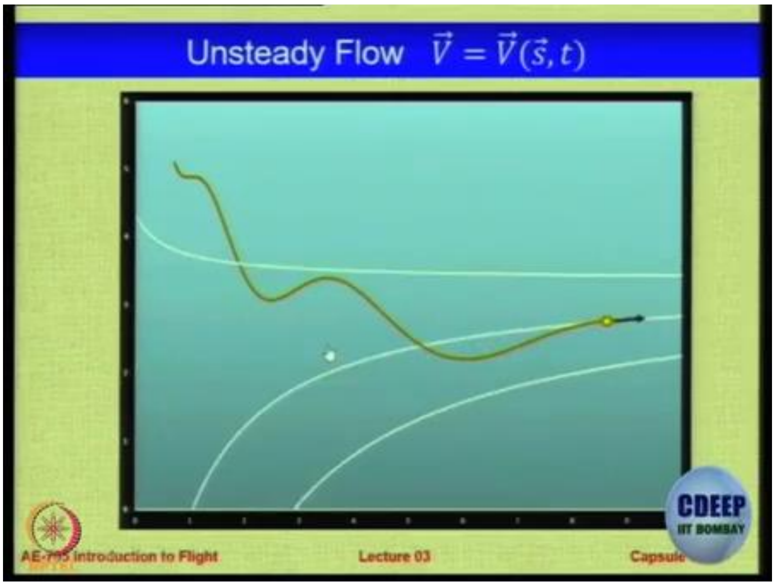


Alright. So let us understand the difference between streamlines and pathlines very clearly. They are not equal too. First of all, they will be the same only if it is a steady flow. That means, what is meant by steady flow? When the path lines and streamlines are identical. This helps you define steady flow. If there is, if the flow is invariant with time, so you remove the time. So partial derivative of a property over time is 0. That becomes steady flow. So that is why many of the answers about change in density being equal to 0 as incompressible flow is acceptable in steady flow because there is no change with time. Okay. So that is what I was saying when you are defining rho equal to 0 or almost 0, that is for the fluid, not for the flow and it becomes applicable for steady flow.

In other words, in steady flow the particles follow the streamlines. So this is now the benefit or advantage of streamlines. They help you identify whether the flow is steady or unsteady. Okay. So streamlines are at an instant of time and the pathlines are over a period of time. And if there is no time variance, they are the same. Simple.

(Refer Slide Time: 20:37)





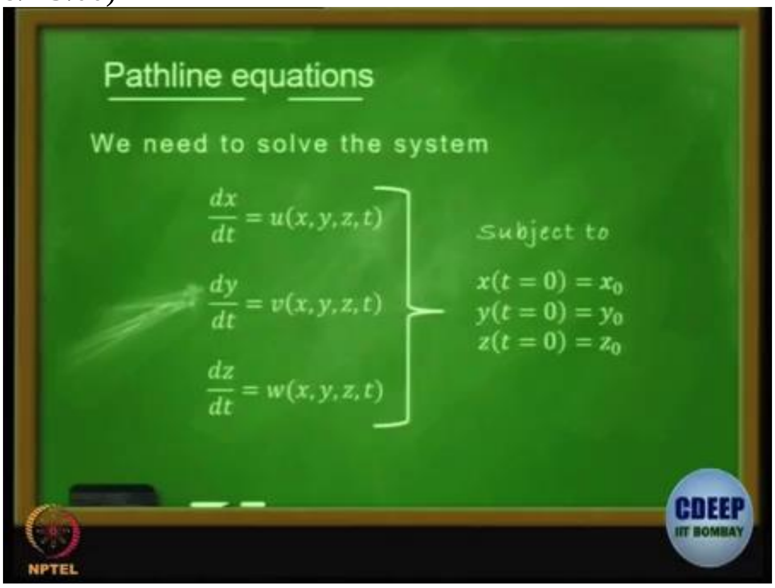
Okay. Let us see an example of pathline and streamline and steady flow. So you can see that because the flow is steady the pathline and the streamline are identical. Because velocity V is a function only of the location and not the time. Okay. Let us look at the unsteady flow. Here the streamlines are changing with time. So you might think that they are changing with the particle. No, if you actually focus only at one place in the space, you will find that the streamlines sometimes go through and sometimes do not go through it.

So the particle is actually following its own path. But the streamlines are changing with time, because the flow is not steady, it is unsteady. The flow is going to be having to be a time. So there is a technical word for this is spatial and temporal. So temporal property means time variant. Spatial means space variant, location variant. Okay. So this is an example of unsteady flow. Now real life flows, most of the cases are unsteady flow.

Okay. But unsteady flow is difficult to numerically as well as physically examine. So steady flow is an assumption and in some practical cases the flow is steady where the streamlines and pathlines are intersecting. But we need to understand unsteady flow and its properties because that is what we will usually encounter in any practical situation. So by with this I think the distinction between the pathlines and the streamlines is clear.

The pathlines are simply a trace of the path taken by the particle. But during its journey, the particle could be, not on the, particle need not be on the same streamline. Or the streamlines could be changing as a function of time.

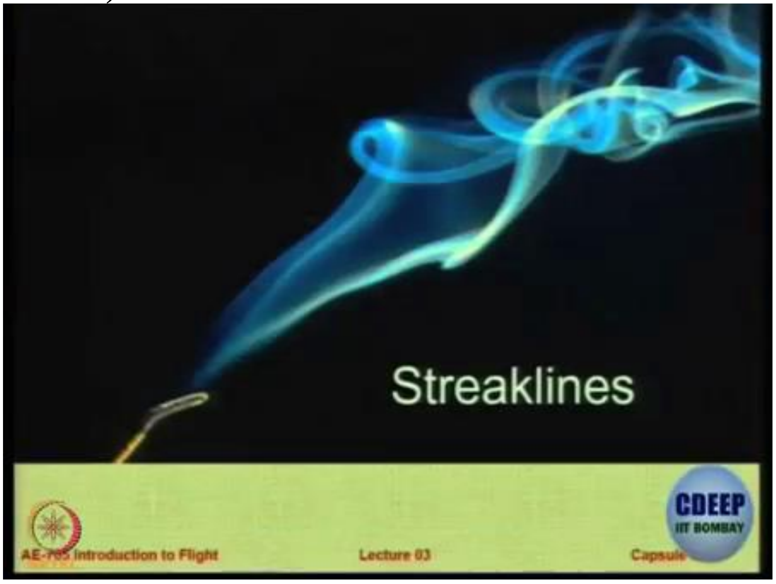
(Refer Slide Time: 23:00)

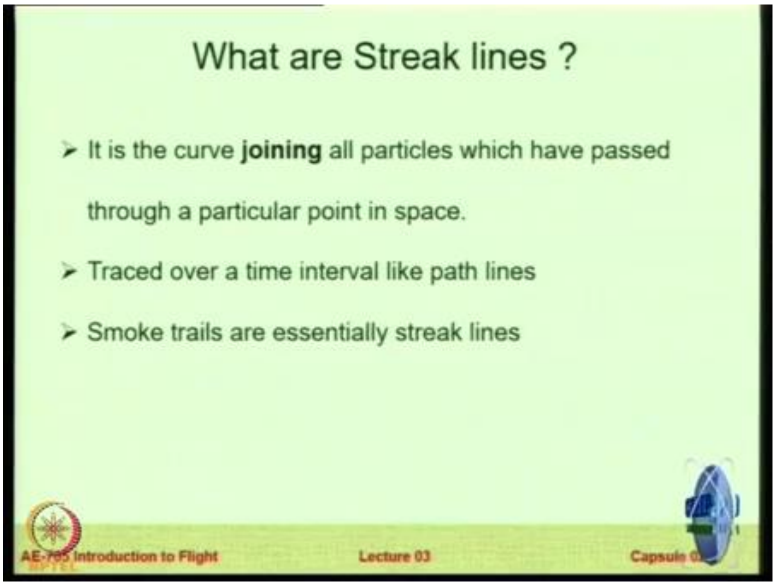


Back to the blackboard. Let us look at the mathematical representation of pathlines. So basically, we have to solve this system where the dx by dt, dy by dt, dz by dt or the xyz components of the velocity, of the position which change with time correspond to the values of u. So u is a function of xyz and t. So not only the location but also time, that is why it is unsteady flow.

So all of these subject to some initial condition, so when you start it at that time the particle was at a place which was x0, y0, t0 at time equal to 0. And from there the particles start moving. As it starts moving, its velocity u is changing in general as the xyz and t change. Similarly v, similarly w. So if you remember our equation of streamlines. So we can take that into consideration and derive the expressions.

(Refer Slide Time: 24:05)

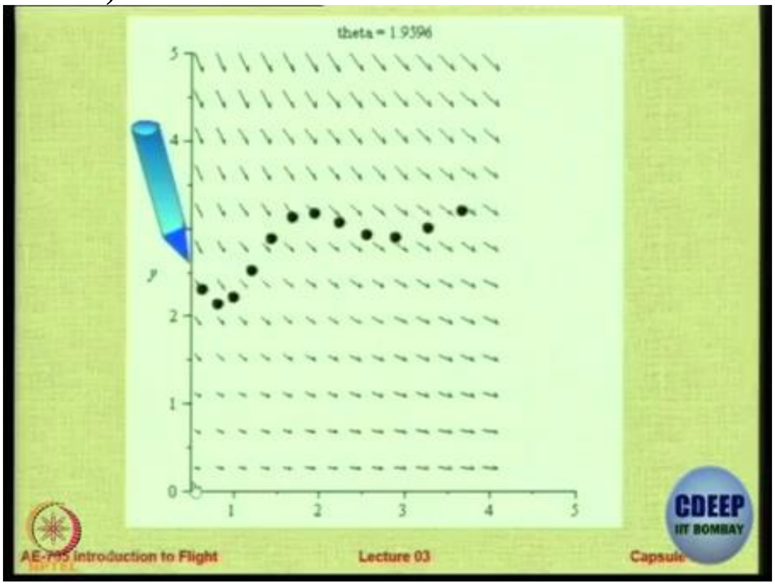




The last thing is the streaklines. So what you see basically are streaklines in this picture. So what is a streakline? If you can visualize the flow, what you see over a function of time or a period of time? Okay. So what are streaklines? It is a curve that joins all the particles which has passed through a particular point. So go back again to our Eulerian policeman. He or she is standing at the IIT main gate. Whichever car is going through that person, you just tag those particles.

Now forget about that person. Now let us assume that there is a small square marked on the ground opposite IIT Bombay. And there is someone who is keeping a track of how many cars going through this square. Okay. So and if you, if a car goes through that square, you tag that particular particle. So then if you plot that you will get what is called as a streakline. So what you saw just now the smoke trails, they are essentially streaklines. So let us give you an example of a streakline.

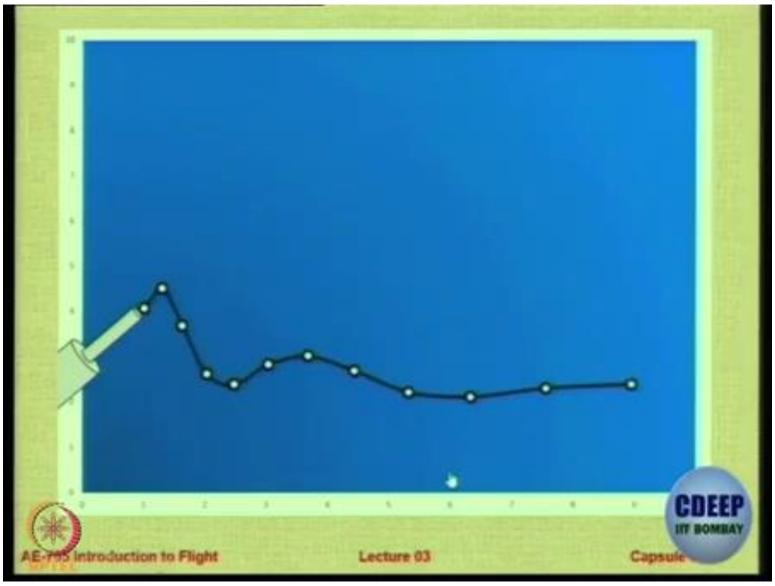
(Refer Slide Time: 25:31)



Look at this video. So this particle has come out. And all the particles which have come out from the point if I join them together, that is a streakline. So all these particles have passed through this point 0, 3. This is the point, fine. They are going somewhere. You have the local velocity marked at every time t. The local velocity changes because of the field, the field has got some property which changes the local velocity. Okay. It could be a hurricane, or some kind of a vortex or something. Because of that there is a local velocity change as a function of time.

Theta is a time step but all the particles which are going through this particular point 3, 2. Once I keep a track on those particles, then the line that joins all particles which have gone through 0, 3 is a straight line. So if you look at a cigarette with smoke coming out or an agarbatti or anything, what you basically see is all the particles that came out from this point only, so that is why what you see is a streakline.

(Refer Slide Time: 27:08)



So let us see another example. This is another good example. Here, we have not only done tracking of the particles but also joined the particles together. Notice that the tagging is not happening at equal time intervals. Not necessary. The gap between two particles coming out need not be the same. It could be the same. It may not be the same. Also notice that this particular line can also evolve with time.

It will not be a symmetric or a fixed pattern. So since from any chimney, everything that comes out from the chimney actually has come from the chimney. So, if you can see it as a function of time, it is a streakline because came from the same point and now they are at different places. So generally what we normally see in flow visualization is a streakline. Sometimes, if you track a particle, you can get a pathline. That you can get a path. Streamline is a theoretical concept. You can only see a streamline equal to pathline in steady flow. So with this, I think I have come to the end of the presentation.