Turbo machinery Aerodynamics

Prof. Bhaskar Roy
Prof. A M Pradeep
Department of Aerospace Engineering
Indian Institute of Technology, Bombay

Lecture No. # 33
Tutorial – 6 Centrifugal Compressors

Hello and welcome to lecture number 33 of this lecture series on turbo machinery aerodynamics. As promised in the last lecture, we shall be having a tutorial session in today lecture. But before I start the tutorial, let me quickly recap what we have been discussing in the last two lectures. As you know that last two lectures have been devoted exclusively for centrifugal compressors, so during the first lecture on the series that was on lecture number 31 where we introduce the aspect of centrifugal compressors.

We discussed in detail the thermodynamics of centrifugal compressors, and we also discussed in some detail about why is it that centrifugal compressors have certain amounts of benefits in some cases. And as well as the fact that, there are a lot of disadvantages associated with centrifugal compressors and that is primarily the reason why these compressors are not used in large sized engines.

So, the application of centrifugal compressors are primarily at this momentum limited to smaller sized or smaller thrust class engines, because axial compressors have inherent advantages over centrifugal compressors which makes them more suitable for larger sized engines.

But centrifugal compressors continue to be used in smaller engines. And of course, they were used popularly in the early days of development of the jet engines, when axial compression system was not that developed in those days. And then, we also discussed about different components which constitute a centrifugal compressor like, the inlet part of the centrifugal compressor, the inducer. Inducer is one which guides the flow into the impeller to ensure that there is smooth entry of the flow into the impeller, then the impeller itself which is the rotor of the centrifugal compressor, inducer is often attached to the impeller and it forms the initial part of the impeller itself. Of course, in some of the older generation engines and compressors, inducer was sometimes kept as a separate component and not necessarily a part of the impeller itself.

Impeller flow gets discharged into a vane less space, which is where the diffusion continues from the impeller, it then continues in the vane less space; we have seen the physics behind that. And then, the flow proceeds into the diffuser which could be either vane type diffuser or it could be pipe diffuser, channel diffuser and so on. From the diffuser the flow is collected through what are known as collectors of volute, and then it is discharged from the compressor.

So, these are different components which constitute a centrifugal compressor, we have seen the flow, as it passes through these different components how we can calculate different parameters associated with the flow, so these were some of the things we had discussed in the first lecture which was on during lecture number 31. In the previous lecture, we continued our discussion on this, and many more advanced concepts like the carioles acceleration and what its significance case in reference to centrifugal compressors.

We have seen that carioles acceleration basically leads to a tangential variation in the relative velocity, and at the exit of the impeller this would mean that, there is what is known as a slip. Slip is basically referring to a difference between the tangential components of the absolute velocity to the tip blade speed, that is, c w 2 by u 2. And so, slip factor basically affects the performance in terms of pressure rise of the centrifugal compressor.

We have seen that slip factor is a strong function of the number of blades; therefore, there are empirical correlations from which one can calculate slip factor for different types of impellers. And subsequently, we discussed in detail about the performance characteristics of centrifugal compressor. We have seen that is very similar to that of axial compressors, and we spent some time discussing about the choking aspect of centrifugal compressor and also the fact that choking is different in different components, that is, the way choking is calculated in the impeller or the that is the rotating components or the stationery components like the inlet or the diffuser is quite different.

And also, the fact that one can continue to operate the engine at a higher mass flow than the choking mass flow. If we can increase the speed of the impeller and also the fact that this does not lead to choking of the other components, that is, as long as all the other components are not choked, for a given operating condition if the compressor is operating under choked conditions, if we increase the rotational speed we can actually operated a slightly higher mass flow rates at least theoretically.

These were the different aspects that we had discussed in the last two lectures, and therefore, it is about time that we have a tutorial session, and I think I also mentioned why we are not emphasizing too much on the centrifugal or the radial flow machines like centrifugal compressors or you will see shortly about radial turbines is the fact that these components or these types of turbo machines, though have been used extensively in the past, in the current day scenario their application is very much limited. And most of the modern day jet engines that we are aware of the larger sized ones, definitely use axial flow turbo machines like the compressors and turbines.

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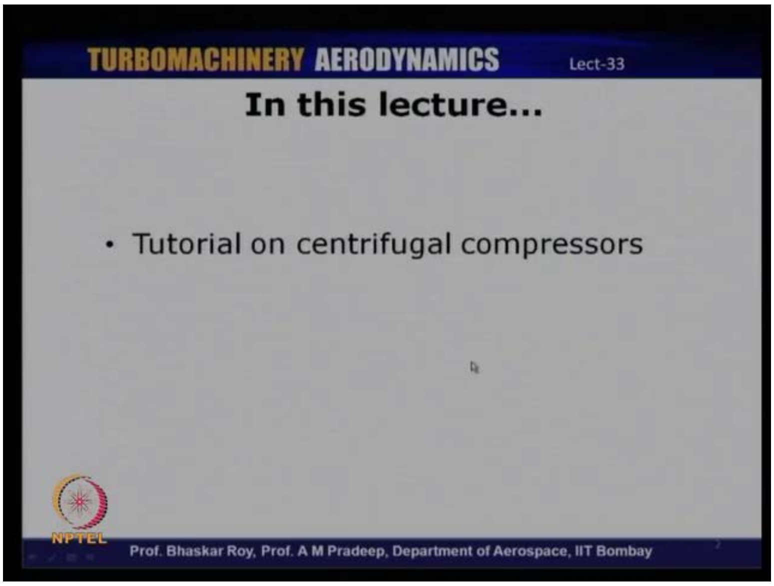

So, today's lecture, we will devote towards centrifugal compressors, basically trying to solve some problems on centrifugal compressors, let us look at basically, have a tutorial session on centrifugal compressor. So, we will have a tutorial on centrifugal compressors, let us take a look at the first problem that we have today.

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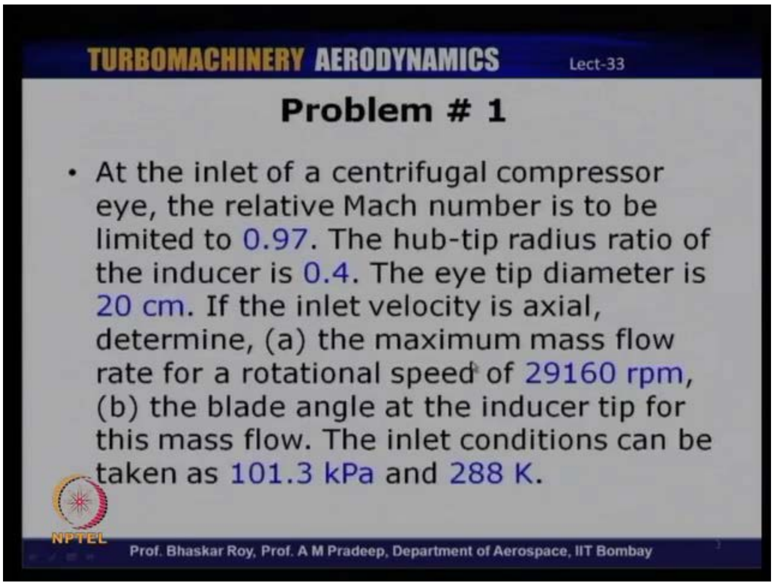

So, the first problem statement is the following, at the inlet of a centrifugal compressor eye, the relative Mach number is to be limited to 0.97. The hub to tip radius ratio of the inducer is 0.4. The eye tip diameter is 20 centimeter and if the inlet velocity is axial, determine, part a - the maximum mass flow rate for a rotational speed of 29160 rpm, part b - the blade angle at the inducer tip for this mass flow. The inlet conditions can be taken as 101.3 kilopascals and 288 kelvin.

So, let us read this question little more carefully what is basically the problem statement referring to, it tells us that there is a relative Mach number at the inlet eye and that is limited to 0.97, that is, we have to keep the Mach number limited to 0.97 and not let it attain supersonic speeds. The hub to tip ratio is given as 0.4, the eye tip diameter is given as 20 centimeter, then we have the blade rotational speed, we need to calculate maximum mass flow and the blade angle.

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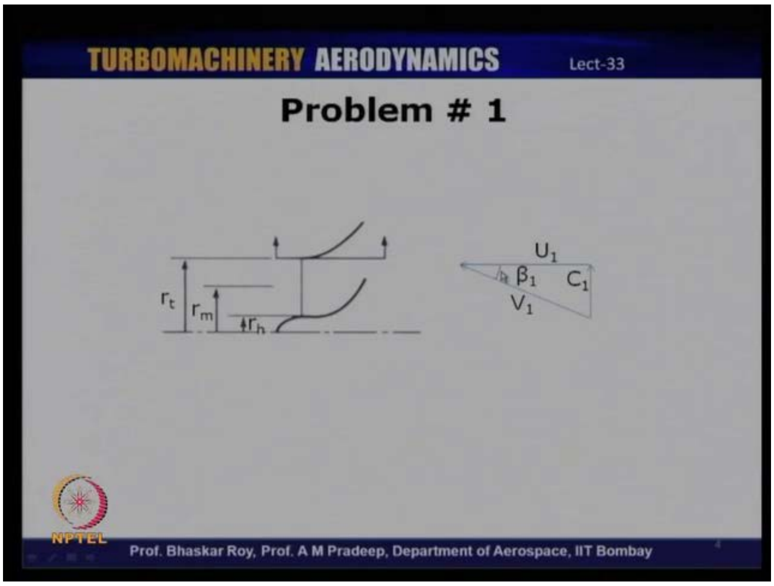

So, let us take a look at the schematic of this particular problem, we have this hub to tip ratio that is been given to us, the radius ratio is 0.4 which means that r h by r t is 0.4. The inlet velocity is given as axial, so C 1 is axial, and then the blade angle beta 1 is one of the parameters we need to calculate, since U 1 is also specified, because we know the tip diameter and the rotational speed, so from there we can calculate the blade angle beta 1.

So, this basically is a problem which refers to a typical centrifugal compressor problem, which has the flow entering the inducer axially, in this case, the flow is indeed axial, and that is one of the parameters which has been given. Rotational speed is given some of the geometrical dimensions with reference to the inducer is also given that the tip diameter, the hub to tip ratio etcetera is given to us. And one of the important statements in the problem is that we need to ensure that the Mach number is limited to 0.97 at the inlet. Actually start solving the problem from there onwards, so we know that the Mach number is to be limited to that and from there we can calculate, let us say, the static temperature because inlet stagnation temperature is given, the inlet velocities basically can be calculated.

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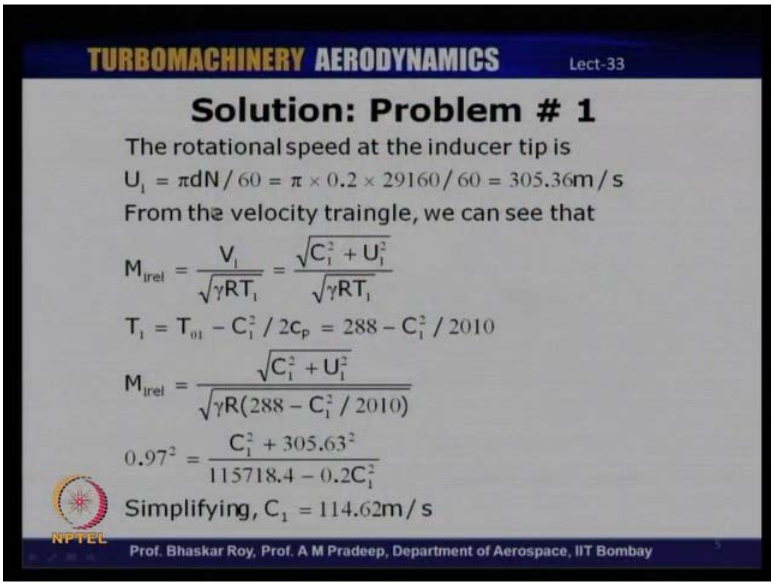

So, if you look at the rotational speed we have been given that the rotational speed is 29160 rpm, diameter is 0.2 - 20 centimeters, so we have phi N by 60 this is 305.36 meters per second. Now, from the velocity triangle we can see that the relative Mach number is basically a relative velocity by square root of gamma R T 1, and at the inlet V 1 is equal to square root of C 1 square plus U 1 square, this divided by square root of gamma R T 1.

So, we also know that T1 is equal to T 01 minus C 1 square by 2 Cp and T 01 is given as 288, C 1 of course is not known, C p we will assume for air as 1005 Joules per kilogram Kelvin and therefore 2 Cp is 2010. So, here we have two equations, one is for Mach number and the other is for static temperature. So, let us substitute this in this equation, relative Mach number become square root of C 1 square plus U 1 square divided by square root of gamma R into the temperature 288 minus C 1 square by 2010. Right hand side involves an a unknown that is C 1 and left hand side is already given that we have to limit the Mach number to 0.97.

So, if you substitute all the values we have 0.97 square is equal to C 1 square plus 305.63 square divide by gamma R into 288 that is 115718.4 minus 0.2 C1 square. So, if we simplify this, it is a simple quadratic equation which we can simplify, and then we can get C 1 is 114.62 meters per second.

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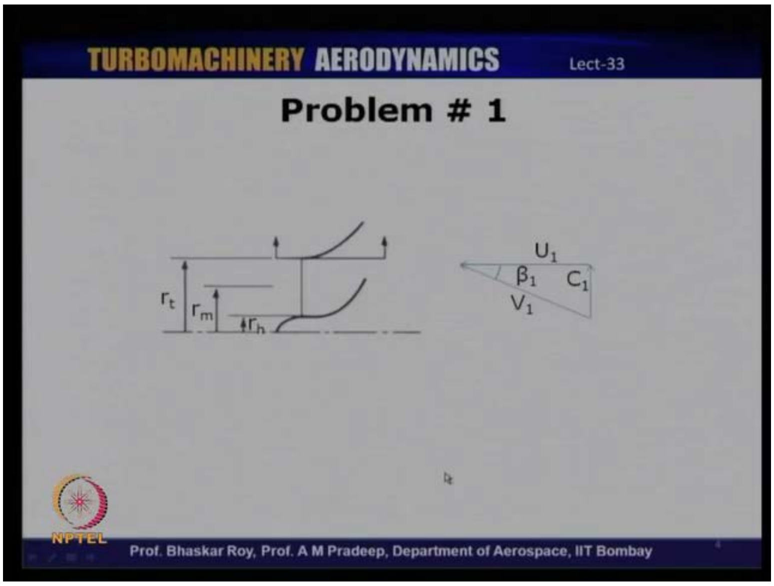

So, if we referred to the velocity triangle we have basically determined C 1, and since U 1 is known you can now calculate beta 1, because it is basically the tan of U 1 and C 1, of course, the first part of the question is also to find the mass flow rate.

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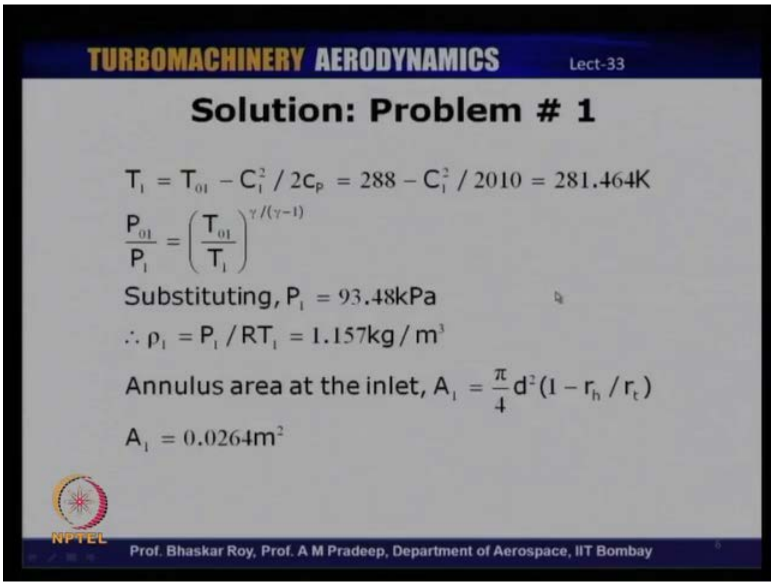

So, let us calculate the mass flow rate, for calculating mass flow rate we need the density, axial velocity is known, we also need the area. Now, the static temperature, you can calculate here because, the stagnation temperature is given, C 1 we have just now calculated, and therefore, static temperature would be 288 minus C 1 square by 2 Cp, C 1 square we have calculated and therefore, it becomes 281.46. And then we have, since no efficiency has actually been specified, we can use isentropic relations here to calculate the corresponding static pressure, because stagnation pressure has been specified, stagnation temperature and static temperature are known. So, if we substitute these values here, we can calculate the static pressure P 1 which is 93.48 kilopascals. Now, once we calculate the pressure and temperature, we can calculate the density which is P 1 by R T1 and that is 1.157 kilograms per meter cube.

So, this is one of the parameters which we need for calculating mass flow rate, the other parameter is the area of a passage of the mass flow rate and that is the annulus area at the inlet for which we have been given the tip diameter and the hub to tip ratio. So, the annulus area would be phi square by 4 into 1 minus r h by r t. So, that is 0.0264 meter square, so this is the annulus area through which the mass flow rate passes, and we have calculated density. Now the third parameter for calculating mass flow rate is the velocity and for mass flow rate we need to known the axial velocity, but at the inlet we know that the absolute velocity itself is axial and therefore, C a1 should be equal to C 1.

So, mass flow rate is quite straight forward, now we just multiply density, annulus area at inlet and the axial velocity, so rho 1 A 1 and C 1, the product of these three would give us the mass flow rate. And so we multiply the density area and absolute velocity, we get the mass flow rate. Then, the second parameter is the blade angle, so how does one calculate the blade angle? Blade angle at the inlet is very straight forward again, because we have already calculated C 1 and we know the blade speed U. So, tan inverse of C by U C 1 by U 1 would give us the angle at the inlet that is beta 1. So, if we substitute those values here, we get mass flow rate as rho 1 A 1 C 1 which is 1.157 into 0.0264 which is annulus area, C 1 is 114.62 meters per second.

So, the product of all the three will give us the mass flow rate which comes out to be 3.5 kilograms per second. Blade angle at the inlet is at the tip is tan beta 1 which is C 1 by 1 therefore, beta 1 is tan inverse C 1 by 1 which comes out to be 20.57 degrees. So, these are two parameters that we have calculated the mass flow rate, which was the first part of the question; second part was for the same mass flow what is the blade angle, so we have also calculated the blade angle for this question.

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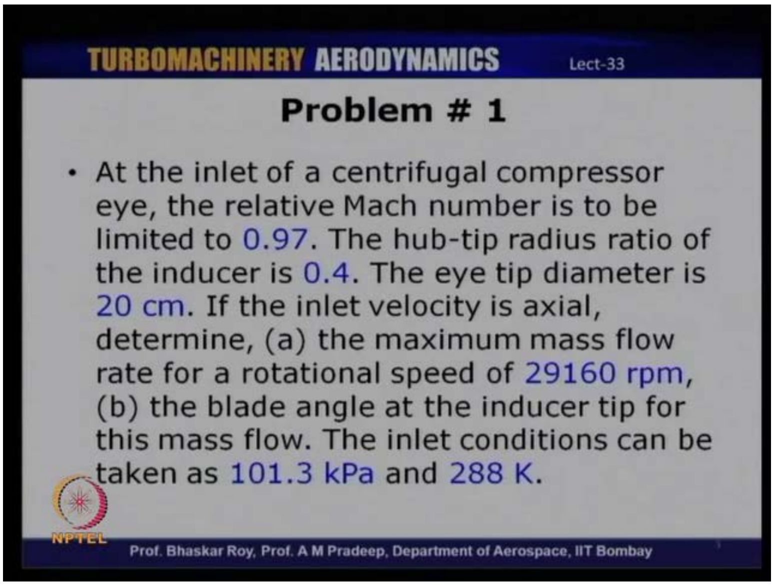

I think, I have mentioned this several times in the past that the key to solving problems to do with turbo machines, the fundamental problems to do with turbo machines is to get the velocity triangles. So, even for centrifugal compressors as we have seen, velocity triangle is the starting point for any of this analysis, so if you start solving the problem once you get the velocity triangle right. And so if the velocity triangle can be set right, then solving the problem is quite easy, because you know what to calculate and how to calculate them.

And so, I would urge you to keep this in mind, whenever you are solving a problem irrespective of whether its axial compressors, axial turbine, centrifugal or radial turbines, whatever be the problem that you are solving, it is very important that you understand the significance of velocity triangles, and that is why you need to understand the physics very well to be able to construct the velocity triangle and that would basically be the starting point for solving any of these problems.

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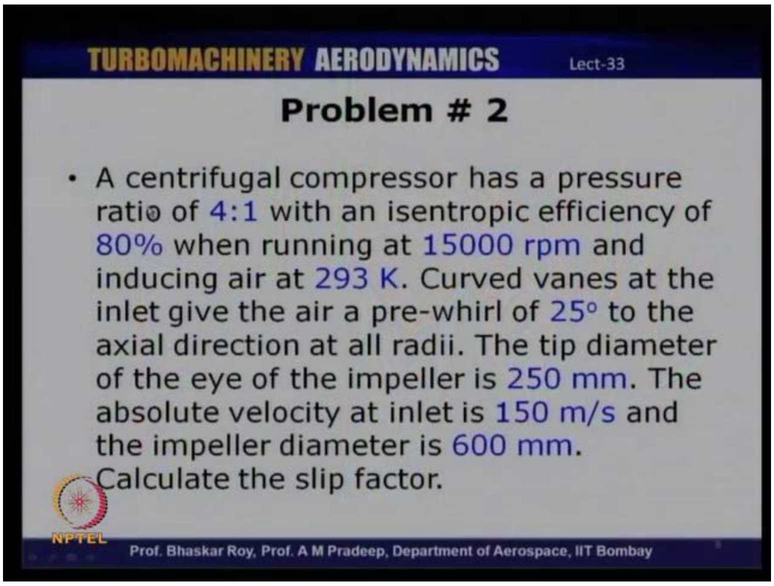

So, let us move on to the next problem now and let us see what this problem statement is, second problem statement is that a centrifugal compressor has a pressure ratio of 4:1 with an isentropic efficiency of 80 percent when running at 15000 rpm and inducing air at 293 Kelvin. Curved vanes at the inlet give the air a pre-whirl of 25 degrees to the axial direction at already eye. The tip diameter of the eye of the impeller is 250 millimeters. The absolute velocity at inlet is 150 meters per second and the impeller diameter is 600 millimeters. Calculate the slip factor.

So, this is in fact, follow up question for this will also for will be given little later, we will also solve that problems is quite similar to this, but with a small twist. So, in this case, the question state that we have a centrifugal compressor which is developing a pressure ratio of 4 : 1 with a certain efficiency and rotational speed. What is to be noted here is that the air is no longer entering axially, unlike the previous question where it was specifically mentioned that the inlet air is axial, here it is not axial it is coming with a pre-whirl, that is, the pre-whirl is like a guide vane ahead of the inducer where the guide vane set the fluid at a certain angle. In this case, the angle has been set at 25 degrees and then we have been given some dimensions of the impeller and some speeds.

So, based on this we need to calculate the slip factor, so slip factor has as we have learnt in the previous lecture, is basically the ratio of the tangential absolute velocity at the impeller exit divided by the corresponding blade speed at the same location.

So, it is C w2 by U 2 which is the slip factor, we are required to find this. Now, since our rotational speed is given and the diameter of the impeller is given, finding the blade speed at the tip is very easy, because U 2 is phi D 2 and by 60, and the D and N both are given to us, so you can calculate the impeller diameter tip speed. But how do you calculate C w2, this of course, will require us to solve the velocity triangle first at the inlet and subsequently at the exit as well to be able to solve this problem.

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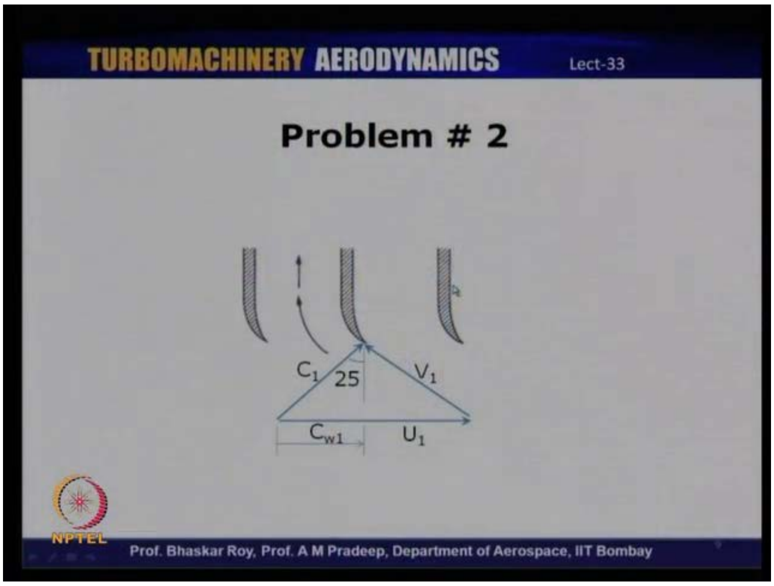

So, let us look at the velocity triangle at the inlet of the inducer, so these are the inducer vanes which are shown and you can see that the flow is not entering the inducer and axial direction. Is in fact, entering at an angle of 25 degrees; that means, that C 1 is not axial, C 1 itself has a tangential components C w1, then the inlet relative velocity is V 1 and the blade speed at the inlet is U 1. So, this is which means that there would be a set of vanes, which will set this flow angle of 25 degrees to ensure that the absolute velocity enters the inducer in this direction and that is called the pre whirl, you can see this word mentioned here it is called pre whirling.

Pre whirl means that there is a whirl in component, which is basically the tangential component of absolute velocity and that is why it is called a pre whirl. In the absence of pre whirl, the flow would have entered axially, but that may not really be an ideal condition for the inducer, because the relative velocity might actually enter the inducer at a different angle of course, this has been designed for a pre whirl.

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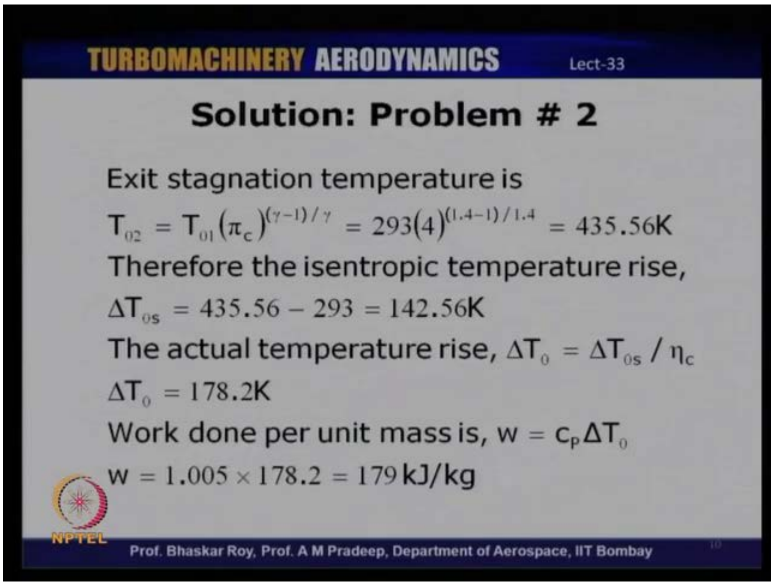

So, let us begin to solve this problem we have the pressure ratio given to us and therefore we can calculate the exit stagnation temperature, exit stagnation temperature is equal to T 01 times the pressure ratio raise to gamma minus 1 by gamma. And therefore, we get 293 into 4 raise to 1.4 minus 1 by 1.4 that is 435.56 Kelvin. So, the isentropic temperature rise would be 435.56 minus T 01 that is 293, we get 142.56 Kelvin, but the actual temperature rise is, isentropic temperature rise divided by the efficiency, which in this case is specified as 80 percent, and therefore, we get delta T not is equal to 1178.2 Kelvin which is 142.56 divided by 0.8.

Therefore, from the temperature rise we can calculate the work done per unit mass, and work done can be calculated in two ways, as we have seen in the past, one is by using the temperature rise and the other method is by using velocity triangles that is U, delta U, C w, will basically give us the work done per unit mass. So, here we know since we know the temperature rise, we can calculate the work done per unit mass, which is basically the enthalpy rise in the compressor and that is C p times delta T not, delta t not we have calculated as 178, this multiplied by C p will give us the work done per unit mass. So, work done here would be C p which is 1.005 into delta T not, delta T not is 178.2, therefore, we get 179 kilo joules per kilo gram.

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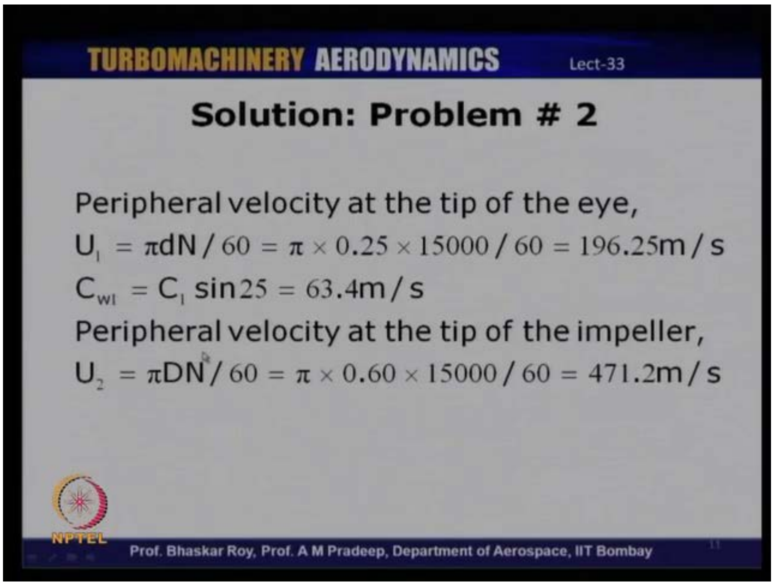

Now, at the exit, we can now calculate some parameters at the exit, because our aim is to calculate the slip factor at the exit and but of course, before that we need to calculate C w1, because that is required for calculating the exit parameters as well. Now, at the inlet of the eye we have U 1 is phi d N by 60 that is phi into the a tip of the eye of the impeller is 0.25 meters into 15000 divided by 60 and that comes out to be 196.25 meters per second. C w1 is C 1 times sin 25, C 1 is given to us and therefore, and also the angle is given alpha, C 1 sine alpha 1 therefore, that comes out to be 63.4 meters per second.

Now, peripheral velocity at the tip of the impeller U 2 would be phi into capital D which is the tip diameter of the impeller that is given as 600 millimeters multiplied by N by 60, so phi into 0.6 into 1500 divided by 60 this is 471.2 meters per second. So, here we have calculated the conditions at the inlet which refers to the C w1, and why we are calculating C w1 is because, we have just now calculated the work done per unit mass, and work done per unit mass is also equal to U 1, well U 2 C w2 minus U 1 C w1, and from there U 1 and U 2 are known C w1 we have just now calculated, and therefore we can calculate C w2. Once you calculate C w2, ratio of C w2 to the blade speed at the tip that is U 2 will give us the slip factor.

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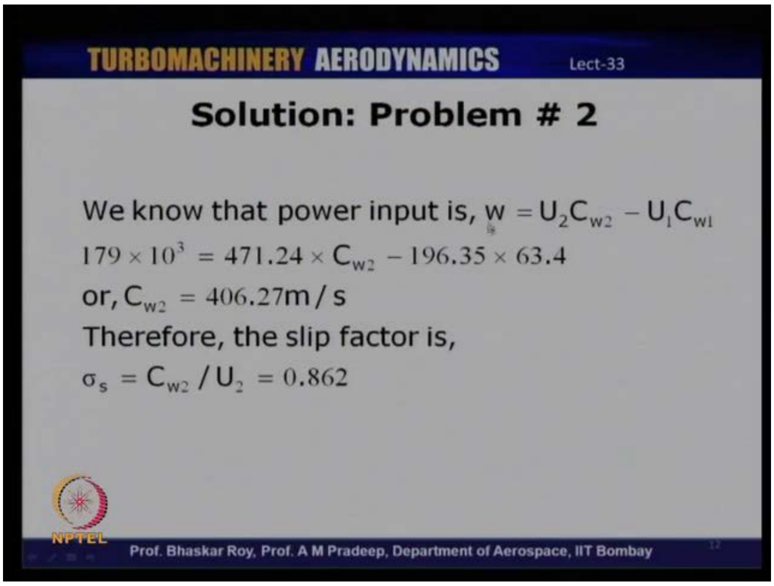

So, let us now calculate C w2 which is the whirl velocity at the tip of the impeller, and since the power work done per unit mass is also equal to U 2 C w2 minus U 1 C w1 and this is equal to work done. We have just calculated earlier 179 kilojoules per kilogram; this is equal to U 2 times C w2 that is 471.24 into C w2 minus U 1 C w1 that is 196.35 into 63.4.

So, C w2, we can calculate as 406.27, and therefore, we calculate slip factor as ratio of C w2 to U 2 and therefore, 406.27 divided by 471.24 that is 0.862. So, here we get a slip factor of 0.86. What are the implications of this well, the implications depends upon the kind of application we have, but the basic implication of having a slip factor much lower than 1, is that it is leads to lower and lower pressure ratios, that is higher the slip factor lower is a pressure ratio for which it is actually been design for.

This is because the tangential velocity at the exit that is basically the swirl velocity at the exit is directly related to the pressure ratio; you can actually derive an expression which relates the pressure rise P O2 by P 01 in terms of C w and U and so on. Therefore, there is a direct correlation between that, so lower the slip factor which means lower is C w2 is comparison T U 2, and therefore, that will affect the pressure rise achieved in a centrifugal compressor.

That is why modern day designers would like to maximize, increase the value of slip factor to keep it as close as possible to 1. We have seen the strong dependence of slip factor on the number of the blades, and that is one of the key optimization challenges because, one can keep increasing number of blades, but then the problem with that would be the fact that increasing number of blades also leads to an increase in these king friction losses, and therefore, that is going to affect the efficiency in some way. The designer does not want to have a poor efficiency with the higher pressure ratio that does not make sense.

So, one need to have a choice of basically a mix of high efficiency as well as higher pressure ratio, and that requires the very intelligent way of trying to optimize this case, where we have on hand one option of increasing number of blades with the risk of in lower efficiency because of losses. Other option is to reduce number of blades and have higher law efficiency, possible higher efficiency because of lower frictional losses, but one may end up with the lower pressure ratio, so there is a trade of required to attain an optimum condition. So, let us now again look at another problem, third problem, also involves slip, let us take a look at what the problem statement is.

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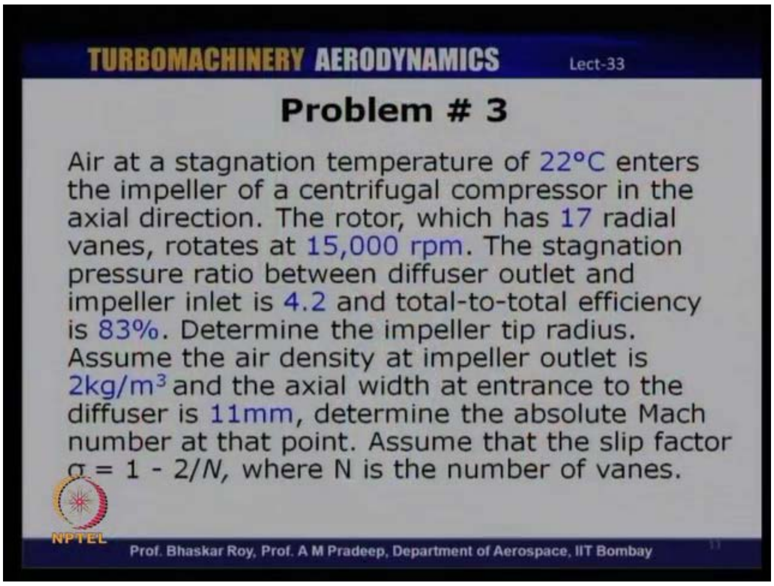

The problem number 3 states that air at a stagnation temperature of 22 degree Celsius enters the impeller of a centrifugal compressor in the axial direction. The rotor, which has 17 radial vanes, rotates at 15000 rpm. The stagnation pressure ratio between the diffuser outlet and the impeller inlet is 4.2 and the total to total efficiency is 0.83 percent or 0.83. Determine the impeller tip radius, assuming that the air density at the impeller outlet is 2 kilograms per meter cube and the axial width at the entrance to the diffuser is 11 millimeters, you also need to determine the absolute Mach number at this point.

We assume that the slip factor is 1 minus 2 by N, where N is the number of vanes. So, we have been permitted to use this standards slip factor, which states that the slip factor is equal to 1 minus 2 by N, N being the number of vanes. So, in this question we have been given the number of blades here 17 radial vanes, rotational speed 15000 rpm and the stagnation pressure ratio 4.2, total to total efficiency 83 percent and then, the density at the impeller outlet and the axial width at the entrance of the diffuser.

So, there are the parameters which are given to us based on which we need to calculate two parameters, one is the impeller tip radius, and the second is, we also need to calculate the Mach number at the exit of the impeller, so we need to of course, calculate the absolute Mach number. So, this is the question which is very similar to one of the questions we have solved earlier, so I will skip the velocity triangle part leaving that to you to figure out the velocity triangle and triangle construct the velocity triangle for that kids.

So, this question involves or requires us to calculate two parameters, one of course the hint given to us is that we can calculate slip factor using the Steinitz formula, they have been giving the number of blades. So, we can actually calculate the slip factor from there, and then we know the pressure ratio and total to total efficiency, and so we can actually calculate the power required from what data has been given to us, the other key information is that the flow enters the impeller in the axial direction, which means that there is no tangential component to the absolute velocity at the inlet. So, C w1 should be 0, and so you can calculate work done based on just U 2 times C w2.

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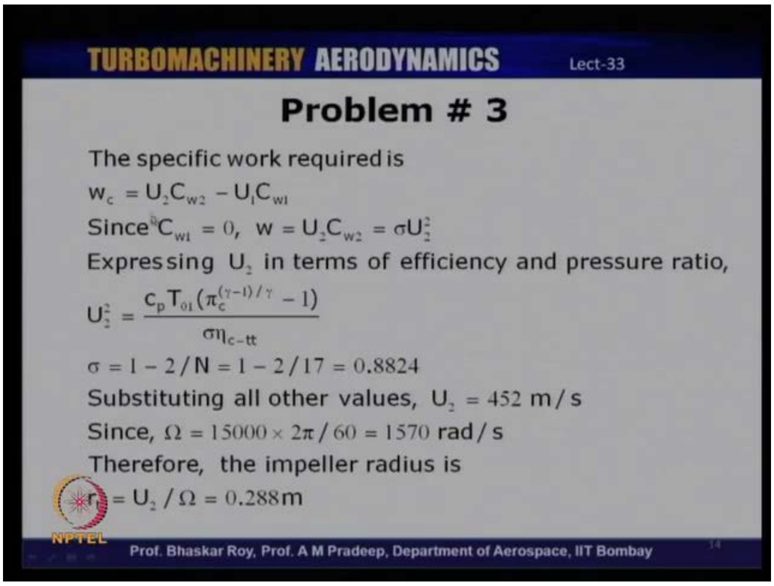

So, the specific work required is U 2 C w2 minus U 1 C w1, C w1 is 0 and therefore, work is U 2 C w2 which is also equal to sigma times U 2 square, because sigma is C w2 by U 2 and so we can express work done in terms of sigma, which is the slip factor and the blade speed at the tip of the impeller U 2, so sigma U 2 square would be the work done - specific work done.

So, we can now express the efficiency in terms of well, U 2 in terms of efficiency and pressure ratio, this comes from the total to total efficiency we have already defined that earlier on with the context of a turbine, so this similar aspect we can also apply for this case. So, when we use the efficiency definition, we have the outlet temperature T 03 minus T 01; T 03 being the isentropic temperature minus T 01 divided by T 03 minus T 01. And then, the pressure ratio is given to us, so the numerator gets converted can be expressed in terms of the pressure ratio, pressure ratio is been given to us as 4.2 and the denominator which is 3 minus T 01 is basically the work done divided by specific heats. So, you can express that in terms of w times w divided by C p, w is sigma into U 2 square, and therefore, we can express U 2 in terms of these parameters which are known to us the inlet stagnation temperature, sigma, the temperature and pressure ratio as well as the efficiency.

So, if we express U 2 in terms of efficiency and pressure ratio, we have U 2 square is C p T 01 multiplied by phi c raise to gamma minus 1 by gamma minus 1 divided by sigma into efficiency. Efficiency is given as 83 percent and sigma given can be calculated based on the number of blades. So, sigma is 1 minus 2 by N, N is 17, we have 17 numbers of vanes or blades, so sigma the slip factor is 0.8824.

So, we know that temperature, we know pressure ratio and the efficiency; if we substituted that we get the blades speed as 452 meters per second and then omega, because the speed is given to us as a 15000 rpm, based on that we can now calculate the tip radius, because tip radius is U 2 or the tip speed is basically equal to omega into r therefore, r t is equal to U 2 by omega and omega here is the rotational speed in radiance per second 15000 into 2 phi by 60, we get 1570 radiance per second.

So, if you substitute for those 452 divided by 1570, we get the tip radius as 0.288 meters. So, this is the first part of the question where we are required to find tip diameter of the impeller, which in this case comes out by 288 millimeters or 0.288 meters. The second part of the question is to find the Mach number, well, the absolute Mach number at the impeller exit, which means that we have to take the ratio of C 2 to the speed of sound at that location that is a 2. So, C 2 by A 2 will gives us the Mach number at station 2 which is impeller exit. So, we need to now find out the absolute Mach number, we also need to know the temperature - static temperature - at the exit of the impeller to be able to calculate the Mach number.

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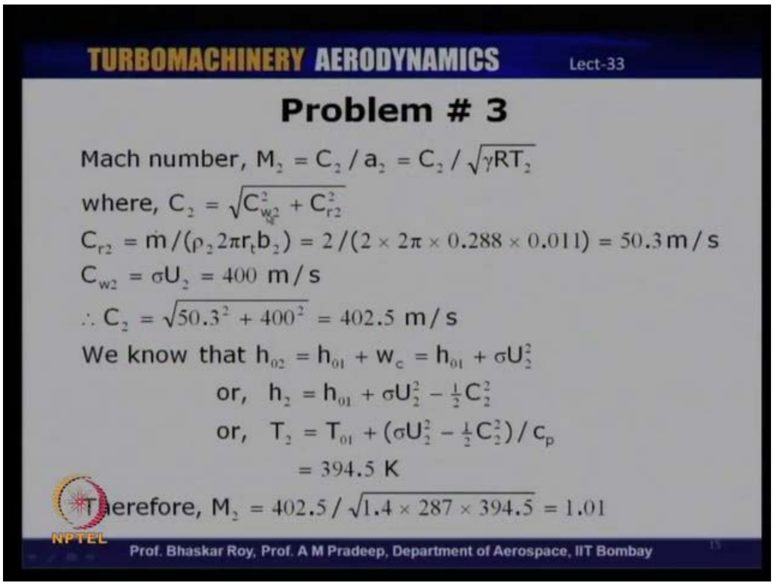

Now, at the exit of the impeller we have C 2, which is root of C w2 square plus C r2 square where C r is the radial velocity, the absolute component of radial velocity, and how do we calculate C r2, C r2 would basically be coming from the mass flow rate, mass flow rate divided by the density times the annulus area. Now, in this case, we have mass flow rate which has been given to us as 2 kilograms per second, density is also given as 2 kilograms per meter cube. Tip radius we have just now calculated 288 millimeters or 0.288 meters, and the axial width is given to us in the question is 11 millimeters, so this 0.011.

So, C r2 which is the radial velocity at the exit of the impeller is mass flow rate divided by density times the annulus area 2 phi into tip radius times the axial width, it gives the annular area, that comes out to be 50.3 meters per second. Similarly, C w2, we can calculate because we know U 2, we also know the slip factor sigma product of those 2 would give us the tangential velocity at the exit C w2, so that is 400 meters per second. So, C 2 is equal to square root of 50.3 squares plus 400 square that is 402.5 meters per second.

So, for calculating Mach number, we now have absolute velocity, we now need to calculate the static temperature as well. So, for calculating the static temperature we will make use of rothalpy, we have already discussed about that earlier on. So, will make use of that concept here, the conservation of rothalpy in the impeller, and based on that we will calculate the static temperature.

So, we know that h 02 which is stagnation temperature at the exit of the impeller would be equal to inlet stagnation temperature plus the work done, that is W c, so h 01 plus W c will be h 02, and therefore, h 01 plus sigma U 2 square, because W c. We know in this case is sigma U 2 square, and therefore, h 2 would be equal to h 01 plus sigma U 2 square minus half C 2 square, because h 2 plus half C 2 square is h 02, this we will convert in terms of temperatures. Since h 2 is equal to h 01 plus sigma U 2 square minus half C 2 square correspondingly in terms of temperature we get T 2 is equal to T 01 plus sigma U 2 square minus half C 2 square divide by C p.

So, we substitute all these values and we get temperature, static temperature at the impeller exit as 394.5 Kelvin, therefore, the Mach number at the exit of the impeller is 402, which is the absolute velocity divided by square root of 1.4 into r 287 into 394.5, Mach number comes out be 1.01. So, we can see that the absolute Mach number is just about sonic, it has just crossed the sonic Mach number, and in fact, in the relative frame of reference the Mach number can in fact be higher than what you have calculated for the absolute case here.

So, the third problem we just now solved concerned was basically about calculating, Firstly the Mach number we have seen how to calculate the Mach number as well as how do you calculate, based on the parameters like the slip factor in this case to be approximated as 1 minus 2 by N. We can also calculate the mass flow rate as we have done in this question. Now, let us take up one more problem which is in some sense identical to what we have solved in the second question and partly also identical to what we have just now solved for the third question, so we will require understanding of how those two problems were solved to be able to solve this question. So, we will be calculating the slip factor as well as the Mach number in this question that we were going to solve.

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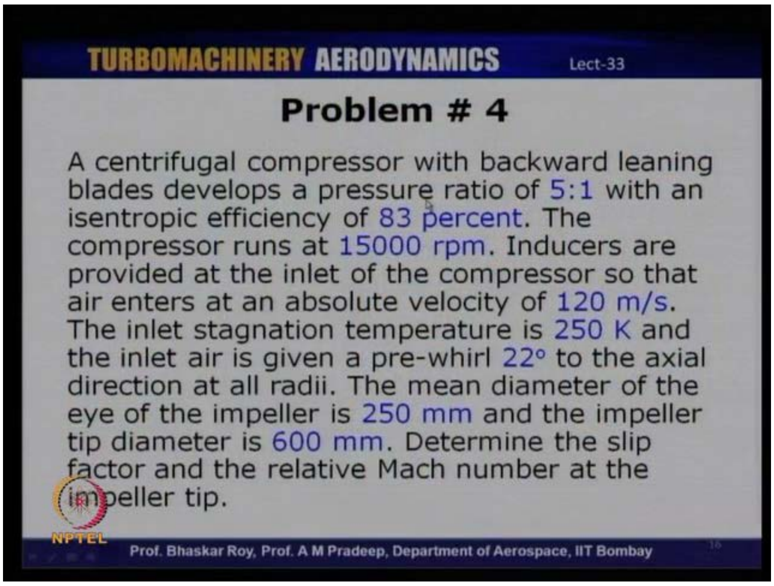

Let us take a look at the problem statement for this question, a centrifugal compressor with a backward leaning blades develops a pressure ratio of 5:1 with an isentropic efficiency of 83 percent. The compressor runs at 15000 rpm. Inducers are provided at the inlet of the compressor. So, that air enters at an absolute velocity of 120 meters per second. The inlet stagnation temperature is 250 Kelvin and the inlet air is given a pre whirl of 22 to the axial direction at all radii. The mean diameter of the eye of the impeller is 250 millimeters and the impeller tip diameter is 600 millimeters. Determine the slip factor and the relative Mach number at the impeller tip.

So, you can immediately see that it has components of both the second question as well as third question, second question we actually calculated the slip factor, and in the third question we calculated the absolute Mach number of course, in this case we are required to calculate the a relative Mach number.

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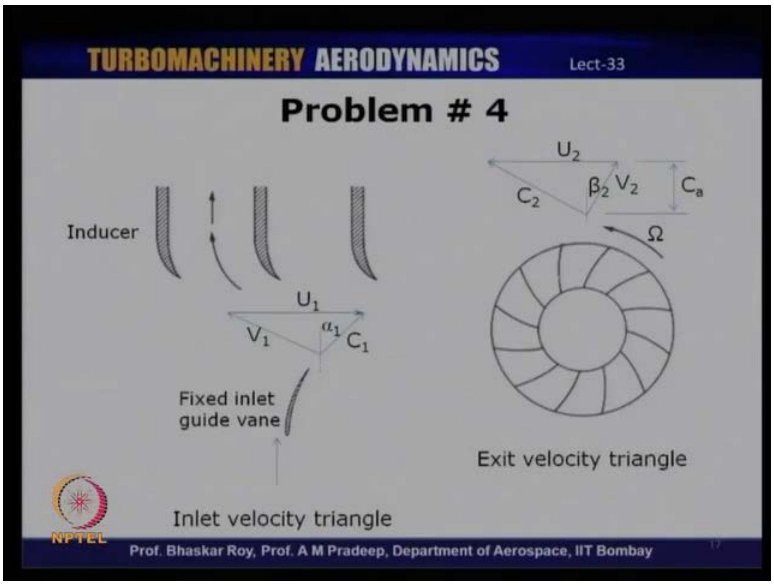

Let us look at the velocity triangles first, both at the inlet as well as the exit, this is the velocity triangle that you should get at the inlet and you have the schematic of the inducer and also fixed inlet guide when which gives a pre whirl to the inlet flow which is entering the inducer.

So, these guide vanes would which are set at an angle of alpha 1 gives a pre-whirl which is given in this case as 22 degrees causing the absolute velocity not to be axial, and therefore, the absolute velocity C 1 is not axial unlike some of the questions we have solved earlier on, and that causes the velocity triangle at the inlet to look like what is shown here, this is C 1 the relative velocity V 1 and the blades speed V 1. At the exit on the other hand, these are backward leaning blades, and so the velocity triangle for a backward leaning blade should look like this. Since the blades are leaning like this, the relative velocity leaves the blades in this direction with an angle of beta 2, and this is the absolute velocity C 2 and the blade speed U 2 at the tip of the impeller, the axial velocity component is shown here as C a and the rotational speed is omega which is given in this case as 15000 rpm. Let us first try to solve the inducer part of the question and then, we will move towards solving the second part.

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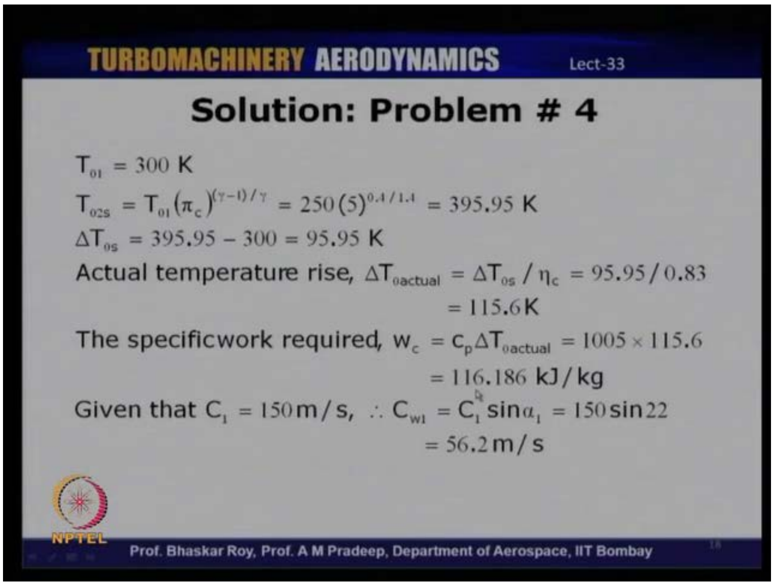

Now, the inlet temperature is given as 300 Kelvin and therefore, based on the pressure ratio and isentropic relation we can calculated the exit stagnation temperature isentropic and that is T 02 s, which is equal to T 01 into phi C raise to gamma minus 1 by gamma and this is 250 multiplied by 5 raise to 0.4 by 1.4, that is, gamma minus 1 is 0.4 divided by 1.4, this basically 395.95 Kelvin. Therefore, the stagnation temperature isentropic is 395.95 minus 300 that is 95.95 Kelvin, but the actual temperature rise is this divided by the efficiency, in this case, efficiency is given as 0.83 and therefore, T 0 s actual basically would be equal to this divided by efficiency that is 95.95 divided by 0.83 that is 115.6 Kelvin.

So, the specific work required would be C p times delta t naught that is 1005 into 115.6, this is 116.886 kilojoules per kilogram, now since it is given that C 1 is 150 we can calculate C w1 which is the whirl component of the absolute velocity that is C 1 times sine alpha 1, alpha 1 is 22.

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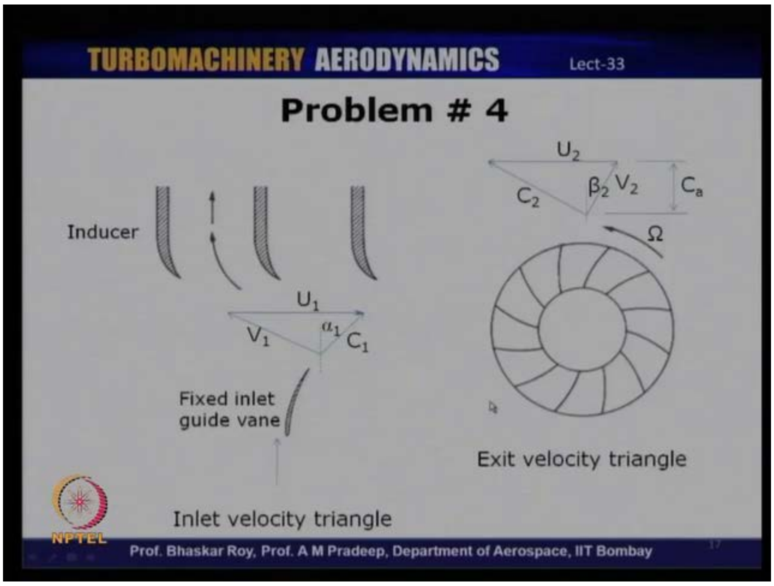

Let me go back to the velocity triangle here, C 1 times sine alpha 1 would give us C w1 which is this component, so that comes out to be 56.2. Now, here what we going to do is, since we know the specific work done from the temperature rise, we also know that specific work done is equal to the product of U 2 times C w2 minus U 1 times C w1. C w1 we have calculated, we can now calculate U 1 and U 2, therefore, we get C w2 and once we know C w2 and we can calculate the slip factor which is C w2 by U 2.

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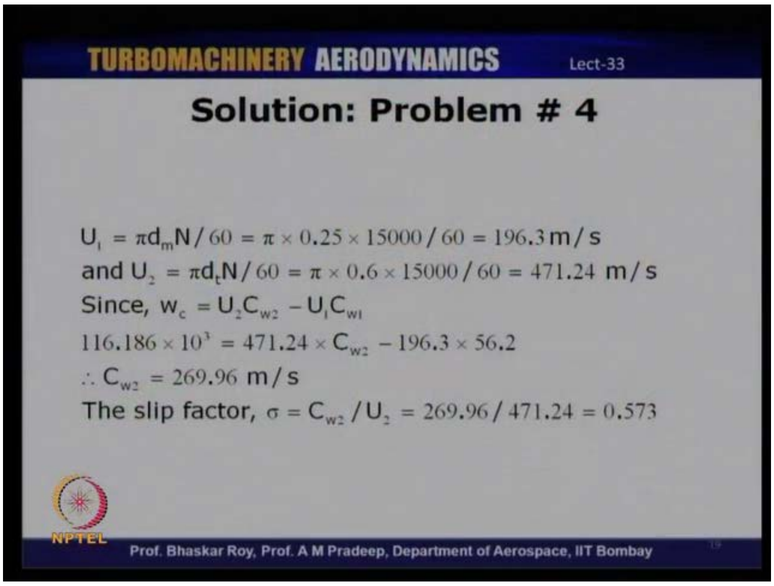

So, let us first calculate U 1 and U 2, U 1 is phi mean of the eye of the impeller into the rotational speed by 60 that is given as phi into 0.25 into 15000 by 60, this is 196.3 meters per second. And U 2 is the blade speed at the tip of the impeller, phi t into N by 60 that is phi into 0.6 into 15000 by 60 is 471.24 meters per second. Now, specific work done as we have seen, we calculated that already from the previous calculations specific work is C p delta T not that is 116.186, specific work is 116.186 into 10 raise to 3 is equal U 2 which is 471.24 into C w2 minus U 1 which is 196.3 into C w1 56.2.

So, from this we can calculate C w2 that is 269.6 meters per second, therefore, slip factor is the ratio of this C w2 divided by U 2 that is 269.96 divided by 471.24 that is 0.573. We can see that the slip factor is a very low number here, one would normally expect relatively higher slip factor of the order of 0.7, 0.8 or even higher than that, this is the very low slip factor. And we have already discussed the disadvantages of having very low by use of slip factor, basically affecting the pressure rise of the centrifugal compressor for a given rotational speed. So, the next part of the question is to calculate the Mach number at the tip in the relative frame.

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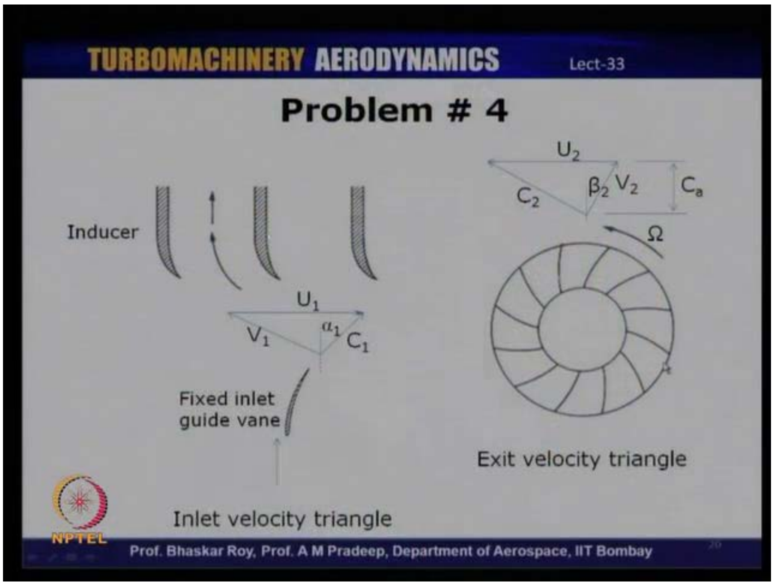

So, the relative Mach number at the tip of the compiler for which we will refer to the velocity triangles once again, let us look at just the velocity exit velocity triangle, for the backward leaning blade we have seen that the velocity triangle would look like this. We have the absolute velocity C 2 and the relative V 2 which is leaving the blades tangentially, and so this is however, the exit velocity triangle would look like as we need to calculate V 2 and also the temperature at the exit of the impeller to calculate the Mach number.

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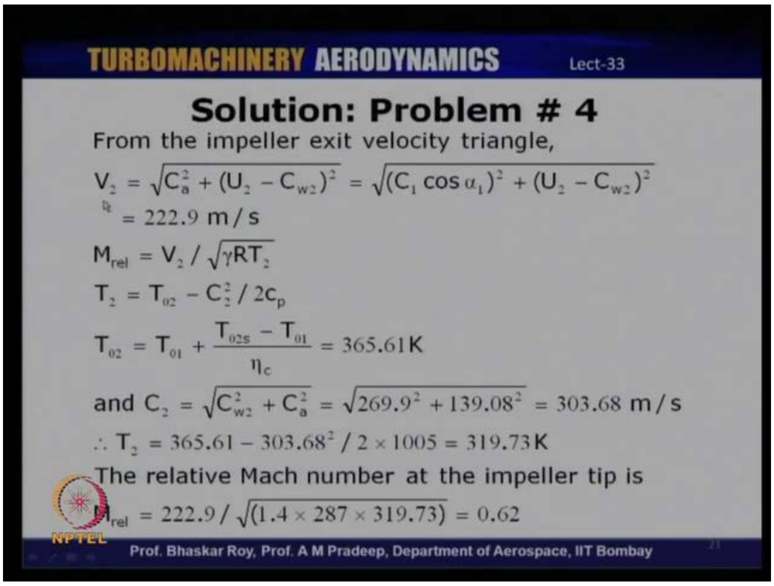

So, from the velocity triangle we see that V 2 is equal to square root of C a square plus U 2 minus C w2 the whole square. So, this C a at the inlet of course, we are assuming that axial velocity does not really change, as it passes through this impeller, this is equal to C a square that is C 1 cos alpha 1 the whole square minus, well, plus U 2 minus C w2 the whole square.

Since all these parameters are known we substituted that we get the relative velocity 222.9 meters per second. And then, we need to also calculate the static temperature at the tip, T 2 which is T 02 minus C 2 square by 2 C p, T 02 we can calculate because, we know the efficiency and the pressure ratio. So, from there we calculate T 02 which is T 01 plus T 02 s minus T 01 by efficiency and that comes out to be 365.61.

And we also need to calculate C 2, from the velocity triangle we see that C 2 is C 2 square is equal to the this component C w2 square plus C a square. So, C w2 we have already calculated that is 269.9, that is, square plus C a square which we can calculate from this C 1 cos alpha 1 that is 139.08 square.

So, C 2 comes out to be 303.68 meters per second, therefore the static temperature is equal to 365.61 minus C 2 square by 2 C p, therefore T 2 comes out to be 319.73 Kelvin. Therefore, the relative Mach number is, the relative velocity divided by square root of gamma R T 2, and this if we substitute we get relative Mach number of 0.62. So, relative Mach number at the impeller tip in this case, is calculated as 0.62. So, this completes four problems that we have solved in today class, we will started off with a very simple problem which just involved solving the velocity triangle and calculating the velocities and also the angles involved.

Second question was to calculate basically the slip, third question involved calculating the Mach number in the absolute frame and the last question was combination of the second and third calculating the slip factor as well as the Mach number at the tip. So, these were four question that we have solved in today lecture, I now have a few exercise problems which you can take up and solve based on our discussion today as well as what we have discussed during the lectures.

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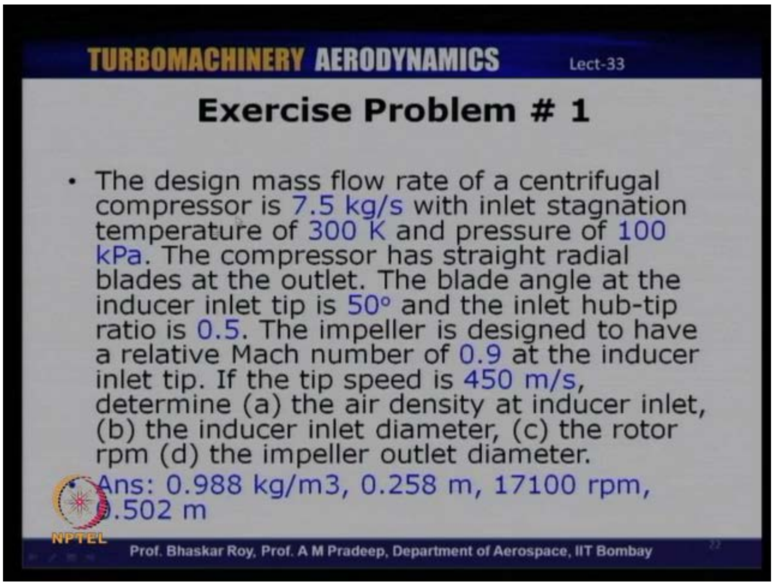

Let us take a look at the first exercise problem, the first exercise problem states that the design mass flow rate of a centrifugal compressor is 7.5 kilograms per second with inlet stagnation temperature of 300 Kelvin and pressure of 100 kilopascal. The compressor has straight radial blades at the outlet, the blade angle at the inducer inlet tip is 50 degrees and the inlet hub to tip ratio is 0.5, the impeller is designed to have a relative Mach number of 0.9 at the inducer inlet tip.

If the tip speed is 450 meters per second, determine part a -the air density at inducer inlet, part b - inducer inlet diameter, part c - the rotor rpm and part d - the impeller outlet diameter. The answers to these four different parts of the questions are density should come out to be 0.988 kilograms per meter cube, inducer inlet diameter is 0.258, rotor rpm is 17100 rpm and impeller outlet diameter is 0.502 meters.

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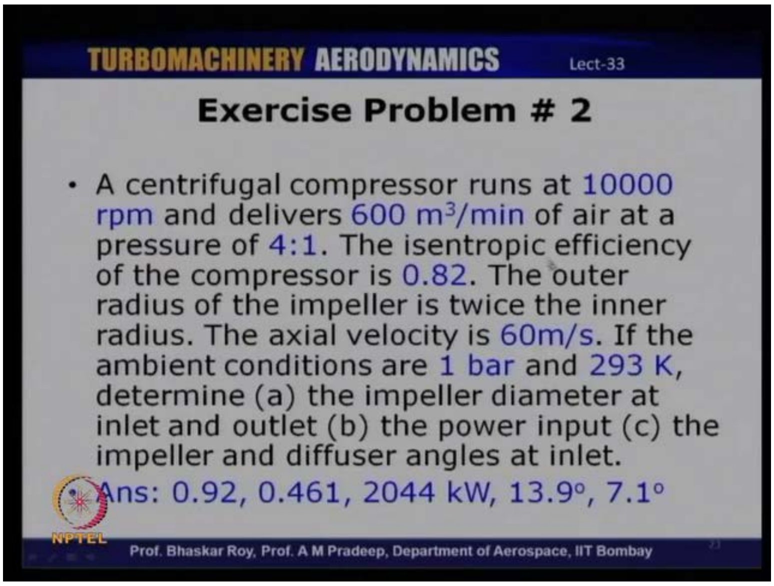

The second exercise question is, a centrifugal compressor runs at 10000 rpm and delivers 600 meter cube per minute of air at a pressure ratio of 4 is to 1. The isentropic efficiency of the compressor is 0.82. The outlet radius of the impeller is twice the inner radius. The axial velocity is 60 meters per second if the ambient conditions are 1 bar and 293 Kelvin, determine part a - the impeller diameter at inlet and outlet the power input, and the impeller angles at the inlet. The answer to the questions are impeller diameter at the inlet should be 0.92 meters and outlet 0.461, and the power input is 2044 kilowatts impeller and diffuser angles at the inlet are 13.9 degrees and 7.1 degrees.

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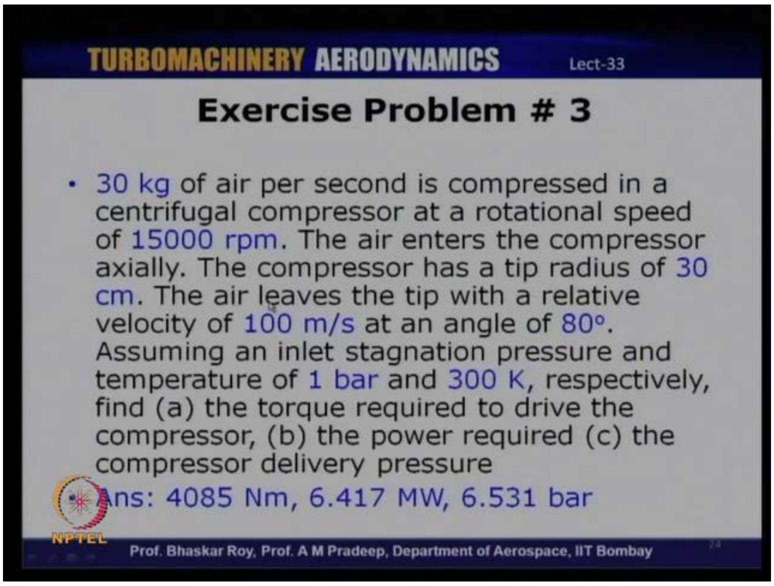

Third exercise problem is 30 kilograms of air per second is compressed in a centrifugal compressor at a rotational speed of 15000 rpm. The air enters the compressor axially. The compressor has a tip radius of 30 centimeters. The air leaves the tip with a relative velocity of 100 meters per second at an angle of 80 degrees. Assuming an inlet stagnation pressure and temperature of 1 bar and 300 Kelvin, respectively, find part a - the torque required to derive the compressor, the power required and the compressor delivery pressure. So, the torque in this case will be 4085 Newton meters, power required 6.417 megawatts and compressor delivery pressure is 6.531 bar.

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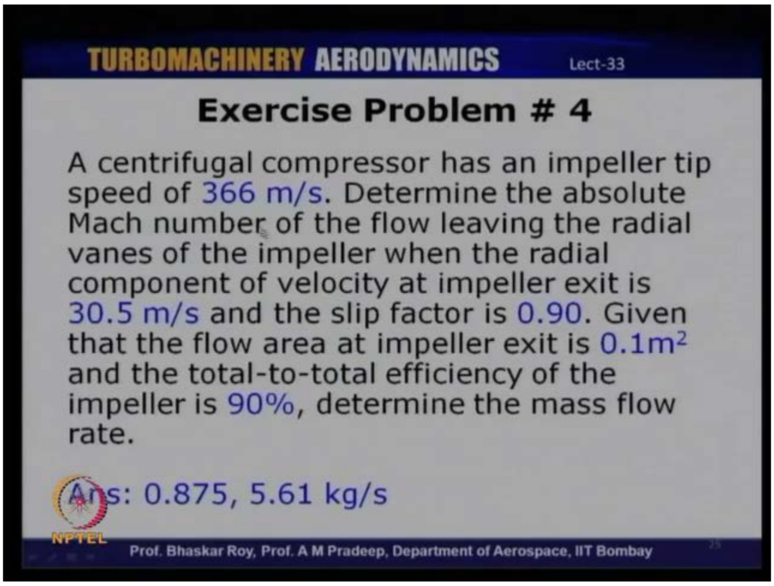

And the last problem is a centrifugal compressor has an impeller tip speed of 366 meters per second. Determine the absolute Mach number of the flow leaving the radial vanes of the impeller when the radial component of velocity at impeller exit is 30.5 meters per second and the slip factor is 0.90. Given that the flow area at the impeller exit is 0.1 meters square and the total to total efficiency is 90 percent, determine the mass flow rate. So, in this case we have the absolute Mach number as 0.875 and mass flow rate as 5.61 kilograms per second.

So, these are four exercise problems that you can solve based on what we have discussed in the last three lectures including today's, and I hope based on this discussions you will be able to solve these four exercise problems that we have for you today. And we will continue our discussion on some of these topics especially, some of the radial flow machines we will probably be taking up the radial flow turbines in the coming lectures. So, we will discuss more to do with radial flow machines in some of the coming lectures, in the next few lectures.