

Turbomachinery Aerodynamics
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Lecture No. # 32
Centrifugal Compressors: Characteristics, Stall, Surge Problems

Hello and welcome to lecture number thirty-two of this lecture series on turbomachinery aerodynamics. We have been talking about the different types of turbo-machines and with stronger bias or emphasis towards the axial flow turbo-machines, basically the axial compressors and axial turbines. We have already elaborated the reasons, why we are sort of giving little more weightage to the axial flow turbo-machines, primarily due to the fact, that modern day jet engines operate primarily with these types of turbo-machines, the axial compressors and the axial turbines due to some of their inherent advantages. Of course, it is not to say, that the radial flow counterparts are not being used at all.

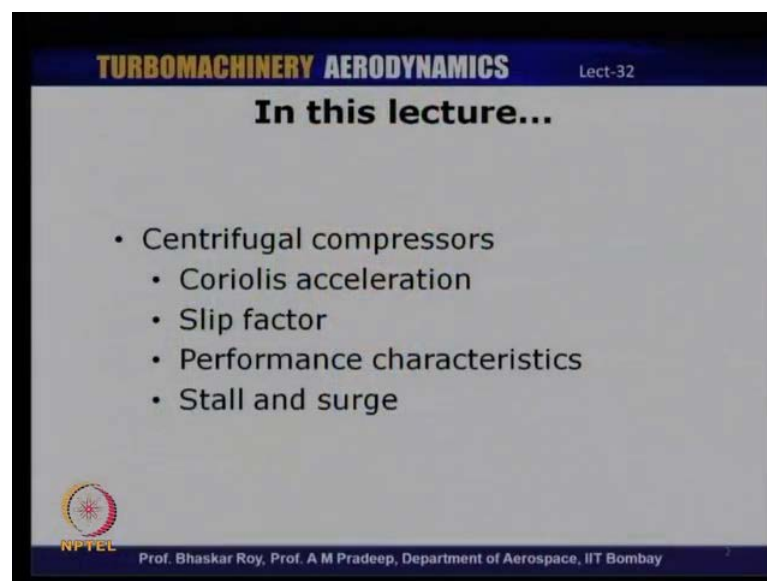
These are also having applications in, in certain specific areas, primarily to do with the smaller sized engines and, and various other applications and therefore, with this in mind, we also initiated some lectures on the radial flow machines and we started off with the centrifugal compressors. In fact, it must be kept in mind, that the earliest jet engines actually operated with centrifugal compressors and for a long time, centrifugal compressors were used in most of the jet engines. And of course, once the axial compressors were developed and designed, they sort of, slowly replaced centrifugal compressors, especially in the larger sized engines. But when you look at the smaller class engines, the thrust class, smaller thrust class engines, centrifugal compressors continue to be used in such applications.

So, in, with this in mind we had started the previous lecture with discussion on centrifugal compressors and lecture 31 was devoted towards an introduction towards centrifugal compressors. We discussed the thermodynamics of centrifugal compression process and also the work done and we had looked at power calculations and so on, and the governing equations, which are involved in centrifugal compressor design and

calculations and analysis. We also had a quick look at the different components, which constitute centrifugal compressor, like the inlet part of, or the intake of the centrifugal compressor, the inducer, the impeller, then the diffuser vanes and so on. So, we also discussed about how one can make calculations and analysis of these different components of a centrifugal compressor. So, that was what we had discussed in the previous class.

What we will do to, is to continue some of our discussion, which we had, had in the last class and sort of wind up our discussions on centrifugal compressor with this lecture. And the next lecture we would obviously, be having a tutorial because having undergone two lectures and an overview of centrifugal compressors it is essential, that we understand how one can make analysis and calculations on centrifugal compressors. So, we will devote the next lecture towards the tutorial on centrifugal compressor.

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In today's class, we will basically be taking up on a few important concepts. We will start our discussion with, what is known as, the Coriolis acceleration. Then, we will discuss about the slip factor, the performance characteristics and also, stall surge and choking associated with centrifugal compressors.

Now, we will start our discussion with Coriolis acceleration. I am sure you must have learnt, at least heard about this term, called Coriolis acceleration, or called Coriolis forces, in your high school classes. In, in your physics side, guess you must have learnt

about Coriolis forces and Coriolis acceleration in sort of a very general perspective. We are going to use some of those principles in, in relation to our current topic of discussion, that is, the centrifugal compressors.

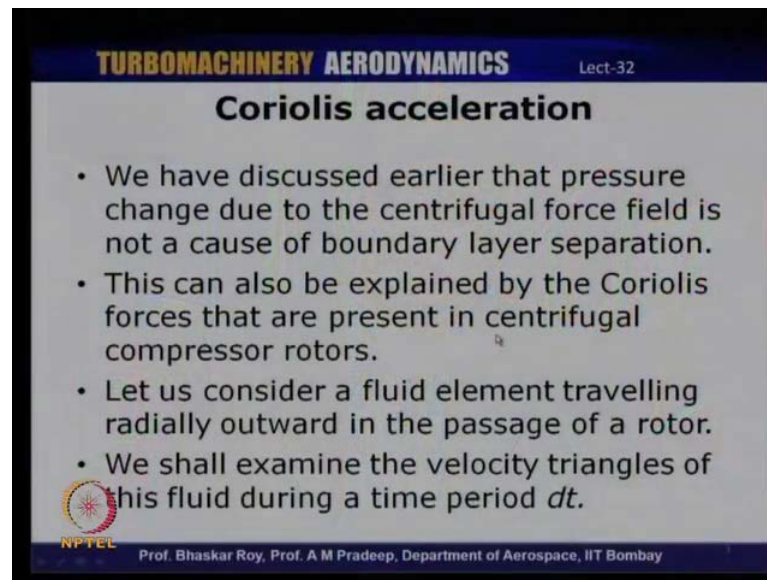
Now, if you remember, in one of the slides, which I had flashed in my previous class, I mentioned, that one of the aspects, that distinguishes a centrifugal compressor from an axial compressor is the fact, that in a centrifugal compressor the pressure rise mechanism is slightly different from that of an axial compressor, in a sense, that in a centrifugal compressor, pressure rise can also occur because of, or it primarily occurs because of displacement of the **centrifugal**, centripetal force field, and because of that there is a pressure rise taking place as a result of diffusion in the centrifugal compressor. So, there are at least two components, which contribute towards the overall pressure rise in a centrifugal compressor.

I think I mentioned that the problems associated with the boundary layers flows are not that severe in a centrifugal compressor. This is indeed true, that boundary layer separation is not, it is still matter of concern, but it is not the primary matter of concern, like we have in axial compressors, which also partly explains the fact, that axial compressors can develop much lower pressure ratio per stage as compared to centrifugal compressor, primarily because of the fact, that axial flow performance is impeded by boundary layer characteristics. So, we will also try to explain in the context of centrifugal compressors.

Let us also try to look at the pressure rise mechanism and deceleration or diffusion in the passages through Coriolis acceleration as a possible means. So, let us try to analyze what Coriolis acceleration does to this overall performance of a centrifugal compressor.

Now, what we will see very soon is the fact, that Coriolis acceleration is going to lead towards a certain discrepancy in the velocity triangle from the ideal characteristics, that is, the velocity triangle at the exit would be slightly different from what it should have been and that is attributed to, well, Coriolis acceleration and because of that it leads to certain amount of pressure loss as we will see it little later.

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Coriolis acceleration

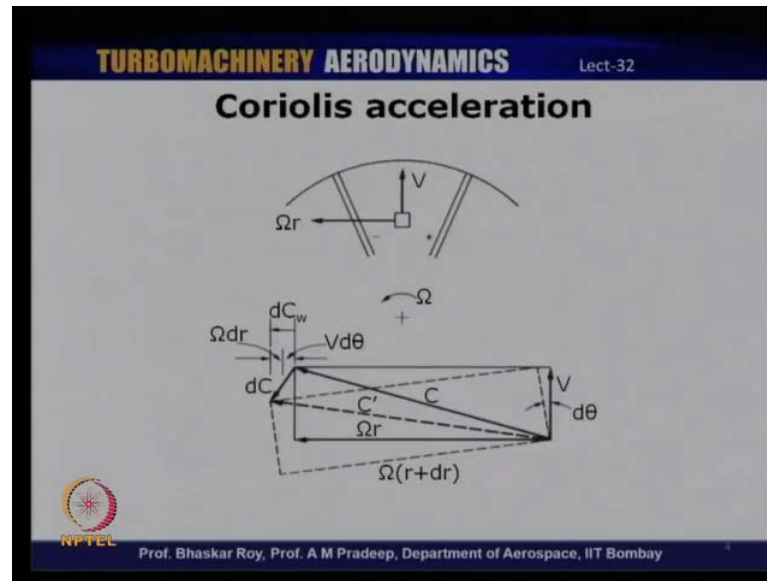
- We have discussed earlier that pressure change due to the centrifugal force field is not a cause of boundary layer separation.
- This can also be explained by the Coriolis forces that are present in centrifugal compressor rotors.
- Let us consider a fluid element travelling radially outward in the passage of a rotor.
- We shall examine the velocity triangles of this fluid during a time period dt .

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So, what we are going to do is that we will consider a certain fluid element, which is travelling radially outward, in the passage of a rotor. So, before this let me just quickly go through these two bullets. I have written here one is to do with centrifugal flow field, which is primarily, not a result of the boundary layer separation and basically, that the fact, that the pressure change due to centrifugal force field is not really, a cause of boundary layer separation. We will try to explain that with Coriolis forces.

So, if you consider a certain fluid element, which is, let us say travelling radially outward in the passage, and we will look at the velocity triangles of this particular fluid element during a certain time period dt .

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So, this is the fluid element that I am referring to. Let us consider a fluid element, which is passing through these straight radially vanes. Of course, you can see that this is the impeller and these are straight radial vanes. A fluid element is passing through the radial vanes, which are straight. So, the fluid element, obviously, has a relative velocity of V and a blade speed or rotational velocity ωr , where r is the radius at which this fluid element is currently located and ω is the rotational speed.

So, if you look at the velocity triangles for this fluid element in addition to V and ω , the absolute velocity c is given by the resultant of V and ωr . So, (()) is the basic velocity triangle is shown by the solid lines and you can see, V and ωr and its resultant is the absolute velocity C .

Now, after a certain time period, well, the impeller is rotating, that also displaces this fluid element by a certain distance. If we consider the fact, that this fluid element is, is being rotated by this center, as I showed here, then after certain time period dt , the fluid element deflects and therefore, the new velocity triangle is shown here by the dotted lines. So, new velocity triangle corresponds to a radial location, which is equal to r plus dr , where dr is the differential change in the radius with time dt .

So, the new speed rotational speed becomes ω . ω is unchanged; ω multiplied by r times dr and let us assume the fact, that there is negligible change in the relative velocity during this time, but because ω has changed, c prime also changes,

that is, the absolute velocity changes and it takes a new value, which is C' . Therefore, the net change in absolute velocity is given by dC , which is, as you can see, a tangential component of, which is shown by dC_w , which has two contributions. One contribution is because of the change in the radius, $\omega \times dr$ and the other contribution is because of $V \times d\theta$, where $d\theta$ is this angular deflection.

So, here, what you need to understand is that there are two distinct aspects, which has led to this change in the absolute velocity or let us say the tangential component of the absolute velocity. So, there are two contributions here, one of them is from the fact, that the radius has now changed to a differential. There is a differential change in the radius or radial location of the fluid element, which is given by dr and that leads to a change in the peripheral velocity, which is $\omega \times dr$.

There is also a change, our contribution from the fact, that relative velocity being remaining unchanged for the fact, that we will assume, that it is unchanged, because it changes much smaller compared to the other components. So, $V \times d\theta$, where $d\theta$ is the angular deflection angle through which the fluid element was deflected.

So, these two components put together result in a change in the tangential component of the absolute velocity and at the moment we are going to be looking at this change in the tangential component and its change with reference to time. So, dC_w / dt is basically, rate of change of the tangential component of velocity with time, that is acceleration, that is basically the Coriolis acceleration, and that is primarily because of the fact, that there is a rotation given to the fluid element.

If the fluid element was displaced simply radially upward without any angular deflection, Coriolis forces would be negligible; there would be hardly any Coriolis force acting on the fluid element. But because of the fact, that in addition to the fact, that the fluid element is getting displaced radially outwards, there is also an angular displacement of the radial element. Both of these contributions put together results in this acceleration, which we will denote as the Coriolis acceleration.

Let me recap the velocity triangles once again. So, here, in these velocity triangles the default velocity triangle is this, this is all shown by the solid lines V and ωr ; V is the relative velocity of the fluid element, ωr is the tangential component or the

blade speed peripheral velocity, resultant of that is C . When it is displaced, we will assume negligible change in V because this magnitude is much smaller than any other magnitudes here. And so, we have a new absolute velocity, which we have denoted by C prime, which is a resultant of $\omega r + dr$ and V . And this change in the absolute component has tangential component, which is dC_w , which has two contributions ωdr and $V d\theta$. So, the sum of this two gives us the dC_θ . So, let us add up these two.

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Coriolis acceleration

- The magnitude of the relative velocity is unchanged, but the particle has suffered an absolute change of velocity.

$$dC_w = \Omega dr + V d\theta$$

$$\text{or, } dC_w = \Omega V dt + V \Omega dt,$$

Thus, the Coriolis acceleration, $a_\theta = 2\Omega V$
and it requires a pressure gradient in the tangential direction of magnitude, $\frac{1}{r} \frac{\partial P}{\partial \theta} = -2\rho\Omega V$

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Now, the magnitude of this change in the tangential component dC_w is basically sum of ωdr plus $V d\theta$ or dC_w is ω into dr , we have expressed as V times dt plus $d\theta$ can be expressed as ω into dt , so V into ω into dt . So, this basically would be equal to 2 into ωV into dt and therefore, dC_w by dt is acceleration a_θ , that is, acceleration in the tangential direction or a_w or a_θ . As it is called, it is the Coriolis acceleration is equal to twice ωV , that is, the Coriolis acceleration is directly a function of the relative velocity and the rotational speed. So, higher the rotational speed, higher the Coriolis acceleration and higher the relative velocity. Obviously, that also changes the Coriolis acceleration.

Now, this basically requires a certain amount of pressure gradient. Why is there a Coriolis acceleration in the first place? Firstly, there is a rotation given to the fluid element, there is also a certain amount of pressure gradient, which is leading to this

Coriolis acceleration. So, if you were to look at the pressure gradient, we can express the pressure gradient in the tangential direction as $\frac{1}{r} \frac{dP}{d\theta}$, that is the rate of change of pressure in the tangential direction, which is basically, equal to twice $\rho \Omega V$.

So, the radial pressure or tangential pressure gradient can be expressed in terms of, twice $\rho \Omega V$, is basically referring to the fact, that the pressure gradient is direction dependent and depends on which direction the rotor is rotating. So, minus $2 \rho \Omega V$. So, this is the amount of pressure gradient, which is acting in the tangential direction and that leads to this much amount of Coriolis acceleration. So, the Coriolis acceleration, basically, requires certain amount of pressure gradient.

So, what we will do is to locate the rate of change of relative velocity in the tangential direction and see how it is related to the, or whether it is indeed, related to the Coriolis acceleration or not? We have seen, that the tangential velocity, well, Coriolis acceleration is, in fact, directly proportional to the relative velocity. Let us see the rate of change, or is there a rate of change of the relative velocity in the tangential direction?

(Refer Slide Time: 15:43)

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Coriolis acceleration

- The existence of the tangential pressure gradient means that there will be a positive gradient of V in the tangential direction.

$$\frac{1}{\rho} \frac{dP}{r d\theta} = - \frac{d(V^2 / 2)}{r d\theta} = - \frac{V dV}{r d\theta}$$

Therefore, $\frac{1}{r} \frac{dV}{d\theta} = 2\Omega$

- This means that there will be a tangential variation in relative velocity.

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So, if we look at the fact, that there is a tangential pressure gradient, then tangential pressure gradient needs to result in a positive gradient of V in the tangential direction. Because if there is a pressure gradient in a certain direction, that needs to also being the

fact, that there has to be a velocity gradient in the same direction and therefore, the pressure gradient in the tangential direction.

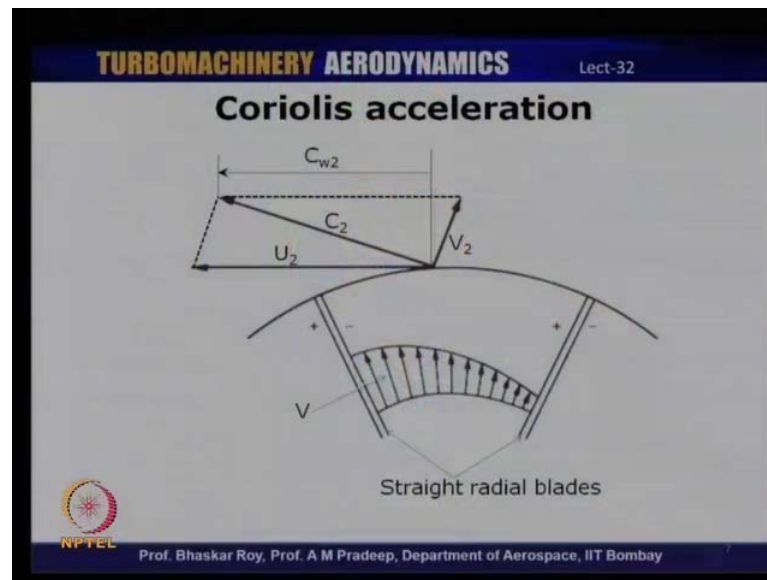
We have expressed in the previous slide, we can equate that with the Coriolis acceleration and then, we will see, that $\frac{1}{r} \frac{dP}{d\theta}$ is basically equal to $-\frac{V^2}{r}$, which is $-\frac{V}{r} \frac{dV}{d\theta}$.

So, this if you look at, if you compare this with the Coriolis acceleration, we can basically infer, that $\frac{1}{r} \frac{dV}{d\theta}$ is equal to 2ω . And what does this basically tell us? This tells us the fact, that there would be a change in the relative velocity in the tangential direction as well.

Now, that was in, in an idealized scenario one would not have expected any change in relative velocity in the tangential direction. But, we can now see from our analysis, that in addition to the fact that the relative velocity will keep changing in the radial direction because that is what the centrifugal compressor does. There will also be a change in the relative velocity in the tangential direction and that is of very crucial piece of information for us because we will very soon see that this leads to a change in the velocity triangle at the exit of the impeller from what it should have been.

So, from our understanding of the Coriolis acceleration we have seen, that as a result of the Coriolis acceleration, an eventual outcome of the fact, that there will be a Coriolis acceleration due to the tangential pressure gradient. Tangential pressure gradient, in turn, leads to a tangential velocity gradient and this velocity would be affected the component of velocity, which is affected is the relative velocity. So, the relative velocity would have a tangential gradient, it will keep changing with the tangential direction.

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So, if you look at this aspect in a schematic sense, I mentioned, that let us consider a same impeller; this is the same impeller we were talking about. These are the two straight radial blades and as an outcome of the analysis, we just have seen, there will be a tangential variation in the relative velocities. So, these vectors, which are shown here, are basically the velocities, relative velocities. There is a gradient of this relative velocity in the tangential direction.

Here, plus and minus indicate the pressure or higher pressure and this would be the pressure surface and suction surface, let us say on, in the case of an axial compressor blade. So, there is an increase in pressure gradient and correspondingly, a change in the relative velocity in the tangential direction.

So, what happens because of this is, that as the fluid element begins to leave the impeller, that is, if you trace the fluid element, which was let us say here, and as it leaves the impeller, by the time it reaches the tip of the impeller, because of this tangential velocity gradient in the relative velocity, you can see, that there relative velocity is lagging behind with the radial direction. So, the relative velocity is actually, now pointing in this direction because it is having a gradient in the tangential direction.

So, because of the gradient, the vector of the relative velocity leaving the impeller is inclined at an angle. It is lagging behind the radial direction and therefore, what is the outcome of this? The outcome of this is the fact, that ideally, one would have assumed a

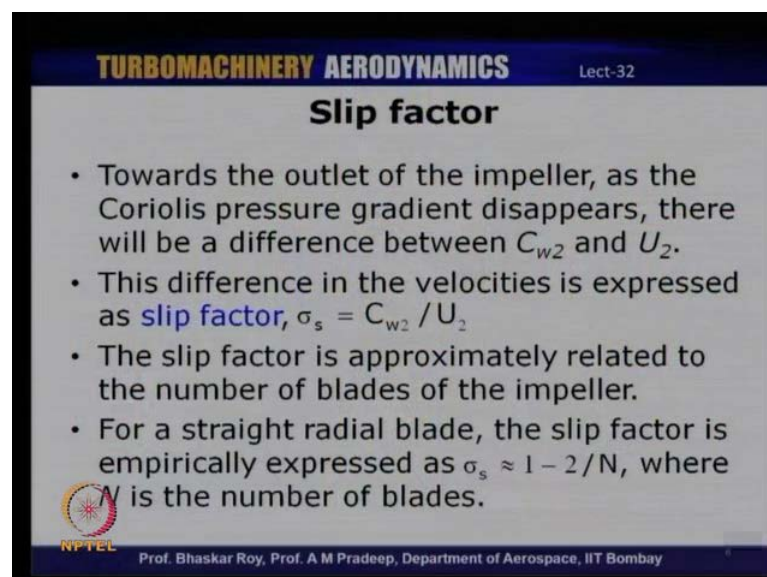
radial velocity; relative velocity is being equal to radial velocity; that is no longer true. Therefore, one has tangential component in the absolute velocities scale, that is, C_{w2} , which is not equal to u_2 .

So, if you recall velocity triangles that I had shown in the previous class, where I had shown three different types of impellers: forward leaning, straight radial and the backward leaning blades. For the straight radial blades, please go back and take a look at the velocity triangle. You will see, that I had drawn the velocity triangle with the relative velocity V or V_2 in the radial direction, which means, that C_{w2} would have been equal to u_2 . That is an ideal scenario.

Now, here we have seen that due to the Coriolis acceleration and its defect and the tangential velocity gradient, there will not be radial, relative velocity will not really, will not necessarily leave the impeller radially. So, there is a change or difference between the tangential component of the absolute velocity, that is, C_{w2} and the blade speed at the impeller exit u_2 .

So, this change in our difference in C_{w2} and u_2 is captured by a parameter, which is referred to as the slip factor while slip factor basically tells us the fact, that at the exit of the impeller the relative velocity is lagging behind the radial direction, resulting in a change or difference in the tangential component of absolute velocity C_{w2} as it should have been equal to u_2 . So, the ratio of C_{w2} to u_2 is called the slip factor.

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Slip factor

- Towards the outlet of the impeller, as the Coriolis pressure gradient disappears, there will be a difference between C_{w2} and U_2 .
- This difference in the velocities is expressed as **slip factor**, $\sigma_s = C_{w2} / U_2$
- The slip factor is approximately related to the number of blades of the impeller.
- For a straight radial blade, the slip factor is empirically expressed as $\sigma_s \approx 1 - 2/N$, where N is the number of blades.

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And this difference in the velocities is basically, expressed as a fraction, that is, $C w^2$ by u^2 , which is usually denoted by σ and the σ is, is a factor of actual blades. It will be less than 1 and one would like to have a slip factor close to 1. So, that means, $C w^2$ will be equal to u^2 and it will lead to much higher, let us say, efficiency and pressure ratios for, for the same rotational speeds and impeller diameter.

Now, slip factor is also a function, a strong function of the number of blades. You can see, I have shown two straight radial blades and you have seen that from one blade to another, there is a tangential velocity gradient. That means, the larger the distance between the blades, the velocity gradient will keep increasing and therefore, the slip factor would be or the difference between $C w^2$ and u^2 would be higher. As we increase the blades' spacing, lower the number of blades, the greater would be the difference and as you keep increasing the number of blades, the tangential velocity gradient is lower and therefore, the slip factor is also likely to be low.

So, slip factor being strongly related to the number of blades, people have come up with empirical correlations for calculating slip factor based on the number of blades. One of the most commonly used empirical correlations is given by **Standards** and that known as the **Standards** slip factor. It is equal to $1 - \frac{2}{n}$, where n is the number of blades, which means, if there is, let us say, ten blades for straight radial blades, the slip factor would be, **1 by**, $1 - \frac{2}{10}$, that is, $1 - 0.2$ and that is 0.8.

And if we increase the number of blades to let us say, twenty, then you can see that the slip factor becomes 0.9, and so on. And that is, as we increase the number of blades, the radial velocity of the, change in the tangential direction of the relative velocity becomes lesser and therefore, that would be lesser difference between $C w^2$ and u^2 . Whereas for larger spacing or lesser number of blades, the variation in the relative velocity in the tangential direction would be larger and larger leading to much larger difference between $C w^2$ and u^2 , leading to poor values of slip factor and so, slip factor being a strong correlation, strongly correlated to the number of blades.

There are different parameters or different researches have come up with different empirical correlation, the most common one is what one I have shown, which is strictly applicable for a straight radial blade. For backward leaning blades, the slip factor is calculated in a different way and there are different other correlations, which are used for

such blades and you will find in literature, there are many more empirical correlations, **which are**, which are in some way or the other related to the number of blades.

So, the basic effect of slip factor is the fact, that it reduces the magnitude of the swirl velocity or tangential component of velocity leaving the impeller and since pressure rise is a function of this tangential velocity for, for lower values of tangential velocity, the pressure rise also decreases. Therefore, slip factor directly affects the pressure ratio of the centrifugal compressor, which obviously, is not a good thing, that designer would want to keep for a given rotational speed and impeller diameter, try to maximize the pressure ratio and presence of slip factor, can bring down the pressure ratio, but because of the fact, that it affect the tangential velocity.

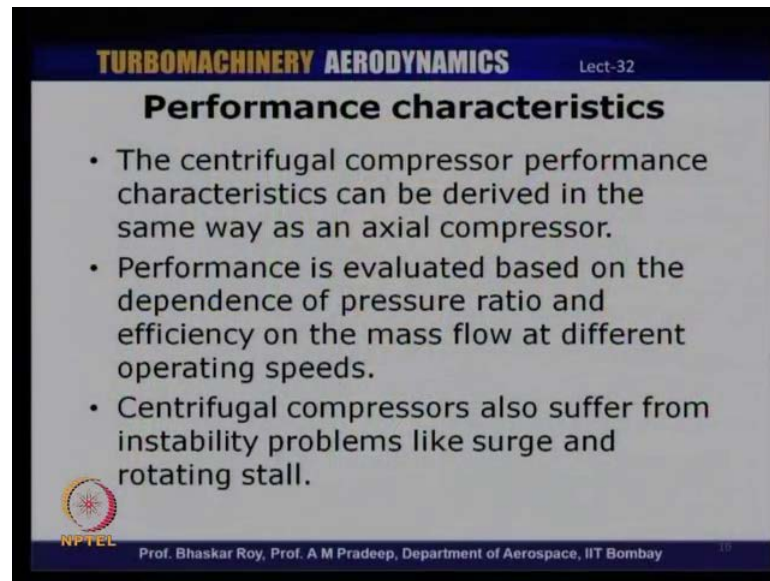
So, way out, the way out of this is to use more number of blades. Well, more number of blades means, you would, obviously, have higher frictional losses and therefore, it is probably not a good idea to keep increasing the number of blades or other option is to increase the impeller diameter, which is for normal application. A land based application may not be a big deal, you can keep increasing the diameter to some levels, but not for an aero-engineer application, where you know, it will increase the drag as well.

But larger the diameter of the impeller, the stress on the impeller blades also goes up, which means, then if they will have to invest in better materials, which can withstand higher stress or the other option is to increase the rotational speed. And again, if you increase the rotational speed for the same diameter, that also affects the stresses on the, on the blades and therefore, that again is a constraint. So, you can see, that the all kinds of constrains here in terms of stresses on the blades or frictional losses. So, there is lot of scope for an optimization to be carried out here and to determine what is the best possible configuration for number of blades versus impeller diameter versus the rotational speed and that can give us the best possible efficiency, as well as, the pressure ratio.

So, having understood slip factor and its effect on centrifugal compressor performance, let us now talk about the performance of a centrifugal compressor in general. We have already discussed about the performance characteristic of axial compressors, as well as, axial turbines. Centrifugal compressor performance characteristic would look in some way, similar to what we have already discussed for an axial compressor, but problems

related to choking is a little more severe in a, in a centrifugal compressor as compared to an axial compressor. Also, centrifugal compressors undergo similar problems that we have seen for axial compressors like stall and surge. And so, let us look at the performance characteristics of a centrifugal compressor and try to understand how we can estimate the performance of a centrifugal compressor.

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The slide is titled "TURBOMACHINERY AERODYNAMICS" and "Lect-32". The main heading is "Performance characteristics". It contains three bullet points: "The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.", "Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.", and "Centrifugal compressors also suffer from instability problems like surge and rotating stall." The slide also features the NPTEL logo and the names of the professors, Prof. Bhaskar Roy and Prof. A M Pradeep, from the Department of Aerospace, IIT Bombay.

TURBOMACHINERY AERODYNAMICS Lect-32

Performance characteristics

- The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.
- Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.
- Centrifugal compressors also suffer from instability problems like surge and rotating stall.

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So, we will evaluate the performance of a centrifugal compressor in same way as we have done for an axial compressor. We will look at the pressure ratio of the, dependence of the pressure ratio and efficiency on mass flow rate, all of which of course, pressure ratio and mass flow rate, mass flow rate being non-dimensionalized for different non-dimensionalized operating speed. And we will very soon realize that compressors, centrifugal compressors also suffer from instability problems, like surge and rotating stall.

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TURBOMACHINERY AERODYNAMICS Lect-32

Performance characteristics

- The compressor outlet pressure, P_{02} , and the isentropic efficiency, η_c , depend upon several physical variables

$$P_{02}, \eta_c = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non - dimensionless parameters,

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\gamma RT_{01}}}{P_{01} D^2}, \frac{\Omega D}{\sqrt{\gamma RT_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

The above reduces to $\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$

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So, I will make this a little quick because we have already done this for an axial compressor, it is exactly the same procedure. The non-dimensional groups are derived from a dimensional analysis, the exit pressure ratio or exit, total pressure P_{02} and efficiency are functions of variety of parameters, like mass flow rate, inlet stagnation pressure, inlet stagnation temperature, rotational speed, ratio of specific, it is gamma, the gas constant or the viscosity, the design, as well as, diameter.

So, if we non-dimensionalize this, we get these many non-dimensional groups. P_{02} by P_{01} , that is, the pressure-ratio efficiency being functions of $\dot{m} \sqrt{\gamma RT_{01}}$ by $P_{01} D^2$, ΩD by square root of γRT_{01} , ΩD^2 by ν , gamma and design.

Now, for a given design and given diameter, we can drop a lot of these terms and basically, the pressure ratio and efficiency becomes function of mass flow rate, $\dot{m} \sqrt{T_{01}}$ by P_{01} and N by $\sqrt{T_{01}}$. This is the non dimensional speed and this is the non dimensional mass flow rate.

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TURBOMACHINERY AERODYNAMICS Lect-32


Performance characteristics

Usually, this is further processed in terms of the standard day pressure and temperature.

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)$$

Where, $\theta = \frac{T_{01}}{(T_{01})_{\text{Std. day}}}$ and $\delta = \frac{P_{01}}{(P_{01})_{\text{Std. day}}}$

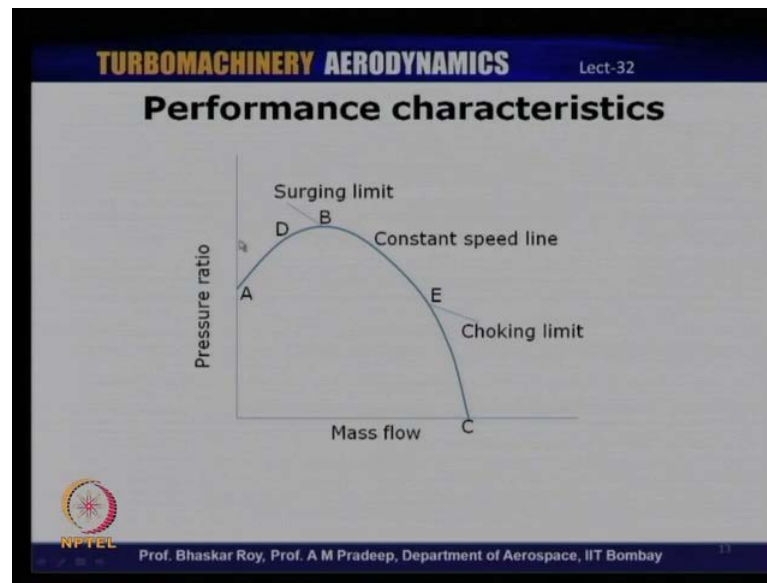
$(T_{01})_{\text{Std. day}} = 288.15 \text{ K}$ and $(P_{01})_{\text{Std. day}} = 101.325 \text{ kPa}$

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This we will further process for a standard day pressure and temperature. So, we get P_{02}/P_{01} efficiency be functions of $\dot{m}\sqrt{\theta}/\delta$ and $N/\sqrt{\theta}$ where, θ is $T_{01}/T_{01, \text{std. day}}$ and δ is $P_{01}/P_{01, \text{std. day}}$ typical values taken for this are $T_{01, \text{std. day}}$ is two eighty eight point one five kelvin which is basically twenty five degree celsius and pressure of standard day is one zero one point three two five kilopascal.

So, with this set of non dimensional parameters pressure ratio and efficiency being functions mass flow rate non dimensionalized as well as the non dimensional speed we will now, look at how the performance characteristics can be plotted. Before that, let us first look at a very general characteristics which, is applicable for any centrifugal compressor and then we will look at a typical performance characteristics in general.

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So, if we look at the variation of pressure ratio versus mass flow rate, one can trace characteristic like this; ideally, one could trace a characteristic like this. There are several salient points, which I marked here, point A, B, C, D and E.

Now, let us take a look at what happens as, let us say, the compressor was operating at some point E and then, as the mass flow is reduced, as we throttled the compressor and decrease the mass flow, the pressure ratio across the compressor increases. And as it increases, it reaches its peak and eventually, you will see that after it reaches its peak, the pressure ratio drops and this could, this drop, of course, could be very drastic as well in some compressors.

In the case of axial compressors we have already seen what really happens and the fact, that when the throttle characteristics become, well, intersect the pressure ratio characteristics beyond a certain level, the compressor undergoes, what are known as, instabilities.

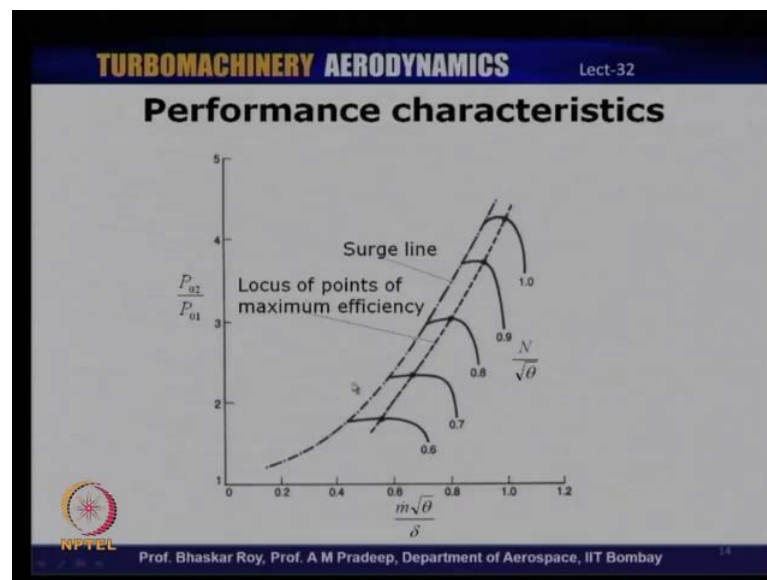
So, that exact same thing happens even in the case of centrifugal compressor beyond this point E, where the slope of the pressure ratio, mass flow characteristics is positive, the centrifugal compressor undergoes instabilities and that is why, B is referred to as the surging limit. And any point after the, on the left hand side of B, between A and B, let us say point D, would be considered unstable point and the compressor cannot really operate in a stable condition there.

We have discussed the stability in reference to axial compressors in quite detail and all those arguments are very much valid even for a centrifugal compressor. And on the other hand, here we have the, what is known as, the choking limit. Beyond point E and little later, the slope becomes extremely sharp and there is a very sharp drop in the pressure ratio characteristic and mass flow remains more or less constant and that is referred to as the choking limit.

So, we will discuss choking in little more detail in relation to centrifugal compressor because these compressors tend to be affected more by choking and as compared to axial compressors. And so, we will discuss choking in little more detail and limit our discussion on surge and stall because that we have already discussed in relation to axial compressors.

So, let us now look at an actual centrifugal compressor map, performance map, in terms of pressure ratio, non-dimensional mass flow rate and for a different non-dimensional speed.

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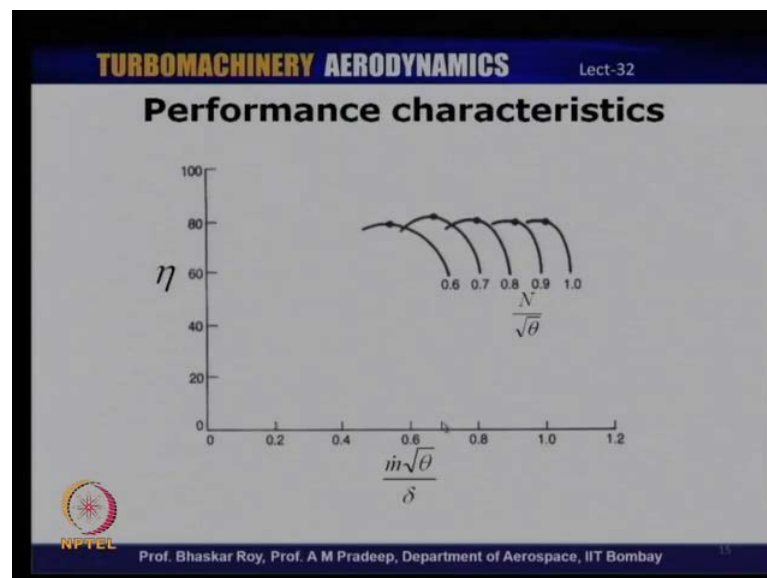


So, if we look at the pressure ratio characteristics versus mass flow rate characteristic, so the previous slide I had shown an idealized curve. Here, we have seen that the curve on the left hand side is not possible and beyond this it chokes. So, the actual performance characteristic is limited between these two points B and E and that is what is shown here by these different lines.

So, as you keep changing the speeds, the performance characteristics also change and you can say, that as speed reaches its max of the design speeds, the performance characteristics become sharper and sharper. For lower speeds, one can see, one can notice a shallower pressure ratio versus mass flow characteristic, that becomes steeper as one proceeds for higher and higher speeds and all these lines are terminated on the left hand side by the surge line and on the right hand side by the choking line.

And if we join all the points of maximum efficiency, we get the dotted line that is I have shown here. And one could ideally want to operate the compressor very close to this maximum efficiency line provided, that of course, this line is not very close to the surge line, which of course, can put the compressor into the risk of surging.

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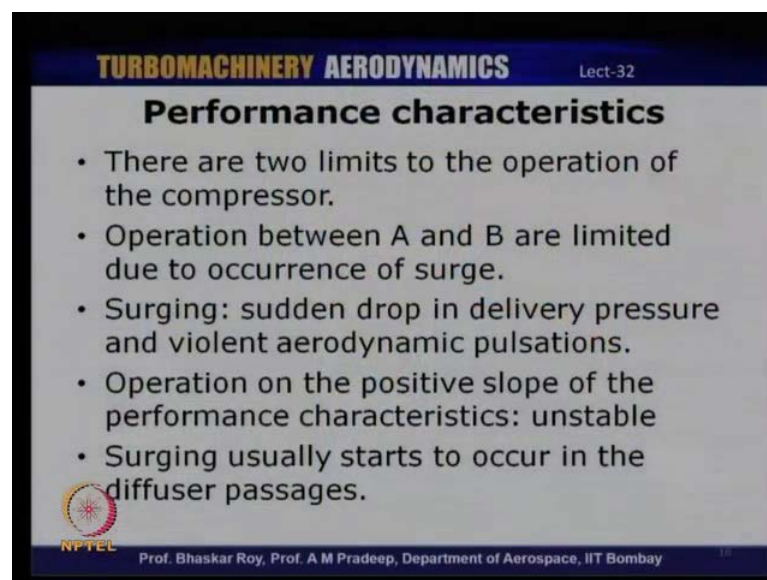
Now, if you look at the efficiency characteristic, again very similar to the axial compressor that we discussed. Efficiency versus mass flow rate for non-dimensional speeds and with increase in speed you can say, that the range of high efficiency becomes narrower.

For higher speeds we have a narrower band of operation, where the efficiency is high and it becomes very sensitive to the mass flow rate. For lower speeds, of course, efficiency is not, is relatively lesser sensitive to the mass flow rate and there is a wider range of operation possible with higher slightly higher efficiencies.

In comparison with an axial compressor we can see, that even centrifugal compressors have two aspects of two lines, which basically define the performance. On left hand side we have the surge line; on the right hand side we have the choking line. Now, between those two points on the map, that I shown, that is, point A and B, the compressor may undergo instabilities. It could be a rotating stall, which eventually leads to surge.

Now, we have already discussed these instability mechanisms in fairly good detail and just quickly mentioned what happens while a compressor undergoes some of these instabilities. So, basically, the operation of a compressor in the positive slope of pressure ratio versus mass flow rate is unstable operation and the compressor cannot operate in that region in a stable manner.

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TURBOMACHINERY AERODYNAMICS Lect-32

Performance characteristics

- There are two limits to the operation of the compressor.
- Operation between A and B are limited due to occurrence of surge.
- Surging: sudden drop in delivery pressure and violent aerodynamic pulsations.
- Operation on the positive slope of the performance characteristics: unstable
- Surging usually starts to occur in the diffuser passages.

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One of the instabilities, that affect the performance is rotating stall and the more severe one is the surge, wherein there is a sudden drop in the delivery pressure and of course, violent aerodynamics pulsations. And in the case of centrifugal compressor it has been noticed, that in general, the surging begins in the diffuser passages, because I think I mentioned in the last class, that diffuser passages are significantly affected by boundary layer performance and so, that is one of the weak links in a centrifugal compressor, where the performance is very sensitive to boundary layer flow and therefore, surging has been, in general, observed to initiate, get initiated in the diffuser passages.

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The slide is titled "TURBOMACHINERY AERODYNAMICS" in yellow text on a blue background, with "Lect-32" in white text to the right. Below this, the main title "Performance characteristics" is in bold black text. A bulleted list follows: the first point states that pressure ratio or temperature rise in a centrifugal compressor depends on blade shaping; the second lists three blade shapes: forward leaning, straight radial, and backward leaning; the third states that forward leaning blading theoretically produces a higher pressure ratio for a given flow coefficient; and the fourth states that such blading has inherent dynamic instability. Below the list, a concluding sentence says "Therefore, straight radial or backward leaning blades are popularly used." To the left of this sentence is a small circular diagram showing a fan-like structure with multiple blades. At the bottom left is the NPTEL logo, and at the bottom center is the text "Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay". A small number "17" is in the bottom right corner.

TURBOMACHINERY AERODYNAMICS Lect-32

Performance characteristics

- The pressure ratio or the temperature rise in a centrifugal compressor also depends upon the blade shaping.
- There are three possible types of blade shapes: forward leaning, straight radial and backward leaning.
- Theoretically, the forward leaning blading produces higher pressure ratio for a given flow coefficient.
- However such a blading has inherent dynamic instability.

Therefore, straight radial or backward leaning blades are popularly used.

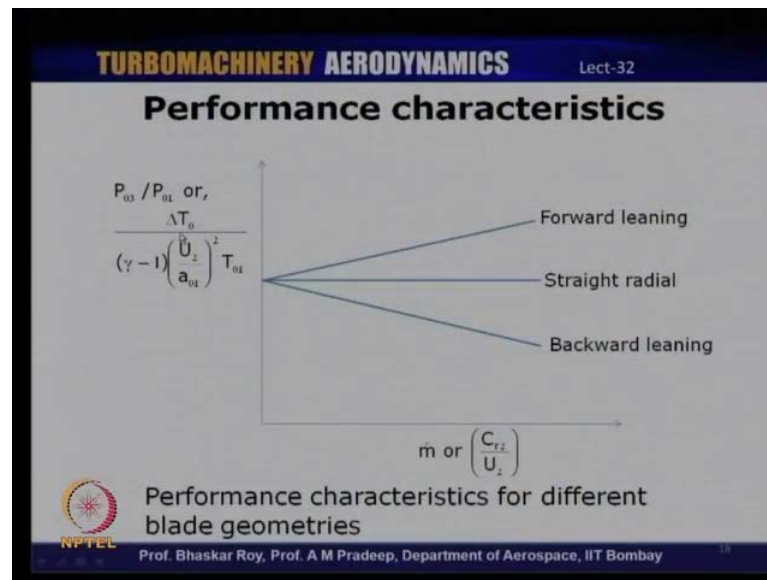
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Now, in a centrifugal compressor the pressure ratio or the performance is also a type of function of the, type of blade, that is used in a centrifugal compressor.

In the last class, we discussed three different possible blades, blade configurations: a forward leaning type, straight radial and backward leaning. Now, if you look at the performance of these three different types of blades and analyze the velocity triangles at the exit, as urge you to go back to the previous lecture and take up that slide, where I had shown the velocity triangle for these three cases, you will notice, that theoretically, forward leaning blades produce higher pressure ratio. Because if we look at the velocity triangle at the exit, you would appreciate this aspect and one would expect forward leaning blades to be much better in terms of performance.

But what is interesting to notice is the fact, that forward leaning blades have an inherent instability, because if you look at pressure ratio versus mass flow characteristic for forward leaning blade, the characteristic always has a positive slope. And, we have just now discussed, that operation on the positive slope of a pressure ratio mass flow characteristic is inherently unstable and that is the reason, I think I mentioned in the last class, that forward leaning blades are not really used in centrifugal compressors because of the fact, that they are inherently unstable, that is why straight radial and backward leaning blades are commonly used in modern day centrifugal compressors.

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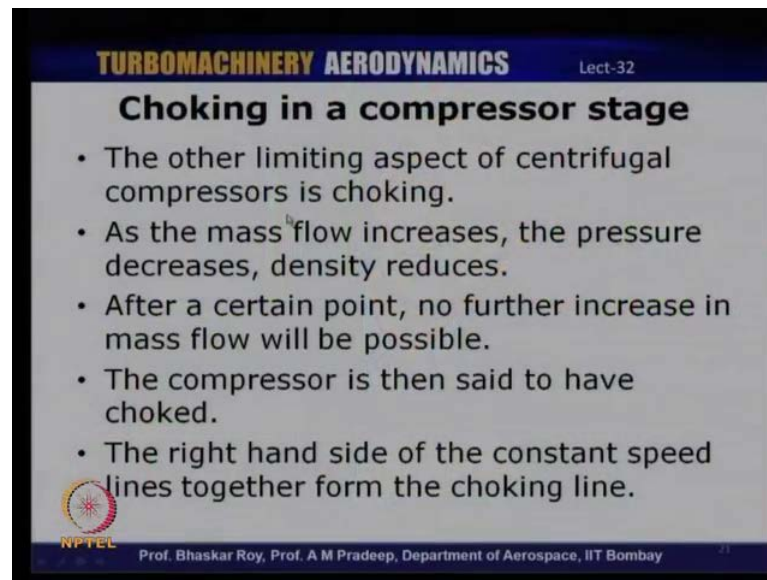


So, if we look at the performance characteristics, either in terms of pressure ratio or the temperature rise versus either mass flow rate or the flow coefficient, a forward leaning blade would have a characteristic, which has positive slope throughout.

So, this is like the left hand side curve of a centrifugal compressor characteristic, where of course, this is idealized, but one would still get a positive slope throughout for a forward leaning blade, which means, that this blade is going to have instabilities, irrespective of the mass flow rate and therefore, this is not a favorable type of blade, that can be used even though the pressure ratio performance is much better than straight radial or backward leaning. So, these two configurations are the ones, which are commonly used, the straight radial and the backward leaning blades.

So, what I will do next is to discuss about a problem of choking, which is what is probably more severe in the case of centrifugal compressors as compared to axial compressors, whereas rotating stall and surge are still the limiting performance parameters on one side. On the other side, we also have the choking problem associated with trying to increase mass flow rate and beyond a certain level of mass flow rate, compressibility effects prevent us from operating the compressor beyond a certain level of mass flow rate.

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TURBOMACHINERY AERODYNAMICS Lect-32

Choking in a compressor stage

- The other limiting aspect of centrifugal compressors is choking.
- As the mass flow increases, the pressure decreases, density reduces.
- After a certain point, no further increase in mass flow will be possible.
- The compressor is then said to have choked.
- The right hand side of the constant speed lines together form the choking line.

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So, let us take closer look at choking in more detail because you already discussed about the other instabilities in relation to axial compressors.

So, as the mass flow increases in, in the case of centrifugal compressors, the pressure ratio will decrease, as we have seen the performance characteristic and therefore, that also reduces the density. After a certain point one would not be able to increase the mass flow beyond a certain value, the compressor is then said to have choked, that is on this, that is, probably we have reached the right hand side of the performance characteristic, wherein the mass flow rate has reached its maxima and we are not able to increase mass flow rate beyond that level and that is when the compressor is said to have choked.

So, in a centrifugal compressor we have seen that different components, which constitute a centrifugal compressor. We have the inlet, the impeller and the diffuser vanes. So, calculation of the choking mass flow is different depending upon whether it is the stationary component or the rotating components. So, we will take a look at the choking mass flow as applicable for inlet, the impeller and diffuses vanes and see their dependence on the upstream parameters.

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The slide is titled "TURBOMACHINERY AERODYNAMICS" in yellow text on a blue background, with "Lect-32" in white text to the right. Below this, the main title "Choking in a compressor stage" is in bold black text. The content includes a bulleted list: "Choking behaviour for rotating passages is different from that of stationary passages." and "Inlet: Choking takes place when $M=1$ ". Below this, the temperature ratio is given as $\frac{T}{T_0} = \frac{2}{\gamma + 1}$. A line of text states: "Assuming an isentropic flow, the choking mass flow rate is". This is followed by the equation $\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}$. Another bullet point states: "Since ρ_0, a_0 refer to the inlet stagnation conditions and are constant, the mass flow rate is also a constant: choking mass flow." At the bottom left is the NPTEL logo, and at the bottom center is the text "Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay".

TURBOMACHINERY AERODYNAMICS Lect-32

Choking in a compressor stage

- Choking behaviour for rotating passages is different from that of stationary passages.
- Inlet:
 - Choking takes place when $M=1$

$$\frac{T}{T_0} = \frac{2}{\gamma + 1}$$

Assuming an isentropic flow, the choking mass flow rate is

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}$$

- Since ρ_0, a_0 refer to the inlet stagnation conditions and are constant, the mass flow rate is also a constant: choking mass flow.

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So, choking behavior, because it is different for rotating passages from that of stationary passages, so if you consider the inlet, we know, that of course choking, irrespective of whether it is inlet or impeller, takes place when Mach, the Mach number reaches 1.

So, when Mach number is unity, the ratio of static temperature to total temperature T by T_0 is equal to 2 by gamma plus 1 because Mach number, if the isentropic relation with the x equates M is equal to 1, we get this relation between static and stagnation temperature. So, for the moment if you assume an isentropic flow, then the choking mass flow rate is basically, \dot{m} dot A by per unit area is equal to $\rho_0 a_0$ into 2 by gamma plus 1 raise to gamma plus 1 by 2 into gamma minus 1. So, this basically comes from mass flow rate being equal to $\rho a v$.

And we have expressed ρ in terms of stagnation density and velocity in terms of the speed of sound and here we can see, that this mass flow rate is expressed purely in terms of parameters, which are the upstream parameters of the inlet stagnation conditions, which remain constant. So, as we keep changing the operating condition, the inlet conditions are still fixed, which means, that the mass flow rate also has to remain constant and that is why, when Mach number becomes 1, what you can see is that the right hand side of the mass flow rate equation as all the parameters, which are basically constants. And therefore, when Mach number becomes 1, one can actually calculate the choking mass flow based on the inlet stagnation condition.

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TURBOMACHINERY AERODYNAMICS
Lect-32

Choking in a compressor stage

- **Impeller:**
 - In rotating passages, the flow conditions are referred through rothalpy, I.
 - During choking, it is the relative velocity, V, that becomes equal to the speed of sound.

$$I = h + \frac{1}{2}(V^2 - U^2) \rightarrow T_{01} = T + (\gamma RT / 2C_p) - (U^2 / 2C_p)$$

$$\therefore \frac{T}{T_{01}} = \left(\frac{2}{\gamma + 1} \right) \left(1 + \frac{U^2}{2C_p T_{01}} \right) \text{ and } \frac{\dot{m}}{A} = \rho_{01} a_{01} \left(\frac{T}{T_{01}} \right)^{(\gamma+1)/2(\gamma-1)}$$

or, $\frac{\dot{m}}{A} = \rho_{01} a_{01} \left[\frac{2 + (\gamma - 1)U^2 / a_{01}^2}{\gamma + 1} \right]^{(\gamma+1)/2(\gamma-1)}$

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23

The next component that we will take a look at is the impeller in a rotating passage. As we have seen, the flow conditions are usually referred through the rothalpy, which I had discussed in the last class.

And during choking, in the case of an impeller, it is the relative velocity, which basically becomes equal to this period of sound when Mach number becomes unity and so, if look at the expression for rothalpy we have, I is equal to h plus half into V square minus U square and so, and we know, that stagnation temperature can be expressed in terms of the corresponding static temperature and the speed of sound.

So, here we, if we express enthalpy in terms of stagnation temperature and V square, because when Mach number is equal to 1, V becomes equal to the speed of sound and therefore, that becomes gamma RT divided by 2C p because enthalpy is T naught times C p minus U square by 2C p. So, we have divided this by the speed of sound and therefore, we get T naught 1 is equal to T plus gamma RT by 2C p minus U square by 2C p.

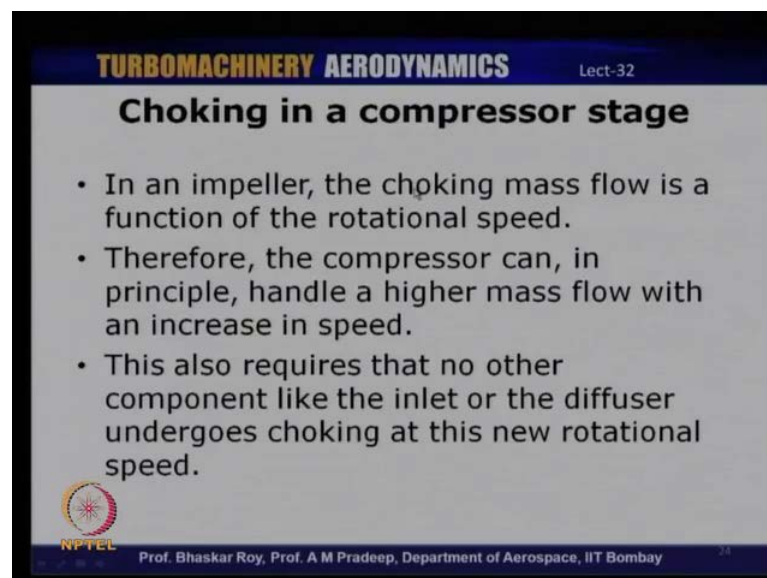
So, if we simplify, that we get T by T naught 1 is equal to 2 by gamma plus 1 into 1 plus U square by 2C p T 01 and therefore, mass flow rate, which is rho a V, again gets expressed as rho 01 a 01 into T by T 01 raised to gamma plus 1 by 2 into gamma minus 1, which can be simplified further and what you see is, that mass flow rate is equal to rho 01 a 01 into 2 plus gamma minus 1 U square by a 01 square by gamma plus 1 raised to

$\gamma + 1$ by 2 into $\gamma - 1$. So, this is rather complex expression for the mass flow rate, once again tell us, that in addition to the inlet condition.

In the case of impeller we also see, that it is a function of the rotational speed and in the case of the inlet we have seen, that mass flow rate is purely a function of the inlet parameters and that, therefore mass flow rate gets fixed once the inlet conditions are fixed. In the case of impeller, besides the inlet conditions you also have the rotational speed, that means, that in principle it should be possible for us to operate the compressor at a different mass flow, at higher mass flow, than the choking mass flow for a higher rotational speed.

Of course, this will also require, that if once you start operating the compressor at a higher rotational speed, the other components do not choke because if the other components choke, the compressor will still remain at the choke condition. So, other components per meeting, like the inlet or the diffuser, the impeller performance or impeller choking because it is a function also, of the rotational speed operating the compressor at higher rotational speed, may permit us to operate or deliver a higher mass flow, provided all other components also, are able to operate at this new operating point.

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TURBOMACHINERY AERODYNAMICS Lect-32

Choking in a compressor stage

- In an impeller, the choking mass flow is a function of the rotational speed.
- Therefore, the compressor can, in principle, handle a higher mass flow with an increase in speed.
- This also requires that no other component like the inlet or the diffuser undergoes choking at this new rotational speed.

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Choking mass flow in an impeller is basically, function of the rotational speed and compressor in principle should be able to handle a higher mass flow with an increasing

speed provided, that other components, like inlet or diffuser does not really undergo choking for this rotational speed.

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TURBOMACHINERY AERODYNAMICS Lect-32

Choking in a compressor stage

- Diffuser:
 - The choking mass flow in a diffuser has an equation similar to that of an inlet:

$$\frac{\dot{m}}{A} = \rho_0 a_0 \left(\frac{2}{\gamma + 1} \right)^{(\gamma + 1) / 2(\gamma - 1)} a_0$$
 - The stagnation conditions at the inlet of diffuser depend upon the impeller exit conditions.
 - It can be shown that the choking mass flow is a function of the rotational speed and therefore can be varied by changing the rotational speed.

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Now, the last component, if the diffuser, the choking mass flow in a diffuser has an expression the same as that of inlet, so it is again a function of the inlet condition for the diffuser. So, mass flow rate is function of $\rho_0 a_0$ into 2 by $\gamma + 1$ raise to $\gamma + 1$ by 2 into $\gamma - 1$.

Now, here, you can see that the stagnation conditions at the inlet of the diffuser depend upon the inlet of the diffuser itself, which is the impeller exit and therefore, mass flow rate can basically be related to the rotational speed of the impeller and therefore, you can see, that the operation of the diffuser and impeller are sort of coupled because the diffuser, choking mass flow, is a function of the impeller exit conditions. And impeller exit conditions can basically be changed by changing the rotational speed and so, there is a certain amount of coupling between the impeller operation and the diffuser operation.

And so, so with this analysis it should be possible for us to calculate under what conditions centrifugal compressor is likely to choke and how is still, that we can operate, still operate the compressor at a probably, slightly a higher mass flow by looking at the other components involved, like the inlet and the diffuser, and ensuring, that all the three components can operate in under the new rotational speed without undergoing a choke.

So, let me now quickly recap our discussion in today's class. We had discussed few aspects, which are an extension of what we had discussed in the last class. We started our lecture today with detailed discussion on the Coriolis acceleration and we have derived an expression for Coriolis acceleration. We have seen, that Coriolis acceleration leads to tangential velocity gradient, velocity gradient in the tangential direction and that is what leads to a difference in the velocity, that is, leaving the impeller that is, it leads to certain amount of difference between $C w^2$, which is the tangential component of absolute velocity and U^2 and that difference is what is referred to as the slip factor.

We have seen, that slip factor is a strong function of the number of blades. Lesser the number of blades, the greater the spacing between the blades, the greater is the tangential velocity gradient and therefore, the slip factor becomes lower and lower. So, this can be, that is, with higher number of blades one can reduce the tangential variation in relative velocity and therefore, one can achieve higher values of slip factors. So, slip factors and its dependents on the number of blades.

We have seen the $\left(\frac{1}{n} \right)$ formula, where one can approximate slip factor as $1 - \frac{2}{n}$, where n is the number of blades and of course, that is true for straight radial blades.

We have then discussed about the performance characteristics and of a centrifugal compressor. Of course, we did not discuss too much of the details of the performance characteristics because it is very similar to that of an axial compressor. We spent much less time discussing about surge and stall because we already discussed that and we have discussed slightly more details about the choking conditions, which is affecting, which might affect centrifugal compressor performance, and how one can estimate the choking mass flow for the inlet, the impeller and the diffuser and the interrelation between choking conditions for these three different components.

So, these were the topics, that we had discussed in today's class and so this will sort of wind up our discussion on centrifugal compressors and therefore, the next lecture, as I mentioned in the beginning, we will basically take up few problems for solving.

So, we will have a tutorial session in the next class. We will discuss about how we can solve problems obtaining centrifugal compressors and at the end of the tutorial we will also have a few exercise problems for you to solve based on the discussion during the

last few lectures. So, we will take up a tutorial in the next lecture, which will be lecture number thirty three.