

Jet Aircraft Propulsion
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Lecture No. #33

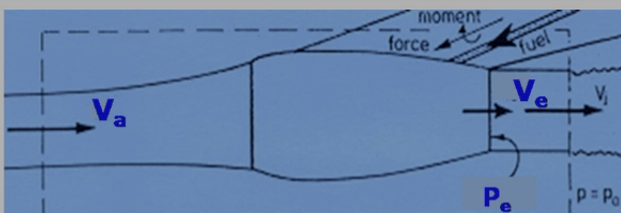
Aircraft Engine Component Matching: Dimensional Analysis

We are talking about off design operation of the jet aircraft engine that is what we talked about in the last class. In today's class, we will talk about matching of these engine components for various design and off design operating conditions, and to begin with of course, we will see how they are matched at least at the design point, when the engine is initially designed. So, today's lecture is about matching of engine components.


Let us take a look at some of the issues that are related to matching of the engine components. There are large number of components as we know we have intakes compressors, combustion chamber, turbine, and nozzle all of them put together constitute an engine. And we will have to see, how these various components are matched together into one single unit which we call a jet aircraft engine.

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- Any variable in the engine is expressed as a function of \dot{m}_f , P_a , T_a and V .
- Because the engine is self contained; fixing one set of engine non-dimensional parameters fixes all others.


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Let us take a look at a typical schematic of what is considered an aircraft engine, and how various parameters influence the matching of these components. If you have an engine that operates at let us say incoming flow velocity V_a , and outgoing velocity V_e . What happens is that we normally express a various functions of the engine in terms of the incoming pressure, and temperature and the velocity which is shown here. The engine performance is quite often expressed as function of these parameters along with the fuel flow \dot{m}_f . Now, the fact that the engine taken all these components together is a one single unit and it is expected to be and suppose to be a self contained unit. One set of engine parameters essentially influences and instantaneously fixes all the other parameters. So, if there is a change of parameter in anywhere in the engine it tends it influence the parameters at other components of the engine almost instantaneously.

So, this is where the matching comes in that we keep an eye on the change of these parameters. So, that when they tend to influence each other they influence each other in a matched manner and that is important issue that we are disusing in today's lecture.

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- Instantaneous cycle temperature ratio, T_{03}/T_{01} fixes the cycle pressure ratio π_c , the instantaneous fuel flow \dot{m}_f and the instantaneous rotational speed, n .
- Similarly in a multi-shaft engine the ratio between the shaft speeds, n_2/n_1 is also fixed by T_{03}/T_{01} .
- For most operating conditions of the engine the final propelling nozzle for the core flow and the bypass flow will be choiced, by design, for maximizing, thrust.
- Thus the engine responds to the inlet stagnation conditions but is unaware of the forward speed.

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Let us take a look at some of the parameters that fundamental parameters that we would need to keep our eyes on. The instantaneous cycle temperature ratio T_{03}/T_{01} tends to fix the cycle pressure ratio given the engine instantaneous fuel flow \dot{m}_f and the instantaneous rotational speed r_p in rpm. Now, all these things are to be matched; that means, these parameters which we call the performance parameters at any particular instant they are

connected to each other. Essentially, if any one of them changes the other three would automatically need to be changed or would change and that matching is an important issue at any given instant of operation of the engine. If we have multi spool of multi shaft engine the ratio between the shaft speeds n_2 by n_1 also is getting fixed by the temperature ratio T_0_3 by T_0_1 .

So, that is another issue if we have a multi spool engine; different spools run at different speeds and then they come into the picture again and they would also need instantaneous change to confound to the changes of the four fundamental parameters that we have talked about. The basic operating condition of the engine, the propelling nozzle that we have for the main cold flow or the hot flow and we have another nozzle for the bypass flow which is a cold flow. Typically, it is assumed that for most of the time of operation of the engine this is choked and this is by design and the purpose of doing this is by design is to keep the thrust as high as possible for every operating condition not that you get back from thrust all the time, but what you get is for that operating condition you get maximum thrust. So, keeping the flow choked or maximizing a mass flow ensures for that particular instantaneous operating condition you are getting the maximum thrust.

So, the engine always responds to the inlet stagnation condition; however, once the flow has gone inside the engine, it is a self contained unit they respond to the other parameters within the engine and they become somewhat unresponsive or unaware of the forward speed of the engine and the aircraft. So, once the engine starts operating and it starts moving, the internal parameters of the engine influence each other instantaneously are not so much as the forward speed of the aircraft.

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Non-dimensional Variables of the Engine

1) Consider mass flow of air through the engine. This can be written, as a function of the rotational speed N of the shafts


$$\dot{m}_a = f(N, P_{01}, T_{01})$$

or, in terms of turbine inlet temperature

$$\dot{m}_a = f(T_{03}, P_{01}, T_{01})$$

or, in terms of fuel flow rate

$$\dot{m}_a = f(\dot{m}_f, P_{01}, T_{01})$$

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So, that is another important issue that we need to keep our eyes on that the internal parameters are more influential rather than the forward speed of the aircraft. Let us look at: what are the non-dimensional variables of the engine that we would probably need to keep our eyes on and these non-dimensional variables are what really influence each other profoundly. So, we need to figure out what these non-dimensional parameters are. First let us consider the mass flow through the engine and this can be written in terms of \dot{m}_a , which can be written in terms of function of rotational speed and as we mentioned the inlet stagnation pressure and stagnation temperature, which contains the velocity field that it has come in with.

The mass flow can also be written in terms of the turbine inlet temperature; that means, they can be written in terms of the T_{03} , P_{01} and T_{01} or they can be written in terms of the fuel flow rate that is function of \dot{m}_f , P_{01} and T_{01} see in a sense all three of them are true that the mass flow through the engine does depend on all these parameters put together; that means, not only P_{01} , T_{01} with which of course, the flow came in, but the other three important parameters that we talked about the rotational speed the turbine inlet temperature and the fuel flow rate all three along with the initial two parameters influence the mass flow rate through the engine.

So, we need to keep our eyes on all these parameters the inlet conditions ,the speed, the turbine inlet temperature and the fuel flow rate all of them together influence what the mass flow through the engine should be at any given instant.


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The non-dimensional mass flow rate is derived from dimensional analysis (e.g. Buckingham Π Theorem), as

$$\bar{\dot{m}} = \frac{\dot{m}_a \sqrt{c_p \cdot T_{01}}}{D^2 \cdot P_{01}}$$

Where, D is a characteristic diameter of the engine, typically the diameter of the inlet of the fan and D^2 denotes a representative area

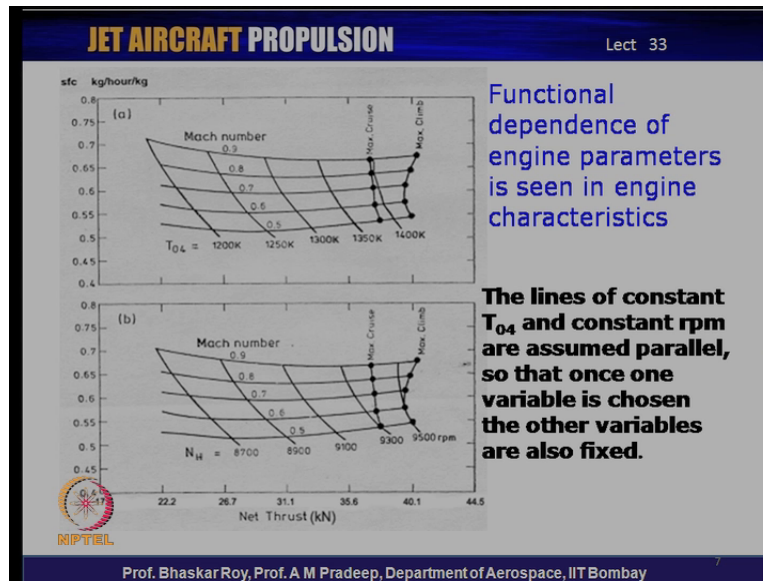
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So, the non-dimensional mass flow can be derived one can sit down and derive it is not a very big problem really from the dimensional analysis, which many of you may have done in your other courses; which is popularly often known as Buckingham pi theorem and if you do that then the non-dimensional mass flow rate can be derived and written down as $\bar{\dot{m}}$; as the non-dimensional mass flow rate in terms of \dot{m}_a into root over as C_p into T_{01} and that divided by D^2 into P_{01} . Now, T_{01} , P_{01} are the inlet condition C_p is the specific heat, and D is the characteristic diameter of the engine.

Typically this is likely to be the diameter of the inlet of the engine and hence this essentially D^2 essentially represents a represented if area of the engine and typically it comes from the inlet area of the fan, which is typically also a likely to be the maximum diameter of the engine at any given time.

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Now, some of the issues related to the non-dimensional mass flow can be now looked at: from slightly different point of view. We can see here in this diagram that the functional dependence of various parameters can be plotted together. We had a look at this diagram in the last lecture also the specific fuel consumption and the net thrust the two axis and the variables in the upper and the lower pictures are shown here. In the upper picture we have the turbine inlet temperature as variable in the lower picture, we have the rotating speed as variable and as we were just discussing all of them together actually impact on what is happening inside the engine. Now, what this picture shows: is that the lines of constant turbine temperature and the constant speed, in the upper and the lower diagrams are can be considered to be essentially parallel to one and other and if that is. So, if that is accepted then once one variable is chosen, the other variables are also fixed

So, diagrammatically it is shown here that the fundamental parameters; that we are talking about essentially influence each other and in this diagram, it is captured together in one characteristic plot.

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Practical Normalizing of Parameters
 For mass flow of air c_p is constant and D^2 is constant for a given engine.

it has units.
$$\bar{m} = \frac{\dot{m}_a \cdot \sqrt{T_{01}}}{P_{01}}$$

Normalized fuel flow the abbreviated form is derived,

$$\bar{m}_f = \frac{\dot{m}_f}{\sqrt{T_{03}} \cdot P_{03}}$$

Non-dimensional speed is abbreviated : $N/(T_{02})^{1/2}$

The corrected mass flow (kg/s) is
$$\bar{m} = \frac{\dot{m}_a \cdot \Theta}{\delta}$$

Where. $\Theta = T_{02}/T_{02ref}$, $\delta = P_{02}/P_{02ref}$,
 and T_{02ref}, P_{02ref} are STP

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So, the dependence or the functional dependence of the engine parameters can be captured with a few simplifying assumptions. Let us take a look at: how do we go about normalizing the parameters, we had a look at the mass flow of the engine as a non-dimensional mass flow; however, that non-dimensional mass flow quite often is not used because once an engine is made and once operating condition is fixed C_p is constant and D^2 is constant. Now, D^2 we mentioned is a characteristic area of an engine at typically at the inlet to the engine near the fan and C_p is the operating specific heat of the operating working medium. Now, if they are held constant and taken out of a definition of the mass flow what we get here is a normalized mass flow of air, which can be written down in terms of just \dot{m}_a into root over T_{01} by P_{01} .

Now, this is what you would see in many of the books and we will also look at some of the diagrams in which this normalized parameter is used as mass flow parameter and not the non-dimensional parameter. So, this normalized parameter is not non-dimensional it has units and it has a which can be written down depending on what unit your using. Correspondingly the normalized fuel flow can also be a written down or and can be derived in exactly the similar manner using dimensional analysis as \bar{m}_f that will be equal to \dot{m}_f divided by root over T_{03} into P_{03} and correspondingly the non- dimensional speed is can be written down as N divided by root over T_{01} root over T_{01} .

Now, this essentially gives us the so called a normalized values, which quite often people use; however, quite often for mass flow configuration a corrected mass flow which has units of normal kilo grams per second can be written down in terms of $\dot{m} \sqrt{T_0}$ into capital theta divided by a delta. Now, capital theta is the temperature ratio of the operating temperature to the inlet to the let us say compressor to the reference temperature. Now, this reference temperature is often as per the standard temperature and pressure and hence these values are essentially referring to the standard temperature and pressure as used in international usages.

The density we are talking about is again with respect to a reference density and again with respect to the standard temperature and pressure. These values are normally used internationally as standard values. So, one can define corrected mass flow with reference to those standard temperatures and pressures. Hence, all mass flows under all operating conditions can be corrected for one standard value of temperature one set of standard value of temperature and pressure and this is pretty much a done thing in many of the engine characteristic plots or graphs that are used for characterizing the engines or the components of the engines like compressors or turbines. So, one is you can normalize the values another is you can use corrected a mass flows for correcting it to standard temperature and pressure.

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Matching Procedure :

- 1) Select operating point (Altitude, Flight Condition)
- P_a , T_a , and M
- 2) From the ambient condition obtain - P_{01} , T_{01}
- 3) Select max Turbine entry temp. T_{03}
- 4) Select Rotational speed, N – then obtain
- 5) Obtain - $N/\sqrt{T_{01}}$, and $N/\sqrt{T_{03}}$
- 6) Select a compressor pressure ratio , $\pi_{0c} = P_{02}/P_{01}$
- 7) Obtain mass flow parameter,
$$\frac{\dot{m} \sqrt{T_{01}}}{P_{01}}$$
- 8) The parameters in (5), (6) and (7) completely define the compressor operation point

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Let us look at: what are the possible ways one can go about creating a matched engine the first step of course, is selecting the operating point, which is often the altitude and the flight condition many of the transport aircraft. For example, engines might be having operating

point which is a takeoff condition, but many of the military aircraft the operating point selected here could be a flight condition with a very high Mach number at some altitude. So, that needs to be selected first to begin the matching procedure.

Now, from the ambient condition; that means, the above three figures: pressure, temperature and Mach number. One can get the total pressure at the entry to the compressor and that is P_{01} and the total temperature T_{01} then one needs to select the maximum turbine entry temperature, which is often selected from the state of art of turbine design and depending on what is the temperature the turbine can withstand that is one of the considerations. Another of course, is a cycle design which one needs to do a priori and from these two one gets an idea what the value of T_{03} should be for, which we would be proceeding towards creating a matched engine.

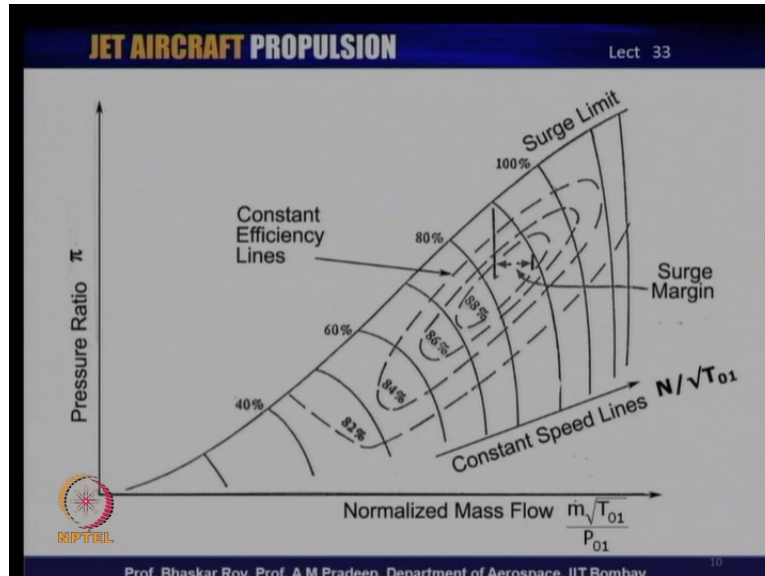
Next is the rotation speed, which also comes from the basic engine considerations various kinds of engines, having various kinds of typical rotational speeds? As, we have seen before for turbo shaft they could be of the order of thirty forty thousand r p m, whereas for military aircraft engine. They are of the order of fifteen to eighteen thousand r p m, whereas for transport aircraft engine civil aircraft engines. They are of the order of ten thousand r p m for the H P spools the L P should be lower than that.

Then of course, you get two normalized speed parameters: one with reference to the compressor temperature and other with reference to the turbine temperature. The first one is normally used for characterizing the compressor performance; the second one is normally used for characterizing the turbine performance. One needs to select the compressor pressure ratio, which normally would come from detail cycle analysis and the cycle design and that should fix the value of compressor pressure ratio P_{02} by P_{01} and then from, which one can find out the mass flow parameter; the normalized parameter, which is what one should be using for characterizing the various engine and compressor performances.

So, these parameters together now, define what the compressor operation point would be. As, we have discussed before you have done in compressor chapter and we have discussed in the earlier lectures that compressor performance has the minimum operating zone or a range of a mass flow. Hence, compressor operation is one of the first things that we need to have fixed before fixing the others, because typically every almost every engine the range of operation is fixed

by the compressors range of operation and hence we need to fix the compressor operation before the others.

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If, you look at the compressor map as we were talking about we see the mass flow parameter that we have defined, we see the pressure ratio on the y axis and then the constant speed lines which are now in terms of N by root over T_{01} . So, these speed lines are taken out the temperature, which could vary from operating point. And of course, typical compressor a performance graph is shown over here. So, once we have these parameters fixed, we would know where the design point is likely to be, which is somewhere on the 100 percent line, and that would define where our matching should start. And rest of the operating points as we have seen before are the so called off design operating point.

First, we have to do the matching at the design point and then we need to do matching at many of the off design important off; design operating points like cruise. So, that we have a matched engine and we have a matched compressor and one of the things, which I was just saying that typically compressor has the lowest operating range in terms of mass flow most of the other components like intakes or turbines or nozzles would have higher operating mass flow ranges. So, quite often the engine gets restricted by the compressor range rather than any other component.

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9) Actual mass flow through the compressor is :

$$\dot{m}_c = \frac{\dot{m}_c \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{T_{01}}$$

10) The turbine mass flow is :

$$\dot{m}_T = \dot{m}_c \cdot (1 + f - b)$$

where, f - is fuel/air ratio, b = air bleed
and Turbine entry Pressure, $P_{03} = P_{02} (1 - \Delta P_{cc})$

11) Based on the actual mass flows through the compressor and the turbine, the work done for these mass flows need to be equated (turbojet engine). The same may be done spool-wise.

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Now, if we proceed towards with matching steps the actual mass flow through the engine can be written down in terms at any given instant can be written down in terms of \dot{m}_c , which is the flow through the compressor and that can be now corrected for the actual operating P_{01} and T_{01} from the normalized value and then the turbine mass flow can be written in terms of, whatever the fuel has been pumped in or injected into the combustion chamber and Certain amount of air that may have been bleeding out from the compressor towards normally done from towards the rear of the compressor for various services. If, we do that we get the turbine mass flow and then the turbine entry pressure can be written down in terms of P_{03} , which would be taken in to account the pressure loss in the combustion chamber and then that gives us the turbine entry pressure.

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12) Work equivalence :

$$\dot{m}_C \cdot \bar{W}_C = \dot{m}_T \cdot \bar{W}_T \cdot \eta_{\text{mech}}$$

for a turbojet engine, or for a spool, i.e.


$$\dot{m}_{LPC} \cdot \bar{W}_{LPC} = \dot{m}_{LPT} \cdot \bar{W}_{LPT} \cdot \eta_{\text{mech-LP}} \quad \text{for LP spool}$$

$$\dot{m}_{Fan} \cdot \bar{W}_{Fan} = \dot{m}_{LPT2} \cdot \bar{W}_{LPT2} \cdot \eta_{\text{mech-FT}} \quad \text{for fan-turbine}$$

or

Where η_{mech} is the Mechanical efficiency of the shaft

$$\dot{m}_{HPC} \cdot \bar{W}_{HPC} = \dot{m}_{HPT} \cdot \bar{W}_{HPT} \cdot \eta_{\text{mech-HP}} \quad \text{for HP spool}$$

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Now, based on the actual mass flow through the compressor and turbine. We can now have the work balance or work done for these mass flows to be equated. So, that is the work balance that we need to do for every engine and this may also need to be done spool wise, if we have multi spool engines. So, if we do that we get a work equivalents to begin with the on the left hand side, you have the compressor work and on the right hand side you have the turbine work together matched with the mechanical efficiency of the shaft connecting the turbine and the compressor. If, we have a simple turbo jet engine for a spool for example, if we take the L P spool; the L P spool work of the compressor is on the left hand side and on the right hand side.

You we have the L P turbines work again multiplied by the mechanical efficiency of the L P shaft. If we have let us say a fan turbine then the work done by the fan, which is on the left hand side, has to be matched with the other L P spool. For example, if we have which runs the fan and the mechanical efficiency of the. So, called fan turbine and of course, if we have the HP spool which is the core of the engine where HP compressor work on the left hand side had to be matched to the H P turbine work on the right hand side again supplied through the shaft which is a mechanical efficiency of the shaft. Now, all the spools now are having matched work between the turbine and the compressor and we need to remember that this matching has to be done at every instant of the working of the engine.

At every instant of the working of the engine all these spools must have matched work between the compressor and the turbine. If, we do not have a matched work what we are going to have is that particular spool either will tend to over speed; if the turbine is supplying more work; if the compressor requires more work and turbine is unable to supply that work. It will settle down to a lower operating speed or rotating speed. So, every spool must have matching of this kind at every instant of working of the engine.

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13) Compressor /Fan specific work (per unit mass flow)

$$\bar{W}_c = \frac{C_{p\text{-air}} \Delta T_{012}}{\eta_c} = \frac{C_{p\text{-air}} T_{01}}{\eta_c} \left[\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma_{\text{air}}-1}{\gamma_{\text{air}}}} - 1 \right]$$

14) Turbine Specific work (per unit mass) :

$$\bar{W}_T = \eta_T \cdot C_{p\text{-gas}} \Delta T_{034} = \eta_T C_{p\text{-gas}} T_{03} \left[1 - \frac{1}{\left(\frac{P_{03}}{P_{04}} \right)^{\frac{\gamma_{\text{gas}}-1}{\gamma_{\text{gas}}}}} \right]$$

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If you look at: the work to be done for the compressor or fan, which we talk about specific work and that is given in terms of C p air into delta T and this is written down in terms of the pressure ratio and these are the relations, which we have done in your compressor chapter earlier. So, we are invoking those relations here again in the matching procedure. The turbine specific work also can be written down in terms of the turbine efficiency the C p, now can be used for the gas and the turbine temperature change and again using the turbine pressure ratio. We can write down the work connecting it to the turbine pressure ratio.

So, what we are trying to do is we are trying to work write down the instantaneous work that is to be done by the compressor and the turbine and the instantaneous pressure ratio of the compressor and the instantaneous pressure ratio of the turbine would have to be either measured or computed and the matching needs to be done between these two for the instantaneous values of the pressure ratio operational at that instant across the compressor and across the turbine.

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15) Turbine mass flow parameter can be determined from :

$$\frac{\dot{m}_{\text{gas}} \sqrt{T_{03}}}{P_{03}} = \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{P_{02}} \cdot \frac{P_{02}}{P_{03}} \cdot \frac{\dot{m}_{\text{gas}} \sqrt{T_{03}}}{\dot{m}_{\text{air}} \sqrt{T_{01}}}$$

16) The net turbine – compressor excess power (if any) may now be decided by:

$$P_{\text{engine}} = \dot{m}_{\text{gas}} \cdot \bar{w}_T - \frac{\dot{m}_{\text{air}} \cdot \bar{w}_c}{\eta_{\text{mech}}}$$

Where η_{mech} is the Mechanical efficiency of the shaft

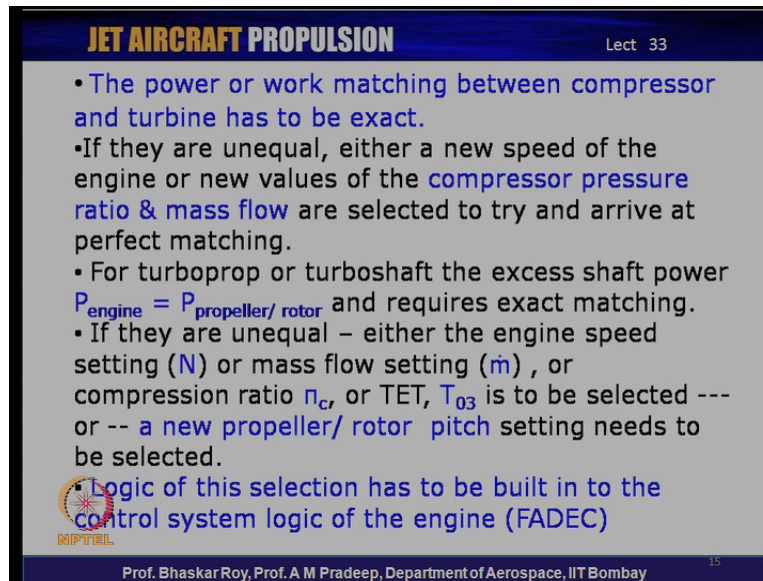
For pure turbojet $P_{\text{engine}} = 0$,
 For multi-spool turbofan $P_{\text{spool}} = 0$

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The turbine mass flow parameter, now can be written down in terms of if we use the matching procedure starting with the on the left hand side: you have the turbine normalized mass flow and on the right hand side: you have first the compressor normalized mass flow and then multiply that with the pressure ratio across the compressor, then pressure ratio across the combustion chamber and then the mass flow ratio between the compressor and the turbine that is air and gas and then the temperature ratio across the engine cycle temperature ratio. So, to say this together gives us the turbine mass flow parameter the so called normalized mass flow parameter operational turbine.

The next step would be to find that if at all there is an excess power that is available or that is somehow happening between the turbine and the compressor. So, the net turbine compressor excess power can be written down in terms of the actual power, in terms of the \dot{m} dot gas, which is the turbine mass flow into the work done by the turbine specific work and the \dot{m} dot air, which is the mass flow through the compressor and the work done by the compressor divided by the mechanical efficiency of the shaft. Now, for a pure turbo jet engine, it is necessary that every instant this net turbine compressor power is zero; that means, there is no net power that needs to be catered to and this should be zero; that means, there should be exact matching between the turbine and the compressor in case of multi spool turbo fan engine each spool LP spool as well as HP spool the net power should be zero. So, in a typical turbo jet or turbo fan there is no scope for any excess power available from the net turbine compressor matching.

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- The power or work matching between compressor and turbine has to be exact.
- If they are unequal, either a new speed of the engine or new values of the compressor pressure ratio & mass flow are selected to try and arrive at perfect matching.
- For turboprop or turboshaft the excess shaft power $P_{\text{engine}} = P_{\text{propeller/rotor}}$ and requires exact matching.
- If they are unequal – either the engine speed setting (N) or mass flow setting (\dot{m}), or compression ratio π_c , or TET, T_{03} is to be selected --- or -- a new propeller/rotor pitch setting needs to be selected.
- Logic of this selection has to be built in to the control system logic of the engine (FADEC)

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So, the power or the work matching between the compressor and turbine as we see now from these calculation steps need to be exact at every instant of operation of the engine and if they are unequal either a new speed of the engine or the spool or a new set of values of compressor pressure ratio and mass flow are to be selected to try and arrive at perfect matching. So, one when one is doing the matching exercise at the design or at the time of design of the engine. Either you choose a new rotating speed of the engine or you opt for a new compressor pressure ratio and mass flow which means you need to redefine your cycle definition and your redefine your cycle analysis. So, that you arrive at a perfect matching between the compressor and turbine for turbo prop or turbo shaft the excess shaft power is typically the power that you need to run the propeller or the rotor is of course, for the shaft engines.

So, the P_{engine} here is not going to be zero. Now, this has to be the power that you supply to the propeller or rotor and this also needs exact matching. Now, this power that is required by the propeller and a rotor is to be decided by the propeller rotor designer and is to be supplied to the engine designer or the propeller rotor designer has to design exactly for the amount of power that is available to him for a selected engine. So, this again requires exact matching at every instant of operation of the engine.

Again, if they are unequal either the engine speed will settle down to a different value or the mass flow setting will go on to different mass flow or the compression ratio or the turbine

inlet temperature are to be newly selected or when you have a propeller or a rotor a new propeller rotor, which setting needs to be selected. So, which means a propeller and rotor should have a variable pitch mechanism available with it for selection of pitch depending on the power availability from the engine and as we know most of the propellers and rotors operational, today do have variable pitch mechanism available with them.

All the things that we have been talking about: So for that means, selection of various component parameters they all need to be built into the control system logic of the engine, which today is the standard faced control system and this requires to be built into the control logic of the engine and that is how the engine is controlled the various parameters of the engines are controlled the fuel flow is controlled; the nozzle area is controlled and if you have variable stagger compressor stators they are also controlled using this logic which is built into the control system of the engine.

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In case of a choked nozzle, the actual mass flow is invariant with change of other parameters:

$$\frac{\dot{m}_{\text{gas}} \sqrt{T_{03}}}{P_{03}} = \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \cdot \frac{P_{01}}{P_{02}} \cdot \frac{P_{02}}{P_{03}} \cdot \frac{\dot{m}_{\text{gas}}}{\dot{m}_{\text{air}}} \sqrt{\frac{T_{03}}{T_{01}}} = \text{const}$$

Assume that pressure ratio across the combustion chamber, $P_{02}/P_{03} = \text{constant}$ and $\dot{m}_{\text{air}} = \dot{m}_{\text{gas}}$

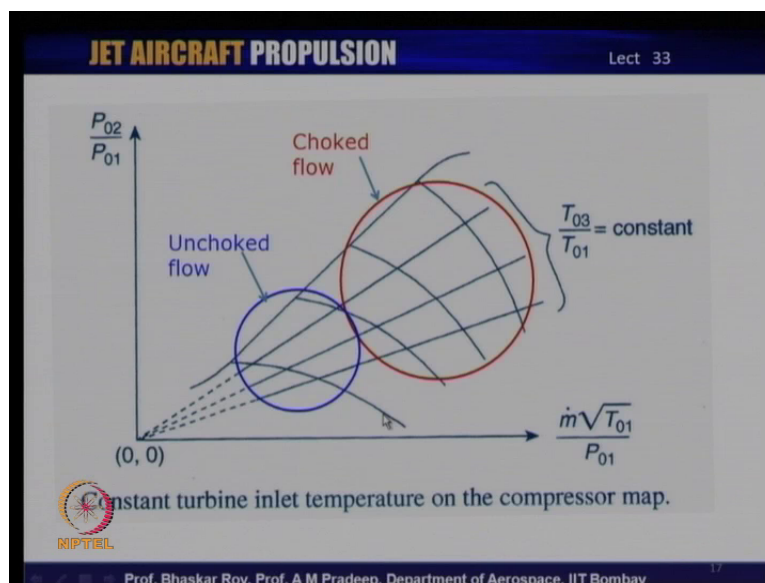
$$\frac{P_{02}}{P_{01}} = K_1 \cdot \frac{\dot{m}_{\text{air}} \sqrt{T_{01}}}{P_{01}} \sqrt{\frac{T_{03}}{T_{01}}}, \quad \text{where } K_1 \text{ is a constant}$$

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Now, if we have a choke nozzle the actual mass flow is invariant with the change of other parameters that is what is of course, the basic understanding of a choke nozzle. So, the flow through the turbine for example, is now going to become constant the earlier part we had written down in the earlier slide and this now becomes a constant value and as we have discussed before the designer tries to design an engine where for most of the time the nozzle is choked most of the operation of the engine the nozzle is indeed choked which means its operating at instantaneous maximum mass flow.

Now, assume that the pressure ratio across the combustion chamber that is P_{02} by P_{03} is also constant and that the compressor mass flow and the turbine mass flow are also equal to each other. Then, what we can write down in the simplified form is that P_{02} by P_{01} is equal to K_1 into the normalized mass flow into the cycle temperature ratio, where K_1 is some constant. So, one can relate the pressure ratio of the engine to the normalized mass flow of the engine to the cycle pressure, temperature ratio of the engine through one single constant K_1 . So, this is a simplified version or simplified way of matching all the three primary parameters of the engine.

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If we look at the diagram which we had looked at in the last class also the pressure ratio versus normalized mass flow versus the cycle temperature ratio and the constant temperature ratio lines are the linear lines cutting right through the compressor map. So, to say and these lines of course, are the speed lines. So, what we see here is this large portion over here, the engine is likely to be operating at choked flow condition whereas, at the lower compression ratios and the lower mass flows the engine is likely to be operating under unchoked flow condition. So, by design a large part of the engine is a large time of the engine operation is done during with choked flow condition through the nozzle and turbine and the compressor is a not choked, but it simply says that the engine is operating under choked flow condition and that is shown here on the compressor map.

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assume that cycle temperature ratio is constant, then

$$\frac{P_{02}}{P_{01}} = K_2 \cdot \frac{\dot{m}_{air} \sqrt{T_{01}}}{P_{01}}, \quad \text{where } K_2 \text{ is another constant}$$

For a straight and level cruise flight

$$\frac{P_{02}}{P_{01}} = K_3 \cdot \sqrt{\frac{T_{03}}{T_{01}}}, \quad \text{where } K_3 \text{ is another constant}$$

If T_{01} / T_{03} is held constant for a cruise flight

$$\frac{P_{02}}{P_{01}} = K_4, \quad \text{where } K_4 \text{ is another constant}$$

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Now, if we move forward and see that the cycle temperature ratio is held constant. If it is held constant then the pressure ratio can be written down in terms of K_2 into the normalized mass flow where K_2 is another constant and the cycle temperature ratio T_{03} by T_{01} has now been taken out of the equation. Now, for a straightened level cruise flight, we can say that the pressure ratio cycle pressure ratio can be related to the cycle temperature ratio through another constant called K_3 ; if however, T_{01} by T_{03} that is a temperature ratio is indeed also held constant for a cruise flight when the mass flow parameter is expected to be constant. So, that is been taken out of the equation and now if we say T_{01} by T_{03} that is a cycle temperature ratio is also held constant for a cruise flight then the P_{02} by P_{01} that is your pressure ratios cycle pressure ratio then also becomes a constant.

So, through a few simplifications, we can see that the matching of the various primary parameters the pressure ratio, the temperature ratio and the normalized mass flow can be related to each other through simple constants K_1 , K_2 , K_3 and K_4 depending on your operational point. So, these are slightly simplified to show that during matching one can arrive at some very straight forward parametric relationship between the fundamental parameters or what we call functional relationship between the fundamental parameters.

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Off-Design Matching of Turbojet Engine

The Intake delivery Pressure and Temperature can be found from

$$\frac{P_{01}}{P_a} = \left(1 + \eta_I \cdot \frac{\gamma_{air} - 1}{2} M_a^2 \right)^{\gamma_{air}/\gamma_{air}-1}$$
$$\frac{T_{01}}{T_a} = \left(1 + \frac{\gamma_{air} - 1}{2} M_a^2 \right)$$

- Higher the flight Mach number, M_a higher would be P_{01} and T_{01} at any constant altitude
- For constant flight Mach number, M_a - the P_{01} and T_{01} decreases with increase of altitude & vice versa

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If you look at the off design matching of a typical turbo jet engine; if you start from the beginning the steps that you have done before in great detail in the earlier lectures. So, we will invoke all those steps over here again one by one at the intake for example, the pressure that is developed through the intake can be written down in terms of the intake efficiency and the mass flow with which the flow is coming in and using the isentropic relationship with efficiency built into it gives us the P_{01} that is the compressor entry pressure. Similarly, we can get the compressor entry temperature from the ambient temperature with which the flow is coming in using the intake flow conditions.

Now, at higher flight Mach number m the value of P_{01} and T_{01} would be higher and higher at any given altitude where P_a and T_a are constant. So, higher the Mach number at which you fly your P_{01} and T_{01} are going to be higher and higher. On the other hand for a constant flight mach number m_a the P_{01} and T_{01} decreases with a increasing altitude and vice versa; that means, as you go higher up in the altitude your P_a and T_a are going down. So, your if your mach number is constant your P_{01} T_{01} are going to come down and of course, vice versa. So, these are the intake conditions with which you start your turbo jet engine configuration.

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- The Ram pressure development in the intake increases/ decreases the compressor inlet and then compressor outlet pressure.
- It then increases / decreases the turbine inlet / outlet pressure
- Thus the pressure ratio across the nozzle increases / decreases
- At high nozzle pressure ratio, the flow is choked – it is then independent of nozzle pressure ratio – and hence from flight speed
- This fixes the turbine operating point w.r.t nozzle choked condition.

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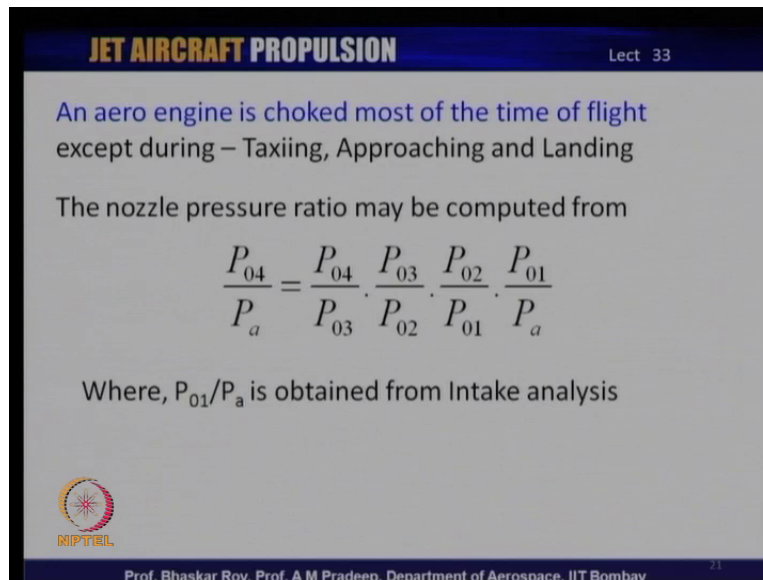
Now, the ram pressure development in the intake which increases and decreases the compressor inlet and then the compressor outlet absolute values of the pressures. Now, these increases and decreases also the turbine inlet and outlet absolute values of the pressures and that is the pressure ratio across the nozzle increases and decreases. So, the absolute value of the pressure delivered to the nozzle entry also increases and decreases with the ram pressure that is happening across the intake. We see here that the absolute values also are important because that absolute value at the intake to the inlet to the nozzle sets up the nozzle pressure ratio, which then operates according to the pressure ratio loss. So, the absolute values also need to be computed and figured out.

As the nozzle high pressure ratio is if the nozzle pressure ratio is high, the flow is choked and it is then independent of the nozzle pressure ratio. So, as soon as it reaches a choking pressure ratio then from there onwards it does not matter what the nozzle inlet pressure is any more and now from there onwards it does not matter what the flight speed is any more hence, once it is choked it is independent of the forward speed of the aircraft and this is what I mentioned earlier in this lecture.

So, which means that most of the engine designers would like to design in such a manner that for most of the operation of the flight the nozzle is operating in choked flow condition, which means the nozzle is independent of the flight speed of the aircraft. Now, that allows us also to fix the turbine operating point with respect to the nozzle choked condition which we

have done in the last class; that means, the turbine and the nozzle have to be matched to each other and this matching requires that if the nozzle is choked it easier to match the turbine with such a choked nozzle.

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An aero engine is choked most of the time of flight except during – Taxiing, Approaching and Landing

The nozzle pressure ratio may be computed from

$$\frac{P_{04}}{P_a} = \frac{P_{04}}{P_{03}} \cdot \frac{P_{03}}{P_{02}} \cdot \frac{P_{02}}{P_{01}} \cdot \frac{P_{01}}{P_a}$$

Where, P_{01}/P_a is obtained from Intake analysis

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So, we see that for most of the time of the operation of the aero engine; it is operating in choked flow condition few times when it is not choked is when the engine is aircraft is taxing or when it is approaching for landing and the landing itself. These are the periods during which the engine is actually unchoked and operating under a low thrust making and low compression ratio and other operating conditions. The nozzle pressure ratio, which we are talking about and which needs to be taken to choking condition can be simply computed from the various pressure ratios that we have looked at the intake pressure ratio; the compressor pressure ratio; the combustion chamber pressure ratio and of course, the turbine pressure ratios all of them lined up together indeed gives you the nozzle pressure ratio and we intend to ensure that for most of the time of operation that remains at choking value. So, as we see all the parameters are kind of connected to each other through simple parametric analysis.

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Thrust of the engine may be written as:

$$F = m_5(V_5 - V_{flight}) + A_5(P_5 - P_a),$$

where flight speed is given by, $V_{flight} = M_a \sqrt{\gamma \cdot R \cdot T_a}$

The gas exhaust speed V_5 depends on the flow condition at nozzle exit, and with choking reaches the maximum for that condition. For example, some time during the climb operation if the nozzle is choked, it will remain so during the climb, with continuous fall in ambient pressure P_a .

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The thrust of an engine can now indeed we have written down in terms of simple thrust equation that we have done earlier in the course in terms of the mass flow through the engine and the velocity the exit velocity which is written here as V_5 is the exit velocity with which is coming out and the difference of that with the flight velocity of the aircraft and the momentum thrust in this equation is also supplemented by the pressure thrust which comes out of the nozzle exit pressure and if there is a residual nozzle exit pressure that has been gives us a pressure thrust and that together gives us the instantaneous thrust of the engine. So, this is going to be the instantaneous thrust of the engine as related to the instantaneous flight speed of the aircraft.

So, if the aircraft is operational such that the gas exhaust speed V_5 or V_e depends on the flow condition at the nozzle exit and when it reaches choking that is the maximum mass flow that you can have through the engine and all if all the parameters are then operational in unition as an unit then we instantaneously we get the maximum possible thrust for that particular operating condition and hat is what I mentioned earlier that engine designer tries to ensure that any given instant the engine is producing preferably the maximum possible thrust which as I mentioned is not the maximum thrust of the engine, but for that given operating condition that is the maximum possible thrust.

So, most of the time the engine should be operating under conditions that give maximum thrust for that particular operating condition that is maximizing the use of the engine for the

purpose for which it is created during the climb operation if the nozzle is choked it will remain.

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During the entire climb with continuous fall in ambient pressure now, this is something which we just saw that if you are in the last equation. If your ambient pressure is falling your all the other pressures will start to fall actually on the other hand your flight mach number is increasing. So, the pressures may get restored we need to ensure that during this entire flying operation that this nozzle pressure ratio remains at choked value. So, that we have maximized thrust creation during the entire climb operation through all these pressures that are operational inside the engine.

So, this is another thing which the engine designer needs to ensure that all the units are the sub components the intake the compressor the combustion chamber the turbine and nozzle and all of them are matched together in one unit such that for example, during the climb they continue to produce maximum thrust during the entire flight operation even when the altitude is changing the flight mach number is changing but the engine continuous to produce maximum instantaneous thrust during the entire operation of the climb.

Now, this is; this can be done only if you have a matched engine only, if you have an engine that is taken care of this continuous variation of the parameters that invariably happen and during the entire operation then you can have maximum instantaneous thrust production by the engine.

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For choked nozzle:


$$F = m_5(V_5 - V_{flight}) + A_5(P_5 - P_a),$$

where exhaust velocity is given by,

$$V_5 = \sqrt{\gamma_{gas} \cdot R \cdot T_5} = \sqrt{\frac{2\gamma_{gas} \cdot R \cdot T_{04}}{\gamma_{gas} + 1}}$$

For un-choked nozzle :

$$V_5 = \sqrt{\gamma_{gas} \cdot C_{p-gas} \cdot (T_{04} - T_5)} = \sqrt{2C_{p-gas} \cdot T_{04} \left[1 - \left(\frac{P_a}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

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So, this is what matching essentially accomplishes that it produces maximum thrust instantaneous thrust at any given point of operation. If, we look at a choke nozzle operation, the exhaust velocity is given by the instantaneous temperature that is operation at the exit of the nozzle. This is assuming that the flow there is choked and it is a convergent nozzle we get a sonic velocity as the exhaust velocity. For an unchoked nozzle, we assume that the flow has continuously accelerated towards maximum value and hence we use the isentropic relationship that you are familiar with using the maximum nozzle pressure ratio; that means, assuming that the nozzle. Finally, exhaust flow to the ambient at ambient pressure; that means, maximum acceleration or maximum change of velocity through the nozzle has taken place to the ambient pressure and this is the unchoked nozzle that one can get without getting choked.

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For choked nozzle:

$$P_5 = P_c = P_{04} \left(1 - \frac{1}{\eta_{nozzle}} \cdot \frac{\gamma_{gas} - 1}{\gamma_{gas} + 1} \right)^{\frac{\gamma_{gas}}{\gamma_{gas} - 1}}$$

For un-choked nozzle :

$$P_5 = P_a$$

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Now, for the choked nozzle: We know that it reaches the critical pressure written here as P_c . This is given in terms of the nozzle operating condition starting with: let us say P_{04} coming from the turbine and then the nozzle efficiency and using the component specific ratio of the gas operational at the nozzle. We can find the choking pressure that is operational at the nozzle exit. For the unchoked nozzle, as I mentioned a typically, it is assumed that it is a fully accelerated to the ambient pressure P_a .

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- The engine performance seems to be decided by engine normalized speed $N/\sqrt{T_{01}}$, but the maximum performance is capped by the engine speed, N_{max} .
- This max speed is decided by stress limits of rotating components.
- At $N_{max}/\sqrt{T_{01}}$ as the ambient temperature increases thrust will decrease, but engine speed cannot be increased much more.
- The engines are often designed for 15°C , 288K . They are bound to give lower performance in tropical (hot) atmospheres & higher performance in colder climates.

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Now, if you do that the engine performance, as you can see now over the entire procedure that we have gone through is decided by engine normalized speed to begin with N by root over T_0 , but the maximum performance is kept by the engine maximum speed design speed that is N_{max} , for which the engine components have been designed taking into account the structural and many other issues and hence there is no way an engine is going to operate beyond N_{max} . So, the engine maximum speed is decided typically by the stress limits of the rotating components of compressor and turbine and in turbine you have an additional issue of high temperature and the issues like creep also come in.

So, those things finally, decide what the maximum speed rotational speed of a particular spool should be combination of compressor and turbine and then this N_{max} divided by root over T_0 is the normalized maximum speed parameter, but as the ambient temperature increases this thrust will decrease and the engine speed cannot be increased any more. Now, this is where the problem is we know as the ambient temperature increases in a hot day or in tropical countries like India, the engine thrust starts falling to compensate for that the only way you can do that is to engine **the rotate** the rotating components at a higher speed to increase mass flow and to increase the engine performance, but if it is already reached the maximum speed it cannot be increased anymore. The control algorithm of the engine will stop it from going to higher speeds and hence the engine thrust cannot be increased any more.

So, engine does reach its maximum thrust under those operating conditions. Most of the engines, even today tend to be designed for the international standard temperature and pressure which is 15 degree centigrade and 288 K. Hence, they are bound to give lower performance in tropical hot atmospheres as in countries like India and correspondingly slightly higher performance in the colder climates in the northern hemisphere. This is something, which is on unavoidable at the movement, because most engines do tend to use these parameters as the starting design parameters. This is something, which requires to be factored into the normalizing parameters that we have used.

Hence, we have defined the corrected mass flow and such corrections need to be applied to ensure that during the process of matching. The matching is also done with reference to the reference temperature and pressure, which we talked about earlier, because those are the values at which the engine is indeed designed. So, we see that we have a large number of issues here that need to be taken care of during the process of engine design all the components of the engine would have to be matched together, and only then you have a

matched unit which we call engine, which supplies power or thrust to the aircraft. And to cater to the aircraft at every instant of the flight of the aircraft, the instantaneous performance of the engine has to be a matched performance of all the components inside the engine.

So, we need to ensure that happens by design and do not live it to chance. So, these are the simple procedures that is innovatively needs to be done during the process of engine design. In the next class, we will look at the component matching, and how these components sometimes need to be sized to ensure good matching, and this is what we will do in the next class, the component matching and sizing of the engine components of aircraft engines.