Jet Aircraft Propulsion

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Lecture No. # 31
Tutorial – 5

Hello and welcome to lecture number 31 of this lecture series on jet aircraft propulsion. We have covered quite some distance in terms of discussing various aspects of jet engines. We have discussed in detail about the ideal and real cycles of jet engines. Subsequently, we have taken up individual components like the intakes, the compressors and fans, combustion chambers, turbines and finally the nozzle. And so, all these components as individual entities where analyzed. And we have discussed about the geometric construction of these different components and also different performance parameters which are associated with these components.

And at the last few lectures, we were discussing about two of the other important components, which ofcourse form part of the whole engine; the air intakes and the diffuser and the nozzles. We have in detail discussed about different types of air intakes the subsonic, the supersonic and the variable geometry intakes and so on. We have also discussed about performance parameters associated with air intakes like distortion pressure recovery and so on. Subsequently, we also discussed about the nozzles; what are the different types of nozzles; how is the subsonic nozzle different from a supersonic nozzle and why should there be difference between these two types of nozzles and again how to analyze performance of different nozzles.

So, these are these were some of the topics that we have been talking about in the last few lectures. So, I think it is just the right time that we also start discussing about how we can solve some problems on intakes and nozzles; that is based on what we have discussed so far. Can we now take up a few numerical problems and try and solve them. So, that we get a better idea of what is actually happening in in the nozzle and and the diffuser and so that, we get some feel of the numbers associated with diffusers and nozzles or intakes and nozzles. So, today's lecture, we are going to devote entirely to towards a tutorial on intakes as well as nozzles.

So, we are going to talk about different… We will infact I have sorted out four different problems for you. The first two problems are to do with intakes. One is for subsonic; the other second problem would be for a supersonic intake involved shocks. And subsequently for nozzles, I have first the convergent nozzle which basically a subsonic nozzle. Then I would be taking up a problem which is on a convergent-divergent nozzle which is basically a supersonic nozzle. So, after this I will also list out a few exercise problems for you; which you can solve or you can attempt to solve based on what we have been discussing in the last few lectures as well as based on the tutorial that we are going to have today.

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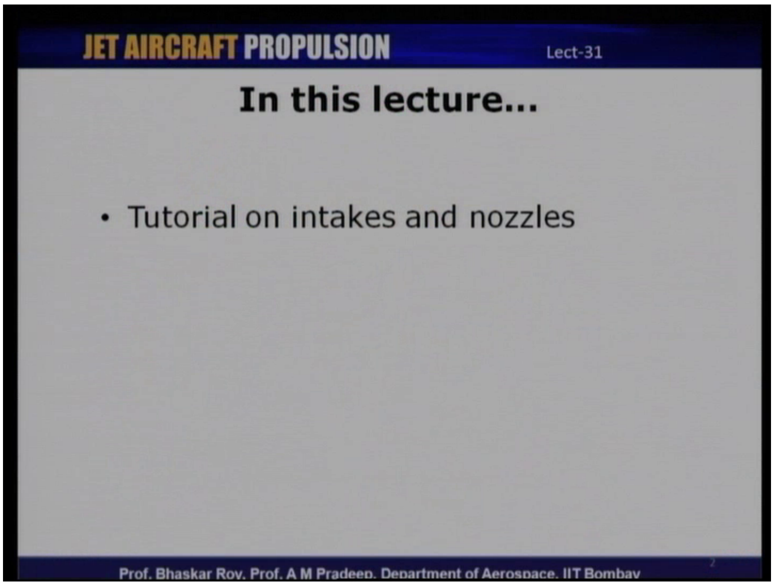

So, today's lecture is basically going to be a tutorial session. We are going to talk about intakes and nozzles and different problems associated with these two cases.

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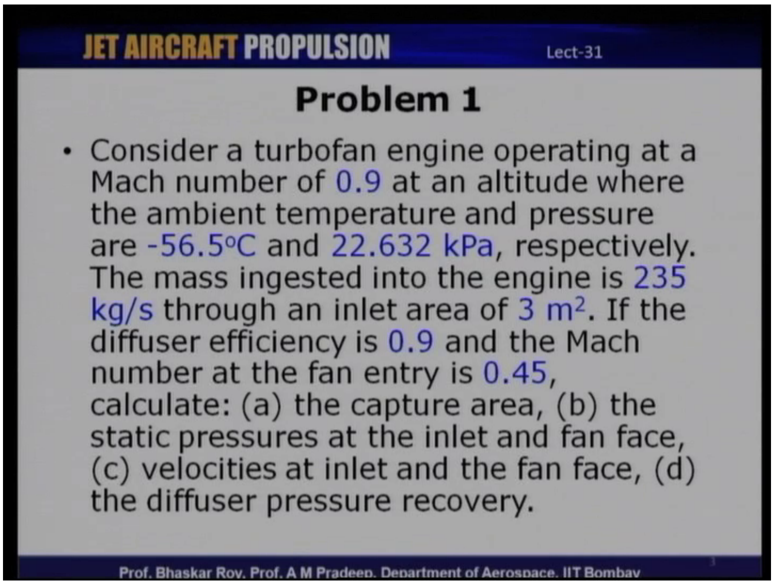

So, as I said, the first problem that we are going to discuss about will be on subsonic intake. So, basically it is related to a turbo fan engine which is what is used in normal civil air civil transport aircraft and the turbofan engine, which operates around at a Mach number of 0.9 at an altitude, where the ambient temperature and pressure are minus 56.5 degree Celsius and 22.632 kilo Pascal respectively. The mass ingested in the in to the engine is 235 kilo grams per second through an inlet area of 3 meters square. If the diffuser efficiency is 0.9 and the Mach number at the fan entry is 0.45, calculate the capture area; part b, static pressures at the inlet and the fan face; the velocities part c is velocities at the inlet and the fan face and part d is the diffuser pressure recovery.

So, this is a problem which is for purely a subsonic aircraft. Therefore, this is normal subsonic intake that we have seen. So, subsonic intake as we have discussed is like a very simple diffuser, which is used in subsonic flow. It will basically involve a diverging or an increasing area along the axis. So, in this subsonic intake, we have been given a few parameters like the altitude at which the aircraft is operating in the Mach number. Then we also have the intake area, the inlet area as such and also the Mach number at the fan face. So, based on the on these data that has been provided, we are required to find out a variety of other things like static pressure, temperature, then the capture area and so on.

So, we will first begin with calculating what is the corresponding Mach number at… well Mach number is given as 0.9. So, can we not calculate the velocity associated with this Mach number? Once we calculate velocity, we also know the free stream density; because the temperature and pressure are given. So, density can be calculated and therefore, mass flow rate well the capture area can be calculated; because mass flow rate is given. So, mass flow rate divided by the product of density and velocity gives us the capture area and then we will see how that compares with the inlet area, which has been specified to us.

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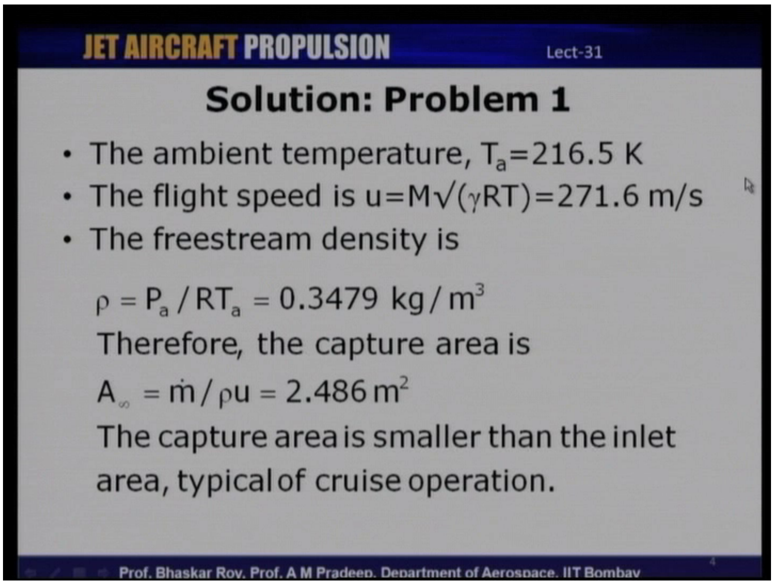

So, let us do this calculation first. So, ambient temperature is given as 216.5 Kelvin. Therefore, from this, we can calculate the flight speed which is u; Mach number is known 0.9 into square root of gamma R T, which is the speed of sound. So, Mach number times the speed of sound which is in terms of static temperature. So, we can calculate speed of sound square root of gamma R T is the speed of sound multiplied by Mach number and therefore, the flight speed can be calculated as 271.6 meters per second. Now, we can now calculate the free stream density; because the ambient pressure is given; the temperature is also given and gas constant for air we can assume as 289 well 287 joules per kilogram Kelvin.

So, from this, we calculate density; pressure is known; temperature and gas constants. So, we calculate this as 0.3479 kilograms per meter cube. Therefore, we can now calculate the capture area. Capture area is simply the ratio of the mass flow rate times density and velocity. Mass flow rate is given as 235 divided by density which is 0.3479 multiplied by 271.6. So, if you calculate this, you would get the capture area as 2.486 meter square. So, you can immediately see that this capture area is smaller than the inlet area. Inlet area is specified in the problem as 3 meter square. Capture area we have calculated as 2.486.

Now, if you recollect our discussion on subsonic intakes, where I had discussed about two operations or two different regimes of operations of the air intake, one was during takeoff and the other was during the cruise. And during cruise as we know, the thrust requirement is not as high as it is during takeoff; which means that the capture area upstream which will provide the necessary mass flow to the intake or the engine would be smaller than the air intake inlet area itself. So, here in this case, the air intake inlet area is given as 3 meter square. We have just now calculated the capture which is about 2.5 meters square. You can clearly see that the capture area is smaller than the intake entry area.

This is typical of a subsonic intake operating during cruise operation, which is high speed and low mass flow requirement; because thrust requirement is lower. This is unlike during takeoff. During takeoff, it is the other way round wherein the mass flow requirement would be high; because the engine needs to generate the maximum thrust during takeoff. So, because the mass flow requirement is high and the velocity is not high enough, the capture area would be much higher than the intake entry area itself; in in which means that before the inlet begins, there is actually an acceleration of the flow; because the capture area and the stream tube will now converge towards the intake entry area. So, this is how a subsonic intake would be operating during cruise. So, let us proceed further and calculate the other parameters.

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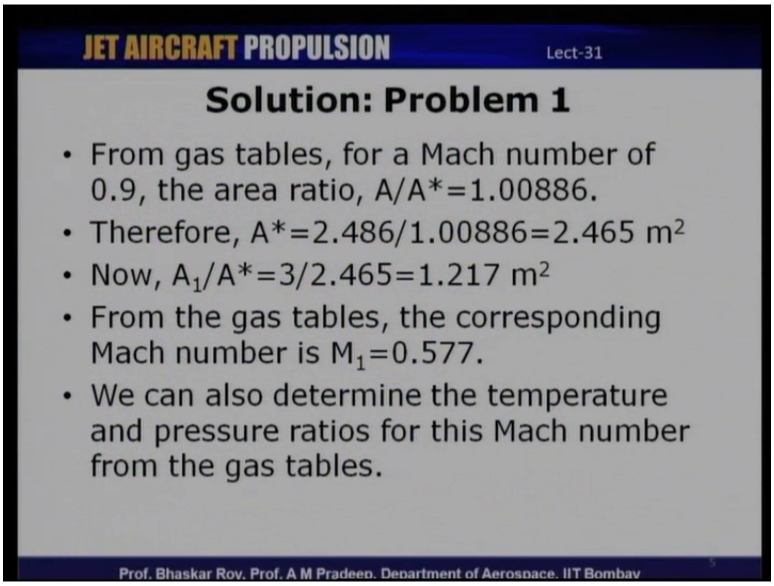

Now, during there is one thing that I mentioned here in today's tutorial, you would be requiring the gas tables and the shock tables. So, I would suggest that you have the gas tables and the shock tables handy; because our problem solving in today's tutorial will involve use extensive use of gas tables and shock tables. So, if you have the gas tables with you, you will need to fill pages and see that for a Mach number of 0.5, you will find that the area ratio corresponding to this is 1.00886. Area ratio is A by A star which is where A star is the fort area. So, this is 1.00886. So, this would be available in any gas table or infact in some of the text books also, you will find gas tables given towards the end of the text book.

So, here A star can now be calculated which is 2.486, which is the capture area divided by 1.00886. So, if we calculate this, we would get 2.465 meter square. So, we also have the inlet entry area which has been given to us. So, we can also calculate A1 by A star. A 1 is given as 3 meter square; A star we have just now calculated as 2.465. So, 3 by 2.465 is 1.21 square meters 217 meter square. So, this is the ratio of the intake entry area to the throat area. So, from the gas tables again for this corresponding area ratio, we can calculate that the Mach number associated with this area ratio is 0.577. So, this will be the Mach number which we can calculate for the intake entry. Now, once we know this Mach number, we can also calculate the corresponding temperature pressure ratio etcetera again from the gas tables.

I will also discuss a little later after we finish this that we can also solve the same problem using theoretical expressions. I will derive some equations which you can use to solve the same problem in a in a same way. But it is not using the gas tables; but using simple formulae. After all the gas table is based on all these formulae, so either we know the formulae and solve the problem or we simply use the gas tables. So, gas table is just for convenience that uses all these theoretical expressions, analytical expressions and they have been tabulated for different Mach numbers corresponding temperature pressure ratios. So, we will I will also explain that procedure after we have finished this. So, for this Mach number, we have just now calculated which is 0.577. We can now calculate the temperature and pressure ratios.

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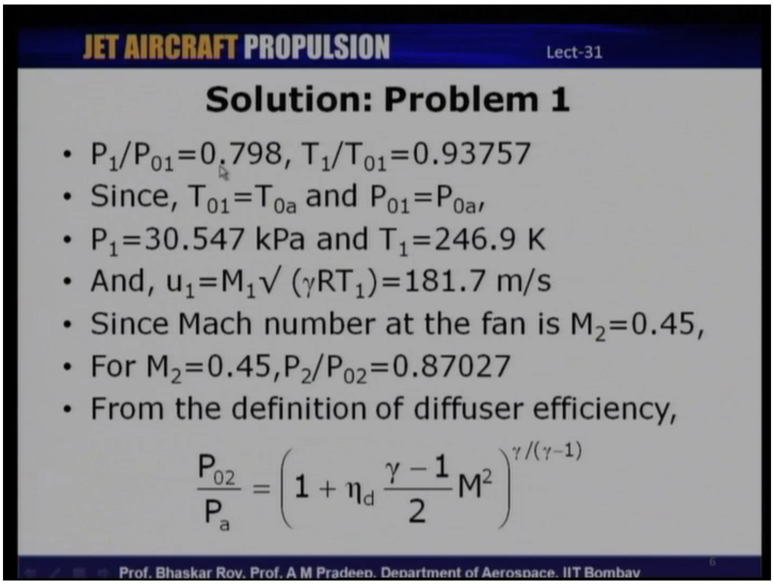

So, if you go to the gas table, again you will find the pressure ratio that is static to stagnation pressure ratio P 1 by P 0 1 which is 0.798. The corresponding temperature ratio for Mach number that we have seen is T 1 by T 0 1 that is 0.93757. Now, we are going to… because it is an adiabatic flow, we can safely assume that and it is subsonic. So, it can be safely assumed that T 0 1 is equal to T 0 a; that is the stagnation temperature at the intake entry is equal to the free stream stagnation temperature. Similarly, P 0 1 will be equal to P 0 a. So, having calculated these; that is the stagnation temperature and pressure; since the ratio is known, we can now calculate the static pressure and the static temperature.

So, P 1 is because this ambient pressure is known, we can calculate P 1 which is 30.547 kilo pascal. Similarly, T 1 is 246.9 Kelvin. So, having calculated this static temperature and we also know the Mach number at station 1, we can now calculate the speed of velocity that is u 1 at the intake entry. So, u 1 would be M 1 into square root of gamma R T 1. M 1 is known as 0.577 which we have calculated in the previous slide. So, that multiplied by the speed of sound at station 1 square root of gamma R T 1; this is 181.7 meters per second.

Now, it is also given that the Mach number at the fan phase is 0.45, which is typical of subsonic aircraft. Infact, the Mach number at the fan phase is usually of this order would be around 0.4 to 0.5; between 0.4 and 0.5. So, that has been specified. So, for a Mach number of 0.45, we can calculate the pressure ratio which is again from the tables. So, we get P 2 by P 0 2 as 0.87027. Now, in this question, we have also been given the diffuser efficiency and we have we have earlier related the diffuser efficiency to the pressure ratio P 0 2 by P a is 1 plus eta d which is the diffuser efficiency multiplied by gamma minus 1 by 2 M square the whole raise to gamma by gamma minus 1.

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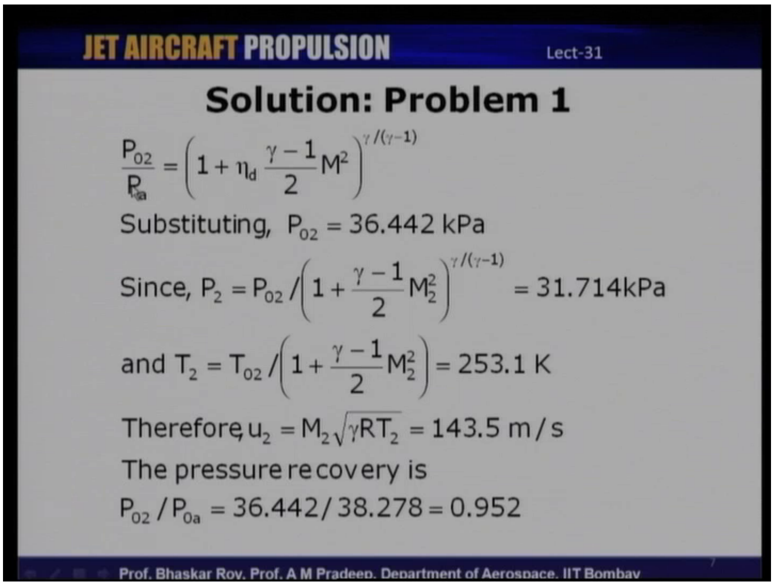

So, from this, we can calculate P 0 2 by substituting the values; we know the M 2; we also know P a. So, we can calculate P 0 2; efficiency has also been given as 0.9. So, if we substitute those values, we calculate P 0 2 and that comes out to be 36.442 kilopascals. And we also know that P 2 and P 0 2 that is the stagnation pressure and temperature are related to the isentropic relation. Therefore, P 2 is P 0 2 divided by 1 plus gamma minus 1 by 2 M 2 square raise to gamma by gamma minus 1. We again substitute these values here; we get the static pressure at station 2 as 31.714 kilopascal. Similarly, we can also find the static temperature at station 2 which comes out to be 253.1 Kelvin.

Now, once we are calculated static temperature at station 2, we can calculate the flight speed or the velocity which is Mach number time square root of gamma R T 2, where Mach number at station 2 is 0.45. So, this multiplied by T 2 which is 253.1.We get u 2 as 143.5 meters per second. So, once we have calculated u 2 well that was one of the aspects we need, we were required to calculate velocity at station 2. The final thing that we are required to calculate is the pressure recovery. Pressure recovery as we know it is the ratio of the stagnation pressure at the intake exit to the stagnation pressure at the entry. Between P 0 a and P 0 1, there are no losses.

Therefore, P 0 1 should be equal to P 0 a and therefore, pressure recovery is P 0 2 divided by P 0 a. P 0 2 we have calculated as 36.442; P 0 a we know is given that is 38.278. So, this pressure recovery is 0.952. So, this is one way of solving this problem that we have just now discussed which was using the gas tables. So, it is a very simple way of solving; because if you have the gas tables with you, you can effectively make use of the gas tables to solve such problems. What you notice is that for a subsonic intake that we have just now solved.

The pressure recovery is of the order of.9 5 which means that there is still a pressure loss in in terms of stagnation pressure loss occurring in the diffuser, which is of the order of 5 percent. So, this pressure loss is purely because of the frictional effects as we have discussed earlier and ofcourse in most of the intakes, it would be very low. It is probably on the higher side that we have just calculated. Usually, it is 2 to 3 percent that goes into the pressure loss alone, unless ofcourse there is a flow separation. So, we will now proceed towards solving the same problem using slightly different approach.

But effectively they are one and the same; because the first approach that we have just now solved is using the gas tables. Now, what we are going to use is we will basically be deriving equations which we have also we discussed earlier on for calculating some of these properties. Infact, the tables would be using exactly the same equations; but they have already been calculated and tabulated at the form of tables. So, you could adopt either of these approaches and you should be getting exactly the same answers; because they are one and the same in some sense of the other. So, let us take a quick look at how the same problem can be solved in a slightly different way.

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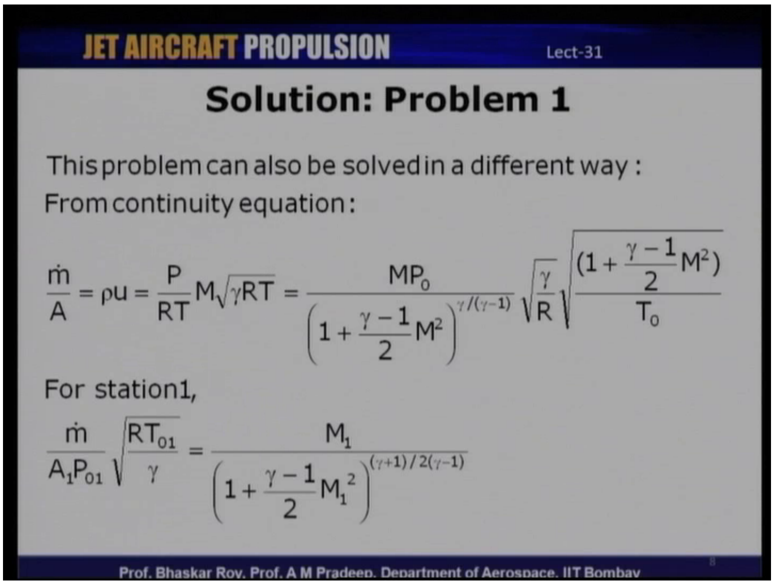

So, in during if you want to use equations or expressions to solve this problem, let us now calculate the mass flow rate for example. So, this ratio that is m dot by A is basically equal to density times velocity; density we know is P by R T and velocity is M into square root of gamma R T. So, this we will express in terms of total temperatures and pressures. So, we have M times P naught by 1 plus gamma minus 1 by 2 M square raise to gamma by gamma minus 1; that is the pressure into square root of gamma by R; because there is a gamma here and R square root gamma R here and another R here. So, that becomes square root of gamma by R.

Temperature will express in terms of stagnation temperature. So, this becomes square root of 1 plus gamma minus 1 by 2 M square divided by T naught. Let us simplify or rearrange this and simplify; we have m dot by A 1 P 0 1 multiplied by square root of R T 0 1 by gamma. This is equal to M 1 divided by 1 plus gamma minus 1 by 2 M square raise to gamma plus 1 by 2 into gamma minus 1. So, these we have use the same equation. But this has been applied for station 1; which is why you see subscripts of 1 here for temperature and pressure and as well as Mach number. So, this relates the mass flow rate and the inlet stagnation pressures and temperatures to the corresponding Mach number.

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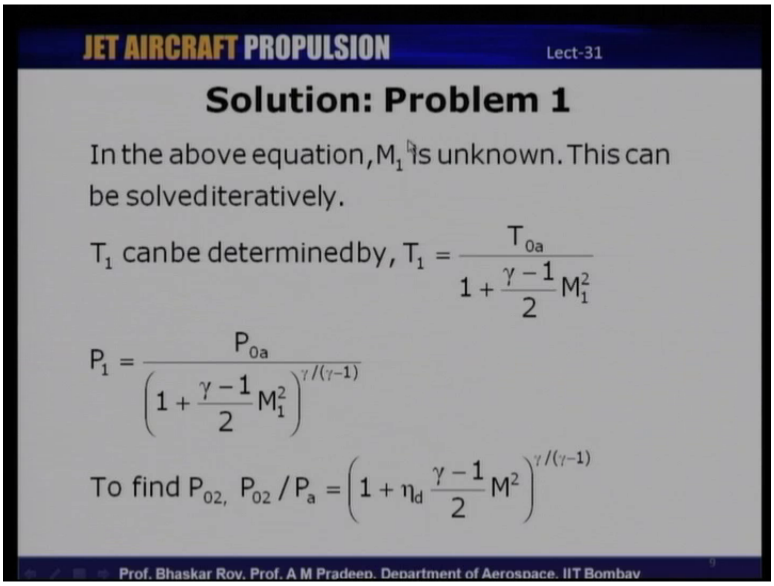

So, in this equation that we have seen this equation that we have just now seen; all the parameters on the left hand side are known. We know m dot; we know A 1, P 0 1, T 0 1; all these are known; Mach number is not known. So, Mach number obviously has to be solved a little iteratively and once we do that, we can determine the Mach number in station 1 iteratively. So, once we calculate Mach number, the static temperature and pressure can be easily calculated. T 1 can be determined by T 1 is T 0 a because T 0 1 and T 0 a are the same divided by 1 plus gamma minus 1 by 2 M 1 square. Similarly, P 1 is P 0 a divided by 1 plus gamma minus 1 by 2 M 1 square raise to gamma by gamma minus 1. Now, to find properties at station 2, we have the efficiency definition. So, we have P 0 2 divided by P a is equal to 1 plus eta d into gamma minus 1 by 2 M square raise to gamma by gamma minus 1.

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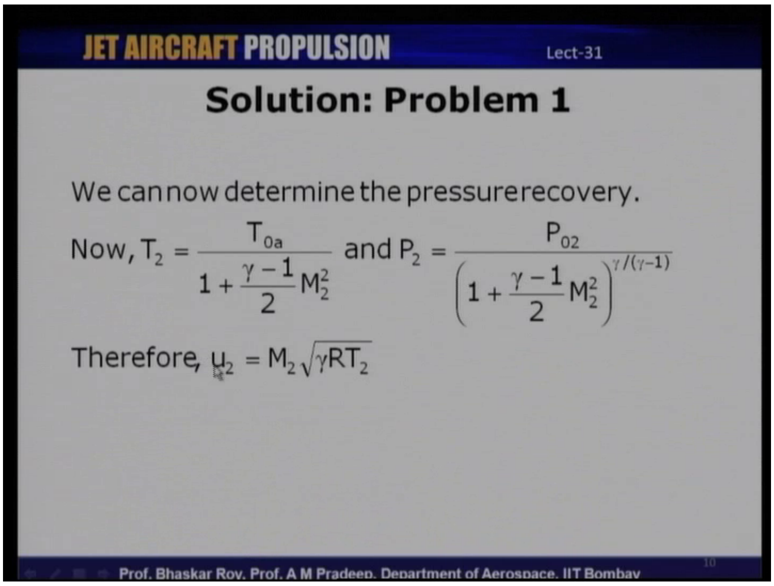

So, we can once we find P 0 2, we can find pressure recovery involved all we need to find is the velocity at station 2. Therefore, T 2 that is static temperature at station 2 is T 0 a divided by 1 plus gamma minus 1 by 2 M 2 square; M 2 is given to us. Similarly, P 2 is P 0 2 by 1 plus gamma minus 1 by 2 M 2 square raise to gamma by gamma minus 1. So, u 2 can be calculated; because M 2 is known, T 2 is also known. So, we can calculate the velocity at station 2 and which is Mach number time square root of gamma R T 2. So, in this approach that we have solved, we can also as we have seen ofcourse I have not really substituted for the parameters here; because I assume you can do that because we have already done it during the different approach.

So, you can use either of these approaches; either the gas tables or the second approach as you can see. If you start from the fundamentals if you start from the fundamental principles, you can easily solve this problem; because Mach number you can equate mass flow rate to density times velocity and then continue to express them in terms of static pressures and then stagnation pressures and temperatures. And therefore, you have expression for mass flow rate related to stagnation pressure, stagnation temperature and the Mach number. And therefore, one can easily solve this problem also by using these expressions.

So, it is up to you to choose and decide which option you would like to exercise and solve this problem for based on which. So, either of these procedures that you use methods that you use, you should get exactly the same answers. So, the first problem that we have just now solved is pertaining to a subsonic diffuser. We will now take up a supersonic diffuser which will invariably involved the presence of shocks; which means that for the solving this problem, you will need the shock tables and possibly the shock chart or the theta beta m or delta beta m as it is in some books, relation or the plots.

Because that will be required for calculating lot of the parameters associated with the shock. I am assuming that you have already undergone a course in gas dynamics; wherein you have had some exposure to shock flows and you know how to solve flow properties across a shock; because that will be essential to understanding of this question. If you have not undergone, I would urge you to just go through any text book on gas dynamics, where they deal with flow through oblique shocks and normal shocks and how to use these normal shock and oblique shock tables. So, let us take a look at the second problem that we have for today.

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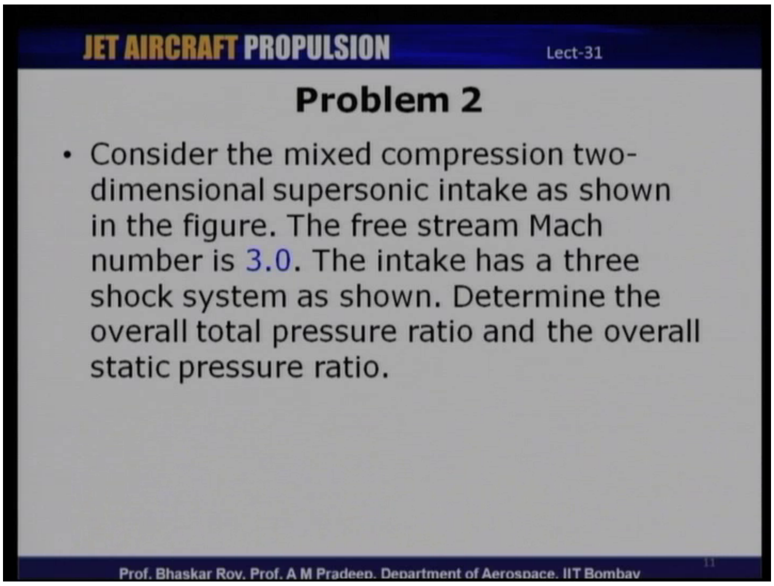

So, this is a mixed compression two-dimensional intake which is supersonic and there is a figure for that. So, consider the mixed compression two-dimensional supersonic intake as shown in the figure. The free stream Mach number is 3 and the intake has three shock systems as shown in the figure. Determine the overall pressure ratio, total pressure ratio and the overall static pressure ratio.

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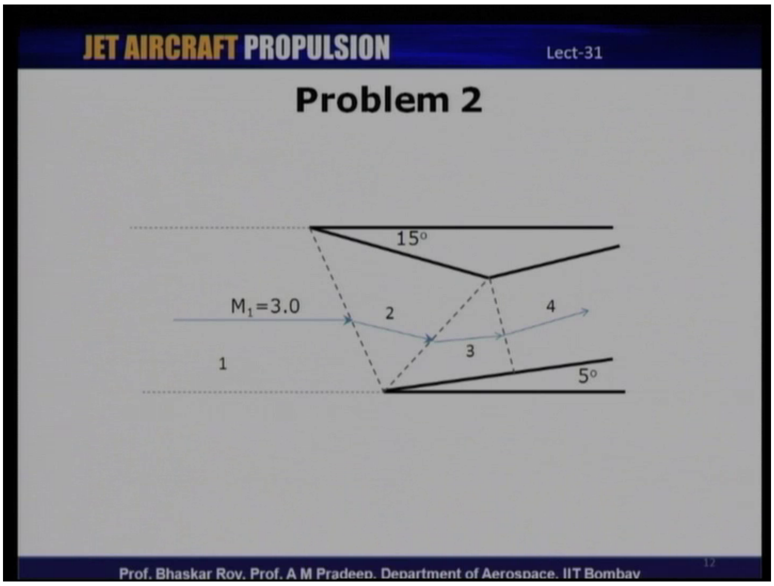

So, this is the problem that is at hand for us now. This is the supersonic intake. It is a mixed compression intake; because there is one shock which is just at the exit. This is just outside the intake entry itself. So, this is a mixed compression intake. There are three shocks which are used to decelerate the flow; two of them are oblique shocks. This and this shock these are two oblique shocks and then there is a normal shock, which converts a supersonic flow to a subsonic flow and these are the wedge angles given it is a two-dimensional intake. So, there is a ramp angle which is of 5 degrees here and there is also a wedge angle at this side.

The four body angle which is about 15 degrees. The free stream Mach number is 3. So, let us denote station just before the first oblique shock at station 1; just after the oblique shock first oblique shock as station 2, which also happens to be the upstream of the second oblique shock. Downstream of the second oblique shock is station 3; downstream of the normal shock is station 4. So, you can see that these oblique shocks basically deflect the flow in such a way that the flow can take this turn and eventually, the 4 flow takes apart which is parallel to the surfaces.

So, the oblique shocks there are two of them will decelerate the flow in steps; from Mach 3, it will reduce to a supersonic Mach number which is less than 3. Across the second oblique shock, the flow is further decelerated. It will still be supersonic and after the normal shock, the flow becomes subsonic and that is at station 4, it would be a subsonic flow. So, to solve this problem, we will make use of the shock tables and so am assuming that you have the shock tables with you at present. So that, you can understand what is been discussed about in this problem.

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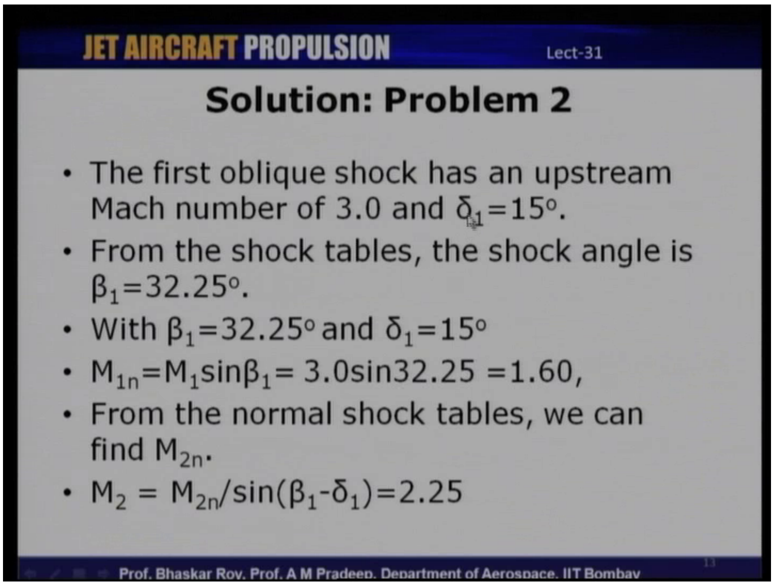

So, the first oblique shock has an upstream Mach number of 3 and it has a wedge angle of or deflection angle of 15 degrees. So, deflection I have denoted here as delta. So, delta at station 1; therefore delta 1 is 15 degrees. So, if you take up the shock tables you will or the shock chart, you will find that for a deflection angle of 15 degrees and the Mach number of 3. The shock angle the corresponding shock angle which is beta 1 would be 32.25 degrees. So, we have we now have the shock angles as well as the deflection angle. Therefore, we can calculate the Mach number the component of the Mach number, which is normal to the oblique shock. So, M 1 n would be M 1 sin beta 1. So, M 1 n will be M 1 sin beta 1 which is 3 into sin 32.25 which is 1.6.

And so from the normal shock tables, for this Mach number which is the normal component of the incoming Mach number at station 1. So, M for M 1 n is equal to 1.6, we can find out the downstream Mach number M 2 n which will be a subsonic number. So, it will be less than 1. From there, we can calculate M 2 which will be M 2 n divided by sin beta 1 minus delta 1. Therefore, we can get the downstream Mach number at station 2 which is 2.25. When you look at this shock tables, you will see that for this Mach number 1.6, you will get downstream Mach number which is normal to the oblique shock which will be subsonic. But the actual

Mach number will continue to be supersonic; because the actual Mach number is M 2 n divided by sin beta 1 minus delta 1. So, you get a Mach number of 2.25.

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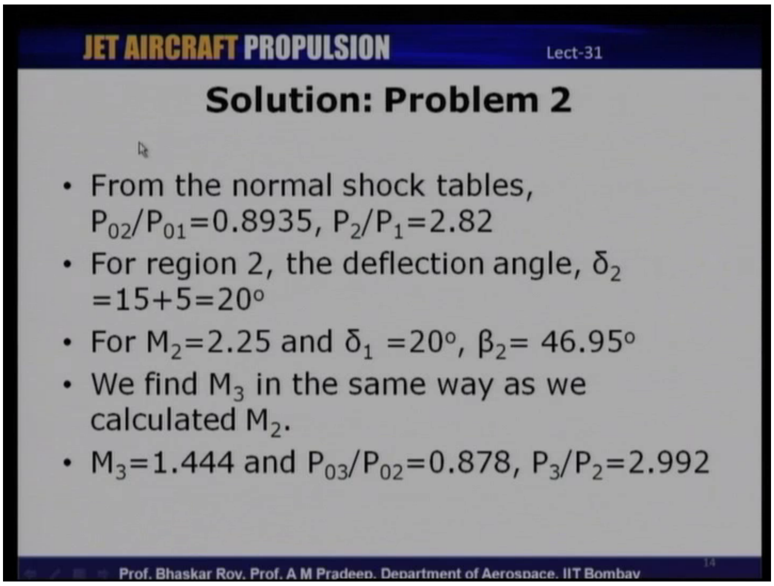

So, for this Mach number that we have seen from the normal shock tables, we can also find out the corresponding stagnation pressure ratios as well as it stagnation where static pressure ratios. So, P 0 2 by P 0 1 which is 0.8935 and P 2 by P 1 is 2.82. So, this will also be given in the normal shock tables as you solve the problem. Now, for the second region, we now know that the deflection angle delta 2 will be a sum of the first two angles. So, let us go back to that chart again. (Refer Slide Time: 24:22) So, here the flow has been deflected by 15 degrees in this case; because there is a deflection from here as well as a deflection because of this. The effective deflection became becomes 15 plus 5 degrees and that is 20 degrees; because the both these deflections will have an effect on the second oblique shock.

So, for region 2, the deflection angle delta 2 is 15 plus 5 which is 20 degrees and so we know the Mach number absolute Mach number at station 2 is 2.25. So, for a deflection of 20 degrees, we can calculate the shock angle as 46.95. So, having calculated the shock angle, we now proceed to find the absolute Mach number M 3 in the same way as we calculated M 2. What we will do is we will resolve M 2 into the normal component that is M 2 n, which is M 2 sin beta. And then we calculate the downstream Mach number from the normal shock tables for the M 2 and we get M 3 and which is the normal component of the Mach number at station 3. And therefore, absolute Mach number M 3 would be M 3 n divided by sin beta 2 minus delta beta minus delta 2.

So, from there, we get the absolute Mach number. Having calculated the absolute Mach number, we can also now calculate the static pressure ratio as well as a stagnation pressure ratio. For the third shock which is the normal shock, we can use the Mach number as it is from the normal shock tables. We do not have to resolve it anymore; because the normal shock is 1, where the incoming Mach number will be at an angle of 90 degrees to the flow. So, across from station 3 to 4, it is a normal shock; we just use the normal shock tables as it is. So, having done that, we can calculate M 3 in the same way as we calculated M 2. M 3 will come to be 1.444 and the corresponding stagnation pressure ratio is 0.878 and the static pressure ratio is 2.992.

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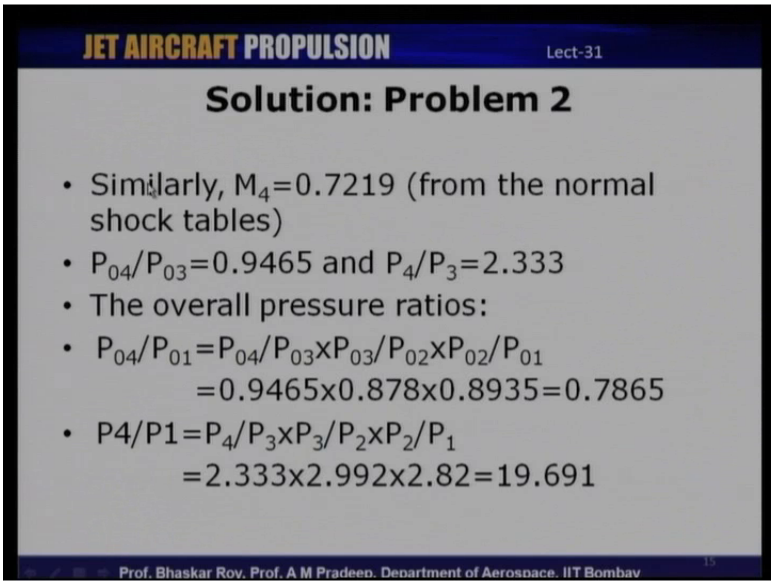

So, from the shock normal shock tables for the same upstream Mach number of 1.144, we use the normal shock tables and interpolate and we get M 4 as 0.7219. Corresponding stagnation pressure and static pressure ratios are P 0 4 by P 0 3 is 0.9465 and static pressure ratio P 4 by P 3 is 2.333. So, the overall pressure ratio the static pressure ratio well a stagnation pressure ratio P 0 4 by P 0 1 is the product of the three pressure ratios. We get P 0 4 by P 0 3 multiplied by P 0 3 by P 0 2 multiplied by P 0 2 by P 0 1; that is 0.9465 into 0.878 into 0.8935; that is 0.7865. Similarly, the static pressure ratio, we get P 4 by P 3 multiplied by P 3 by P 2 multiplied by P 2 by 1. So, this product comes out to be 19.691.

So, what you see here is that between the exit of the intake and the inlet, there is a drastic reduction in the stagnation pressure close to 122 percent drop in the stagnation pressure. Which is why, the exit stagnation pressure is only 0.78 times the inlet stagnation pressure. On the other hand, there is a substantial increase in the static pressure. Exit static pressure is almost 19.7 times that of the inlet static pressure. So, there is a substantial raise in static pressure across the shock system. At the same time, there is also a loss in stagnation stagnation pressure across the shock system. If you had calculated the same thing for a single Mach number, I would leave that as an exercise for you.

So, instead of three shocks that is two oblique shocks and a normal shock, let us say we had only one normal shock and the upstream Mach number is 3. And so for this normal shock if you were to calculate and find out what is the static pressure raise and stagnation pressure raise, you can compare that with this and you will see the difference. If you were to do that, you will see that the stagnation pressure loss across such such a system which involves just a normal shock would be substantially higher. So, if you had only one normal shock which was decelerating a very high Mach number flow of Mach 3 to low subsonic, the stagnation pressure loss can be tremendously high.

Which is a reason why, one would not want to use a single normal shock not just because it leads to lot of stagnation pressure, there are also other issues like the stability of the shock itself in terms of the shock having to be located right at the intake entry. Because if it is upstream of the intake, there will be spillage drag; if it is downstream, then there are other issues associated with it. And so besides this, ofcourse stagnation pressure loss can be substantially high; which means there is lot of loss of thrust. As a result of this, drop in stagnation pressure. So, we have so far now solved two problems. One was related to a low subsonic intake; the first one which was used for a civil aircraft.

Second one was a mixed compression intake which involves two shocks and two oblique shocks and a normal shock and we have seen how we can go about solving this problem. As I suggested, you should be using the tables; you should have the tables handy with you. The shock tables as well as isentropic tables to be able to solve these problems effectively. And for those who have not really undergone a course in gas dynamics, I would suggest that you go through a book on gas dynamics and see how you can solve flow across oblique and normal shocks. Let us now take a look at the next problem that we have it is a nozzle problem. We will first take up a problem on a convergent nozzle. Subsequently, we will take up a problem for a supersonic nozzle or a convergent-divergent nozzle.

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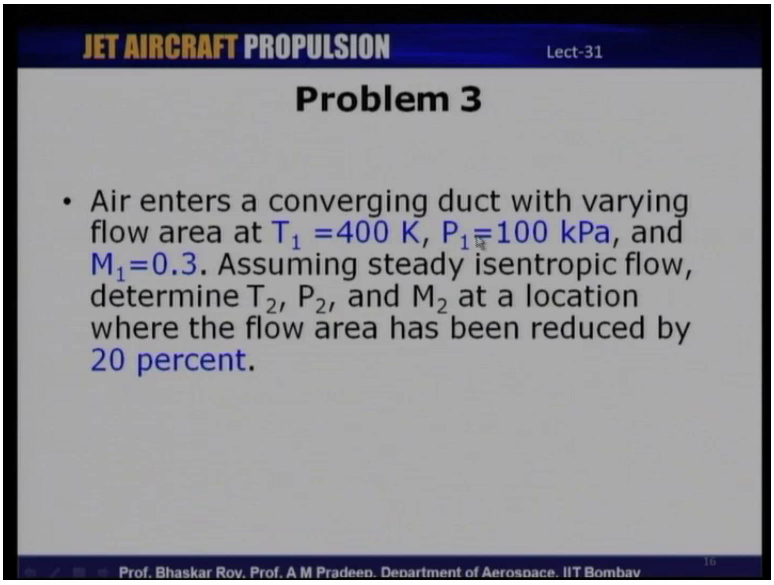

So, the first problem on the nozzle that we have is a convergent nozzle which states that air enters a converging duct with a varying flow area and the temperature at the inlet of the duct is static temperature is 400 Kelvin and static pressure is 100 kilo pascal and there is the inlet Mach number which is given as 0.3. So, if we assume steady isentropic flow, determine the exit static temperature and pressure as well as the Mach number at a location, where the flow area has been reduced by 20 percent. So, it is given that as the flow exceeds the nozzle, the flow area has been reduced by 20 percent; that is, area at the exit of the nozzle is 80 percent the area at the inlet.

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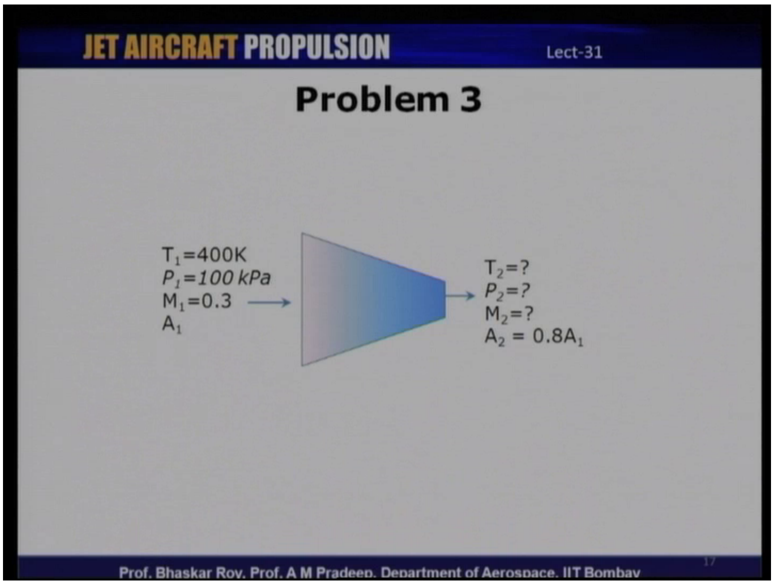

So, this is the problem statement. We have a convergent nozzle, inlet static temperature, static pressure and the Mach number are specified and the exit area is given as 0.8 times the inlet area. We are required to find out the exit static temperature pressure and the Mach number.

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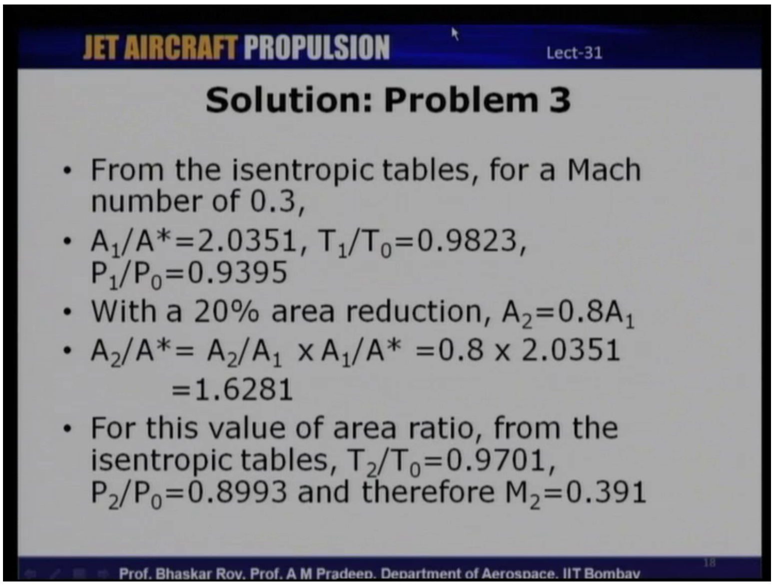

So, as I said, we will again need the tables the isentropic tables to be able to solve these problems. You can also use the equations as we have discussed in the first problem on diffusers. But the tables would make it much more convenient and quicker to solve such problems. So, from the isentropic tables for a Mach number of 0.3, you will see that the area ratio that is A 1 by A star, where A star corresponds to the throat area is 2.0351. We can also see from the isentropic tables, the temperature ratio T 1 by T naught as 0.9823 and P 1 by P naught as 0.9395. So, it is given to us that at the exit of this nozzle, area has been reduced by 20 percent.

So, A 2 which is the nozzle exit area is 0.8 times A 1. So, we know that A 2 by A star can be written as A 2 by A 1 multiplied by A 1 by A star. A 2 by A 1 is known; it is given a 0.8 and A 1 by A star we have just now calculated from determine from the isentropic tables as 2.0351. So, if we multiply these, 2 we get A 2 by A star; that is 1.6281. So, if we now have to calculate what are the exit conditions, we can see that for this value of area ratio which again go back to the isentropic tables and then look for this area ratio.

For this area ratio of 1.6281, the corresponding temperature and pressure ratios are T 2 by T naught as 0.9701; P 2 by P naught as 0.8993 and the corresponding Mach number as 0.391. So, we have already determined the exit Mach number; that is 0.391. You can immediately see that inlet Mach number was 0.3. With a 20 percent reduction area for a subsonic nozzle, we get an exit Mach number of 0.391. So, there is an acceleration of flow in this nozzle where the area has been reduced by 20 percent. Now, let us now calculate the exit static temperature and static pressure.

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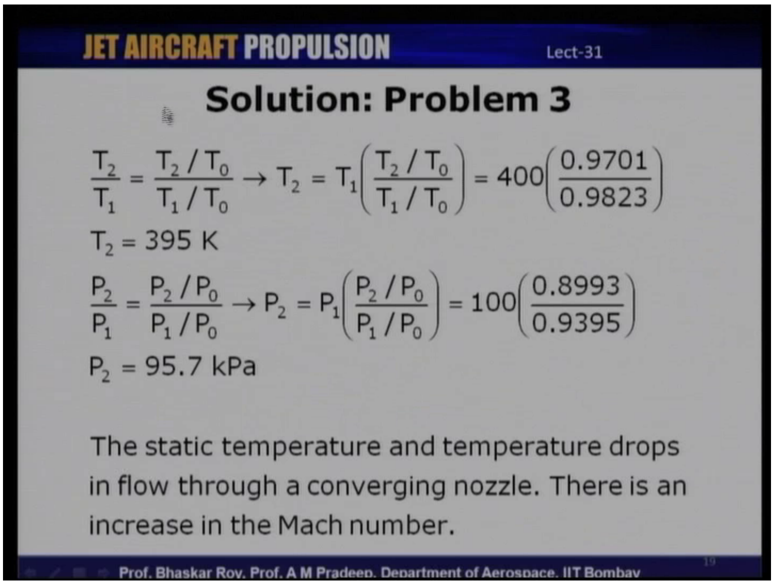

So, T 2 by T 1 is can be this ratio temperature ratio can be rewritten as T 2 by T naught divided by T 1 by T naught or static temperature T 2 is T 1 multiplied by T 2 by T naught divided by T 1 by T naught. We have already calculated these ratios T 2 by T naught and T 1 by T naught. (Refer Slide Time: 36:30) T 1 by T naught is 9 0.9823. T 2 by T naught is 0.9701. So, we get an inlet temperature is already given as 400 Kelvin. So, 400 multiplied by these temperature ratios 0.9701 divided by 0.9823. So, exit static temperature is 395 Kelvin. Similarly, exit static pressure can be calculated from the pressure ratios. P 2 is equal to P 1 into P 2 by P naught divided by P 1 by P naught, which is 100 multiplied by P 2 by P naught is 0.8993 and P 1 by P naught is 0.9395.

So, if we substitute these pressure ratios and temperature ratios, we can calculate the exit static pressure as 95.7 kilo pascal. So, you can see immediately that at the exit of the nozzle, these static pressure and static temperature are lower than the inlet static pressure and inlet static temperature. Inlet static temperature was 400 Kelvin that has now reduced to 395 Kelvin. Similarly, inlet static pressure was 100 kilo pascal. At the exit of the nozzle, it becomes 95.7 kilo pascal. This is accompanied by a corresponding increase in Mach number. So, if we do not assume any losses in the nozzle, this drop in static pressure is basically the reason why we have an increase in Mach number; that is the stagnation parameters will be conserved.

Stagnation pressure at the inlet and exit will remain the same, if there are no pressure losses. And if it is in adiabatic flow, the static the stagnation temperature also remains the same between the inlet and the exit. But ofcourse there is a change in static temperature; there is a drop in static temperature from the inlet to the exit. Correspondingly, there is a drop in static pressure as well between the inlet of the nozzle and the exit of the nozzle. So, this problem that we have solved right now is a very simple problem that pertains to a converging nozzle. We will now take up a converging-diverging nozzle, which is basically the nozzle that one would use to accelerate to supersonic speeds; because a converging nozzle cannot really accelerate to supersonic speeds.

The max the maximum speed that you can attain there is Mach 1. And so if we have to accelerate to a speed beyond that, we will need an area which is expanding or an increase in area after the throat of a converging nozzle; So that, the Mach number can now increase; because we have already discussed the principle behind this. That is in supersonic flow, acceleration can take place only with an increase in area and therefore, we need a converging-diverging nozzle to be able to accelerate the flow to supersonic speeds. So, the next problem that we will solve is for a nozzle which is a converging diverging nozzle and it has a supersonic Mach number at its exit. So, let us see what the problem is.

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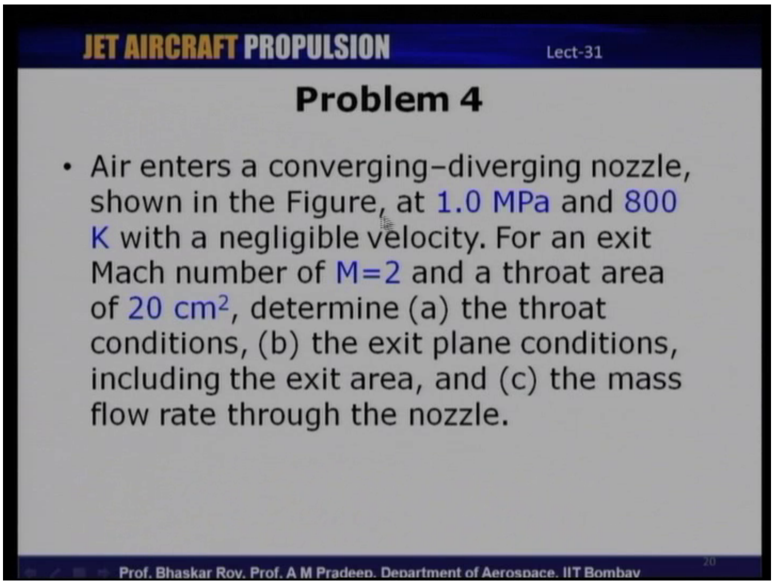

Air enters a converging-diverging nozzle as shown in the figure. So, that we will explain what is shown there at a Mach number at a inlet pressure of 1 Mega pascal and temperature of 800 Kelvin with a negligible velocity. For an exit Mach number of 2, Mach number at the exit is 2 and the throat area of 20 centimeter square, determine the throat conditions; part b, the exit plane conditions including the exit area and part c, the mass flow rate through the nozzle. So, in this problem, we have a converging-diverging nozzle. Inlet conditions are given; the stagnation pressure and temperature are given.

We can assume the pressure and temperature to be stagnation; because it is mentioned that the velocity is negligible. It is also given that the throat area has a certain dimension. It is 20 centimeter square and if the exit Mach number has to be 2 Mach 2, then we are required to find out different conditions at the throat, at the exit, like the throat area etcetera. So, we will first take a look at the problem statement itself in terms of an illustration and then we will see how we can solve this problem based on what we have discussed in the last few lectures.

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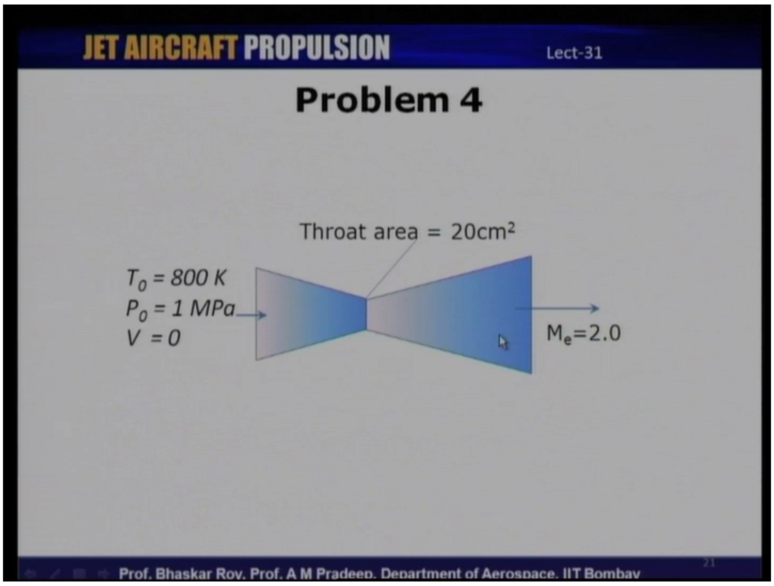

So, this is the nozzle a convergent divergent an illustration of a convergent-divergent nozzle. Inlet conditions are temperature of 800 Kelvin; pressure, 1 mega pascal; velocity is 0; throat area is given as 20 centimeter square and Mach number at the exit is given as Mach 2. (Refer Slide Time: 42:29) We are required to find throat conditions, the exit plane conditions including the area and also the mass flow rate through the nozzle. So, let us take up the problem one by one. We will solve the problem for the throat conditions first; then the exit conditions and subsequently the Mach number.

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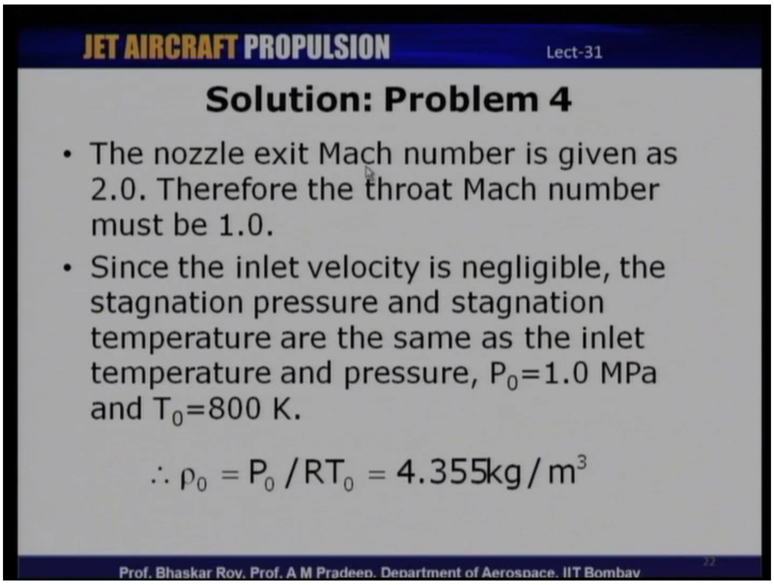

So, the nozzle exit area we have nozzle exit Mach number is given as Mach 2. So, if the Mach number at the exit is 2, then at the throat the Mach number has to be 1; because in a convergent-divergent nozzle, the flow accelerates to supersonic speeds only when it reaches Mach number of 1 at the throat of the nozzle. So, the throat Mach number would be 1. So, inlet velocity is given as negligible. So, inlet stagnation pressure and the stagnation temperature are same as that of a static and stagnation condition. So, we have P naught which is stagnation pressure as 1 mega pascal and temperature as 800 Kelvin. Therefore, density at the inlet is 4.355 kilo grams per meter cube; that is pressure divided by P by R T. We get the density that is 4.355 kg per kilo grams per meter cube.

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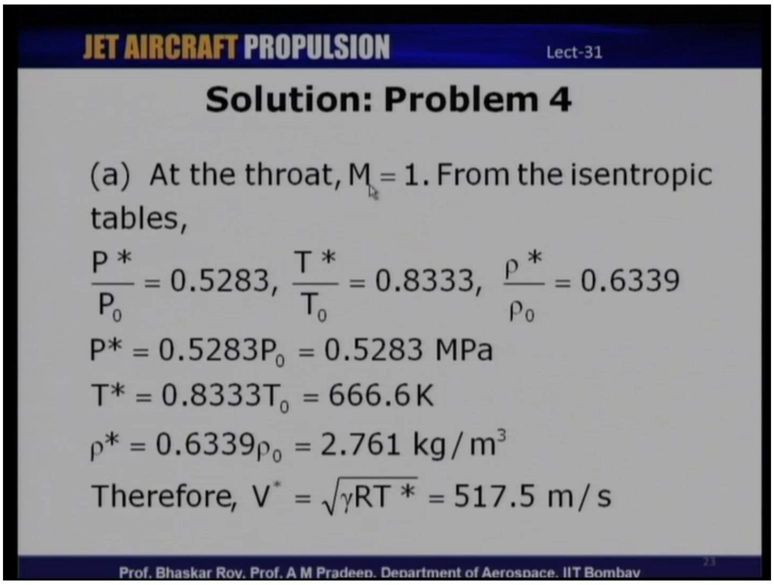

Part a of the question is to find the conditions at the throat. Since at the throat we know Mach number is 1, from the isentropic tables we can calculate the corresponding temperature and pressure ratios. P star by P naught is will be for Mach number of 1 would be 0.5283. T star by T naught is 0.8333 and rho star by rho naught the density ratio is 0.6339. So, this you can see from an any isentropic tables for a Mach number of 1, this these ratios can be easily found out. Therefore, the pressure at the throat P star would be 0.5283 multiplied by the pressure at the inlet P naught. So, that is 0.5283 mega pascal.

Temperature is 0.8333 times T naught which is 666.6 Kelvin and density is 0.6339 times the inlet density that is 2.7 61 kilo grams per meter cube. So, we can also calculate the velocity at the throat; because Mach number is 1. Velocity of the throat will basically be equal to the speed of sound at the throat; that is square root of gamma R T, where T is equal to T star. And so if you substitute for gamma R and T star which you have calculated, we can calculate the velocity of the throat as 517.5 meters per second. So, this is the velocity of the throat which is also the speed of sound at the throat; because Mach number is 1.

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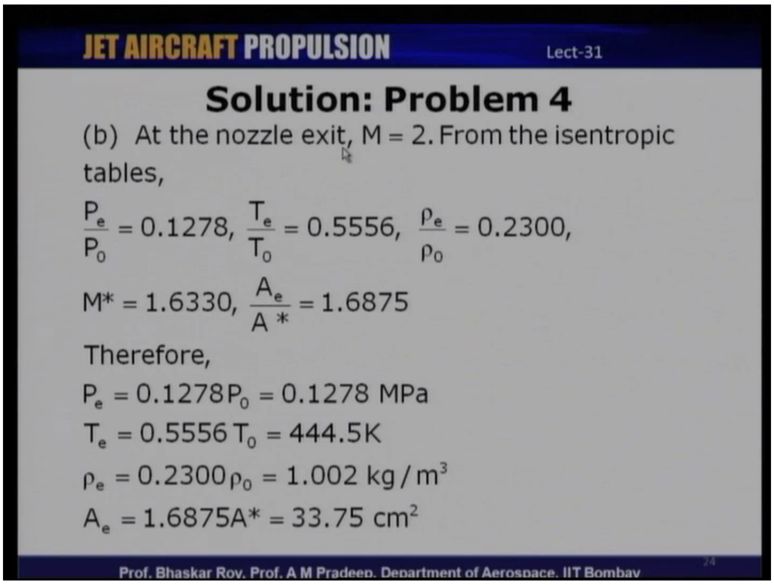

Second part of the question is to find out the conditions at the exit of the nozzle including the nozzle exit area. Mach number at the exit is given as Mach 2. Again from the isentropic tables for Mach number of 2, we can calculate the pressure and temperature ratios which would be P e by P naught as would be given as 0.1278. T e by T naught is 0.5556 and density ratio rho e by rho naught is 0.23. So, from the shock table well the isentropic tables, you will also be seeing the critical Mach number there as M star which is 1.6333 and also the area ratio A e by A star as 1.6875. So, in a rho, you would find all these parameters which have been specified.

Now, the since we are not assuming any losses; it is not mentioned that there are any losses in stagnation pressure, we will assume that the inlet stagnation pressure is valid even at the exit. So, and ofcourse there are no shocks present in the divergent portion of the nozzle. So, inlet stagnation pressure is still the same. So, exit pressure P e is equal to 0.1278 times the stagnation pressure, which is 1 mega pascal and therefore, P e is 0.1278 mega mega pascal. T e is 0.5556 multiplied by stagnation temperature and that is 0. multiplied by the temperature which comes out to be 444.5 Kelvin. Similarly, the density rho e is 1.002 kilograms per meter cube. Now, A e by A star the ratio has been specified in the problem. So, we find the A e the exit area is 1.6875 multiplied by A star, which is 33.75 centimeter square; because A star is given as a 20 centimeter square.

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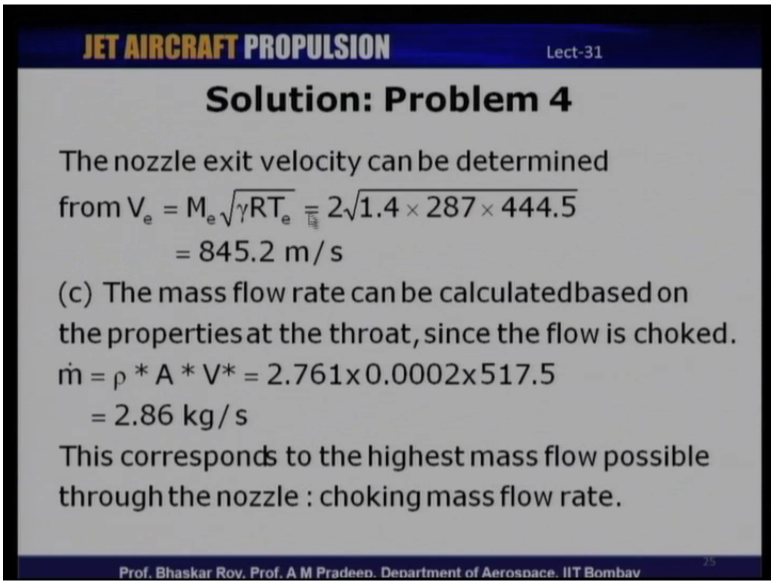

So, we can also calculate the exit velocity which is Mach number time square root of gamma R T. Mach number is given as 2. So, square root of gamma R T multiplied by the Mach number, we get the exit velocity as 845.2 meters per second. We now have to calculate the mass flow rate. So, mass flow rate can be calculated based on the throat; because the flow is chocked. So, we know the parameters at the throat. We can also calculate this based on the exit conditions; because we know the velocity, density and area at the exit. Either way, we calculate the mass flow rate comes out to be the same; it is 2.86 kilo grams per second.

So, this is basically the chocking mass flow; that is a maximum mass flow which the nozzle can handle; because it is chocked at the throat of the nozzle. So, we have now solved four different problems; two of them pertaining to intakes. One was a subsonic intake; the other was a supersonic intake which involved two oblique shocks in a normal shock. We also solved two problems pertaining to nozzles; a subsonic nozzle a converging nozzle and also a converging-diverging nozzle. So, I have a few exercise problems for you based on diffusers or intakes and nozzles, which you can solve based on what we have discussed today and also our discussion in the last few lectures.

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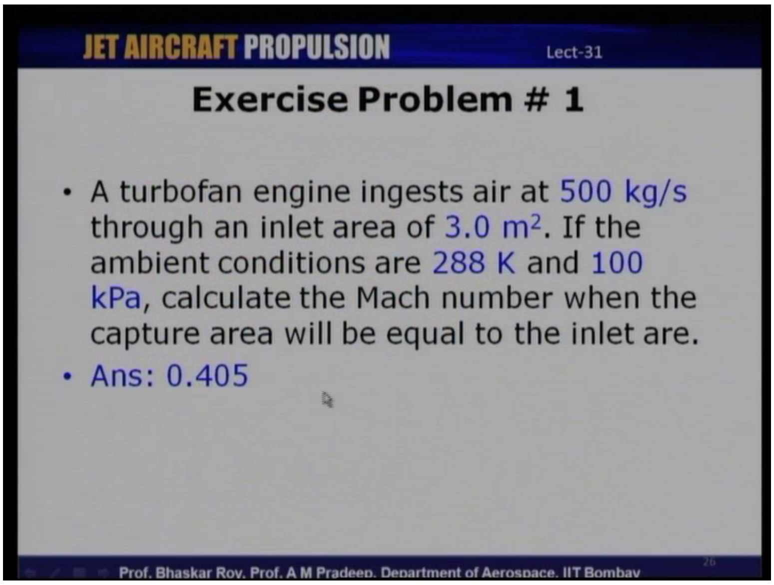

So, the first exercise problem is on a subsonic intake. Turbofan engine ingests air at 500 kilograms per second through an inlet area of 3 meter square. If the ambient conditions are 288 Kelvin and 1 kilo pascal, calculate the Mach number when the capture area will be equal to the inlet area. So, when capture area is equal to the inlet area, what is the corresponding Mach number? So, this is the first problem which is very similar to what we have solved in today's class; the first exercise; first tutorial problem we had solved.

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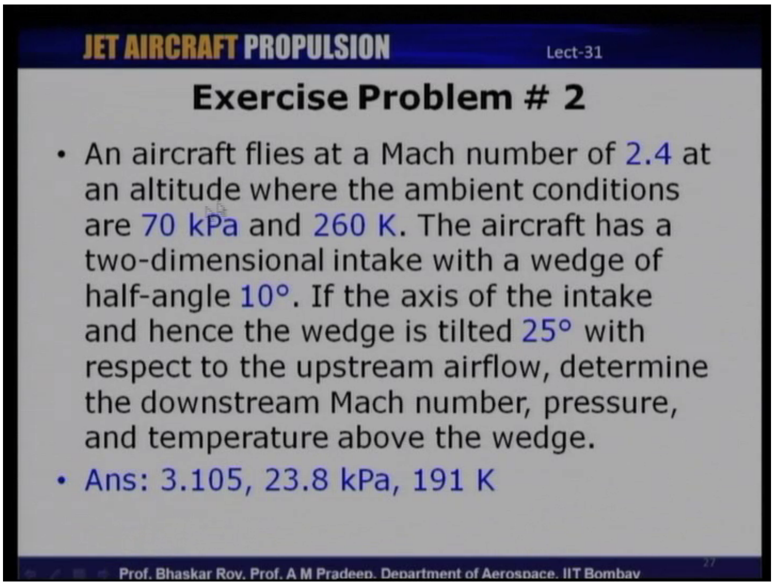

The second question is an aircraft flies at a Mach number of 2.4 at an altitude, where the ambient conditions are 70 kilo pascal and 260 Kelvin. The aircraft has a two-dimensional intake with a wedge angle of 10 degrees. If the axis of the intake and hence the wedge is tilted by 25 degrees with respect to the upstream air flow, determine the downstream Mach number, pressure, temperature as compared to the wedge above the wedge. So, this is an intake which has an half angle of 10 degrees and if this is tilted by 25 tilted to 25 degrees, for the same Mach number you are required to calculate the corresponding conditions downstream of the wedge. So, answer to this question would be 3.105; the pressure is 23.8 kilo pascal and temperature is 191 Kelvin.

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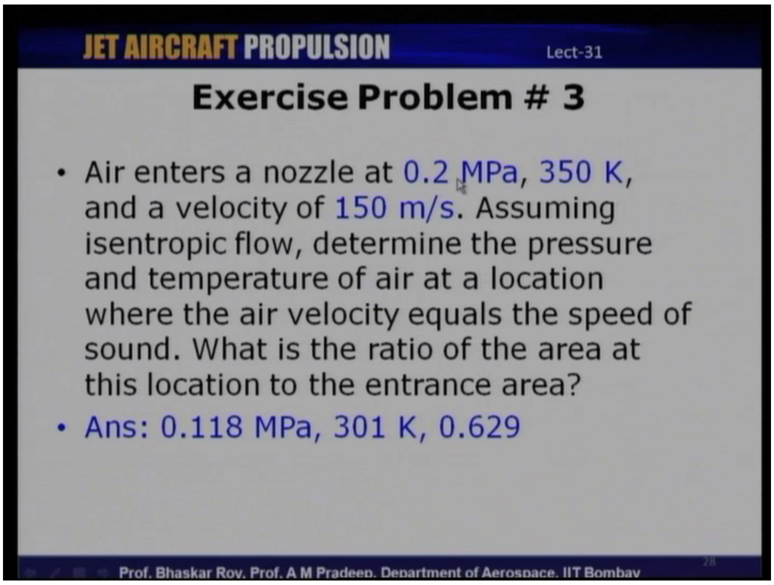

Third question is on a subsonic nozzle. Air enters a nozzle at 0.2 mega pascal, 350 Kelvin and a velocity of 150 meters per second. Assuming isentropic flow, determine the pressure and temperature of air at a location, where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area? So, we need to find pressure and temperature, when the velocity is equal to speed of sound; that is it is chocking. So, that condition, the pressure would be 0.118 mega pascal; temperature is 301 Kelvin and the ratio of the area at this location; ratio of area at the throat to the entrance area is 0.629.

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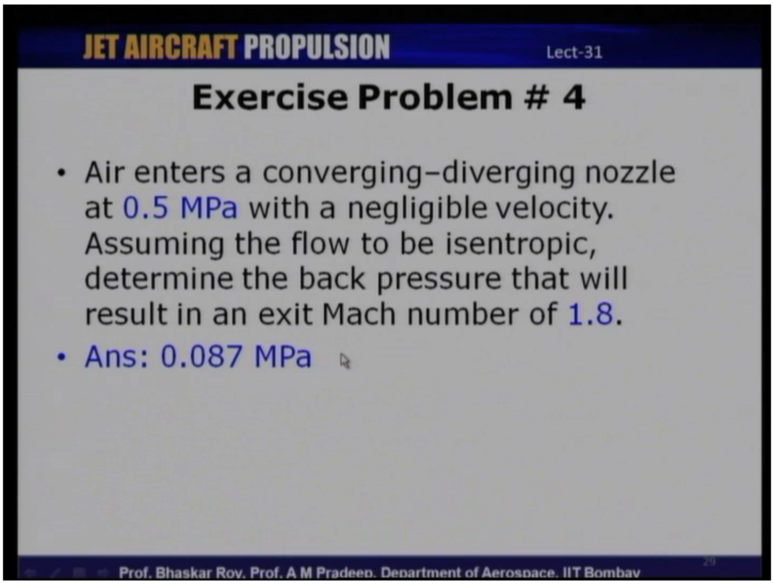

The fourth problem is a converging-diverging nozzle. Air enters a converging-diverging nozzle at 0.5 mega pascal with negligible velocity. Assuming the flow to be isentropic, determine the back pressure that will result in an exit Mach number of 1.8. So, you need to find out what is the exit pressure for which the exit Mach number becomes 1.8. So, the exit Mach number exit back pressure should be 0.087 mega pascal. So, we will need to use the shock table or isentropic tables to be able to solve some of these problems.

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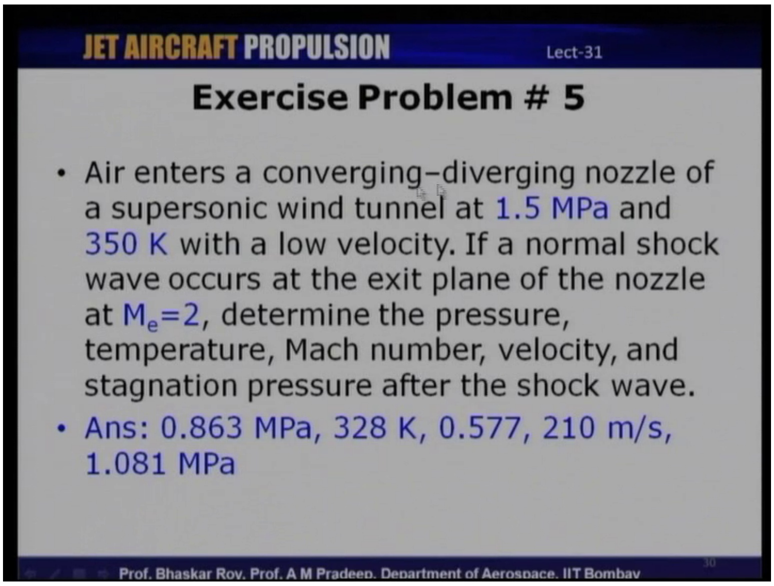

And the last problem is air enters a converging-diverging nozzle of a supersonic wind tunnel at 1.5 mega pascal and 350 Kelvin with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at Mach number of 2, determine the pressure, temperature, Mach number, velocity and stagnation pressure after the shock wave. So, in this question there is a shock a normal shock in the divergent section of the nozzle, we need to find out the conditions after the shock wave. The pressure is 0.863 mega pascal; temperature would be 328 Kelvin; the Mach number is 0.577; velocity is 210 meters per second and the stagnation pressure is 1.081 mega pascal. So, these are a few exercise problems that you can solve. I would assume that you can solve these problems based on what we have discussed today.

And as I said, you will need both the isentropic tables as well as the shock tables to be able to solve these questions. Not just for the tutorial which we have solved, you will also need them for the exercise problems that I have listed for you. So, that brings us an end to an end to this tutorial which was on intakes and nozzles. We will then in the next lecture, we will be discussing about some of the some different aspects of air breathing engines, which pertain to ram jets and pulse jets. So, we will take up a discussion on some of these new concepts in the next lecture. And so, we are winding up our chapter on intakes and nozzles with this tutorial. And I hope based on our discussion during the lecture as well as during today's tutorial, you would be able to appreciate and solve problems pertaining to some of these topics.