**Jet Aircraft Propulsion**

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**Lecture No. # 30**

**Working of Convergent and C-D Nozzles**

Hello and welcome to lecture number 30 of this lecture series on introduction in on jet aircraft propulsion. We have been discussing about the components of aircraft engines and the last lecture, we had devoted exclusively for discussion on one of the components which constitute an aircraft engine that is the nozzle. And I think during the last lecture, we had some detailed discussion on different types of nozzles. We have discussed about subsonic nozzles and supersonic nozzles and various types of these nozzles like fixed geometry, variable geometry, axisymmetric nozzles, and two-dimensional nozzles and so on.

We also had some chance to discuss about what are the functions of a nozzle. Besides the fact that nozzles are primarily meant to generate thrust, modern day nozzles have various other functions; Some of which are for thrust reversal which is used for breaking an aircraft, when it is landing. Then thrust vectoring used popularly in compact aircraft and besides that, it also has to keep noise under control and some of the aircraft have to like the military aircraft also have to keep in mind that the nozzle exhaust should have as low and IR signature; that is infrared signature as possible. So, that it can escape enemy radar from being detected.

So, these are some of the functions that a modern day nozzle has to satisfy. With the fact that, it has to also work as a unit as a single unit along with the rest of the other components like the intake, the compressor the fan and the compressor, the combustion chamber, the turbine and the jet pipe and so on. So, all these components put together constitute the engine and when an aircraft is operational, it has to operate over a variety of operating ranges and all these components need to be matched. And so, you would also be looking at matching process between various components in the next few lectures.

You would also be discussing about matching of these components separately and matching is obviously a very important aspect. Even though individual components may be designed for highest possible efficiency, it should be ensured that when they are all put together and when they form an engine, then they all work as optimum as possible and that is where, the importance of matching comes in to picture. So, nozzle obviously is another component; the last component of an aircraft engine which also needs to be matched with the other components.

So, in today's lecture, we are going to discuss about the function. While we have already discussed the functions, we are going to discuss about the working of these nozzles that different types of nozzles from a very fundamental thermodynamic sense. If you have undergone thermodynamics or gas dynamics course, you probably would have discussed about… you you would have already learnt little bit about how different nozzles work; that is subsonic nozzle and a supersonic nozzle work and how the pressure ratio across these nozzles can be controlled and shown. We will discuss some of these aspects in today's lecture.

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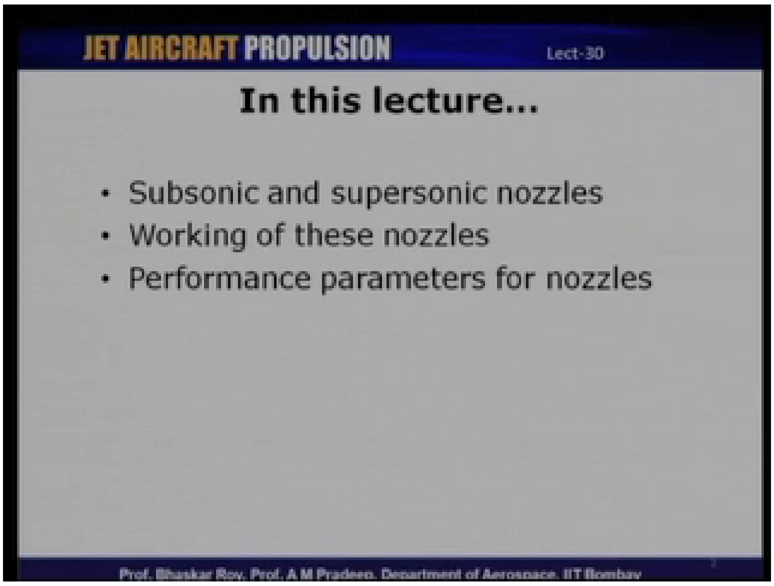


So, today's lecture we shall be taking up some of these topics. We shall talk about the working of subsonic and supersonic nozzles. We will be very briefly discussing about the performance parameters of these nozzles. And towards the end of the lecture, we also discuss some of the issues which nozzle expansion will have like if it is not correctly expanded or fully expanded, then there could be some problem with the flow that is coming out of the nozzle. So, before I start discussing details about the subsonic or the convergent nozzle and subsequently the convergent-divergent nozzle, let us review some of the concepts which I presumed.

You would have learnt in either the gas dynamics course or if you have undergone one of our earlier courses on introduction to aerospace propulsion. Some of these topics have already been covered there in detail. We will quickly review some of these concepts before proceeding towards understanding of the working mechanism of these nozzles. Now, in the last class, I think I mention that in a convergent nozzle, there is certain limitation to the Mach number that can be achieved. That is, if we keep varying keep reducing the downstream pressure the back pressure so called the back pressure, the flow will accelerate in a subsonic flow.

It will accelerate in the nozzle and what is the maximum Mach number that it can reach. The maximum Mach number it can reach in a convergent nozzle as you probably know is mach 1. That occurs when the flow or the flow rate has reached its maximum level at the throat of the nozzle. Throat is the area that is the minimum cross sectional area of of a convergent nozzle. When the mass flow reaches its maximum, then that is instance the Mach number at the throat would be unity and under this condition, the flow or the nozzle is said to be choked.

Now, if you want to reduce the Mach number further, one might intuitively want to reduce the area further; that is we are attach let say another convergent nozzle to that. What you will notice obviously is that that is not going to help; it is not going to reduce the… it is not going to increase the Mach number further. If a Mach number increase has to indeed take place after the throat of a convergent nozzle, we will need a divergent section. And that is why we have convergent-divergent nozzles wherein we can achieve supersonic flow, which is not possible in conventional simple convergent nozzle.

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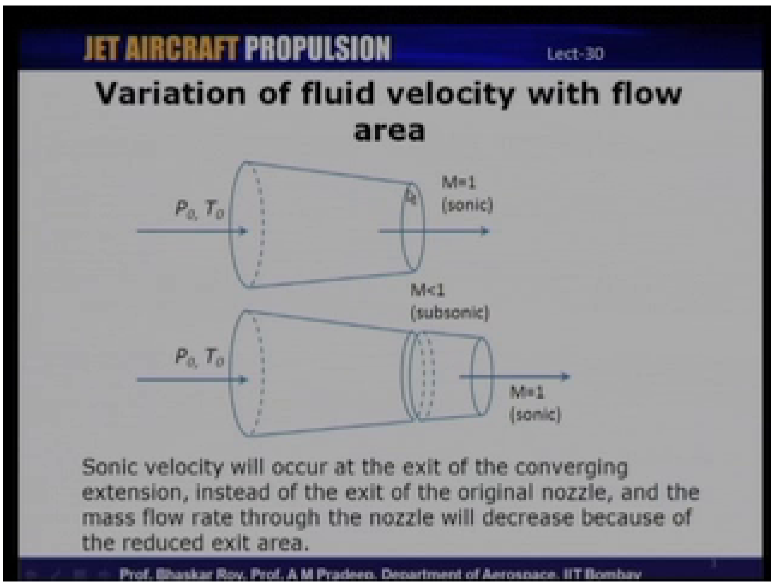

So, in a convergent nozzle, as I am going to illustrate here now this is an illustration of is convergent nozzle. So, in the first one, we have a convergent nozzle which has upstream pressure stagnation pressure P 0 or p naught and stagnation temperature, T naught. As the flow accelerates through the nozzle, it reaches sonic Mach number that is mach 1 at the throat. And ofcourse, this occurs only for certain back pressures; it does not occur always. If there is a sufficient pressure ratio, then the nozzle can be choked and you get sonic velocity here. Now, if you want to increase the Mach number further and after; that is, after this we want to supersonic Mach number, it will not help if you put in another convergent nozzle here.

Let us say we put another convergent nozzle with this as the inlet area of that nozzle and an area which is much smaller than this at the exit. What will happen is that the sonic point of the throat will shift. So, as you attach this additional portion here, the throat will shift to a new location; which means that the location which was the throat for the first nozzle, where you had Mach number unity will no longer be sonic Mach number. You would have a subsonic mach number here. Sonic Mach number would have shifted. Ofcourse, this will not keep happening if you keep increasing the area I mean the area ratio further, then it will obviously result in loses.

Then there will be increased loses in the nozzle and it is not a feasible proposition. So, this requires that we attach a divergent section after the throat. Attaching a divergent section is the only solution that we can get a supersonic mach number in a nozzle that is convergent section ending in a throat followed by a divergent section. Now, in a gas dynamics course, you must have derived what is known as the area velocity Mach number relation right, where we must have seen that when mach number is less than 1. Then when there is an increase in area, we have decrease in velocity; that is we have a diffuser.

And if there is a decrease in area, we have an increase in velocity; whereas, if it is supersonic Mach number, then the trend is reversed and increase in area will actually give an increase in velocity and vice versa. That is, decrease in area will result in decrease in the velocity. This is for a supersonic flow which means that if that is to be true, for this convergent nozzle which has attained a sonic Mach number at its throat. You can achieve a supersonic Mach number further, if we attach a divergent section; that is in at the end of a convergent section where it has reached its throat, if you attach a divergent section there, then it is possible that we can get supersonic Mach number in the divergent section.

And little later we will see that it does not always happen. It requires certain conditions to be satisfied like the pressure ratio the back pressure has to be low enough for you to achieve supersonic flow even in the divergent section; that we will discuss a little later. Now, in the next slide what I have shall show you is probably something you would have learnt in gas dynamics or in the previous course which is a direct outcome of the area Mach number velocity relation, which is what I had just described that what happens for subsonic flow and what happens for supersonic flow are entirely opposite, when it comes to nozzles and diffusers.

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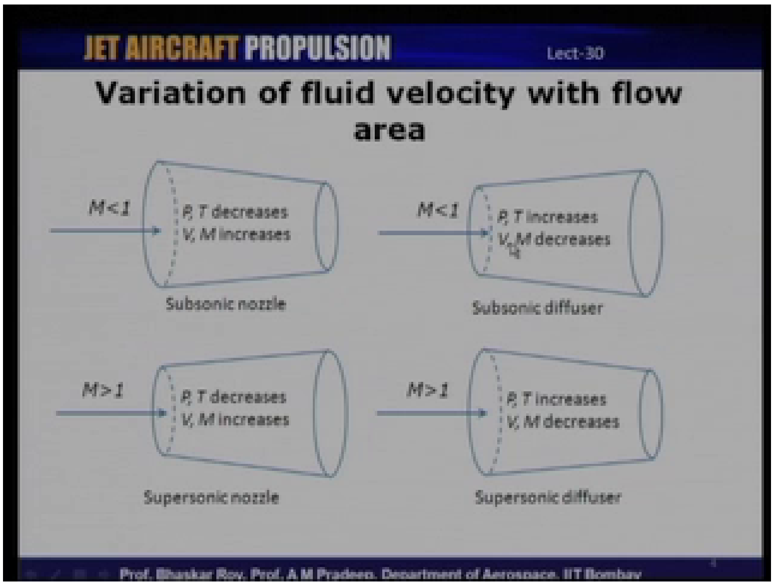


So, let us take a look at what happens there. Now, in a subsonic flow, let us say the first two diagrams the top two pictures or illustrations are true for a subsonic flow. Subsonic flow mach number upstream Mach number is less than 1. Decrease in area results in decrease in pressure and temperature, since static pressure and temperature; yet at the same time results in increase in velocity and hence the Mach number. If we put the nozzle the other way round, that's Mach number is still less than 1 upstream; but area is now increasing. Then the pressure and temperature static pressure and temperature increases, velocity and therefore the mach number decreases.

Therefore, the first one is known as a subsonic nozzle; second one is known as a subsonic diffuser. Now, the reverse exact reverse of this happens, when the flow is supersonic. If Mach number is greater than 1, then an increase in area as you can see here. We will result in decrease in static pressure and temperature and therefore velocity and Mach number and therefore, this is supersonic nozzle. And Mach number greater than 1 that is in supersonic flow, if there is a decrease in area, it results in increase in static pressure and temperature and therefore, it will it results in increase decrease in velocity and hence mach number.

So, this is basically supersonic diffuser. Say nozzle in a subsonic flow acts like a diffuser in a supersonic flow. A diffuser in a subsonic flow acts like a nozzle in supersonic flow. So, this one important aspect that you need to understand here that what a nozzle for a subsonic flow is will act like a diffuser for a supersonic flow and what acts like a diffuser in a subsonic flow acts like a nozzle in a supersonic flow. So, this gives us some idea on how we can proceed now. We are struck with the fact that in a in a convergent section. We can only achieve sonic Mach number. So, what we do if we have to get a supersonic Mach number beyond the throat?

So, after the throat now if we need to get a supersonic flow, the best way to do that is or well rather the only way to do that is to attach a divergent section. So, a divergent section as we have seen now will get us a supersonic flow; because a divergent section in a supersonic flow is a nozzle for supersonic flow. So, if we have to get achieve supersonic flow, we will need a divergent section. So, all supersonic nozzles will have a convergent section which will end in a throat, which is the minimum area of that entire nozzle followed by a divergent section, where we can get supersonic flow.

So, in a supersonic nozzle, we normally referred to supersonic nozzles as convergent-divergent nozzle; because it as a convergent section, it also has a separate divergent section and in between these two, there is a throat where the area is minimum. So, supersonic nozzles will be basically convergent-divergent nozzle. So, today's lecture, we will discuss in detail about how a supersonic nozzle can work. And we will see very soon that it cannot work in under any circumstance that needs to be certain pressure ratio across the nozzle which has to be maintained. So, let us begin our discussion with a discussion on the subsonic nozzle first. Let us see what happens in the case of a convergent nozzle.

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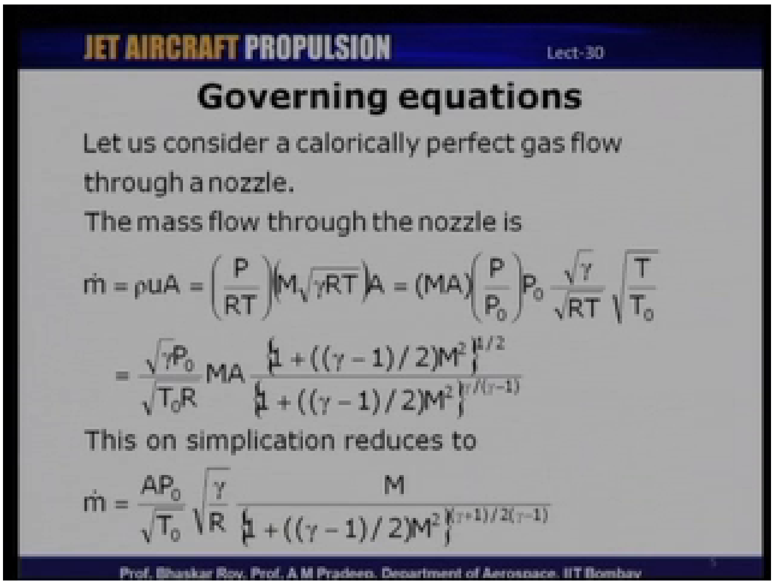


So, in a convergent nozzle, what we will discuss begin with is that convergent nozzle as we have seen can give as the maximum of Mach number of 1. So, beyond that, obviously we cannot increase the Mach number; that is a convergent nozzle. So, before we look at the convergent nozzle, let us look at one set of equations which will be used in both the cases where whether it is subsonic or supersonic flow. So, we will look at one of the governing equations for this; where we will derive an expression for the mass flow rate which can pass through these geometries whether it is convergent or convergent-divergent nozzle. So, mass flow rate as we know it from continuity is equal to the product of the density, the velocity and the area.

So, from product of these three, we can calculate mass flow rate. So, let us express mass flow rate in terms of parameters like the upstream parameters, the stagnation pressure and temperature and some of the other parameters including Mach number. Let us express or derive an expression for mass flow rate which we can use in our subsequent discussion on convergent as well as C-D nozzles; that is convergent divergent nozzle. So, let us basically consider flow of calorically perfect gas through a nozzle, which could be either be a convergent nozzle or convergent-divergent nozzle. So, mass flow rate as we know it is product of density, velocity and area. Density we know is pressure divided by R T; P by R T.

Velocity is Mach number time square root of gamma R T and area. So, here we have we are going to simplify this. We have now Mach number times area P by P naught, which is P naught is stagnation pressure multiplied by P naught in to square root of gamma divided by square root of R T in to square root of T by T naught. This basically comes from this expression here; because we have as a square root of R T here and R T. So, this becomes square root of R T and T by T naught. So, this again we can simplify. We now have square root of gamma in to P naught which is stagnation pressure divided by square root of T naught in to R multiplied by Mach number in to area. In the numerator, we have 1 plus gamma minus 1 by 2 M square and where does this come?

This comes from the temperature ratio; that is T T naught by T is 1 plus gamma minus 1 by 2 M square. And that is why, we have the square root 1 plus gamma minus 1 by 2 M square rise to 1 by 2; because it is square root of this. Similarly, pressure we have expressed in terms of Mach number. We have 1 plus gamma minus 1 by 2 M square rise to gamma by gamma minus 1. This we can again simplify. This after we simplified we be we get an expression mass flow rate is equal to area time stagnation pressure divided by stagnation time pressure square root square root of gamma by R which are properties of the gas multiplied by Mach number M divided by 1 plus gamma minus 1 by 2 M square whole rise to gamma plus 1 divided by 2 in to gamma minus 1.

So, this is a simple expression which relates mass flow rate and some of the other parameters which are involved; mass flow rate, then the inlet stagnation pressure temperature and Mach number. Besides ofcourse, the gas properties like the ratio of specific heats gamma and the gas constant R. So, what we have here is an expression which tells us that what the mass flow rate which a given area can give. So, here again the area comes in to picture, once we know the stagnation pressure temperature and the Mach number or vice versa. If we know the mass flow rate and ofcourse the area, we can actually calculate what Mach number we are operating at and so from this expression which can be used for both subsonic and supersonic nozzles.

We can basically relate mass flow rate with the other parameters like stagnation pressure temperature, Mach number and so on. So, with this background in mind, we will now try to analyze flow through convergent nozzle. But we will look at an isentropic flow through a convergent nozzle. When we talk about isentropic flow, we are inherently assuming that the flow is adiabatic and ofcourse, it is reversible. Well, adiabatic flow is true in this case; because the amount of change in pressure that occurs across the nozzle is much larger than the amount of heat transfer, which may take place across the walls of the nozzle. So, with that assumption in mind, we can keep we can safely assume that the flow through these nozzles are adiabatic.

There is no heat transfer really. Even though there would be heat transfer, that is very small compared to the pressure drop which occurs in the nozzle. Therefore it safe to assume that the flow is adiabatic and so with for an isentropic nozzle flow or isentropic flow across a nozzle, we will now look at what happens in a convergent nozzle; how the pressure changes, how the mass flow rate changes and so on in convergent nozzle, which is basically a subsonic nozzle; which we will very soon seen that the max Mach number which we can get and reasons behind that is unity. You can get a Mach number of 1 at the throat of the nozzle.

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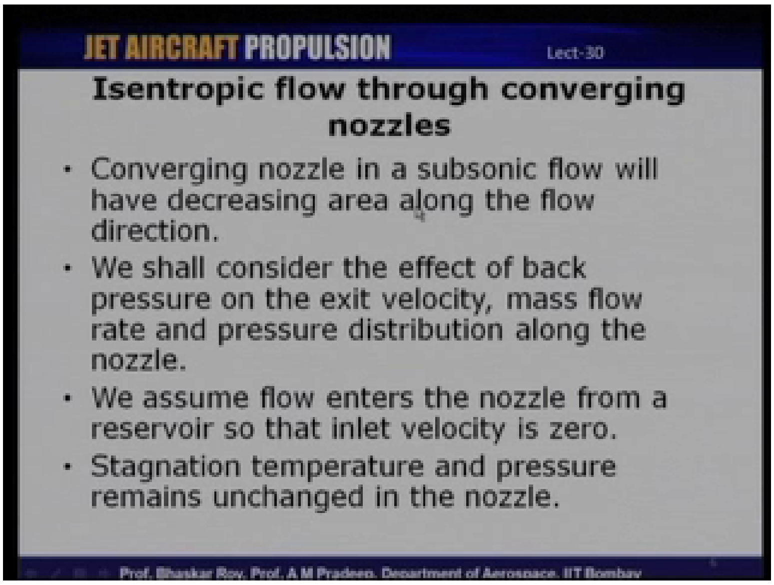


So, we will look at a convergent nozzle first in a subsonic flow, which means the convergent nozzle in a subsonic flow will have to have a decreasing area along the flow direction. We will now consider the effect of back pressure on the exit velocity, mass flow rate, pressure distribution etcetera. And we will also assume that the flow is entering the nozzle from a reservoir in such a way that, its stagnation pressure there therefore inlet velocity will be zero and stagnation pressure and temperature will not change in the nozzle. That is stagnation temperature cannot change; because it is adiabatic. There is no heat transfer across the nozzle valves and the stagnation pressure we will assume does not change; because we will assume that there are no pressure loses.

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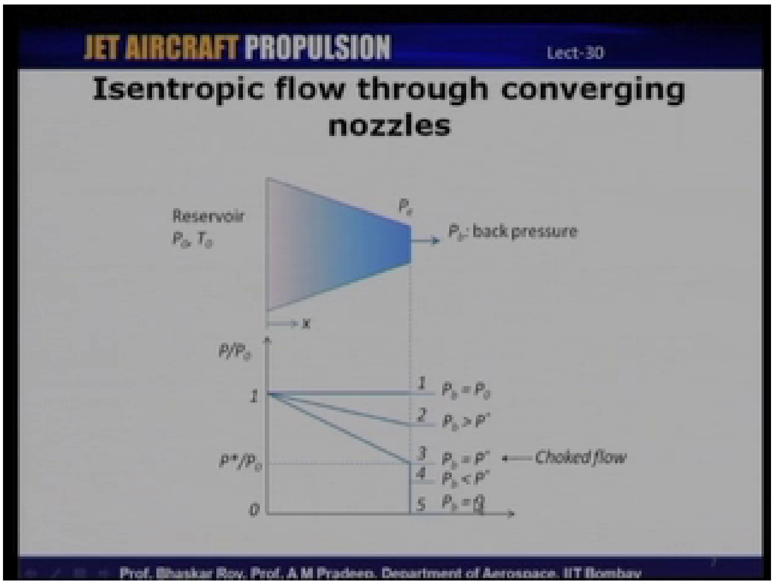


Let us take a look at what happens. So, I have here a very simple schematic of a convergent nozzle. Though this is a convergent nozzle in subsonic flow, it has to be increasing area. We will now see the effect of back pressure which I have denoted by P subscript b on the flow properties in the nozzle. The upstream pressure that the reservoir pressure is P naught, which is stagnation pressure; it also has stagnation temperature of T naught. The nozzle exit pressure is denoted by P subscript e or P e. Let us look at the ratio P by P naught that is a pressure at any instant versus the reservoir pressure for any value of back pressure.

The first instance is if back pressure is equal to this reservoir pressure, what happens. Well, if the pressures at both ends are same, obviously there is no flow taking place across the nozzle. So, there will not be any flow. Let us see what happens if we decrease the back pressure. We have decrease the back pressure now; but it is still greater than the critical pressure. Critical pressure is denoted by P star. Critical pressure tells us or denotes that pressure at which we get a sonic flow or choked flow at the exit of the nozzle. So, second instance we have P b which is greater than p star.

But it is still less than this reservoir pressure. So, P b is less than p naught as you can see here. This is the location where P b by P naught is equal to 1. Now, we have P b by P naught less than 1 there. But it is still greater than the critical pressure. Now, if you further reduce it, we reduce back pressure and now it is equal to the critical pressure. Critical pressure refers to the pressure, where we get sonic fluid at the throat. Now, when P b is equal to P star; that is when the flow has attain the maximum mass flow rate that it can pass through, we get choked flow here. Mach number is equal to 1 and mass flow rate reaches the maximum value possible.

So, that is why P by P naught; when p P b is equal to P star that pressure ratio P star by P naught corresponds to a choked flow. Any further reduction in P b, if it is P b less than P star, it will still lead us to the same flow rate that we are talking about. Whatever be the flow rate here, we continue to get the same mass flow that we get at this particular point. So, it does not change the pressure characteristic; it is the same. Because the way the pressure drops in the nozzle will continue to remain the same, even after we reduce back pressure any further than the critical pressure. So, this is how pressure ratio varies across the nozzle.

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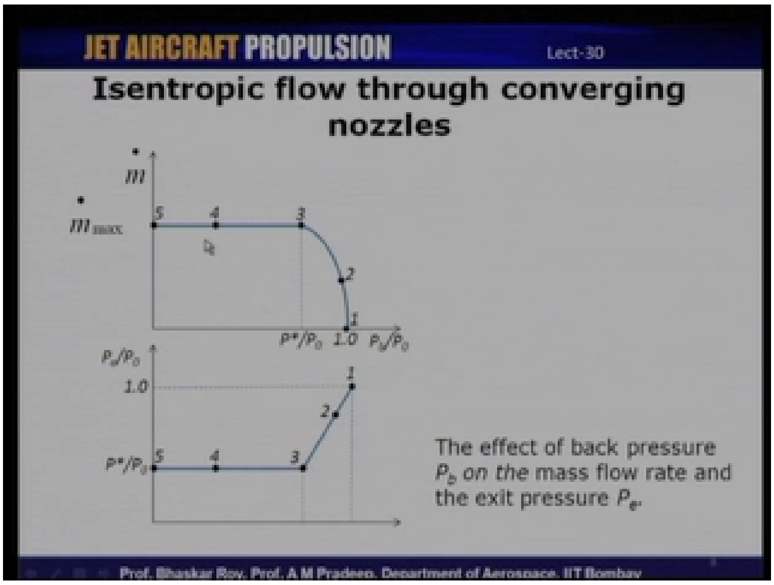


Let us look at how mass flow rate varies. We have discussed about pressure ratio already. The first chart here is for the mass flow rate; the second one is the pressure ratio exit to the inlet stagnation pressure. So, we has beginning our flow right here, where mass flow rate is 0 which occurs when P b is equal to P naught; that is ratio P b by P naught is equal to 1; that is when we have the zero mass flow rate. Now, as the ratio decreases as P b by P naught is reducing, mass flow rate is increasing. As you can see here from 1, it goes to 2. When P b is equal to P star which is the critical pressure, then we have the mass mass flow rate which reaches its maximum value, which is m dot max.

If you look at the corresponding pressure ratio plot here, we starting from here where P by P naught is zero is equal to 1 and as that ratio continuously decreases, we reach here which is P star by P naught. Any further reduction does not change the mass flow rate and it does not change the pressure ratio as well. p P star by P naught remains the same; mass flow rate has reached its maximum value. So, this particular condition of the flow which we have seen just now discussed that is at state 3 and beyond is known as the choked flow. We are saying it is choked will literally choked means that whatever we could pass the maximum that one could pass has reached and that is known as choked. We cannot take up anything more than that; that is known as choking.

So, the nozzle has choked means that it has pass the maximum mass flow that it can. And any further increase will not change the mass flow rate that the nozzle can handle; that is known as the choked flow. So, for a subsonic nozzle or convergent nozzle, choked flow is the limit to which a nozzle can operate; beyond which nothing can happen. If we continued to, it will infact lead to more losses or if one tries to increase the mass flow; because mass flow cannot increase beyond that level. So, nozzle when operating under choked condition will have mass flow rate is equal to maximum; Mach number is equal to unity, sonic Mach number; pressure will be equal to the critical pressure, which we have denoted by P star.

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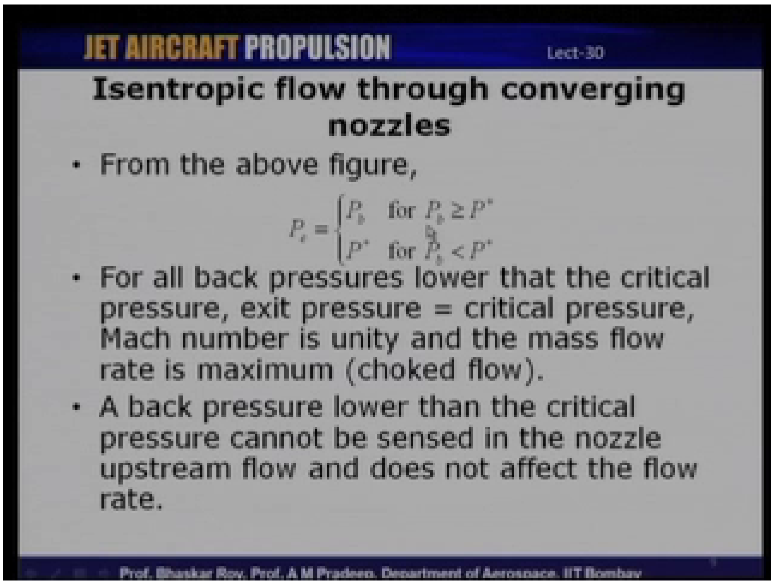


So, from the above figure that we have just discussed now, the back the exit pressure P e will be equal to the back pressure, for all back pressures greater than p star. Exit pressure will be equal to the critical pressure, for any back pressure which is less than the critical pressure itself. So, for all back pressures lower than the critical pressure, exit pressure will be equal to critical pressure; Mach number is unity and mass flow rate is maximum; that is we get a choked flow. Back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and it does not affect the mass flow rate. So, any back pressure which is lower than the critical will not be sensed by the upstream flow and therefore, it does not change the mass flow rate.

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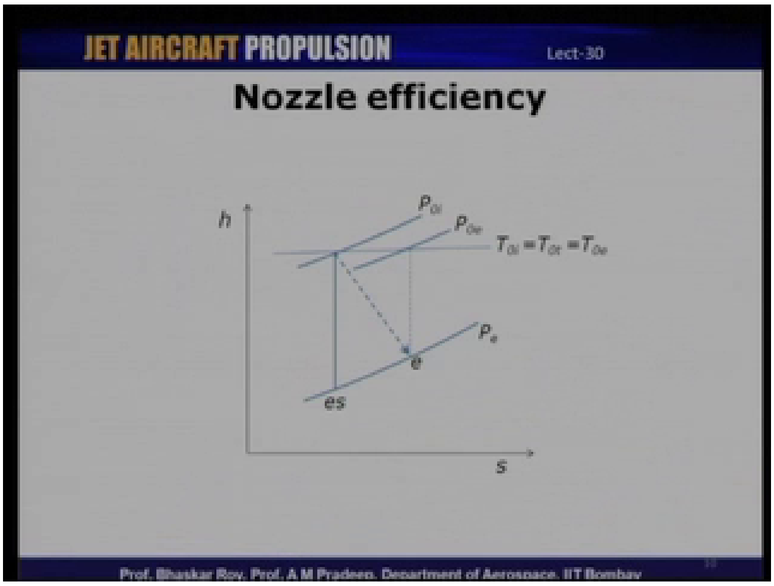


So, before we discussed a little more on how we can analyze the flow, let us look at quickly look at nozzle efficiency. We have already derived this in one of the earlier lectures. But it has been quickly reviewed that nozzle efficiency we are talking about a process. This process which is shown by the dotted line, where the nozzle inlet pressure is been noted by P o that is P naught stagnation inlet and exit condition is denoted by P e which is static pressure. So, this is the process actual process. The solid line here shows the isentropic process which will be denoted by P e s, where s stands for the isentropic flow.

Now, the corresponding exit stagnation pressure would be P o e or p P naught is the stagnation pressure; subscript e is for the exit. Now, what is to be also kept in mind is that there is no change in stagnation temperature; because it is an adiabatic flow. Stagnation temperature cannot change, unless there is heat transfer. So, T o T naught i which is stagnation temperature inlet will be equal to T naught t, which is stagnation temperature at the throat that will also be equal to stagnation temperature at the exit T naught e. So, this is how the process will look like.

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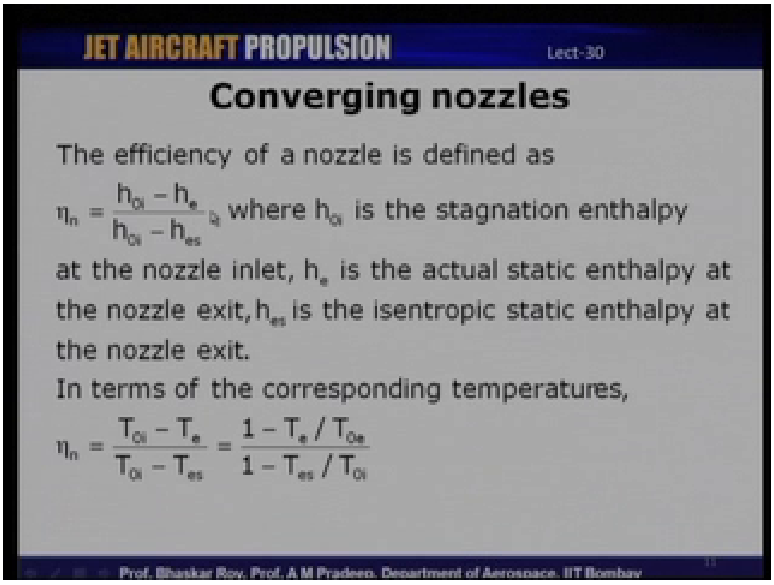


Now, with that in mind, we have defined efficiency earlier. Nozzle efficiency is h naught i which is stagnation enthalpy at the inlet minus h e static enthalpy at the exit divided by h naught i minus h e s, which is static enthalpy for an isentropic process; where h e s is static enthalpy for an isentropic process. We express this in terms of temperatures. So, efficiency will be equal to T naught i minus T e divided by T naught i minus T e s which is 1 minus T e by T naught e; because T naught i and T naught t are the same divided by T e s by T naught i. So, this we will now express in terms of the Mach number and the pressure ratios.

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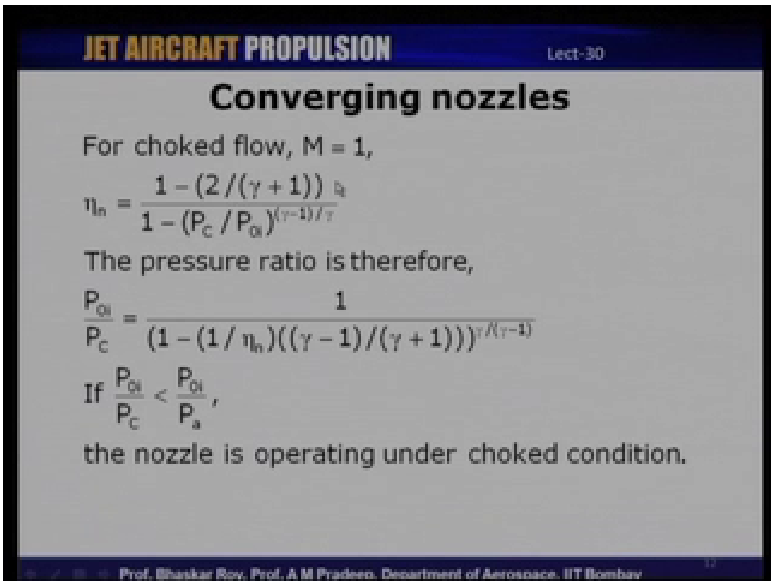


So, for a choked flow what happens is that Mach number is equal to 1. So, this temperature ratio that you see here T naught e by T e is 1 plus gamma minus 1 by 2 M e square. And since

M e is equal to 1, when we equate that equal to 1, we get 1 minus 2 by gamma plus 1 divided by 1 minus T c or critical pressure divided P naught i rise to gamma minus 1 by gamma. So, from this if we simplify, we get the critical pressure ratio or just the pressure ratio P naught i by P c is equal to 1 by 1 minus 1 by eta n the nozzle efficiency multiplied by gamma minus 1 by gamma plus 1 rise to gamma by gamma minus 1.

So, if we see that in this case, when Mach number is equal to unity, the pressure ratio the critical pressure ratio is only a function of the gas and ofcourse the nozzle pressure ratio. So, if we get this number P naught i by P c less than the actual pressure ratio P naught i by P a or the ambient where P a is the ambient pressure, the nozzle is operating under choked condition. So, if this pressure ratio that we have seen with which we can calculate based on the gas properties and nozzle efficiency. If that is less than the pressure ratio corresponding to the ambient pressure, the nozzle is operating under choked condition.

And so, given a certain nozzle geometry and some of its properties, the type of gas which is normally either air or combustion products. From the pressure ratio, we can actually find out whether the nozzle is going to operate under choked condition or not. So, if that pressure ratio which we have just now derived P naught i by P c comes out to be less than P naught i by P ambient, then it means that the nozzle is choked. And most of the aircraft which use subsonic nozzle or convergent nozzles, nozzle will most of the time atleast under the design operating condition be operating under choked condition.

Because that gives as the maximum mass flow which can which is the nozzle can handle. And ofcourse, this choking condition can be changed by changing the area ratio which is one advantage which adjustable or variable nozzles have. So, if the nozzle exit area can be changed, then the choking condition can be changed. So, the nozzle can be unchoked and then depending upon the operating condition, the nozzle area can be changed to get again a choked flow at the exit. So, that is obviously one of the advantages of a variable area nozzle. So, for a convergent nozzle, let me summarize what we have just now discussed.

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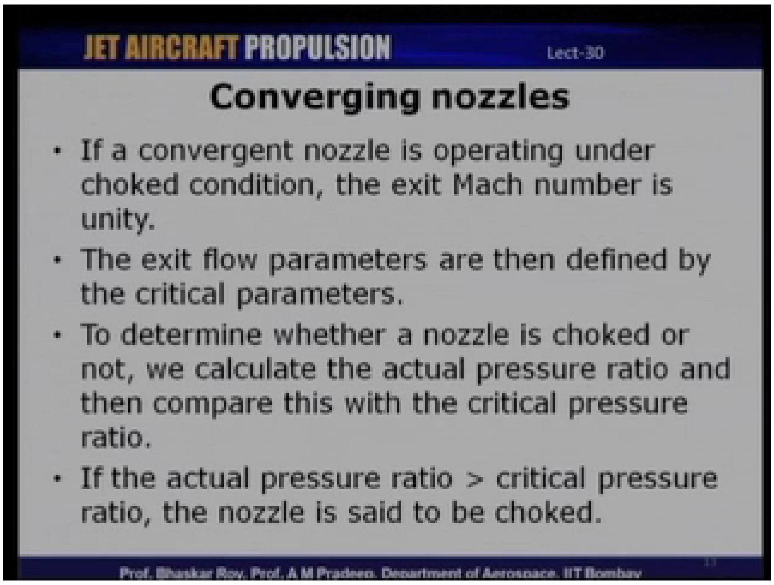


So, in a convergent nozzle, if it is operating under choked condition, the exit Mach number is unity as we have seen. The exit flow parameters will then be defined by the critical parameters; that is the exit pressure will be equal to critical pressure; exit temperature will be equal to critical temperature; exit density is critical density and so on. All exit properties will be equal to critical parameters. This also we have discussed when we were talking about the cycle analysis real cycle analysis or even ideal cycle, where we have actually calculate the pressure ratio and then say just see if the nozzle is choked or not. If it is indeed choked, then we have to equate the exit conditions to be choked parameters.

So, if the pressure ratio is greater than the critical pressure ratio, the nozzle is set to be choked. In which case, the parameters at the exit of the nozzle will be equal to the critical parameters that we have which we can calculate. Because critical temperature for example, T e will basically be a function of gamma; that is it will be 2 by gamma plus 1 in to T naught and so on. Because T naught by T is equal to 1 plus gamma minus 1 by 2 M square; for critical condition, M is equal to 1. So, you get T naught by T e is equal to 2 by gamma plus 1 and so on or gamma plus 1 by 2. So, based on this, we can actually calculate the exit conditions of the nozzle, which will only be purely a function of the upstream stagnation conditions and the ratio of specific heats that is gamma.

So, having understood operation of a subsonic nozzle, let us now take up a supersonic nozzle; that is a convergent-divergent nozzle. Let us now look at how an isentropic flow behaves as it passes through a C-D nozzle, a convergent-divergent nozzle. We will analyze the flow in a very similar manner as we have done for subsonic flow; that is we keep changing the exit back pressure and see how the flow varies in the nozzle. Ofcourse, in this case the nozzle the flow is going to be slightly more complicated. Because of the fact that we are going to deal with supersonic flows, there will be presence of shocks. So, some of these aspects we are going to discuss very soon.

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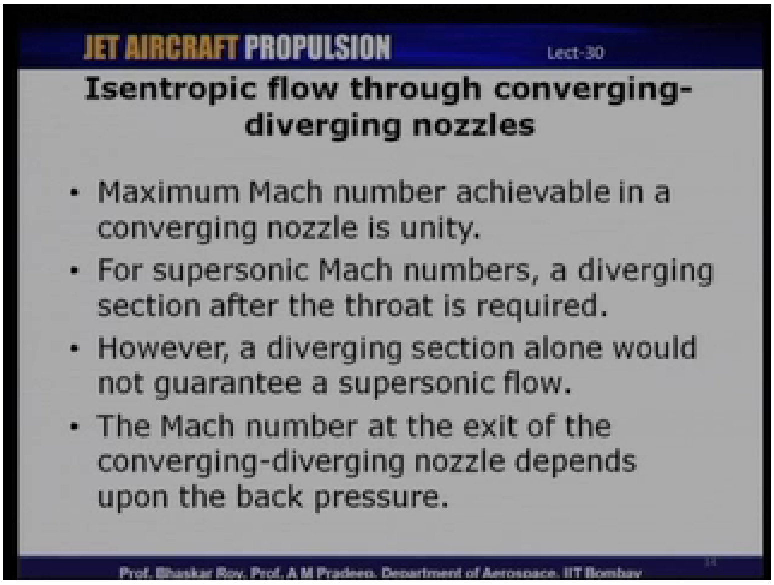


So, in a C-D nozzle, the basic advantage is that we can achieve supersonic Mach number, which can be achieved in the divergent section. Well but divergent section alone cannot guarantee a supersonic flow. We need to ensure certain back pressure. So, that is what we are going to discuss now.

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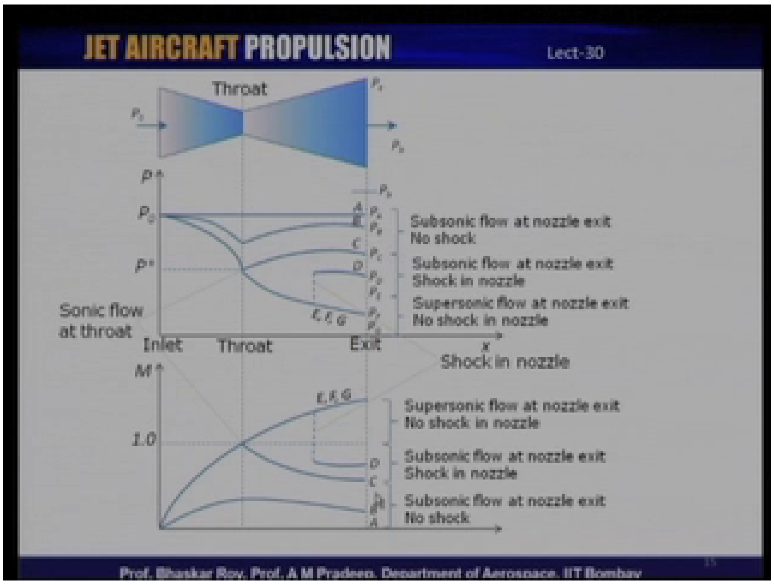

So, for that let us again take up a very simple C-D nozzle. It is a linear C-D nozzle just for the sake of simplicity. We have a convergent section as you can see here, which is very similar to the convergent nozzle we have seen in few slides earlier. Followed by this, this throat this is a throat section, we have the divergent section; convergent nozzle ending in a throat followed by the divergent section. This constitutes a typical convergent-divergent nozzle. Upstream pressure that stagnation pressure, this reservoir stagnation pressure is P naught; exit pressure static pressure is P e; we have a back pressure P b. So, let us now look at how the flow would vary as we keep changing the back pressure.

Initially, when P a is equal to P b which is the equal to the stagnation pressure, there is no flow taking place. As we decrease the back pressure, we have now new pressure which is P b which is less than P naught; but it is still greater than the critical pressure. In a convergent section, the flow will accelerate which means there is a drop in static pressure from inlet to the throat. From the throat till the exit, it is a divergent section in a subsonic flow. So, there has to be diffusion or deceleration leading to increase in static pressure. So, static pressure increases; but it reaches a value which is lower than the stagnation pressure upstream. We further decrease it; we reach a point where which is basically corresponding to the choked condition.

We still have subsonic flow; because when P b is equal to P star which is the critical pressure at the throat, we get sonic flow at the throat and that is indicated by P star. So, you can see that for all these three cases, we still have a subsonic flow at the nozzle exit. Now our objective here is to get supersonic flow which means that we have to now further decrease the back pressure. Now, what happens is for a back pressure which is lower than the critical pressure, there are different conditions which can exits. That is, adds the back pressure is lowered to value which is lower than the critical pressure, which means that the flow is no longer… well it becomes sonic at the throat.

But since the back pressure is lower than critical pressure and we have a divergent section here which will act like a nozzle in a supersonic flow. So, we have a divergent section. So, the flow becomes supersonic which is why you can see the static pressure continues to drop; because the flow is accelerating and that happens in a supersonic flow. So, after the throat, it become flow becomes supersonic and under certain conditions if the back pressure is not low enough, you might have a normal shock within the divergent section of the nozzle itself. So, there could be a normal shock here, which means that downstream of the normal shock the flow is subsonic.

So, after the normal shock, we have a subsonic flow which means that a divergent section will now behave like subsonic flow diffuser. So, you can see there is again an increase of static pressure. So, this occurs till a point where the back pressure is low enough to ensure that there is a supersonic flow. That is, they have to continuously decrease even we are now at the back pressure which is equal to P d. We have to decrease further even if it is P e, we continue to have subsonic flow at the nozzle exit, which is not the objective. So, the back pressure has now is now supposed to be lower than all these pressures.

So, if back pressure is further reduced, we now we can ensure that the normal shock that was existing here can be pushed out of the nozzle. So, that we get a supersonic flow at the nozzle exit, which is basically our objective that we need a supersonic flow at the nozzle exit. So, for back pressures which are not as low as this; but it is lower than critical pressure, we might under having a shock within the divergent section of the nozzle which is obviously not a good thing. So, if you look at the Mach number plot for these conditions, we had the first case. That is for subsonic flow, we have Mach number increasing from the inlet all the way to the throat.

But it is not Mach number equal to 1 at the throat; after which, again the Mach number decreases. For critical pressure, when we have sonic flow at the throat, Mach number increases which is Mach 1 and then again it decreases or decelerates; because it is subsonic. For back pressure is equal to P d, we have Mach number equal to 1 at the throat. It continues to increase, but only up to the shock; after which, it becomes subsonic and it decreases below 1. For Mach numbers for continuous supersonic Mach number all the way up up to the nozzle exit, we need to ensure that the back pressures are low enough; So that, there is no shock within the divergent section of the nozzle.

So, this is a basic operation of C-D nozzle; a convergent-divergent nozzle, where we can achieve supersonic flow under certain operating conditions that is depending upon the nozzle pressure ratio. One can ensure that they can achieve supersonic flow in a C-D nozzle. So, simply fitting a divergent section at the throat of a convergent nozzle will not suffice. We need to ensure that there is sufficient pressure ratio, which can lead to supersonic flow within the nozzle. So, with this background in mind, we shall now derive some expression as we get for convergent nozzle also to evaluate the performance or some of the parameters like mass flow rate through a supersonic nozzle and so on.

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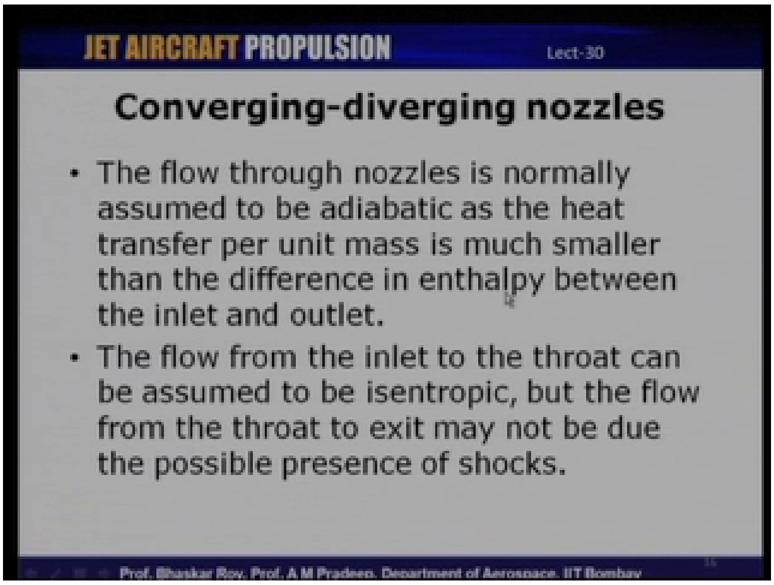


So, what we will do is we are going to assume as we have done for convergent nozzle that the flow is adiabatic because heat transfer per unit mass is much smaller than the difference in enthalpy between inlet and outlet. The flow can be assumed to be isentropic up to the throat. But from the throat to the exit, it may not really the isentropic; because there could be possible shocks especially under certain operating conditions.

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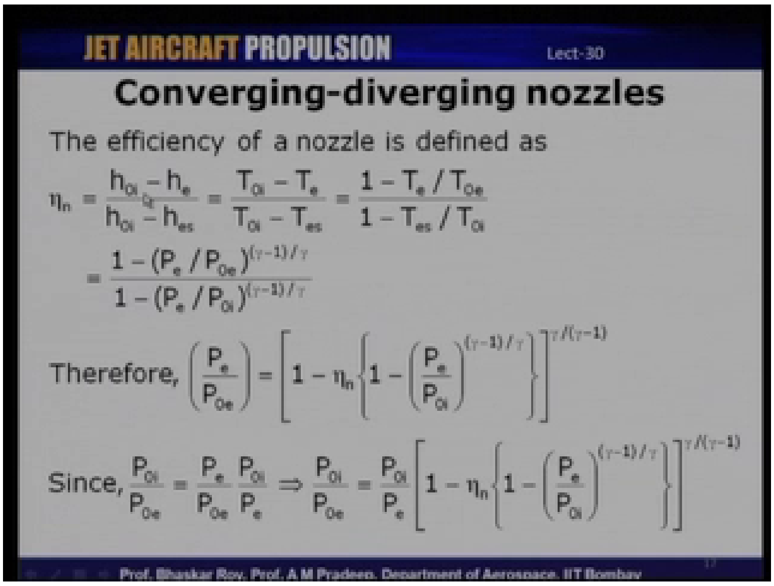


So, nozzle efficiency we have already derived for the subsonic nozzle which is h naught i the inlet stagnation enthalpy minus h e divided by h naught i minus h e s, which we can expressed in terms of temperature ratio is 1 minus T e by T naught e divided by 1 minus T e s by T naught i. This will now be expressed in terms of the pressure ratio. So, we get 1 minus P e by P naught e raise to gamma minus 1 by gamma divided by 1 minus P e by P naught i raise to gamma minus 1 by gamma. So, here we will now express P e by P naught e in terms of the nozzle efficiency and the other pressure ratio.

So, the exit pressure to the exit stagnation pressure ratio is equal to 1 minus eta n which is nozzle efficiency in to 1 minus P e by P naught i raise to gamma minus 1 by gamma, the whole thing raise to gamma by gamma minus 1. Now, we can now express P naught i by P naught e as P e by P naught e multiplied by P naught i by P e. So, from this, we get P naught i by P e P naught e is equal to P 0 i which is string stagnation pressure divided by P e in to this term; that is, 1 minus eta n in to 1 minus P e by P naught i raise to gamma minus 1 by gamma rise to gamma by gamma minus 1.

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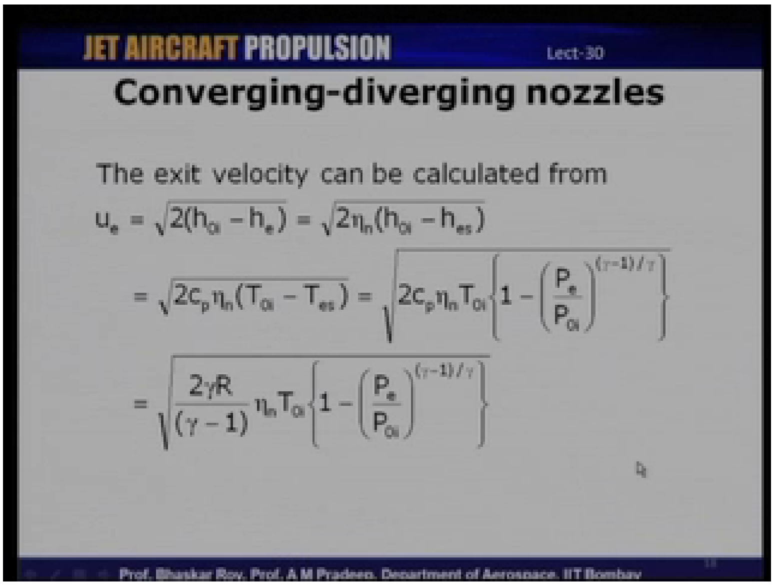


So, the exit velocity; So, which we have also derived for the cycle analysis is u e which is square root of 2 in to h naught i minus h e, which in terms of efficiency becomes 2 in to eta n in to h naught i minus h e s and this again in terms of temperature is 2 c p eta n in to T naught i minus T e s. This is on simplification 2 gamma R by gamma minus 1 in to eta n in to T 0 i multiplied by 1 minus P e by P 0 i raise to gamma minus 1 by gamma.

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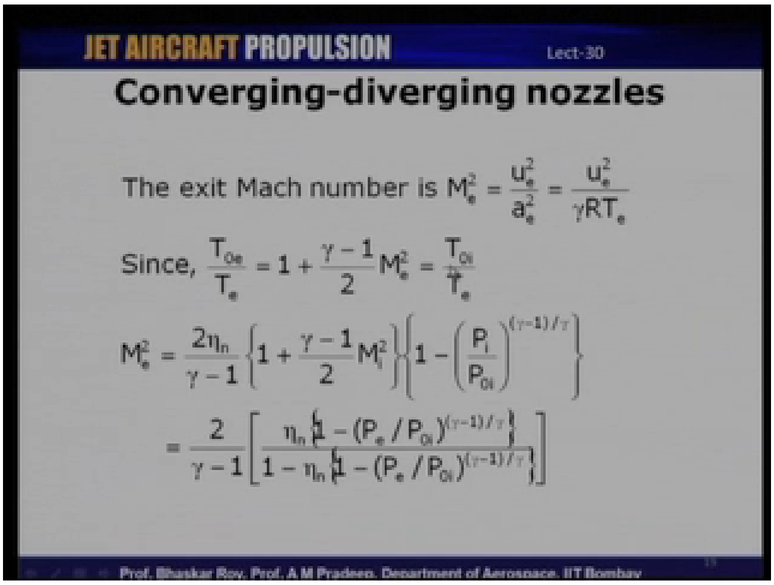


So, this combine with the previous equation that we have seen we will now express that in terms of the Mach number. So, the exit Mach number as we know is M e square of that is u e square by a e square that is u e square by gamma R T e. Now, we know that T e by T naught e by T e is 1 plus gamma minus 1 by 2 M e square which is T naught i by T e. Therefore, we can express M e from what we have seen in the previous equation that is u e square and this equation we get 2 in to eta n by gamma minus 1 multiplied by 1 minus gamma minus 1 by 2 M i square where M i is the inlet Mach number multiplied by 1 minus P i by P naught i raise to gamma minus 1 by gamma.

This ofcourse can be simplified and written here in terms of the pressure ratio 2 by gamma minus 1 in to the nozzle efficiency 1 minus P e by P naught i raise to gamma minus 1 by gamma divided by 1 minus eta n in to 1 minus P e by P naught i raise to gamma minus 1 by gamma. So, here we have expressed the Mach number in terms of the pressure ratio that is the exit pressure to the inlet pressure. And so, this will be useful in determining whether we get a sonic Mach number or supersonic Mach number especially a supersonic nozzle one would want to have supersonic Mach number at the exit. So, we can find out what is the pressure ratio that will be required, if we have to achieve a supersonic Mach number.

Let us say we want to achieve a Mach number of 1.2. So, given the exit Mach number is 1.2, we can now find out what will be the pressure ratio that the nozzle will have to develop, if one one has to achieve the supersonic Mach number with the certain nozzle efficiency. So, nozzle efficiency in mind, one can actually find out what will be the kind of pressure ratio that is needed to achieve that Mach number. So, the second part that we going to look at now is to see how we can also act derive expression for the area ratio. We now just derived the equation for the Mach number. We will now look at how we can derive an expression for the area ratio.

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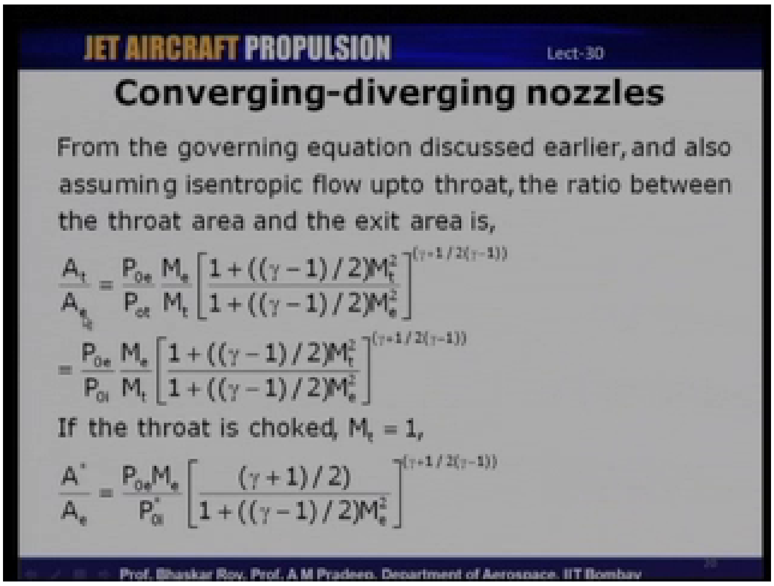


So, we have already discussed about the mass flow rate of the governing equation little earlier. We will assume again isentropic flow up the up to the throat. So, we can express the throat area and the exit area as A t by A e which is throat area by exit area expressed in terms of the corresponding pressures and Mach number P naught e by P naught t which is the stagnation pressure at the throat multiplied by M e by M t Mach number at the throat in to 1 plus gamma minus 1 by 2 M t square divided by 1 plus gamma minus 1 by 2 M e square raise to gamma plus 1 in to divided by 2 in to gamma minus 1. This also can be expressed in terms of the inlet pressure.

Now, if the throat is choked where we have Mach number M t as equal to 1, the throat area becomes A star, which is critical area. A star by A e would now be equal to P naught e which is exit stagnation pressure multiplied by exit Mach number divided by inlet stagnation pressure, which leads to the choking multiplied by gamma plus 1 by 2 by 1 minus gamma minus 1 by 2 M e square the whole raise to gamma plus 1 by 2 in to gamma minus 1. So, from this, we can actually find out what is the critical area ratio. Area ratio between the throat and the exit of the nozzle, which can lead to critical flow that is choked flow at the exit of the nozzle.

So, this is one of the expressions that one can use. We have also seen how we can calculate the pressure ratio that is required to develop a certain supersonic Mach number at the exit of the convergent nozzle. We will also look at what is the mass flow rate that one can achieve. Especially, if it is under choked condition, what how we can calculate the mass flow rate? Because you have already derived general equation for mass flow rate in terms of the inlet stagnation pressure temperature, the Mach number and the area. And we now know that for chocking, the Mach number at the throat becomes 1. So, from that, we can actually calculate the mass flow rate in terms of the throat area and the inlet parameters.

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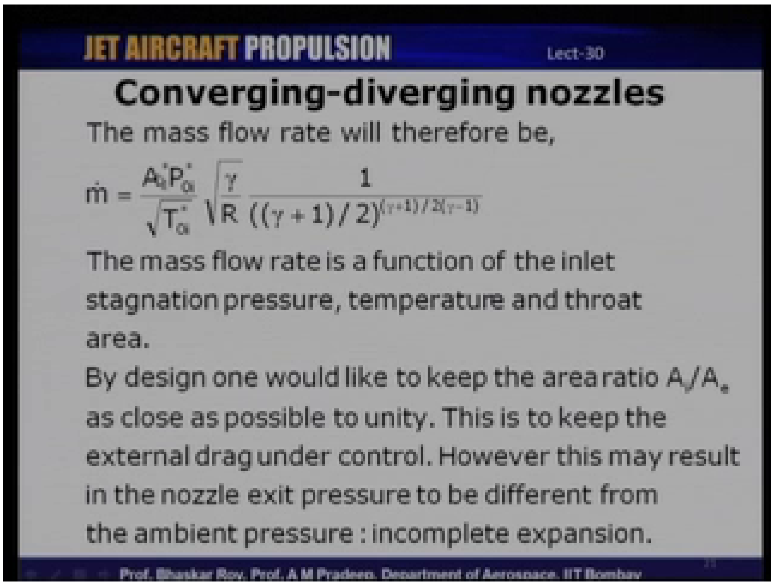


So, mass flow rate we can calculate using the equation we have discussed earlier, where we substitute M is equal to 1. We now get A star which is the throat area multiplied by P naught i inlet stagnation pressure divided by T naught i square root; the gas properties gamma by R square root 1 by 1 plus gamma plus 1 by 2 raise to gamma plus 1 by 2 in to gamma minus 1. So, here we see mass flow rate is function of the inlet stagnation pressure temperature and ofcourse the throat area. Now, another aspect concerning the designer is that he would like to always try to keep the exit area the area ratio that is A i by A e as close as possible to unity.

Because if the exit area is very large, then it has other implications like it might lead to high levels of external drag; because obviously the entire nozzle and the engine is housed in casing in a nasal. So, if the nozzle area is increased very large; if we have an exit area which is much larger than the inlet area, then the possibilities of external drag increasing would be very high. But if we does not do that; if we tries to keep the area ratio as close as possible to unity, then the nozzle may not be operating under what is known as the fully expanded condition.

That is fully expanded condition is one, where the exit pressure is equal to the ambient pressure; P e is equal to P a. If P e is not equal to P a, then the nozzle could be either be operating as an under expanded nozzle or a over expanded nozzle. Both of which has certain issues which we are going to discuss now. That is, if a nozzle is under expanded, what are the problems; if it is over expanded, what are the problems. So, will take a look at how we can how a designer can keep this in mind in under these conditions. So, an incomplete expansion is a result of a difference between the nozzle exit pressure and the ambient pressure.

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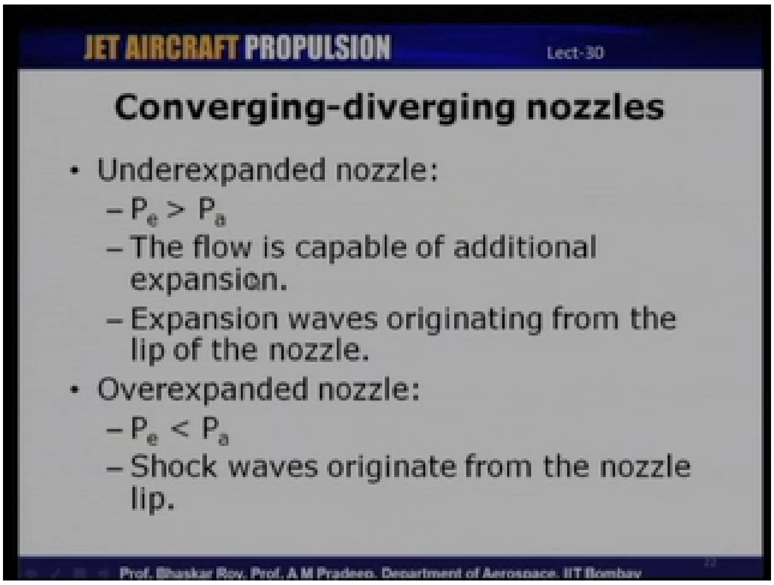


Incomplete expansion could be either over expanded nozzle, where the exit pressure is greater than the ambient pressure. If P e is greater than P a, it means that the flow is capable of additional expansion. That is, the flow has potential for expanding further and this is now carried out using expansion waves, which will originate from the lip of the nozzle. That is, because the flow is having enough energy to further expand and that is why, it will carry out that expansion; because the pressure has to eventually become equal to the ambient pressure.

So, that expansion takes place through nozzles; through the formation of expansion waves. The other extreme case would be over expanded nozzle; that is if P e is less than P a. That is, the nozzle is expanding more than what it should and how can that be taken in to account. Well that can happen if there are I mean in in the instance where the exit pressure is greater is less than ambient pressure, there would be shock waves which would again be originating from the nozzle. So, across the shock wave, the pressure would change increase the static pressure would increase. So, that you have exit pressure finally equal to the ambient pressure.

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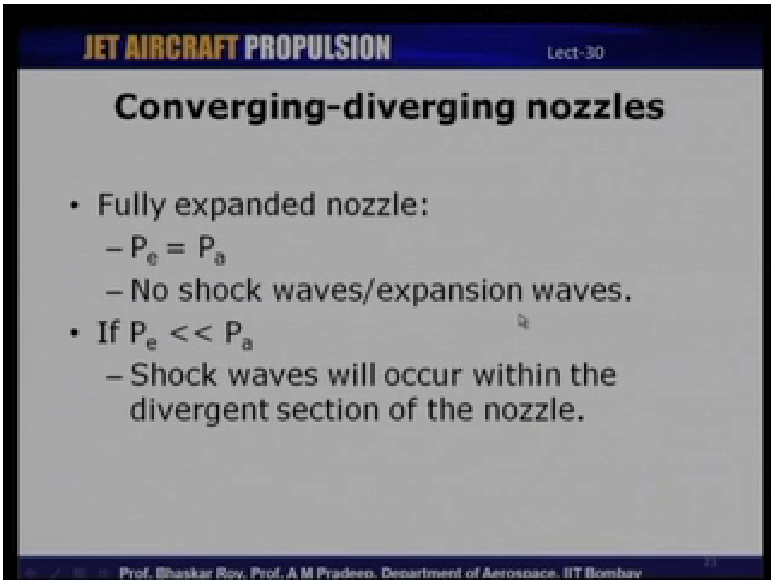


Well, the ideal case one would like to have ofcourse is fully expanded nozzle, where the exit pressure is exactly equal to ambient pressure. So, there are no shock waves or expansion waves here. Now, if the exit pressure is much less than the ambient pressure, then the shock waves will actually occur within the divergent section of the nozzle. We have already seen that when we discussing about the nozzle performance and behavior that if P e much less than P a, shock waves may actually occur within the divergent section of the nozzle.

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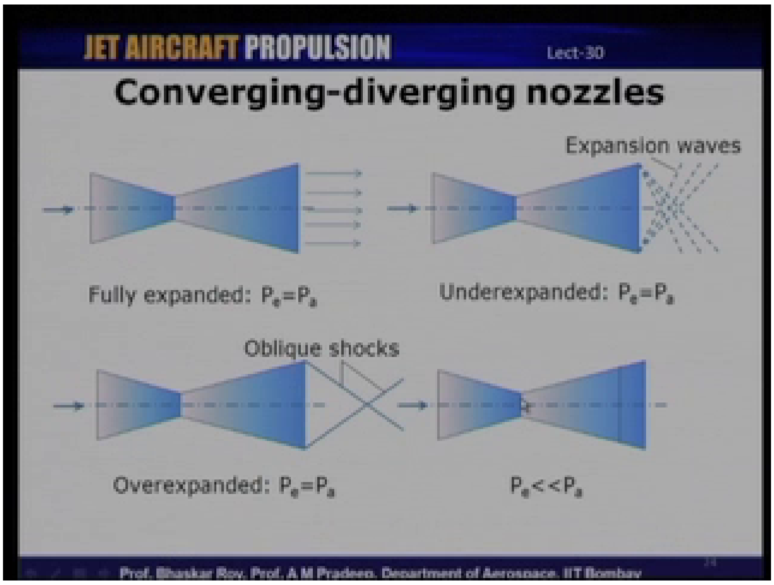


So, these are the illustrations of all these four cases; fully expanded nozzle P e is equal to P a. So, there are no issues within the nozzle itself. If P e is less than P a which means that it is under expanded, so there would be expansion waves as you can see here which originate from the lip of the nozzle. These are the expansion waves through which the flow will further expand and become equal to the ambient pressure. Now, if P e is greater than P a that means it has it is actually expanded more than what it should be, then we have an over expanded nozzle. In which case, the pressure gets finally equalized through what are known as oblique shocks. Because now the pressure has to eventually become equal to the ambient pressure and that happens through what are known as oblique shocks.

You may have oblique shocks originating from the lip of the nozzle and the pressure gets finally equalized there. If P e is much less than P a, then we have an occurrence of an normal shock or which which could occur within the nozzle itself in the divergent section and the pressure gets across the normal shock, we know there is an increase in static pressure. So, across the normal shock, the exit pressure finally becomes equal to that of the ambient pressure. But that is through a normal shock, which obviously means there are going to be loses across such nozzle geometry. So, these are different operating conditions of supersonic nozzle where the depending upon the exit pressure in relation to the ambient pressure, the nozzle may operate under different conditions.

So, let me now quickly summarize what we have discussed in this lecture. We started our lecture today with discussion on subsonic nozzle convergent nozzle. And we have seen that there are certain problems with subsonic nozzle in the sense that there is a limitation to the maximum Mach number, which a subsonic nozzle can deliver and that sonic Mach number that occurs right at the throat. So, we have seen how we can as we change the back pressure, how the flow behaves through a subsonic nozzle. We also derived an equation for analyzing this. We then took up the convergent divergent nozzle. We have discussed in detail about how the flow flow behavior can analyze right from the inlet to the exit as the back pressure keeps changing.

And then we have also derived equations for calculating the pressure ratio pressure ratio across the nozzle required for getting a desired supersonic Mach number at the exit of the nozzle, what will be the mass flow rate corresponding to choked flow and so on. And towards the end, we have discussed about different operating conditions of supersonic nozzle; where depending upon the exit pressure in relation to the ambient pressure, the nozzle may undergo either over expansion or under expansion. Or if the exit pressure is much lower than the ambient, one might also have normal shock within the divergent section of the nozzle. So, these are some of the topics that we have discussed in today’s lecture.

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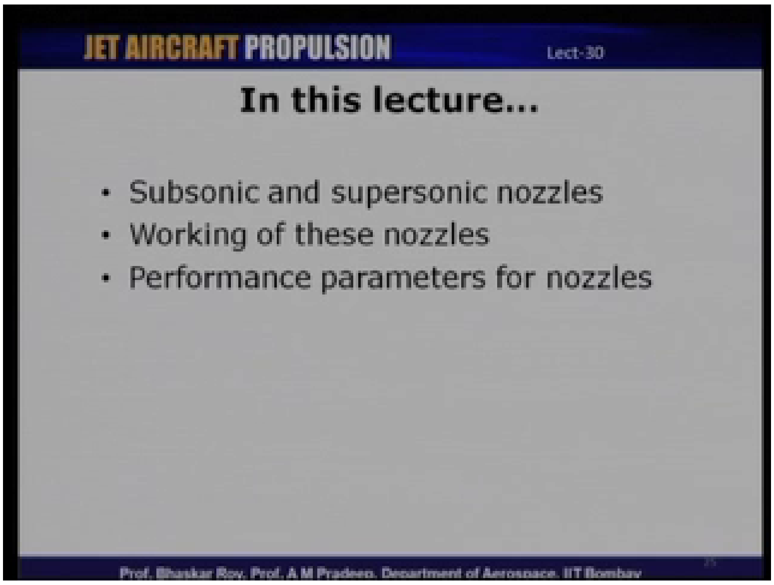


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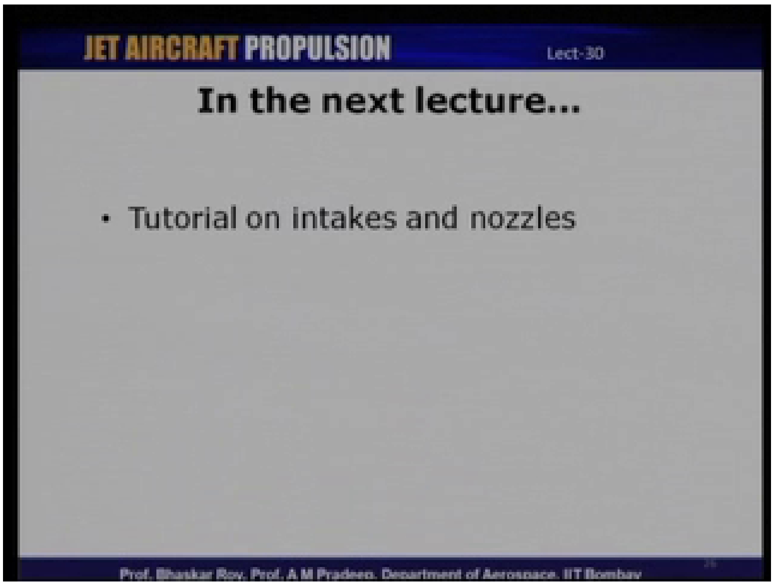


And in the next lecture that we going to discuss, we will basically be having a tutorial wherein we will solve some problems on intakes as well as nozzles. So, we are we have already discussed about intakes. Today's lecture, we conclude our discussion on nozzles and so in the next class, we will take up some simple problems on intakes and nozzles and we will solve them in the class. I will also have a few exercise problems for you which you can solve based on our discussion during the last few lectures as well as our discussion during the tutorial. So, we will take up some of these topics for discussion in the next class.