

Jet Aircraft Propulsion

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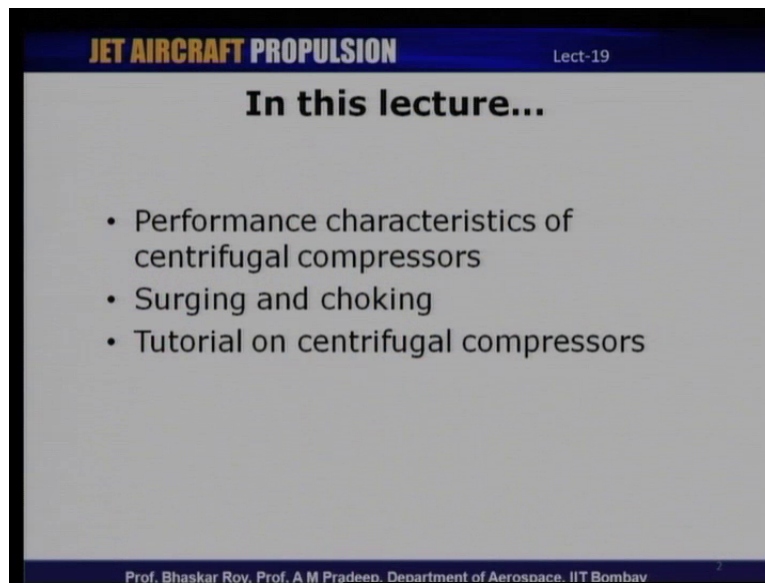
Lecture No. #19

Centrifugal Compressor Characteristics: Surging, Chocking

Hello and welcome to lecture number nineteen of this lecture series on jet aircraft propulsion. We have been discussing about the basic cycles of jet engines, and during the last few lectures we have also initiated discussion on various components of air jet engine. And we have already discussed about axial flow compressors. In the last lecture, we had some discussion on centrifugal compressors, and in today's lecture we are going to continue our discussion on centrifugal compressors. We are going to basically talk about performance characteristics of centrifugal flow compressors, and how we can analyze centrifugal compressors, and we will also be discussing a little bit on the various instabilities that affect the performance of centrifugal compressors.

Basically the mechanisms that we have already discussed for axial compressors are very much valid for this as well, but we will take a look at these mechanisms once again. Subsequent to that we will also have some time for solving a couple of problems related to centrifugal compressors, which means that today we are going to have part of the lecture devoted towards discussion on performance characteristics, and subsequently we will also have a some discussion on solving problems on centrifugal compressors.

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So, let us begin our discussion on performance characteristics, and if you recall during our discussion on axial compressors what we had realized is that the performance of axial compressor are very much is the function of a variety of other parameters. That is the exit the delivery stagnation pressure or the pressure ratio and the efficiency or functions of a host of other parameters like the rotational speed the geometric parameters and so on and of course- the mass flow rate. So, in centrifugal compressors too the performance is very much depend on these parameters, which means that the performance characteristics would be, at least the parameters that are used in evaluating the performance characteristics are identical to what is used in axial compressors.

You would basically be looking at these stagnation pressure ratio across the compressor are developed by the compressor as a function of mass flow rate, for different rotational speeds. At the same time we will also be looking at efficiency verses mass flow rate at different speeds and that would give us an idea of the performance map of centrifugal flow compressors. This is very much what we had done for axial compressor aspect. So, let us take a look at what we had discussed during axial compressors we will discuss that in brief with regard to centrifugal flow compressors.

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Performance characteristics

- The centrifugal compressor performance characteristics can be derived in the same way as an axial compressor.
- Performance is evaluated based on the dependence of pressure ratio and efficiency on the mass flow at different operating speeds.
- Centrifugal compressors also suffer from instability problems like surge and rotating stall.

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Now, in the performance characteristics I think the derivation of performance characteristics is exactly identical to what we had discussed for actual compressors and we would basically be looking at the dependence of pressure ratio and efficiency on mass flow rates, at different speeds and **what we will** very soon realize is that centrifugal compressors also suffer from instability problems like surge and rotating stall. So, that we will also take up for discussion after we talk about the performance characteristics.

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Performance characteristics

- The compressor outlet pressure, P_{02} , and the isentropic efficiency, η_C , depend upon several physical variables

$$P_{02}, \eta_C = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non - dimensionless parameters,

$$\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{\gamma RT_{01}}}{P_{01} D^2}, \frac{\Omega D}{\sqrt{\gamma RT_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

The above reduces to $\frac{P_{02}}{P_{01}}, \eta_C = f\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$

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So, for a centrifugal compressor the outlet pressure that is P_{02} and the Isentropic efficiency depend upon several physical variables, which is what we had discussed for actual compressors as well. Let me just quickly discuss that once again P_{02} and efficiency are functions of mass flow rate and the inlet the stagnation pressure that is P_{01} inlet stagnation temperature, then the rotational speed or the angular velocity ω , the ratio of specific yields γ , the gas constant r , then the viscosity new the design of the compressor itself and the diameter D .

So, these parameters if we were to carry out a dimensional analysis then we can reduce these set of variables to a set of non-dimensional parameters. These are basically the pressure ratio of P_{02} by P_{01} and efficiency as functions of mass flow rate which has been non-dimensionalized \dot{m} into square root of $\gamma R T_{01}$ divided by $P_{01} D$ square and ωD by square root of $\gamma R T_{01}$, ωD square by new γ and design this of course, reduces to what is written here that is pressure ratio. Efficiency, can basically be expressed as functions of \dot{m} into root T_{01} by P_{01} and N by root T_{01} . This is because for a given compressor the diameter does not really change and. So, ωD is n and γ and r are fixed. So, based on that we can reduce this set of non-dimensional parameters for a given design to P_{02} by P_{01} and efficiency as functions of \dot{m} root T_{01} by P_{01} and N by root T_{01} .

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Performance characteristics

Usually, this is further processed in terms of the standard day pressure and temperature.

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)$$

Where, $\theta = \frac{T_{01}}{(T_{01})_{\text{Std. day}}}$ and $\delta = \frac{P_{01}}{(P_{01})_{\text{Std. day}}}$

$(T_{01})_{\text{Std. day}} = 288.15 \text{ K}$ and $(P_{01})_{\text{Std. day}} = 101.325 \text{ kPa}$

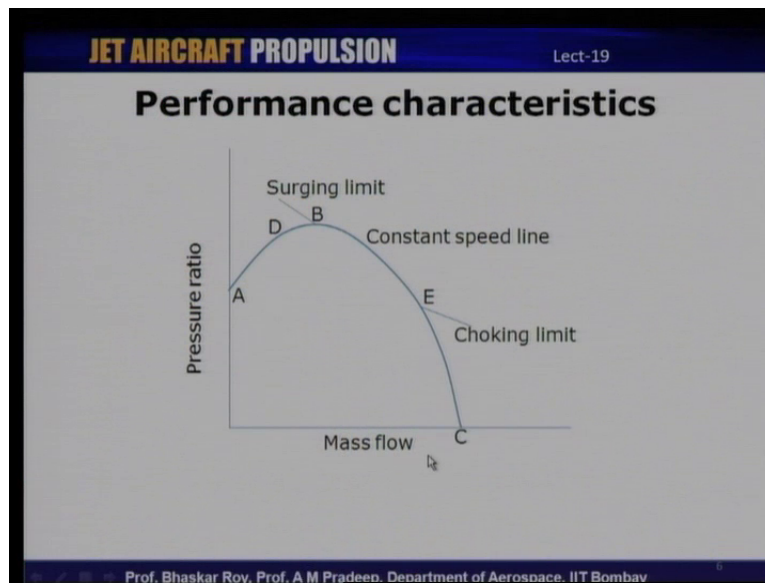
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This we will further reduce and this is basically a process in terms of standard day pressure and temperature that is the temperature is taken with reference to temperature on a standard day and pressure is taken as the ratio of pressure between that as compared to that of a standard day stagnation pressure. So, P_0^2 by P_0^1 and efficiency would become functions of $m \cdot \sqrt{\theta}$ by Δ , where θ is T_0^1 by T_0^1 standard day and N by $\sqrt{\theta}$ where θ is basically the temperature ratios and Δ is the pressure ratio P_0^1 by P_0^1 standard day. And here the standard day temperature and pressure are T_0^1 standard day is usually taken as 288.15 and P_0^1 standard day as 101.325 kilopascals.

So, I think if you recall our discussion for axial compressors this is exactly what we have done for axial compressors as well that we can express the pressure ratio and efficiency which are the performance parameters which we are interested in, in terms of certain parameters which we can associate with the compressor that is mass flow rate and temperature and the speed. So, pressure ratio and efficiency expressed just functions of $m \cdot \sqrt{T_0^1}$ or $m \cdot \sqrt{\theta}$ by Δ and N divided by $\sqrt{\theta}$. So, performance map or performance characteristic would involve, the looking at variations of pressure ratio and efficiency as functions of mass flow rate and the non-dimensional speed and then what we will see very soon is that the performance characteristics are limited by two extreme operations at a given speed: one limiting factor occurs on towards the left hand side of the performance curve that is known as the surge limit and on the right hand side of the performance is limited by maximum mass flow rate and that is known as the choking.

So, these two parameters or these two phenomenon's restricting the operation of centrifugal compressors to a certain band within which the operation is stabled and outside of which; obviously, the operation is not desirable. Let us take first look at the performance characteristic at a given con speed and then subsequently we will take up the discussion for a range of speeds. So, pressure ratio verses mass flow rate for a given speed. Let us take a look at what happens, as we change mass flow rate what happens to the pressure ratio and basically theoretically what is that should happen as we change this.

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So, if we look at mass flow variation of pressure ratio as a function of mass flow rate at a given speed. So, this line this curve that is shown here indicates what is known as the constant speed line. So, for a given speed of a rotation or rotation of the rotor the characteristic would look like this, pressure ratio and mass flow rate. So, I have indicated a few points, salient points on this curve and we will take up these the significance of these points in detail. What happens let us take a look at what happens at the extreme cases at one ca and you can see its marked as C, where we have a certain mass flow but the pressure ratio is zero we will take up that why that is.

So, little later on the other hand we have certain pressure ratio mass flow rate is zero and what I have indicated as two distinct points are these points B and E this point B is basically referred to as the surging limit point E is a known as the chocking limit . Let us take a look at what happens between points A and B. So, if let us say the compressor is operating at point B and then we further reduce mass flow rate. So, as we reduce mass flow rate what should happen is that according to this curve the pressure ratio drops.

So, as the pressure ratio drops, it means that the pressure downstream if it does not reduce corresponding to what is shown in this curve then there is a likelihood that because the pressure of downstream has not reduced as per what is shown here there could be a reverse flow from downstream towards the upstream and as this flow occurs the pressure downstream would have dropped. Immediately there is a movement of mass flow from upstream towards

the downstream and this continues back and forth. This E basically occurs at a fairly high frequency and it affects the entire annulus of the compressor it is not a localized phenomena and it affects the entire annulus.

We had discussed that this occurs even for an axial compressor, and that was referred to as surge for an axial compressor that the same thing happens here as well that under certain operating condition that is on pressure ratio verses mass flow rate curve. On the positive slope of this curve as mass flow reduces there is also a drop in pressure ratio leading to this problem of surge that is surge is associated with a violent fluctuation or oscillation of mass flow rate back and forth. It is access symmetric that is it is all over the annulus of the compressor which means that between these two points A and B operation of the compressor is unstable. So, you would not want to operate your compressor in a range where there is a positive slope for the pressure ratio verses mass flow characteristic.

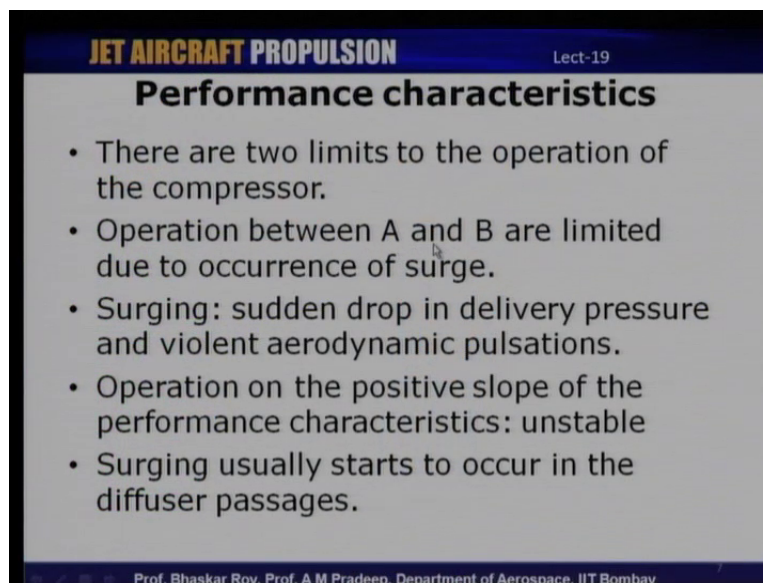
On the other side of the curve that is after the point B towards the right hand side what happens is that as we increase mass flow it is as it is accompanied by a reduced pressure ratio that is reduction in pressure ratio which is fine because the pressure downstream is lower than what is upstream. So, there is not issue of a back flow which is likely to happen as in this case. So, the operation of the compressor from point B onwards does not really have a restriction in terms of what happens between points A and B. But there is also a limit for operation on this curve from B onwards that is from point B onwards I have indicated a limit here which is indicated by letter E and this is known as the choking limit.

So, what basically happens after this point is as we keep reduce increase in mass flow rate there is a drop in pressure ratio and then beyond the certain level what happens is that the maximum amount of mass flow which the compressor can discharge has already taken place and which means that no matter how much you try to increase your mass flow mass flow has reached its maximum level and beyond that point you would not really get an increase in mass flow but; obviously, if we try to do that there could be reduction in pressure ratio leading to loss of performance.

So, there is a certain limit on the right hand side of the stable curve where the operation is supposed to be stable but what limits is the maximum mass flow which the compressor can handle and that is known as choking. Choking on one hand one side of the curve limits the performance surging limits the performance on the other side. So, these are two limiting

parameters that affect the performance of centrifugal flow compressors the same way this was affecting an axial flow compressor as well but what we will see later on little later is the performance characteristics of a typical centrifugal flow compressor for a variety of speeds. So, we will get to few more details of the performance characteristics a little later.

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Performance characteristics

- There are two limits to the operation of the compressor.
- Operation between A and B are limited due to occurrence of surge.
- Surging: sudden drop in delivery pressure and violent aerodynamic pulsations.
- Operation on the positive slope of the performance characteristics: unstable
- Surging usually starts to occur in the diffuser passages.

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As I mentioned there are basically two limits which affect the performance of the compressor operation between points A and B limited due to surge wherein there is a sudden drop in delivery pressure and also accompanied by violent aerodynamic pulsations and which means that operation on the positive slope of the curve is unstable. It is observed that most of the times surging usually begins to occur in the diffuser passages primarily because of the effect of the effect of boundary layer separation on the diffuser passages I think when we discussed about diffusers in the last lecture I specifically mentioned that the performance of diffusers is very sensitive to the occurrence of boundary layer separation and therefore, surges often initiated in the diffuser passages.

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Performance characteristics

- Rotating stall might also affect the compressor performance.
- In this case a stall cell (that might cover one or more adjacent blades) rotates within the annulus.
- Full annulus rotating stall may eventually lead to surge.
- Rotating stall may also lead to aerodynamically induced vibrations and fatigue failure of the compressor components.

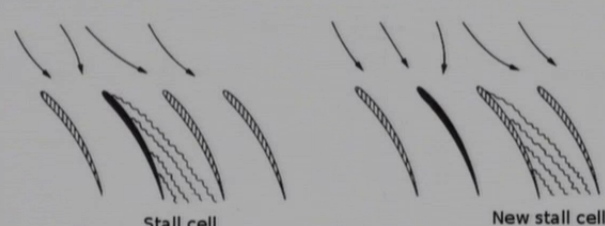
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Now, the other instability that might affect the performance is rotating stall we have already discussed about rotating stall for an axial compressor rotating stall may also affect the performance of a centrifugal compressor. What basically happens is that if one or more adjacent blades are undergoing stall this stall cell will rotate within the annulus and if allowed to grow these stall cells might propagate and affect the entire annulus eventually leading to surge. Rotating stall on its own can itself lead to aerodynamically induced vibrations and eventually if it failure of the compressor components.

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Performance characteristics



Stall cell New stall cell

Propagation of rotating stall

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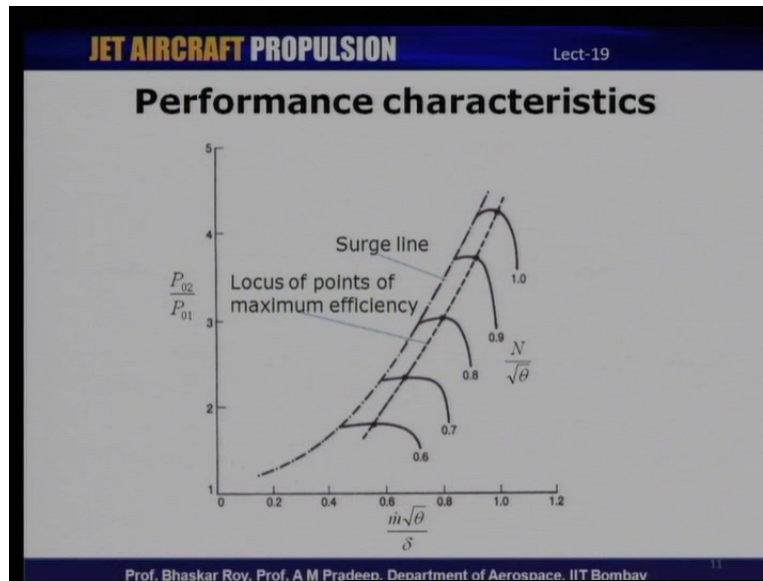
So, rotating stall basically involves, I think we have discussed this in the last class I will just go through it very quickly once again. In rotating stall what happens is that if there is a flow non uniformity which is entering into the compressor in this case the impeller and that non uniformity initiates separation boundary layer separation on one of the vanes of the impeller that would lead to a deflection of the flow approaching the adjacent set of vanes which means that if this is the vane. That is undergoing stall because of the fact that the vanes themselves are rotating and because of the fact that this blade is undergoing stall the flow approaching the adjacent blade also experiences a higher incidence.

So, because of this what happens is that this stall cell now shifts to the adjacent blade or set of blades or vanes and because the vanes are rotating the stall cell continues to rotate along the annulus of the compressor? So, this stall cell continuous to rotate and in some cases, the stall cell might propagate and expand and affect more number of vanes adjacent vanes and eventually the entire annulus can be affected by stall and the compressor might be forced into surge.

So, the difference between what we see here and for a centrifugal compressor is that what I have shown here is basically set of air foil blades which are not really the case in a centrifugal compressor. We normally do not have air foil blades for the impeller, impeller blades are basically vanes and not necessarily air foil sections but the phenomenon is the same that affects a centrifugal compressor as well. So, rotating stall is the other instability that could affect the performance of a centrifugal compressor besides the fact that the surge and choking on the other side that is the performance characteristics gets limited on a account of occurrence of these instabilities.

Choking, I think we have already discussed that on the right hand side of the performance characteristics, the mass flow reach as the maximum level beyond which we cannot really increase mass flow and puts a limit on the performance of the compressor on one side of the comp on one side of the performance characteristic.

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So, let us now look at pressure ratio verses mass flow characteristics of a typical centrifugal compressor. So, what we say here is that on the Y axis we have the pressure ratio - the stagnation pressure ratio, and on the X axis we have the mass flow rate, the non- dimensional mass flow rate \dot{m} into root theta by delta. And the characteristics have been plotted in terms of varying non-dimensional speeds. So, all these lines that we see here for 0.6, 0.7 and so on... Corresponds in two different values of \dot{m} by root theta, and there are two lines which are been shown here, one is the surge line and the other is of course the point of maximum efficiency. Now, surge line is the locus of all the points on these performance characteristics which correspond to the surge point which we have discussed for a single characteristic. So, if we join all those lines we get what is known as the surge line.

And what we also see is that on the other side or the right hand side especially as we go towards higher speeds we have a very sharp sharply drooping characteristics which is because of choking that is as we approach higher and higher speeds the performances affected by choking which means that we have very narrow band of operation between surge and chock for higher speeds which is not the case at lower speeds.

In the lower speeds we can see that the performance characteristics are flatter and there is a larger band of operation of the compressor for the lower speeds. The other thing to notice here is that typically we may have pressure ratios which are much higher than what we get for an axial compressor first stage. So, we can see that the pressure ratios can be on the higher

side something we have discussed in the last class that the first stage pressure ratio for a centrifugal compressors can be higher than an axial compressor for the same mass flow rate. So, this characteristics although looks very similar to what an axial compressor characteristics would look like in of course, both these compressors are affected by surge and rotating stall as well as choking. So, a designer would like to keep the operating line. Operating line is the line which corresponds to the point, where the compressor has been designed to operate.

As close as possible to maximum efficiency and at the same time keep a safe distance between the operating line and the surge line because if the operating line or operating point is very close to this surge line then the possibility that under certain off design conditions the compressor might go into surge line is very high. So, one would like to keep a certain margin between the surge line and the operating line and that is usually referred to as the surge margin which is basically a margin that the designer would like to keep to ensure that the compressor has a safe band of operation and that the risk of the compressor entering into surge is minimal.

The other characteristic which we are also interested in is the efficiency, efficiency versus mass flow rate for different speeds. We can again see that these efficiency points those which have been indicated here are those lines which have been joined and as the locus of points of maximum efficiency. We see that even in the case of efficiency as we approach higher speeds the efficiency characteristics also attain drastic droop which means that the band of high efficiency drastically drops as the compressor is operating at higher speeds and is as it approaches choking whereas, for lower speeds we have relatively higher larger band of high efficiency operation.

So, what we have discussed. So, far are the performance characteristics of centrifugal compressors as well as the performance penalties which a compressor is likely to pay. Especially as it as we change the operating range for a given speed. So, on one side it gets affected by surge or even before that the compressor might enter into rotating stall and also choking which affects the performance on the other end.

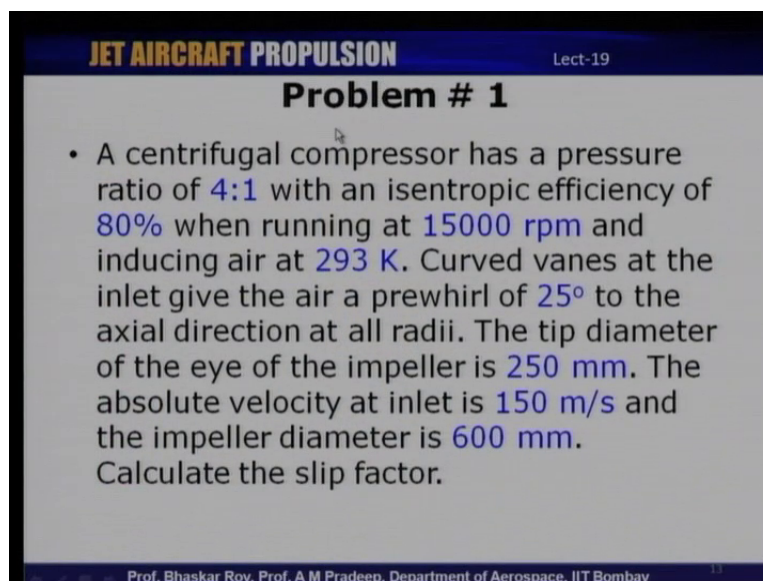
So, a compressor designer; obviously, has to keep these things in mind an especially because the compressor for an aircraft engine has to operate for a variety of speeds and range of operations starting from take off to cruise and then landing. So, the range of operation for

aircraft engines is substantially more than for example, a land based power generating unit where of course, they are designed for a certain set of speeds. So, there the performance characteristics in the case of aircraft engines becomes even more significant because the compressor is usually is likely to go through all these different stages of operations of the compressor for these speeds.

So, the similarities between axial compressor performance characteristics and centrifugal compressor characteristics are quite visible and we know that performance characteristics can be evaluated very much the same way as for an axial compressor and a centrifugal compressor but there are certain inherent differences as well between these two compressors in terms of their operation and also the mechanism of pressure rise in both these compressors. With this, I will wind up the discussion on centrifugal compressors I we have discussed about the components of the centrifugal compressor, the mechanism of pressure rise performance characteristics and also the instability aspects of centrifugal flow compressors.

The next few minutes what we will do is to quickly discuss about a few tutorial problems where I have two problems which I will solve for you and then a few exercise problems which you can take up and solve based on our discussion in today's lecture as well as what we had discussed in the last lecture.

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Problem # 1

- A centrifugal compressor has a pressure ratio of 4:1 with an isentropic efficiency of 80% when running at 15000 rpm and inducing air at 293 K. Curved vanes at the inlet give the air a prewhirl of 25° to the axial direction at all radii. The tip diameter of the eye of the impeller is 250 mm. The absolute velocity at inlet is 150 m/s and the impeller diameter is 600 mm. Calculate the slip factor.

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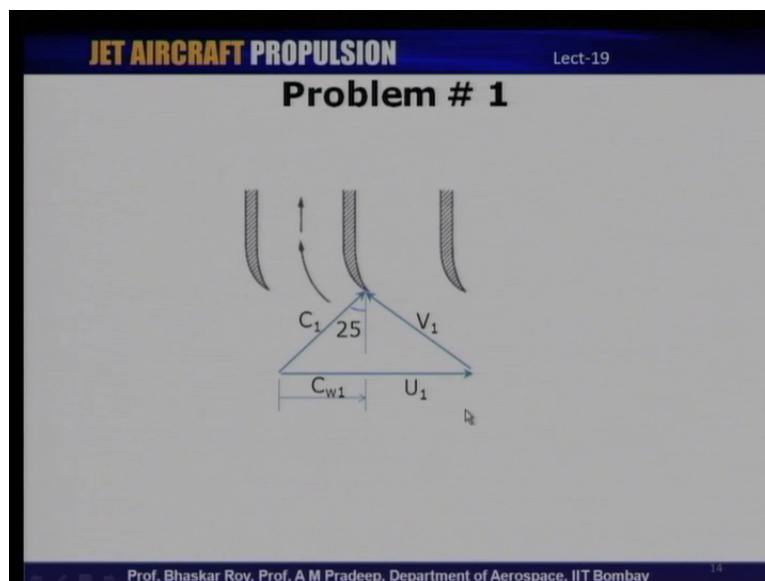
Let us take a look at the first problem that I have for you which we shall solve that is basically a centrifugal compressor with a given pressure ratio, here the pressure ratio is given

as four is to one here the pressure ratio is given as four is to one with an Isentropic efficiency of 80 percent when running at 1500 Revolution Per Minute and inducing air at 293 Kelvin. Curved vanes at the inlet give air a prewhirl of 25 degrees to the axial direction at all radii. The tip diameter of the eye of the impeller is 250 millimeters the absolute velocity at the inlet is 150 meters per second and the impeller diameter is 600 millimeters calculate this slip factor.

So, here we have a centrifugal compressor with a certain pressure ratio its efficiency is given the Revolution per Minute is given and we have some information about the inflow direction of the incoming air some of the geometric parameters are given that is the impeller diameter and the axial velocity these are specified we are required to calculate what is the slip factor. Now, slip factor if you recall is the ratio of the tangential component of the absolute velocity at the outlet of the impeller that is $c_w 2$ divided by the rotor speed or the impeller speed at the outlet that is $u 2$. So, $c_w 2$ by $u 2$ refers to the slip factor.

So, we are required to calculate this based on the data provided to us. So, as always I have mentioned even for an axial compressor the very first step towards solving these problems would be to draw the velocity triangles and then try to see what are the aspects for the velocity triangle which have been specified in the problem and that is that should be the starting point first solving such problems pertaining to axial flow compressors as well as to centrifugal compressors.

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So, let us first take a look at what the velocity triangle should look like at the inlet we based on what information we have been specified. So, at the inlet it is given that the absolute velocity the flow is given a prewhirl of 25 degrees with reference to the axial direction. This is axial direction the flow enters the blade in this direction and it is mentioned that there is a prewhirl given and therefore, this angle which the absolute velocity makes with the axial direction is 25 degrees and therefore, we have the velocity triangle as given here we have C 1 with a 25 degrees U 1 is the blade speed that the high of the impeller and therefore, U plus V is c and that completes the velocity triangle at the inlet.

And this part this component of the absolute velocity is the whirl component or the tangential component C w1 which is this component of c in the tangential direction in a direction of U 1. So, this is the velocity triangle at the inlet and. So, based on this we can base on the velocity and the angles specified we could probably solve the velocity triangle here and then we can also move towards solving the velocity triangle at the outlet.

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Solution: Problem # 1

Exit stagnation temperature is

$$T_{02} = T_{01} (\pi_c)^{(y-1)/y} = 293(4)^{(1.4-1)/1.4} = 435.56K$$

Therefore the isentropic temperature rise,

$$\Delta T_{0s} = 435.56 - 293 = 142.56K$$

The actual temperature rise, $\Delta T_0 = \Delta T_{0s} / \eta_c$

$$\Delta T_0 = 178.2K$$

Work done per unit mass is, $w = c_p \Delta T_0$

$$w = 1.005 \times 178.2 = 179 \text{ kJ/kg}$$

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Now, it is mentioned that the stagnation pressure ratio is 4 and the inlet stagnation temperature is also specified which means that the exit stagnation temperature can be calculated based on isentropic relations therefore; this temperature is basically isentropic temperature. So, that is 293 into 4 which is the pressure ratio rise to gamma minus 1, 1.4 minus 1 by 1.4 this is 435.56 Kelvin. Therefore the Isentropic temperature rises 435.56 minus 293 that is 142.56 Kelvin. So, this is the Isentropic temperature rise, we need to calculate the

actual temperature rise which will be this temperature divided by the Isentropic efficiency therefore, actual temperature rises $\Delta T_{\text{naught S}}$ which is the Isentropic temperature rise divided by efficiency that is η_c . So, that comes out to be 178.2 Kelvin.

So, we can calculate the work done per unit mass which is C_p times ΔT_{actual} and C_p is 1.005 kilojoules per kilogram Kelvin multiplied by $\Delta T_{\text{naught S}}$ that is 178.2 refer work done per unit mass by the compressor is 179 kilojoules per kilogram. what we have just now calculated is based on the pressure ratio and the efficiency is specified we can calculate first the actual temperature rise which is avail the Isentropic temperature rise because we know the stagnation pressure ratio and from Isentropic relations we can calculate the exit stagnation temperature T_{02} is T_{01} into $\pi_c^{\frac{\gamma-1}{\gamma}}$.

Subsequently we can then calculate the actual temperature rise which will be the Isentropic temperature rise $\Delta T_{\text{naught S}}$ that is $T_{02} - T_{01}$ divided by efficiency. So, that we get the actual temperature rise and then work done per unit mass will be specific constant pressure multiplied by the $\Delta T_{\text{naught actual}}$. We get the work done by the compressor per unit mass. Now, the purpose of calculating this is that work done per unit mass is also equal to $U_2 C_{w2} - u_1 C_{w1}$ and. So, we know U_2 and $u_1 C_{w1}$, we can calculate from the velocity triangle at the inlet therefore, we can calculate C_{w2} and therefore, slip factor would be C_{w2} divided by U_2 . We would be equating the work done which have just now calculated that is $C_p \Delta T_{\text{naught}}$ equated to $U_2 C_{w2} - u_1 C_{w1}$.

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Solution: Problem # 1

Peripheral velocity at the tip of the eye,
$$U_1 = \pi dN / 60 = \pi \times 0.25 \times 15000 / 60 = 196.25 \text{ m/s}$$

$$C_{w1} = C_1 \sin 25 = 63.4 \text{ m/s}$$

Peripheral velocity at the tip of the impeller,
$$U_2 = \pi DN / 60 = \pi \times 0.60 \times 15000 / 60 = 471.2 \text{ m/s}$$

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Let us calculate that: the peripheral velocity at the tip of the eye that is a U_1 is $\pi D n$ by sixty, where D is the diameter at the outlet or at the tip of the eye of the impeller, which is given as two fifty millimeters. So, π into 0.25 and N is 15000 divided by 60. So, U_1 is 196.25 meters per second. C_{w1} , we can calculate because C_1 is given as 150 and the angle is given as 25. So, $c_1 \sin 25$ is equal to C_{w1} . So, C_{w1} , we can calculate as 63.4 meters per second. Similarly, we calculate the blade speed at the tip of the impeller U_2 it would be π into capital $D N$ divided by 60, where capital D is the diameter of the tip of the impeller. So, π into 0.60 into 15000 divided by 60. So, that is 471.2 meters per second. This is the blade speed at the tip of the impeller.

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Solution: Problem # 1

We know that power input is, $w = U_2 C_{w2} - U_1 C_{w1}$

$$179 \times 10^3 = 471.24 \times C_{w2} - 196.35 \times 63.4$$

or, $C_{w2} = 406.27 \text{ m/s}$

Therefore, the slip factor is,

$$\sigma_s = C_{w2} / U_2 = 0.862$$

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So, as I mentioned work done is also equal to $U_2 C_{w2} - U_1 C_{w1}$ and we have already calculated work done per unit mass from $C_p \times \Delta T_{\text{naught}}$ which was 179 kilojoules per kilogram. So, work done 179×10^3 is equal to $U_2 C_{w2} - U_1 C_{w1}$ which is $471.24 C_{w2} - 196.35 \times 63.4$. So, from this we can calculate C_{w2} which is the tangential component of the absolute velocity at the outlet of the impeller. Therefore slip factor is basically the ratio of C_{w2} and U_2 . C_{w2} is 406.27 and U_2 we have calculated earlier that is 471.24. So, we can calculate the slip factor as 0.862. So, 0.862 corresponds to the slip factor, which is basically the ratio of the whirl component of the velocity and the blade speed at the impeller outlet.

So, we have already discussed the significance of slip factor. It is basically as a consequence of the disappearance of the Coriolis acceleration towards the outlet of the impeller leading to a tangential variation in the relative velocity. So, there is the difference between the whirl component of absolute velocity and the blade speed at the outlet. If that was not there the whirl component would basically be well C_{w2} will basically be equal to C_{u2} if the coriolis acceleration were not to be considered.

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Problem # 2

- At the inlet of a centrifugal compressor eye, the relative Mach number is to be limited to 0.97. The hub-tip radius ratio of the inducer is 0.4. The eye tip diameter is 20 cm. If the inlet velocity is axial, determine, (a) the maximum mass flow rate for a rotational speed of 29160 rpm, (b) the blade angle at the inducer tip for this mass flow. The inlet conditions can be taken as 101.3 kPa and 288 K.

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So, the next question that we are going to solve is the following, at the inlet of a centrifugal compressor eye, the relative Mach number is to be limited to 0.97. The hub to tip radius ratio of the inducer is 0.4, the eye tip diameter is 20 centimeter. If the inlet velocity is axial, determine the maximum mass flow rate for a rotational speed of 29160 Revolution per minute and part b the blade angel at the inducer tip for this mass flow the inlet conditions can be taken as 101.3 kilopascals and temperature as 288 Kelvin. So, here, what is mentioned is that the relative Mach number has to be limited to 0.97 and we have been given some geometric details like, hub to tip ratio of the inducer as 0.4 the tip diameter at the eye is 20centimeters and also the rotational speed of 29160 Revolution Per minute.

So, given these conditions on a parameters we need to find the maximum mass flow rate if the mach number is to be limited to point nine seven we also need to find the blade angle at the inducer tip for this particular mass flow rate. Let us first start with the velocity triangles as we have been doing always that is kept the velocity triangle first and then proceed towards solving this problem.

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Problem # 2

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So, what is specified in this case is that, this is the inducer inlet and we have been given the radius ratio for the inducer in this case the radius ratio has been given as 0.4 and the eye tip diameter is given as 20 centimeters. So, R/T is given as twenty centimeters radius ratio is specified the revolution, the speed of rotation is specified and the relative Mach number if the tip has to be limited to 0.97. If we look at the velocity triangles, we should be getting a velocity triangle as shown here. At the inlet and we have the blade speed at the inducer inlet, the absolute velocity which is assumed to enter axially and the relative velocity at the inlet of the inducer.

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Solution: Problem # 2

The rotational speed at the inducer tip is

$$U_1 = \pi dN / 60 = \pi \times 0.2 \times 29160 / 60 = 305.36 \text{ m/s}$$

From the velocity triangle, we can see that

$$M_{1rel} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R T_1}}$$

$$T_1 = T_{01} - C_1^2 / 2c_p = 288 - C_1^2 / 2010$$

$$M_{1rel} = \frac{\sqrt{C_1^2 + U_1^2}}{\sqrt{\gamma R (288 - C_1^2 / 2010)}}$$

$$0.97^2 = \frac{C_1^2 + 305.63^2}{115718.4 - 0.2C_1^2}$$

Simplifying, $C_1 = 114.62 \text{ m/s}$

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So, in this case the rotational speed is given the tip diameter of the inducer is given as twenty centimeters and therefore, we can calculate the blade speed U_1 which is $\pi D N$ divided by 60, which is π into 0.2 into 29160 divided by 60, where 29160 is the speed of rotation in rpm. We get the blade speed as 305.63 per second and from the velocity triangle, which I have just shown we can see that the relative Mach number, which is the ratio of the relative speed to speed of sound at the inlet that is V_1 divided by square root of $\gamma R T_1$, which is basically equal to square root of C_1^2 plus U_1^2 . So, V_1^2 is the ratio of these two square roots of U_1^2 plus C_1^2 divided by square root of $\gamma R T_1$.

Now, at the inlet we have the stagnation temperature which is given as two eighty eight Kelvin and. So, T_1 is basically $T_{01} - C_1^2 / 2 C_p$, where T_{01} is 288 C_1 is not known and the specific heated constant pressure is 1005 joules per kilo gram Kelvin. Therefore we get C_1^2 by 2010. So, relative Mach number is equal to square root of C_1^2 plus U_1^2 , where U_1 we have just now calculated 305.36 divided by a square root of γR and we will replace T_1 by $T_{01} - C_1^2 / 2 C_p$ and relative Mach number. We know the limit which is 0.97. So, we substitute for m relative as 0.97 and therefore, we get 0.97 square is equal to C_1^2 plus 305.63 square divided by this γR times 288 that is 115718.4 minus 0.2 C_1^2 . If we simplify this we can calculate what C_1 should be C_1 comes out to be 114.62 meters per second. So, the absolute velocity of the inlet of the inducer is 114.62 per second.

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Solution: Problem # 2

$$T_1 = T_{01} - C_1^2 / 2c_p = 288 - C_1^2 / 2010 = 281.464K$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1} \right)^{\gamma/(\gamma-1)}$$

Substituting, $P_1 = 93.48kPa$

$$\therefore \rho_1 = P_1 / RT_1 = 1.157kg / m^3$$

Annulus area at the inlet, $A_1 = \frac{\pi}{4} d^2 (1 - r_h / r_t)$

$$A_1 = 0.0264m^2$$

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So having calculated that we can now calculate, T_1 at the inlet that is static temperature. At the inlet because we need to calculate density because that will be required for calculating the mass flow rate. So, T_1 is equal to T_0 minus C_1 square by $2 C_p$ and if we substitute for these values here we get T_1 as 281.464 Kelvin and from Isentropic relations we also have P_0 by P_1 is equal to T_0 by T_1 rise to γ by γ minus one P_0 is given T_0 is known T_1 we have just now calculated. So, if we substitute all these three values we can simplify and calculate P_1 , which is static temperature at the inlet and that comes to be 93.48 kilopascals. So, if you substitute for P_1 and T_1 , we can calculate density at the inlet this is 1.157 kilograms per meter cube.

Now, the other aspect we need to calculate is the annulus area for mass flow rate, we need the speed axial velocity which we have calculated as C_1 then density we have just now calculated the next thing is the annulus area. Annulus area at the inlet is πD square by 4 into one minus the hub to tip ratio. So, D is given as 0.2 meters hub to tip ratio is 0.4. So, if you substitute for D and hub to tip ratio, we get the area A_1 as 0.0264 meter square. So, this is the annulus area at the inlet.

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Solution: Problem # 2

Since the flow is axial,
 $C_{ax} = C_1$
 $\therefore \dot{m} = \rho_1 A_1 C_1 = 1.157 \times 0.0264 \times 114.62 = 3.5 \text{ kg/s}$
 The blade inlet angle at the tip is
 $\tan \beta_1 = C_1 / U_1$
 $\therefore \beta_1 = 20.57^\circ$

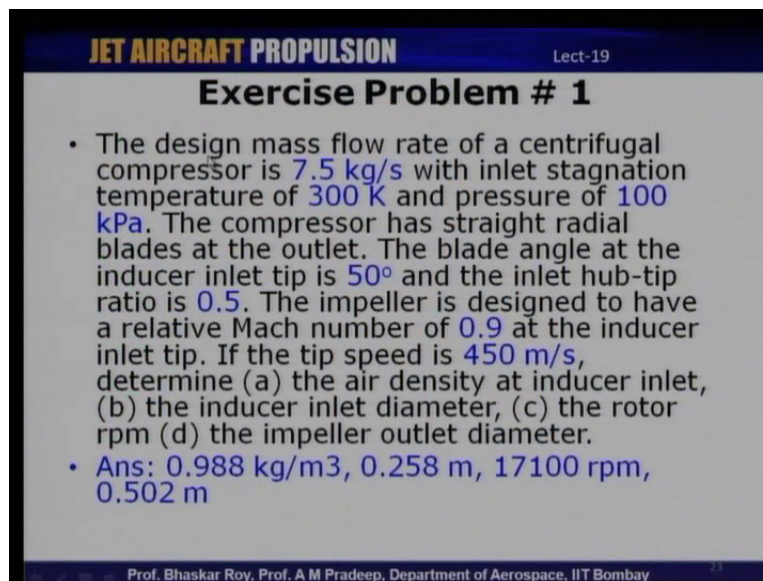
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And since the inlet flow is axial, the axial velocity and the absolute velocity are the same. So, mass flow rate is row times row one a one C_1 , we have calculated all the three now. So, row 1 is 1.157, A_1 is 0.0264 and C_1 is 114.62. So, the mass flow rate can be calculated as 3.5 kilograms per second. The other parameter to be calculated is blade angle at the tip of the eye

which is beta one. So, time beta one from the velocity triangle is C_1 by U_1 if you substitute for both these we can calculate beta one as 20.57 degrees. So, this is the blade angle at the tip of the inducer.

So, in this problem what we have done is this was a problem, which was concerning just the inducer you were not looking at what happens at the outlet of the impeller. In the previous problem we were looking at the slip factor where we are we were required to calculate the impeller exit velocity triangle we were required to solve the exit velocity triangle. In this case we were just solving the inducer of the compressor and we calculated the mass flow rate and also the blade angle at the inlet of the inducer. We have solved two problems today, on centrifugal flow compressor is one pertaining to the impeller and the other pertaining to the inducer of the centrifugal compressor. So, we will now take up a few exercise problems and then, I shall leave it to you for solving these problems is based on what we have discussed just now and also based on our discussion in the last lecture.

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Exercise Problem # 1

- The design mass flow rate of a centrifugal compressor is 7.5 kg/s with inlet stagnation temperature of 300 K and pressure of 100 kPa. The compressor has straight radial blades at the outlet. The blade angle at the inducer inlet tip is 50° and the inlet hub-tip ratio is 0.5. The impeller is designed to have a relative Mach number of 0.9 at the inducer inlet tip. If the tip speed is 450 m/s, determine (a) the air density at inducer inlet, (b) the inducer inlet diameter, (c) the rotor rpm (d) the impeller outlet diameter.
- Ans: 0.988 kg/m³, 0.258 m, 17100 rpm, 0.502 m

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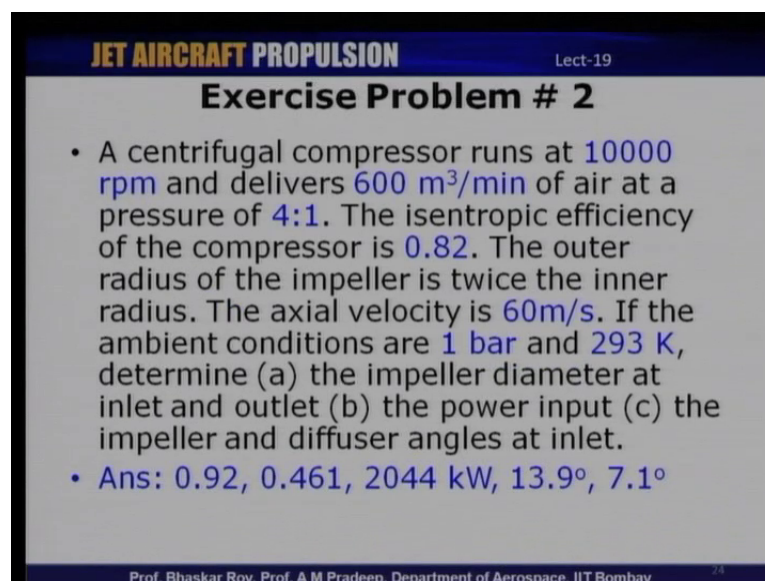
So, the first exercise problem is on a centrifugal compressor is the following: the design mass flow rate of a centrifugal compressor is 7.5 kilo gram per second with an inlet stagnation temperature of 300 Kelvin and a pressure of 100 kilopascals. The compressor has straight radial blades at the outlet the blade angle at the inducer inlet tip is 50 degrees and the hub to tip ratio is 0.5.the impeller is designed to have a relative mach number of 0.9 at the inducer inlet tip if the tip speed is 450 meters per second determine the density air density at the

inducer inlet, the inducer inlet diameter, the rotor Revolution Per Minute and the impeller outlet diameter. So, this is a question very similar to what we just now solved, the second problem that we had solved and here we are required of course, the mass flow rate is specified the inlet conditions are given the angles are also given hub to tip ratio is given Mach number is specified. So, based on this, we are required to calculate the air density, this is as which we you can calculate and it will come out to be 0.988 kilograms per meter cube.

The second part is inducer inlet diameter that would come out to be 0.258 meters, and then the rotor Revolution per Minute would be seventeen thousand one hundred Revolutions per Minute and the impeller outlet diameter that is 0.502 meters.

So, this problem I am sure you will be able to solve because you have already solved the problem on these lines the second one that we have solved was very similar to this.

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Exercise Problem # 2

- A centrifugal compressor runs at 10000 rpm and delivers 600 m³/min of air at a pressure of 4:1. The isentropic efficiency of the compressor is 0.82. The outer radius of the impeller is twice the inner radius. The axial velocity is 60m/s. If the ambient conditions are 1 bar and 293 K, determine (a) the impeller diameter at inlet and outlet (b) the power input (c) the impeller and diffuser angles at inlet.
- Ans: 0.92, 0.461, 2044 kW, 13.9°, 7.1°

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The second question exercise problem is a centrifugal compressor runs at 10000 Revolution Per Minute and delivers 600 meter cube per minute of air at a pressure ratio of 4 is to 1 the Isentropic efficiency of the compressor is 0.82, the outer radius of the impeller is twice the inner radius and the axial velocity is 60 meters per second. If the ambient conditions are 1 bar and 293 Kelvin, determine part an impeller diameter at inlet and outlet part b power input and part c the impeller and diffuser angles at the inlet.

So, in this case we need to calculate, we are required to calculate the impeller diameters at the inlet and outlet and the power output based on the Revolution per Minute and mass flow rate flow rate given pressure ratio is also given. Efficiency of the compressor is known to us and also it is mentioned that the inlet the outer radius of the impeller is twice the inner radius. So, this will help you in calculating the mass flow rate calculating the diameters at the inlet and outlet. So, diameter comes out to be outlet at the outlet it is 0.92 meters at the inlet 0.461 meters power input is 2044 kilo watts impeller and diffuser angles are 13.9 degrees and 7.1 degrees.

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Exercise Problem # 3

- 30 kg of air per second is compressed in a centrifugal compressor at a rotational speed of 15000 rpm. The air enters the compressor axially. The compressor has a tip radius of 30 cm. The air leaves the tip with a relative velocity of 100 m/s at an angle of 80° . Assuming an inlet stagnation pressure and temperature of 1 bar and 300 K, respectively, find (a) the torque required to drive the compressor, (b) the power required (c) the compressor delivery pressure
- Ans: 4085 Nm, 6.417 MW, 6.531 bar

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The third exercise problem that, you can solve is thirty kilograms of air per second is compressed in a centrifugal compressor at a rotational speed of 15000 Revolution Per Minute air enters the compressor axially the compressor has a tip radius of 60 centimeters 30 centimeters the air leaves the tip with a relative velocity of 100 meters per second at an angle of 80 degrees assuming an inlet stagnation pressure and temperature of 1 bar and 300 Kelvin respectively. Find part a: the torque required to drive the compressor, part b: power required and part c the compressor delivery pressure.

So, here in this question the part a and part b are very much related torque required and power required are quite related and straight forward and the mass flow rate is specified the speed of rotation 15000 Revolution Per Minute and the compressor tip radius is also given and the outlet velocity triangle, we have the relative velocity and also the angle which it

makes with the outlet that is also specified. The torque required in this case would be 4085 Newton meters the power required would be 6.417 mega watts and the compressor delivery pressure would be 6.531 bar. So, if you were to solve this problem you, you would be getting these answers that have been specified here.

So, there are three problems that I had given for as an exercise based on the tutorial, we had during part of this lecture that we had today and. So, what we have completed so far is one component of the aircraft engine, that is the axial compressor and then subsequently we are going to take up the combustion chamber then the turbine and also we will be discussing about the intakes and the nozzles of axial compressor jet engine. The next part that, we shall be discussing would be about combustion chambers and then turbines. Subsequently, we will also take up two other important components which constitute a jet engine that is the intake and also the nozzle. So, all these components put together constitute the jet engine as a whole.

So, in the next lecture that we shall be discussing that would be combustion chambers subsequently the turbines and then I shall be talking about air intakes. So, in the next part of my lecture, we would be talking about air intakes and we shall have three lectures on air intakes where we shall be talking about air intakes which are used in transport aircraft, air intakes used in military aircraft and also intakes which are used in subsonic flows and how they are different from supersonic intakes. And what are the geometric differences between these types of intakes.

So, we will take up some of these topics for discussion during our next lecture, where we shall initiate discussion on intakes. So, that brings us to the end of this lecture, and also the end of this chapter on centrifugal compressors. We have already discussed about axial compressors, and so this would be the end of our discussion on centrifugal flow compressors.