# Jet Aircraft Propulsion

**Prof. Bhaskar Roy**
**Prof. A. M. Pradeep**
**Department of Aerospace**
**Indian Institute of Technology, Bombay**

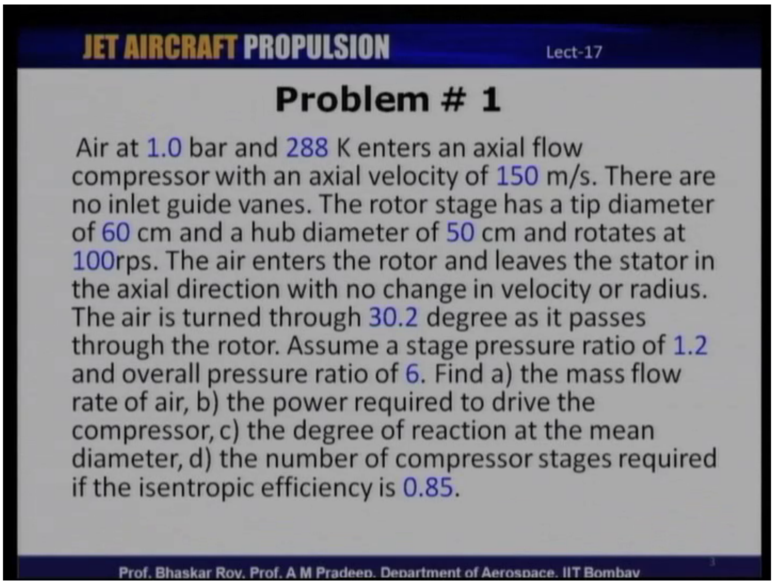
## Lecture No. # 17
## Tutorial - 3

Hello and welcome to lecture number seventeen of this lecture series on jet aircraft propulsion. In our journey so far, we have had quite some interesting revelations about the different components that constitute a gas turbine engine, and besides just the basic thermodynamics of these components. You also had some discussions on how these components different components are working; we of course have not yet covered the rest of the components we are on the axial compressors right now. Subsequently, we will of course be taking up the other components one by one, and towards the end of the course, I am sure you would have a better understanding of how our gas turbine engine works, and how is it that we can initiate a very preliminary design of these components.

So, what we are going to discuss today is basically some solved problems on axial compressors, because we have been discussing about axial compressors in the last three or four lectures. And therefore, it is very much a good idea that we should be able we should try and solve a few problems. And see, how is it that we can apply the principles - the thermodynamic principles that we have studied in the last few lectures on trying to solve a few problems. So that we get some idea about what the numbers look like for a an axial compressor.

So, what I have for you today are a few solved problems, I have basically sorted out three problems, which I shall solve for you. And then I also have listed a few problems towards the end of the lecture, which are basically exercise problem for you to solve based on our discussion today, as well as what we have been discussing in the last few lectures. So, in today’s tutorial we shall be discussing about solved problems on axial compressors, so today’s lecture is basically a tutorial with both solved as well as unsolved problem towards the end of the lecture.

(Refer Slide Time: 02:28)



JET AIRCRAFT PROPULSION Lect-17

## Problem # 1

Air at 1.0 bar and 288 K enters an axial flow compressor with an axial velocity of 150 m/s. There are no inlet guide vanes. The rotor stage has a tip diameter of 60 cm and a hub diameter of 50 cm and rotates at 100rps. The air enters the rotor and leaves the stator in the axial direction with no change in velocity or radius. The air is turned through 30.2 degree as it passes through the rotor. Assume a stage pressure ratio of 1.2 and overall pressure ratio of 6. Find a) the mass flow rate of air, b) the power required to drive the compressor, c) the degree of reaction at the mean diameter, d) the number of compressor stages required if the isentropic efficiency is 0.85.

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Let us, take a look at the first problem that we have or today's lecture. So, this problem is on again an axial compressor, which states that air at 1 bar, and 288 Kelvin enters. An axial flow compressor with an axial velocity of 150 meters per second; there are no inlet guide vanes the rotor stage has a tip diameter of 60, and a hub diameter of 55 centimeters and rotates at 100 revolutions per second.

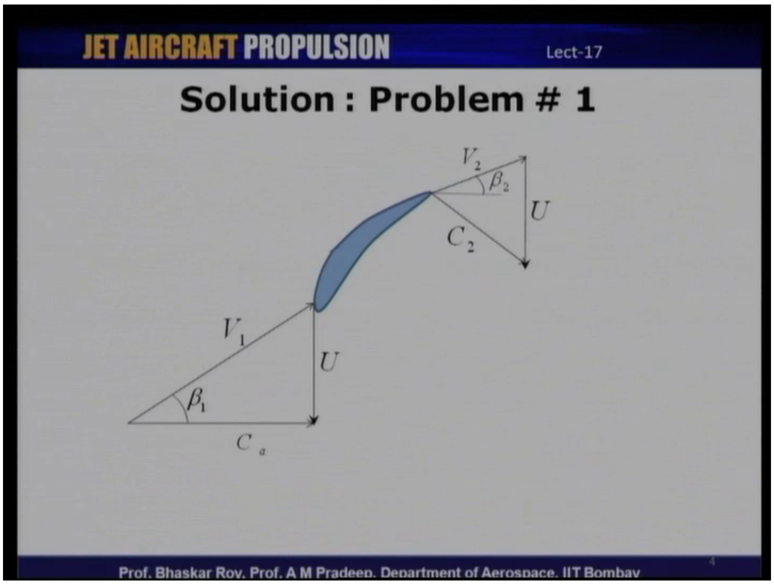
The air enters the rotor, and leaves the stator in an axial direction with no change in velocity or radius air is turned through 30.2 degrees as it passes through the rotor; assume a stage pressure ratio of 1.2 in the overall pressure ratio of 1.6 find part a the mass flow rate of air part b. The power required to drive the compressor part c. The degree of reaction that the mean diameter part d the number of compressor stage required if the isentropic efficiency is 0.85.

So, in this problem it is basically stated that we have some inlet conditions; we have an axial velocity, which has been specified and it is clearly mentioned that there are no inlet guide vanes. So, what does this mean it basically means that the axial velocity, and the absolute velocity at the inlet are in the same direction, which will be clear when we take a look at the velocity triangles as well and besides this we have some geometric details given, and how much is the air deflected through as it passes through the rotor. So, the deflection angle is given, and besides this we have been given the pressure ratios. So, we are required to find the number of parameters the number of stages required power required and so on.

So, I think the first step towards solving any such problem is to draw the velocity triangles. And I think, if you get the velocity triangles right then that solves half of the problem; and that is very important in solving such problems, because from the velocity triangle it will be very clear that what are the different components, which have been specified in the problem, and what are the other components that we need to find to be able to fully solve this problem.

So, you should definitely begin with, drawing the velocity triangle, and that is very important, and based on of course, the information provided for example, in this question it is given that there are no inlet guide vanes, how will the velocity triangle look line. So, we will take a look at what the velocity triangle looks like at the inlet of the rotor that is at the leading edge, and at the trailing edge. Again we will have to figure out what the velocity triangle should look like, and some of the components in the velocity triangle would be known. The blade speed is given as 100 revolutions per second, and the diameter at which we are to calculate these properties are also known so based on that we can calculate the blade speed, and the inlet axial velocity is given and so on.

(Refer Slide Time: 05:49)



JET AIRCRAFT PROPULSION Lect-17

Solution : Problem # 1

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

And also the turning angle through the rotor is specified; so based on the information that we have let us now, construct the velocity triangle. So, velocity triangle for this particular problem would look something like this. Now let me explain why the velocity triangles have been constructed like this.

Now, firstly you know that let us construct. So, we begin with constructing a blade a compressor blade it is typically shown, here a low speed compressor blade is, what is shown here, and the blade speed, which is basically U has been specified in this direction. So, this is U the blade speed, because it is a compressor blade we know the direction of U.

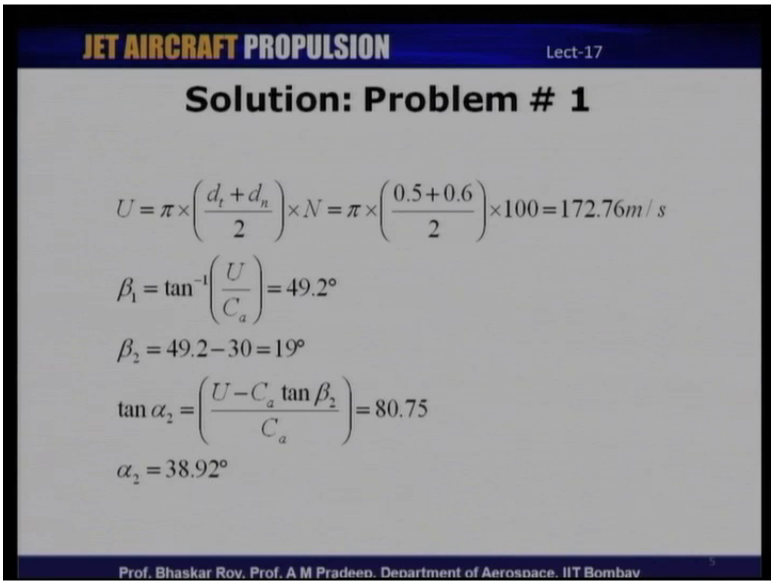
In this question, we have been we have been given that the axial velocity is 150 meters per second at the inlet, and there are no inlet guide vanes. So, if there are no inlet guide vanes what it means is that whatever velocity is coming in it enters the blade axially that is the inlet velocity-the inlet absolute velocity that is C 1 will be equal to C a, and it will be in the same direction this is, because there are no inlet guide vanes.

And velocity V 1 will always be in a direction, which is tangential to the blade leading edge. So, V 1 is tangential there and U is known, and so we know that vectorially U plus V 1 is equal to C, and that is how you get the inlet velocity triangle, and the angle, which V 1 makes with the axial direction is given by beta 1, which is the blade inlet angle.

So, this is how the velocity triangle is constructed at the inlet and what happens at the exit or at the trailing edge at the trailing edge. Again we know that V 2 that is the relative velocity should leave the blade tangentially. So, this is V 2 the direction at of the relative velocity, and we also know the blade speed blade speed is, which is U; which is known the direction is known magnitude is also known; and that completes the exit velocity triangle. Because U plus V is equal to C, so U 2 plus V 2 is equal to C 2 and therefore, that completes this velocity triangle, and beta 2 is the blade angle at the exit that is at the trailing edge. The angle made by the relative velocity with the axial direction; so that is given as beta 2.

So, these are the different components of the velocity triangle. And once, we get the velocity triangle. You would also be able to identify, what are the parameters, which are known. In this case for example, we know the inlet axial velocity that is specified. we know the blade speed; and so based on that we know a certain parameters of this velocity triangle. And we shall now, start to try and solve the problem, and try to find out the other components of the velocity triangle; and subsequently of course, we will need to also find out the other parameters like the pressure ratio and so on and efficiency.

(Refer Slide Time: 08:58)



## Solution: Problem # 1

$$U = \pi \times \left(\frac{d_t + d_n}{2}\right) \times N = \pi \times \left(\frac{0.5+0.6}{2}\right) \times 100 = 172.76 m/s$$

$$\beta_1 = \tan^{-1}\left(\frac{U}{C_a}\right) = 49.2^\circ$$

$$\beta_2 = 49.2 - 30 = 19^\circ$$

$$\tan\alpha_2 = \left(\frac{U - C_a \tan\beta_2}{C_a}\right) = 80.75$$

$$\alpha_2 = 38.92^\circ$$

Let us start to solve this problem; now, how do you find for the first parameter to be calculated is the blade speed at the mean diameter. We have been given the tip diameter and the root diameter. So, based on that we can find out the blade speed blade speed is U, which is pi into mean diameter divided by 2 into n, which is the speed given in a revolutions per second here.

So, here we have pi into d t plus d h, which is the tip diameter plus the hub diameter divided by 2 and multiplied by n. So, we get pi times the tip, and root diameters divided by 2 into 100, which is the blade the revolutions per second. So, if you calculate that we should be able to get the blade speed at the mean diameter as 172.76 meters per second.

(Refer Slide Time: 05:49)

So, we have now found out the blade speed we also know the axial velocity. So, at the inlet velocity triangle; we now know 2 parameters we know the axial velocity, which is 150 meters per second, and U we have just now calculated as 172 meters per second. So, based on this we can find out the angle beta 1, beta 1 tan beta 1 is basically U by C a, and therefore, beta 1 is tan inverse 2 by C a.

(Refer Slide Time: 08:58)

So, if you were to use that find out beta 1 as tan inwards U by C a, we get that as 49.2 degrees.

(Refer Slide Time: 05:49)

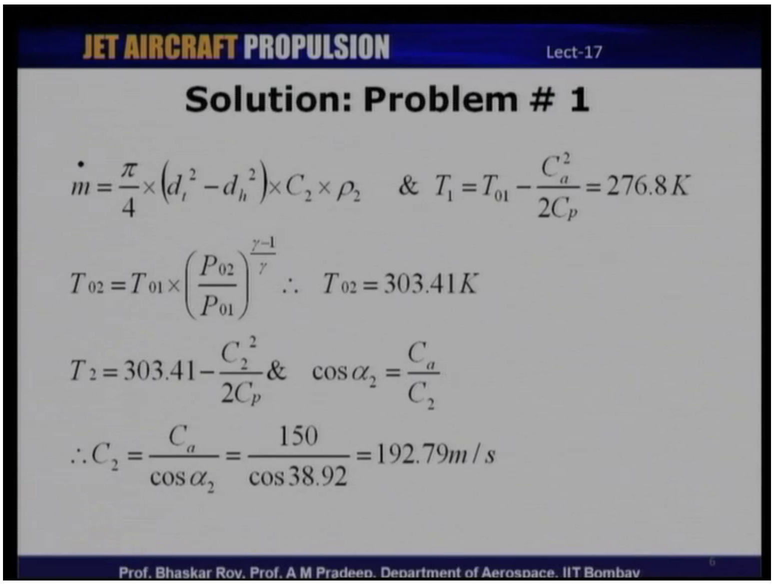
And beta 2 is the exit blade angle, blade angle at the exit of the rotor and in this question it is given that the blade turns the flow by 30.2 degrees, and therefore since beta 1 is 49.2 and the turning angle, which is or deflection angle, which is delta beta delta beta is 30 degrees 30.2 degrees therefore, we have beta 2, which is 49.2 minus 30.2, which is 19 degrees.

So, we have now calculated the blade angle at the inlet and exit of the rotor. We know have beta 1 and beta 2; now, if we know beta 2. We should also be able to calculate some of the other parameters for example, tan alpha 2, which is the angle at the exit of the absolute velocity that is U minus tan alpha 2 is U minus C a into tan beta 2 divided by C a that is this is U tan alpha 2 is this component alpha 2 that is U minus C a into tan beta 2, which is this component divided by C a, because it is given that there is no change in axial velocity as it passes through the rotor.

So, from there we can calculate tan alpha 2 and therefore, alpha 2, which comes out to be 38.92 remember that at the inlet alpha 1 and beta 1 are the same. Because there is no angle made by the absolute velocity with the axial direction, because C a is equal to C 1, so beta 1 and alpha 1 are the same, which is 49.2, so alpha 2 comes out to be 38.92 degrees.

So, we have calculated all the angles we have now calculated beta 1 we now we have calculated beta 2 we have calculated alpha 2. And so we have calculated all the angles required. And so that should be specifying the geometry in terms of the angles. So, let us now, calculate the other parameters that are required. And so the second part of the question was to find out let us, look at what is the second part question requires that we are we are required to find out the mass flow rate of air the we find out the power required at degree of reaction and number of stages.

(Refer Slide Time: 13:00)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 1

$$\dot{m} = \frac{\pi}{4}\times\left(d_t^2 - d_h^2\right)\times C_2\times\rho_2 \quad \& \quad T_1 = T_{01} - \frac{C_a^2}{2C_p} = 276.8K$$

$$T_{02} = T_{01}\times\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad T_{02} = 303.41K$$

$$T_2 = 303.41 - \frac{C_2^2}{2C_p} \quad \& \quad \cos\alpha_2 = \frac{C_a}{C_2}$$

$$\therefore C_2 = \frac{C_a}{\cos\alpha_2} = \frac{150}{\cos 38.92} = 192.79m/s$$

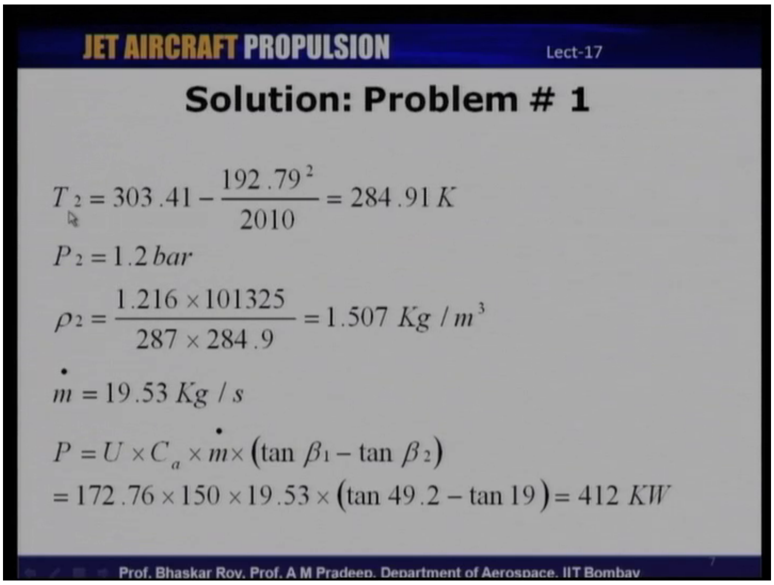
Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

So, let us calculate or determine the mass flow rate, how do we calculate mass flow rate? mass flow rate is given by the, area that is the annular area pi by 4 into d t square minus d h square into velocity in into the density; that is at the exit. So, we can either find out the density at the exit or at the inlet correspondingly use the actual velocities.

So, we have here the temperature T 1, which is equal to T 01 minus C a 2 by 2 C p, so T 01 is given in this question its already been specified C a is already known. So, we can find out T 1 that comes out to be 276.8 Kelvin and the stagnation temperature. We can find by the isentropic relation assuming that, because there is no efficiency specified here we can assume that the process is isentropic. So, T 02 that is stagnation temperature at the exit of the rotor is equal to T 01 into p 02 by p 01 raise to gamma minus 1 by gamma.

And this is the compressor pressure ratio that is already given as six. So, from there we can find out the temperature at the exit of the rotor, and that comes out to be 303.41 Kelvin. So, once we find out T 02 we can find out T 2 that is the static temperature that is T 01 T 02 minus C 20 by 2 C p, and how do we find out C 2 square is not known, and C 2 is basically equal to C a by cos alpha 2. Because let us, go back to the velocity triangle C 2 is this component C 2 is equal to C a that is this divided by cos alpha 2. And cos alpha 2; we have already found out that is 38.92. So, C 2 we can find out as 150 divided by cos 38.92 that 192.79 meters per second.

(Refer Slide Time: 15:11)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 1

$$T_2 = 303.41 - \frac{192.79^2}{2010} = 284.91\,K$$

$$P_2 = 1.2\,bar$$

$$\rho_2 = \frac{1.216 \times 101325}{287 \times 284.9} = 1.507\,Kg/m^3$$

$$\dot{m} = 19.53\,Kg/s$$

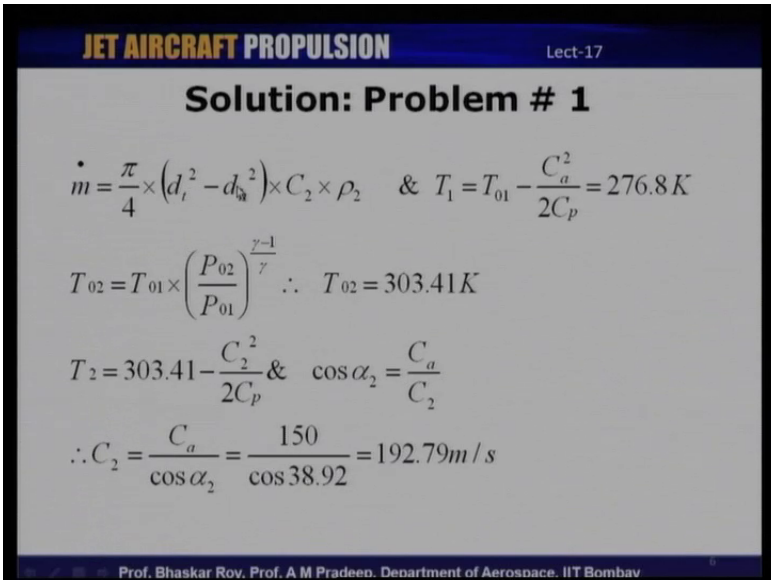
$$P = U \times C_a \times \dot{m} \times (\tan\beta_1 - \tan\beta_2)$$
$$= 172.76 \times 150 \times 19.53 \times (\tan 49.2 - \tan 19) = 412\,KW$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Therefore, we find out T 2 that is static temperature that is 303.41 minus C 2 square by 2 C p that is 192.79 square by 2 into 1005 that is 2010. So, this comes out to be 284.91 Kelvin. Similarly, we find out P 2, which comes out to be 1.2 bar, because the stage pressure ratio is given. And so from there we can find out P 2.

And so once, we find out pressure and temperature we can find out now the density is equal to P by r t, so P we have converted that is in bars we converted that to pascal's divided by r that is gas constant times the temperature. So, the density at the exit of the rotor comes out to be 1.507 kilograms per meter cube.

(Refer Slide Time: 16:10)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 1

$$\dot{m} = \frac{\pi}{4}\times\left(d_t^2 - d_h^2\right)\times C_2\times\rho_2 \quad \& \quad T_1 = T_{01} - \frac{C_a^2}{2C_p} = 276.8K$$

$$T_{02} = T_{01}\times\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \therefore \quad T_{02} = 303.41K$$

$$T_2 = 303.41 - \frac{C_2^2}{2C_p} \& \quad \cos\alpha_2 = \frac{C_a}{C_2}$$

$$\therefore C_2 = \frac{C_a}{\cos\alpha_2} = \frac{150}{\cos 38.92} = 192.79m/s$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

So, from this equation that we have written mass flow rate is basically pi by 4 into d t square minus d h square the tip and hub diameters are already specified c two we have determined, and row 2 also we have calculated. So, if you substitute all those values the mass flow rate comes out to be 19.53 kilograms per second.

(Refer Slide Time: 15:11)

So, that solves the first part of the problem; the second part of the problem was to find out the power required to drive this compressor, and how do we find out power required power required is U into delta C theta multiplied by mass flow rate, and delta C theta is or C w is basically the difference between these two components that is this is C w 2 and this is C w 1.

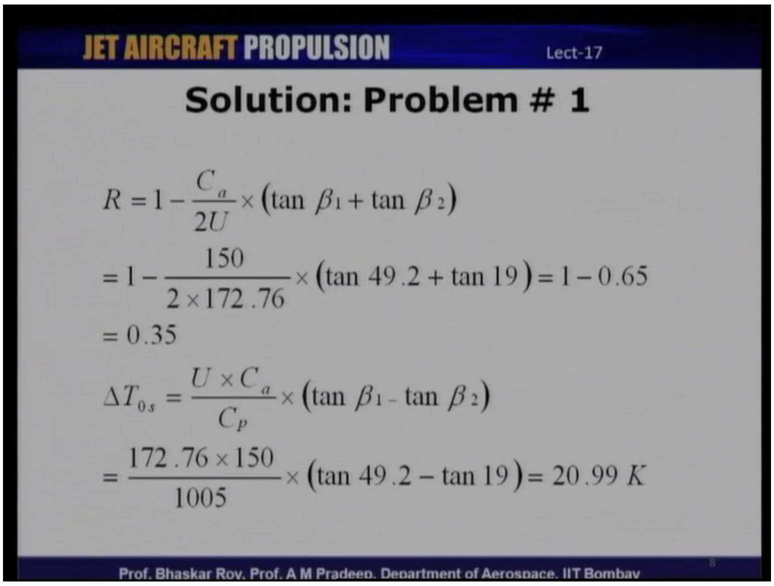
(Refer Slide Time: 15:11)

So, from there basically what happens is we have it its basically the delta C w is equal to C a times tan beta 1 minus tan beta 2 and U is already been calculated 172.76 is known, 150 meters per second mass flow rate is 19.53 and tan beta 1 and tan beta 2 are also known. Therefore, the power required can be calculated by substituting all these values and simplifying so we get the power required as 412 kilo watts.

So, far we have calculated two parts of the question 1 was to find out the mass flow rate, which is basically the annular area times the velocity multiplied by the density. And you could either find that out that at the inlet or exit. So, that is up to you whichever way you can find out, but obviously the mass flow rate will come out to be the same. A second part of the question was to find out the power required and power required is basically equal to mass flow rate multiplied by the blade speed that is U and delta C w in this case it comes out to be C a times tan beta 1 minus tan beta 2. So, from this we find out the power required as well.

So, the next part of the question is to find out the degree of reaction for this particular rotor. Now, if you recall our discussion on degree of reaction a few lectures ago degree of reaction is basically the ratio of the work done by the rotor for the overall compression as part of the stage. So, that is static enthalpy raise in the rotor divided by the stagnation enthalpy raise in this stage. So, that defines the degree of reaction, and we have simplified that basic definition to arrive at an expression, which can be expressed in terms of known parameters like the blade speed or the angles and so on.

(Refer Slide Time: 19:02)



JET AIRCRAFT PROPULSION

Lect-17

**Solution: Problem # 1**

$$R = 1 - \frac{C_a}{2U} \times (\tan \beta_1 + \tan \beta_2)$$

$$= 1 - \frac{150}{2 \times 172.76} \times (\tan 49.2 + \tan 19) = 1 - 0.65$$

$$= 0.35$$

$$\Delta T_{0s} = \frac{U \times C_a}{C_p} \times (\tan \beta_1 - \tan \beta_2)$$

$$= \frac{172.76 \times 150}{1005} \times (\tan 49.2 - \tan 19) = 20.99\ K$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

8

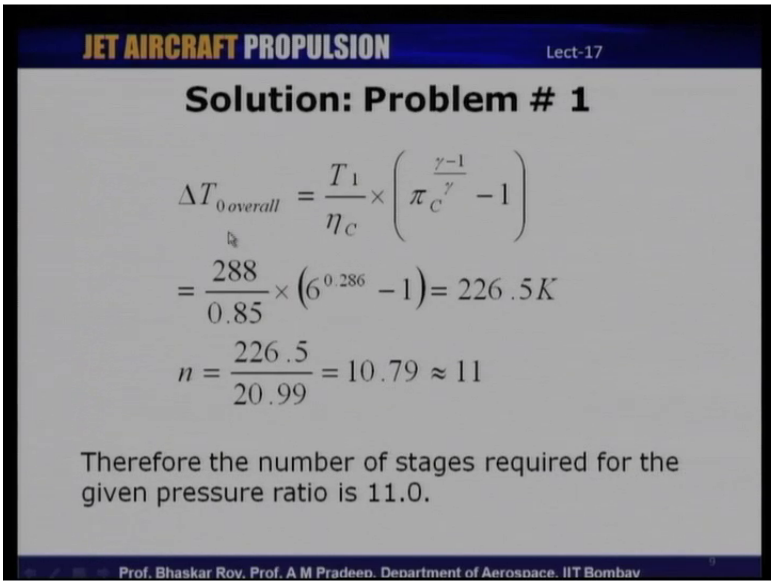
So, if we look at what is degree of reaction degree of reaction is basically r x as we have defined is equal to 1 minus C a by 2 U into tan beta 1 plus tan beta 2 so we have already derived this equation in our earlier lecture. So, if we use that expression substitute all the values. Because all these parameters are known so if you were to substitute those values we have 1 minus C a, which is 150 divided by U or 2 U that is 2 into 172. 76 multiplied by tan beta 1 that is tan 49.2 plus tan beta two that is tan 19. So, this comes out to be 1 minus 0.65 that is 0.35. So, this is the degree of reaction for this particular rotor.

Now, the next part of the question is to find out the number of stages required by this compressor to generate this particular pressure ratio of six. So, for finding out the number of stages, what we are going to do is, which is something, I think you would also be doing later on is that the overall pressure ratio is known. And so we can calculate the overall temperature rise in this particular compressor that is delta T 0 overall, because the pressure ratio is known. We can also find out the per stage temperature rise, because we have already defined per stage pressure raise or temperature raise in terms of the blade speed and the axial velocity and the geometric angles.

So, total temperature raise across the compressor divided by per stage temperature raise will give us, will tell us, how many number of stages are required for generating this pressure ratio. So, first thing we will calculate is delta T 0 for one stage. We have already looked at this equation we have discussed this equation earlier, how we have arrived this at this particular expression. So, delta t 0 for the stage is U times C a divided by C p into tan beta 1 minus tan beta 2 basically this comes from the fact that we equate delta not to U times delta C w so if we equate the two we get the same expression that is C p into delta t 0 should be equal to U times C a into this.

So, let us substitute for all these values when we substitute them we know U that is 172.76 C a is known as 150 C p is already specified tan beta 1, and tan beta 2 are known. So, we get delta t not for the stage as 20.99 kelvin so this is the stagnation temperature rise occurring in one stage of this axial compressor.

(Refer Slide Time: 22:02)



**JET AIRCRAFT PROPULSION** Lect-17

## Solution: Problem # 1

$$\Delta T_{0\,overall} = \frac{T_1}{\eta_C} \times \left(\pi_C^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$= \frac{288}{0.85} \times \left(6^{0.286} - 1\right) = 226.5K$$

$$n = \frac{226.5}{20.99} = 10.79 \approx 11$$

Therefore the number of stages required for the given pressure ratio is 11.0.

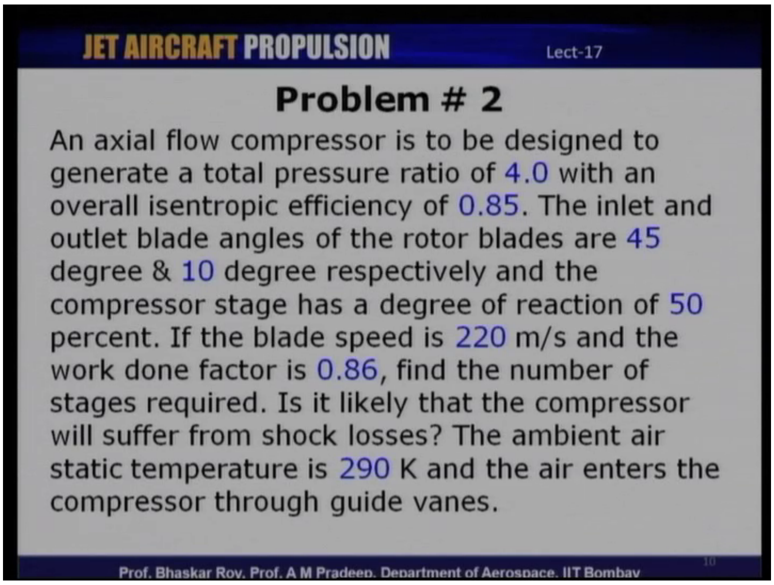
Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

And delta T 0 overall we can find out we basically the delta overall temperature raise is the inlet temperature that is t 01 divided by the efficiency, because it is given as 0.85 multiplied by the pressure ratio overall pressure ratio rise to gamma minus 1 by gamma minus 1. So, there are 288 by efficiency isentropic efficiency is 0.85 multiplied by 6 that is the overall pressure ratio raise to gamma minus 1 by gamma that is 1.4 r minus 1 by 1.4 that is 0.286 minus 1.

So, delta T not overall is 226.5 Kelvin, therefore the number of stages required for generating a pressure ratio of 6 would be overall temperature ratio or overall temperature rise delta T 0 overall divided by delta T 0 for the stage. So, that is 226.5 divided by 20.99 that is 10.79, which is equal to 11. So, 10.799 we have rounded it off to the next higher digit that is 11, because if we use only 10 obviously, you cannot achieve the required pressure ratio of 6. So, that this is a normal practice that if you arrive at a fractional number. Then we round it off to the next higher fraction number, so that we are able to achieve the desired pressure ratio.

So, the numbers of stages we have calculated here by calculating the overall temperature ratio or temperature rise divide that by the temperature rise required for or generated by one stage. So, that gives us the number of stages required for generating this pressure ratio of 6 in this case.

(Refer Slide Time: 23:56)



JET AIRCRAFT PROPULSION Lect-17

## Problem # 2

An axial flow compressor is to be designed to generate a total pressure ratio of 4.0 with an overall isentropic efficiency of 0.85. The inlet and outlet blade angles of the rotor blades are 45 degree & 10 degree respectively and the compressor stage has a degree of reaction of 50 percent. If the blade speed is 220 m/s and the work done factor is 0.86, find the number of stages required. Is it likely that the compressor will suffer from shock losses? The ambient air static temperature is 290 K and the air enters the compressor through guide vanes.

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 10

So, in this particular problem that we have solved we have basically been discussing about the axial compressor and how we can go about calculating the various parameters required in this particular case for example, in the first problem that we have solved we have been specified some of the angles. We have we have been given the inlet flow conditions based on that we have calculated the mass flow rate the power required, and degree of reaction and of course the number of stages required.

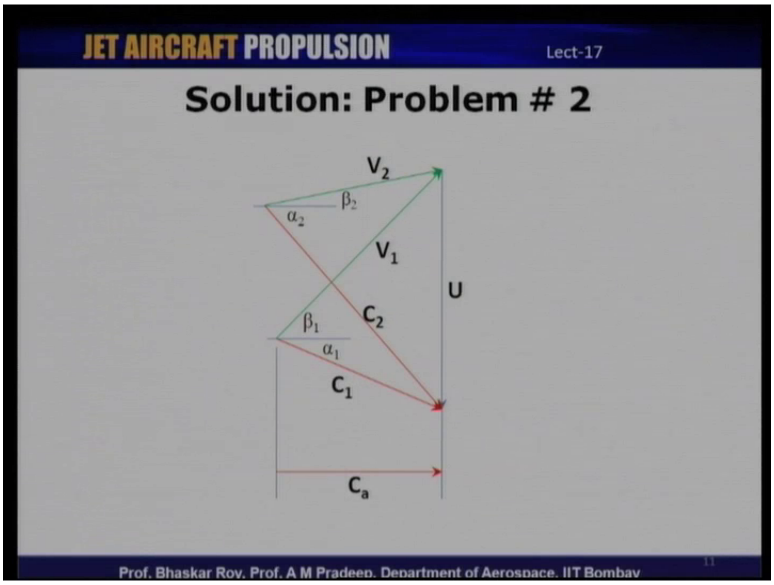
So, we will now take a look at the next problem that we have, which is also pertaining to an axial compressor. Let us take a look at what the problem statement is now this problem, which has been specified here is on axial compressor, which is to be designed to generate a pressure ratio of 4 with an overall isentropic efficiency of 0.85.

The inlet and outlet blade angles of the rotor blades are 45 degree, and 10 degree respectively. And the compressor stage has a degree of reaction of 50 percent if the blade speed is 220 meters per second, and the work done factor is 0.86 find the number of stages required. And you also need to find out is it likely that the compressor will suffer from shock losses.

The ambient air static temperature is 290 kelvin, and the air enters the compressors through guide winds so this question that we have pertains to a rotor, which is having a degree of reaction of 50 percent. Now, what is the property of rotor blade, which has a degree of reaction of 50 percent. We have discussed that if degree of reaction is 50 percent. The velocity triangles will be symmetrical or mirror images that is alpha 1 will be equal to beta 2 alpha 2 will be equal to beta 1, and we have been given 2 of the angles here, which means all the angles of the velocity triangle are already known.

And so based on this we have to now, find out the number of stages required. And we also need to find out whether this particular rotor will be suffering from any shock losses. So, as we discussed in the last question; the first step, towards solving such a problem is to get the velocity triangles. In this case, the velocity triangles are relatively easy, because it is given that it is a 50 percent reaction rotor that is velocity triangles are exactly symmetrical.

(Refer Slide Time: 26:32)


JET AIRCRAFT PROPULSION

Lect-17

Solution: Problem # 2

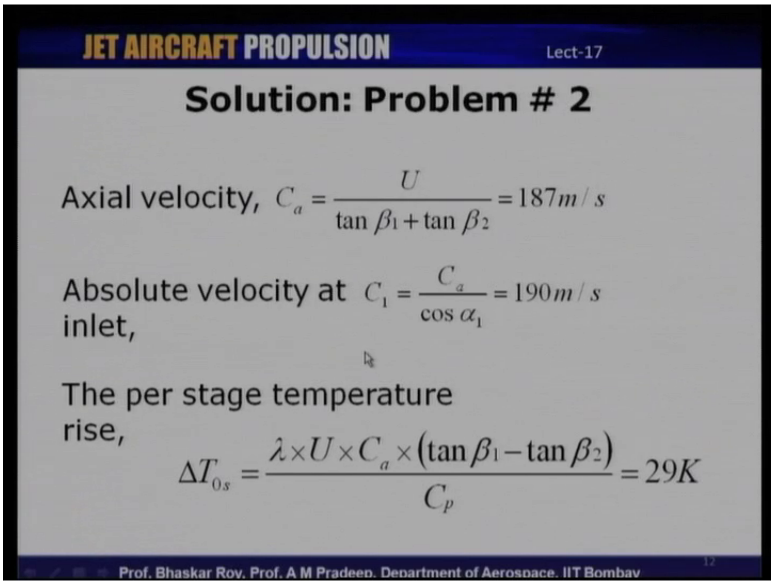
Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

So, with that in mind we have the velocity triangles, which are exactly symmetrical here, and therefore this means that since, the velocity triangles are symmetrical we will have alpha 1, which is equal to beta 2 and beta 1 is equal to alpha 2, and also that will mean that some of these velocity components are also equal C 1 will be equal to V 2 C 2 is equal to V 1 and so on.

(Refer Slide Time: 23:56)

And so we have the inlet and outlet blade angles of the rotor that is beta 1 is 45 and beta 2 is 10, which means the corresponding angles alpha 2 will be 45 and alpha 1 will be 10. The blade speed is already given as 220 meters per second and there is another factor that is come up here that is known as work done factor, I will discuss that shortly, what is meant by work done factor.

(Refer Slide Time: 27:28)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 2

Axial velocity, $C_a = \frac{U}{\tan\beta_1 + \tan\beta_2} = 187 m/s$

Absolute velocity at inlet, $C_1 = \frac{C_a}{\cos\alpha_1} = 190 m/s$

The per stage temperature rise,

$$\Delta T_{0s} = \frac{\lambda \times U \times C_a \times (\tan\beta_1 - \tan\beta_2)}{C_p} = 29K$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

So, axial velocity we have been given blades speed. So, how do we find out axial velocity? So, axial velocity we can find out from the velocity triangles. Now, in the inlet velocity triangles we basically let us, look at the axial velocity at the inlet that are exit both are same. So, C a is this component so tan alpha 1 or tan beta 1 for that matter will be this component and the rest of it will be from what is remaining of U.

So, if we combine both the equations at the inlet velocity triangle and the exit velocity triangle then we get the axial velocity as u divided by tan alpha 1 tan beta 1 plus tan beta 2. Let us, see how that has come up so u is basically equal to C a tan beta 1 plus C a tan beta 2 so if we look at U that is C a is this component, and tan beta 2 or tan beta 1 is this component and tan beta 2 is this component. So, we are adding up these 2 and that is how we get C a is basically U divided by tan beta 1 plus tan beta 2. And so since tan beta 1 or beta and beta 2 are known we can calculate the axial velocity that is 187 meters per second.

(Refer Slide Time: 26:32)

Now, let us also calculate the absolute velocity at the inlet absolute; velocity at the inlet is C a divided by cos alpha 1 let us see how that comes C1 is C a divided by cos alpha 1 cos alpha 1 is or alpha 1 is given as 19 degrees; so if we substitute that if we get C a or c 1 as 190 meters per second. Now, how do we calculate the per stage temperature raise, because we have to now find out the number of stages required for this compressor. So, per stage temperature rise we have earlier defined as U divided by C u into Ca divided by C p multiplied by tan beta 1 minus tan beta 2.

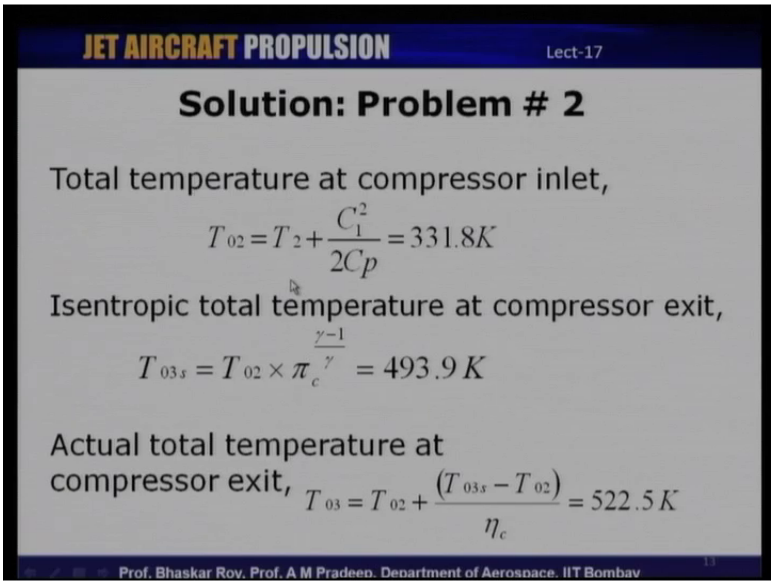
Now, in this case I have included another factor here, which is given by lambda this is basically known as the work done factor. So, work done factor is like an efficiency parameter, which has been added basically, because if you consider a multi stage compressor as we proceed from the inlet of the compressor to the exit of the compressor, I was discussing about annulus laws that is the growth of the boundary layer from the inlet to the exit that is there is a boundary layer growth or development taking place from the inlet or the with exit.

And that means that the effective area of the flow that is available, which is free of the boundary layer will continuously reduce that is the blockage increases as we move from the inlet of the compressor to the exit. There is an increase in blockage as we go from inlet to exit. So, because of this increase in blockage we are accounting for that by using, what is known as a work done factor; that is from as we move from inlet to the exit of the compressor. There is an increase in the boundary layer thickness that means that the effective area that the flow sees will be reduced as we move from inlet to exit. Because there is a growth of boundary layer that is occurring, which means that it is basically appearing as a blockage for the potential flow.

So, one of the ways of accounting for that is to use what is known as a work done factor that is, because of this effective blockage there is per stage temperature rise is reduced and therefore, the per stage pressure ratio is also decreased as a result of this blockage. So, work done factor is basically taking into account the fact that there is a reduction in the area or there is an increase in blockage as the stage proceeds from the inlet to the exit. So, per stage temperature raise the expression for per stage temperature rise will also have this work done factor unless otherwise specified work done factor is assumed to be 0 to be 1. So, if you look at the per stage temperature raise. Now, we have this work done factor lambda multiplied by C a by C p into tan beta 1 minus tan beta 2.

So, if you substitute all the values or work done factor is given U is known C a has already been calculated beta 1 and beta 2 are known, and C p is also known. So, if you substitute all these values here we have the per stage temperature raise, which is 29 Kelvin.

(Refer Slide Time: 32:17)



## Solution: Problem # 2

Total temperature at compressor inlet,

$$T_{02} = T_2 + \frac{C_1^2}{2Cp} = 331.8K$$

Isentropic total temperature at compressor exit,

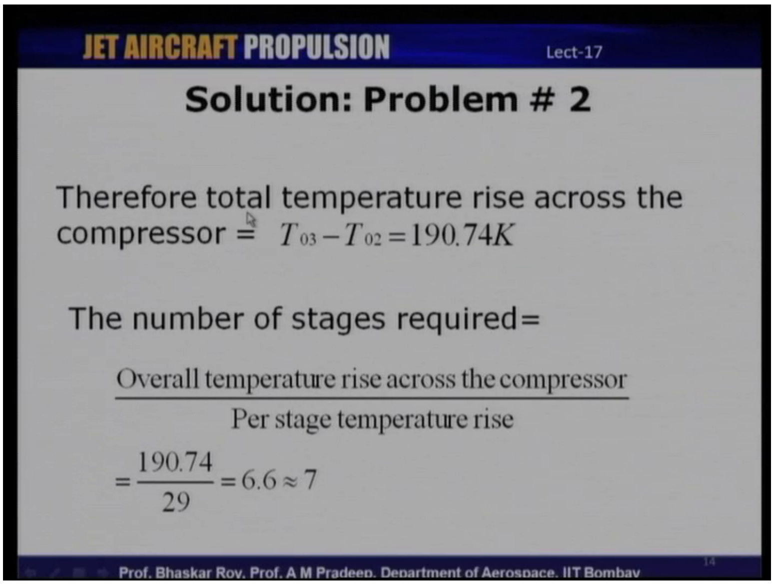
$$T_{03s} = T_{02} \times \pi_c^{\frac{\gamma-1}{\gamma}} = 493.9K$$

Actual total temperature at compressor exit, $T_{03} = T_{02} + \frac{(T_{03s} - T_{02})}{\eta_c} = 522.5K$

Let us now, calculate the actual temperature raise across the compressor. So, here we have the total temperature at the compressor inlet. We have a T 02 that is inlet compressor temperature total temperature, which is static plus C 1 square by 2 C p. So, that is 331.8 Kelvin. Because we know the inlet static temperature; and we also know the absolute velocity at the inlet. So, we calculated the total temperature that is 331.8 Kelvin.

Now, overall pressure ratio is given as 4, which means that the isentropic temperature at the compressor exit can be found out from the isentropic relations that is T 03 is equal to T 02 into 5 C, which is the compressor pressure ratio overall pressure ratio raise to gamma minus 1 by gamma. So, that is 493.9 kelvin, so that is the temperature the total temperature at the exit of the compressor if the compression process was to be isentropic, but we know that it is not isentropic. Because there is an efficiency specified therefore, we can find out the actual total temperature at the compressor exit, which is T 03 that is 302 inlet temperature plus T 03 s minus T 02 divided by efficiency so that comes out to be 522.5 kelvin.

(Refer Slide Time: 33.56)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 2

Therefore total temperature rise across the compressor = $T_{03}-T_{02}=190.74K$

The number of stages required=

$$\frac{\text{Overall temperature rise across the compressor}}{\text{Per stage temperature rise}}$$

$$=\frac{190.74}{29}=6.6\approx 7$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 14

So, this is the actual temperature at the compressor exit. So, we can now find out the total temperature raise across the compressor, which is the difference between the total temperature at the exit, and the total temperature at the inlet of the compressor. So, this is 190.74 Kelvin. So, the across the compressor there is an increase in total temperature of the order of 190.74 kelvin. So, we now have the overall temperature raise across the compressor. We, also have the per stage temperature raise, which we had earlier calculated and therefore, number of stages required would be the ratio of the overall temperature raise to the per stage temperature raise.

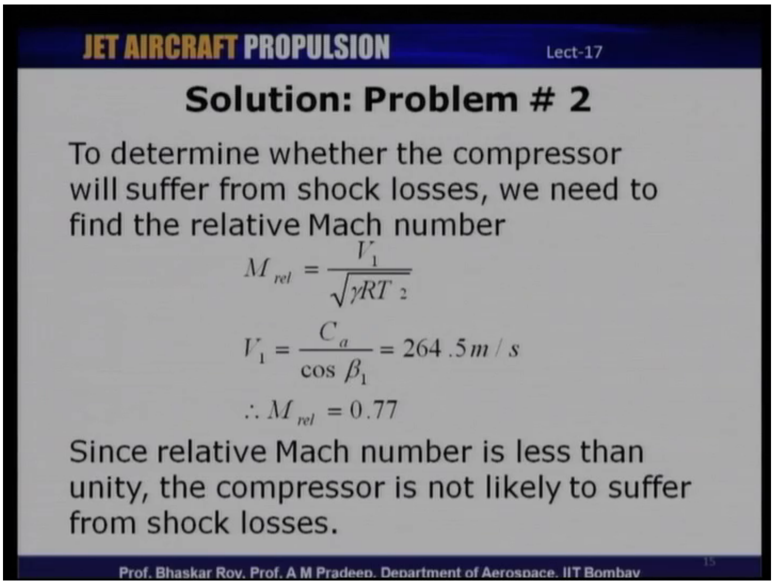
So, that is 190.74 divided by 29 that is 6.6, and as we have been as we have discussed earlier this will be rounded off to the next integer. So, we get number of stages as 7. So, in this case we have the number of stages that are required to generate a pressure ratio of 4 as specified in this problem as 7. So, 7 number of stages are required to generate an overall pressure ratio of 4.

So that was part one of the question to find out, how many stages are required for generating this particular pressure ratio given these parameters, and the velocity triangles and if the degree of reaction was 50 percent. Second part of the question is to find out if this particular rotor is likely to suffer from any shock losses. So, in this case, what we are going to do is that we will basically find out the relative mach number; and see that if the relative mach number has is greater than one point or greater than unity; that means that the flow at the exit is likely to be supper sonic, which means that there is a likely hood that there would be shocks present and therefore, shock losses.

(Refer Slide Time: 26:32)

Let us, go back and look at the velocity triangle; once again so in this velocity triangle. We are going to calculate the Mach number based on the relative velocity. So, that is known as the relative Mach number; and so Mach number based on this would be V 1 divided by square root of gamma r T 1.

(Refer Slide Time: 36:28)



JET AIRCRAFT PROPULSION Lect-17

### Solution: Problem # 2

To determine whether the compressor will suffer from shock losses, we need to find the relative Mach number

$$M_{rel} = \frac{V_1}{\sqrt{\gamma R T_2}}$$

$$V_1 = \frac{C_a}{\cos \beta_1} = 264.5 m/s$$

$$\therefore M_{rel} = 0.77$$

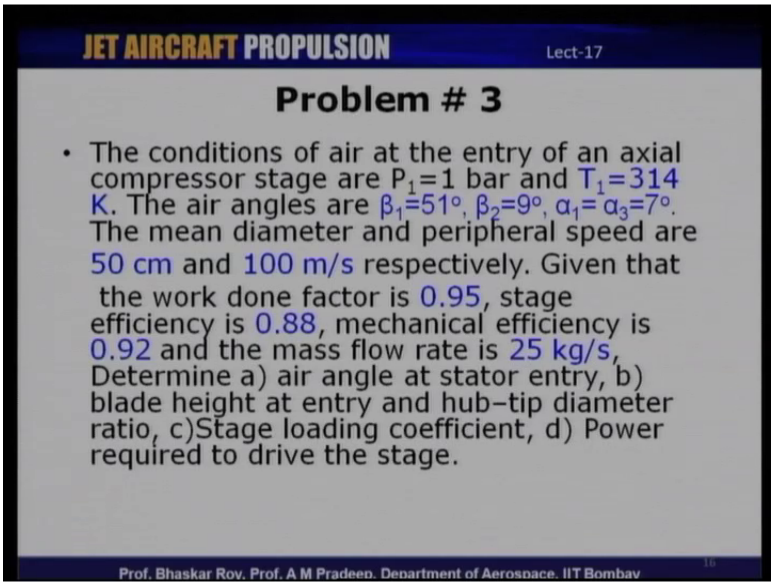
Since relative Mach number is less than unity, the compressor is not likely to suffer from shock losses.

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 15

And if that exceeds one, then we can infer that the flow is supper sonic and there is a likely hood of shock losses there. So, we will calculate the relative Mach number based on the relative velocity. So, relative Mach number is basically the ratio of the relative velocity to the speed of sound and that is square root of gamma R T, and how do we calculate V 1 V 1. We can calculate, because the axial velocity is known the angles are also known from the velocity triangle and therefore, V 1 is the axial velocity divided by cos beta 1, and that is 264.5 meters per second. So, we get the relative velocity at the inlet as 264.5 meters per second therefore, the relative mach number is V 1 divided by square root of gamma r t so that sin 0.77.

So, what we see here is that the relative Mach number is less than unity; and so that is no chance that the this particular compressor at least at the particular diameter, which has been given is likely to suffer from any shock losses. So, it is a normal practice to find out the relative Mach number at the tip of the rotor. In this case, we will not know really that the data given is at least at the tip, but if it is indeed at the tip then relative mach number, which is the tip relative mach number that would be the parameter, which would decide whether this compressor rotor would suffer from any shock losses or not. Because if the tip Mach number exceeds 1, then there is there are chances that the rotor might suffer from shock losses.

(Refer Slide Time: 38:09)



JET AIRCRAFT PROPULSION Lect-17

## Problem # 3

- The conditions of air at the entry of an axial compressor stage are $P_1$=1 bar and $T_1$=314 K. The air angles are $\beta_1$=51°, $\beta_2$=9°, $\alpha_1$= $\alpha_3$=7°. The mean diameter and peripheral speed are 50 cm and 100 m/s respectively. Given that the work done factor is 0.95, stage efficiency is 0.88, mechanical efficiency is 0.92 and the mass flow rate is 25 kg/s, Determine a) air angle at stator entry, b) blade height at entry and hub-tip diameter ratio, c)Stage loading coefficient, d) Power required to drive the stage.

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

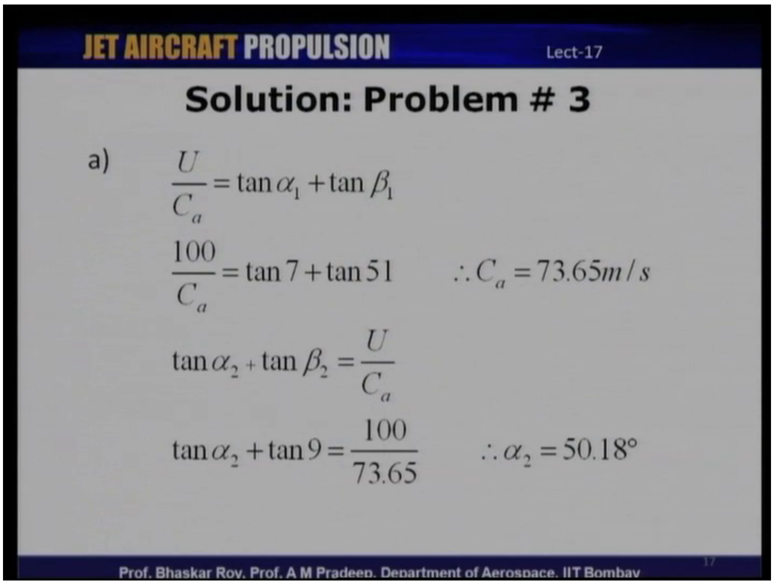
So, this problem that we have just now solved was on a compressor, which was working on a 50 percent degree of reaction stage. Now, we have the third problem for today that is again on an axial compressor. And we have some of the parameters at the inlet, which has been specified that is conditions at the entry of an axial compressor stage or p 1 is 1 bar and T 1 is 314 kelvin the angles at the as specified for this particular rotor are beta 1 is 51 degrees beta 2 is 9 degrees alpha 1 and alpha 3 is equal to 7 degrees.

The mean diameter and the peripheral speed are 50 centimeters and 100 meters per second, respectively given that the work done factor is 0.95 the stage efficiency is 0.88, and the mechanical efficiency is 0.92 the mass flow rate is 25 kg's per second determine part a the stator angle, the air angle at the stator entry that is alpha 2 part b. The blade height at the entry and the hub to tip diameter ratio part c; the stage loading coefficient and part d, the power required to drive the stage.

So, in this question all the angles at the that is the blade angles are specified beta 1, and beta 2 as 51 and 9 degrees. We also have one of the inlet angles that is the alpha 1 that is 7 degrees and that has been given as equal to alpha 3. So, one part of the question is to find out the angle that is alpha 2 at the stator entry. And we need to find out the power required; then we need to find out the stage loading coefficient and so on.

And, we have been specified the mean diameter blade speed and the mass flow rate besides the stage efficiency, and the mechanical efficiency. So, based on the data that has been given we need to find out the angle alpha 2; then the power required and of course, the stage loading coefficient.

(Refer Slide Time: 40:35)



JET AIRCRAFT PROPULSION

Lect-17

**Solution: Problem # 3**

a) $\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$

$\frac{100}{C_a} = \tan 7 + \tan 51 \qquad \therefore C_a = 73.65 m/s$

$\tan\alpha_2 + \tan\beta_2 = \frac{U}{C_a}$

$\tan\alpha_2 + \tan 9 = \frac{100}{73.65} \qquad \therefore \alpha_2 = 50.18°$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

17

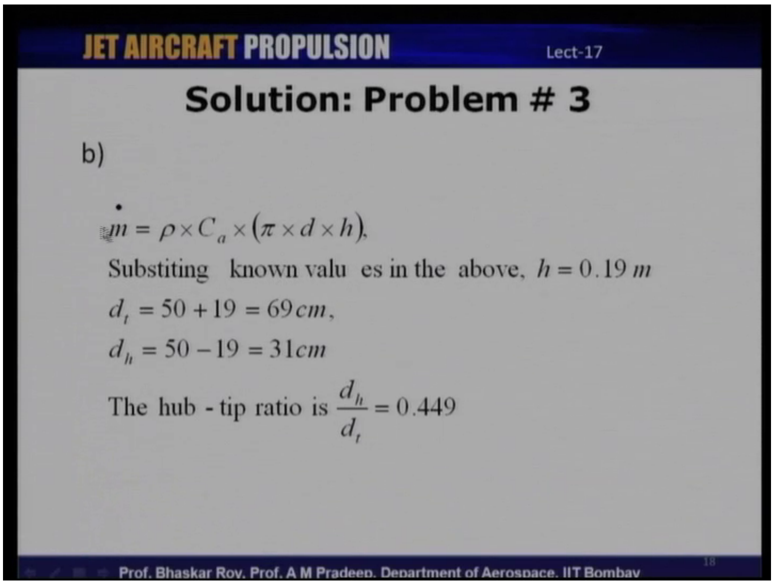
Let us see, how we can solve this problem; so part a is to find out the air angle at the stator entry, which is alpha 2 so from the velocity triangle that we have seen earlier. So, it is a same velocity triangle that we have discussed in problem number 2. So, I have not repeated here. So, we can infer from the velocity triangle that u by C a; that is blade speed divided by the axial velocity is equal to tan alpha 1 plus tan beta 1. In alpha 1 and beta 1 are already specified as 7 and 41 degrees U is given as 100 meters per second and therefore, from this we can find out the axial velocity that is C a. So, C a comes out to be 73.65 meters per second.

So, the same velocity triangle also tells us, that tan alpha 2 plus tan beta 2 is also equal to U by C a tan beta 2 is already specified that is 9 degrees U is known, and C a we have just now calculated, so you substitute for that here we get tan alpha 2 plus tan 9 is equal to 100 by 73.65 therefore, alpha 2 is 50.18 degrees so that is the angle at the stator entry.

(Refer Slide Time: 38:09)

So, we have calculated alpha 2, which is basically the first part of the question to find out the angle at the stator entry. Now, second part of the question is to find out the blade height at the entry, and also the hub to tip diameter ratio.

(Refer Slide Time: 42:17)



JET AIRCRAFT PROPULSION Lect-17

## Solution: Problem # 3

b)

$\dot{m} = \rho \times C_a \times (\pi \times d \times h)$,

Substiting known values in the above, $h = 0.19\,m$

$d_t = 50 + 19 = 69\,cm$,

$d_h = 50 - 19 = 31\,cm$

The hub - tip ratio is $\frac{d_h}{d_t} = 0.449$

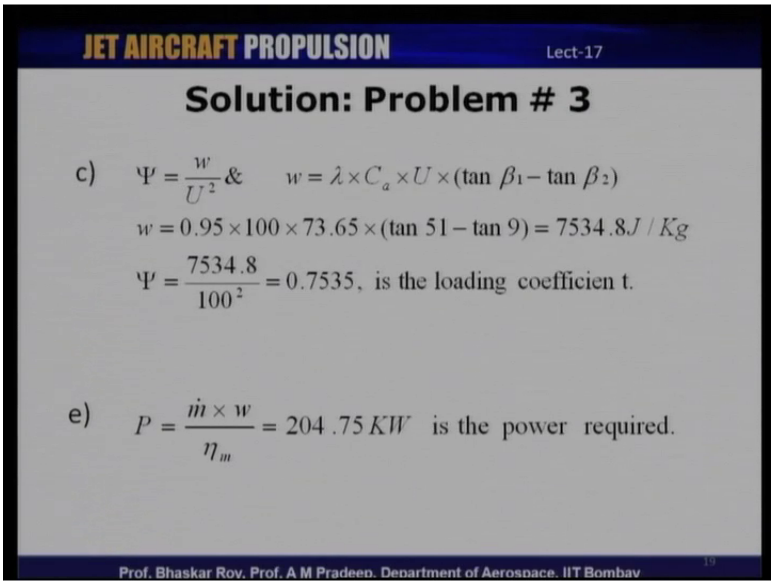
Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 18

Now, to find out the blade height; we have been we already know the mass flow rate; and so mass flow rate is given as a 25 kg's per second. The axial velocity is known the inlet pressure, and temperature are known therefore, we can find out the density mean diameter is known and therefore row times C a into the angular area that is pi into d h is mass flow rate.

So, if you substitute for mass flow rate; the density, axial velocity, and they mean diameter. We can calculate the blade height, which is 0.19 meters; and so we now have the mean diameter and the blade height. And therefore, we can find out the tip diameter, and the hub diameter, and from there that is d t that is the tip diameter comes out to be 69 centimeters. Because mean diameter is given as 50, and the hub diameter is a 31 centimeter, which is again d t minus h.

Therefore the hub to tip diameter ratio is a d h by d t; that is diameter at the hub divided by diameter at the tip that is 0.449 that is basically tells us, what is the angular area from the hub to tip diameter ratio?

(Refer Slide Time: 43:42)



JET AIRCRAFT PROPULSION Lect-17

Solution: Problem # 3

c) $\Psi = \frac{w}{U^2}$ & $w = \lambda \times C_a \times U \times (\tan \beta_1 - \tan \beta_2)$

$w = 0.95 \times 100 \times 73.65 \times (\tan 51 - \tan 9) = 7534.8 J/Kg$

$\Psi = \frac{7534.8}{100^2} = 0.7535$, is the loading coefficien t.

e) $P = \frac{\dot{m} \times w}{\eta_m} = 204.75 KW$ is the power required.

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 19

So, third part of the question is to find out the blade loading coefficient, which is the work done divided by U square; and we can find out work done based on our equation. We had seen earlier that is U times delta C w multiplied by work done factor that is lambda.

So, work done per stage is lambda times C a into U into tan beta 1 minus tan beta 2, which is basically U into delta C w. So, work done factor in this question is given as 0.95 C a; we have calculated as 100 meters per second U is specified as 73.65, and tan beta 1 is tan 51 degrees, and tan beta 2 is tan 9 degrees. And so if you substitute these values we get the work done per stage as 753.8 joules per kilo gram and therefore, the loading coefficient is work done per stage is divided by U square, and that is 753.8 divided by 100 square. So, 0.7535 is the loading coefficient for this particular rotor, which has been specified.

So, loading coefficient is work done divided by U square and therefore, we have 7534.8 divided by hundred square that is 750.7535, and the last part of the question that is part d is power required to drive this compressors. So, power required is mass flow rate into the work done per stage divided by the mechanical efficiency.

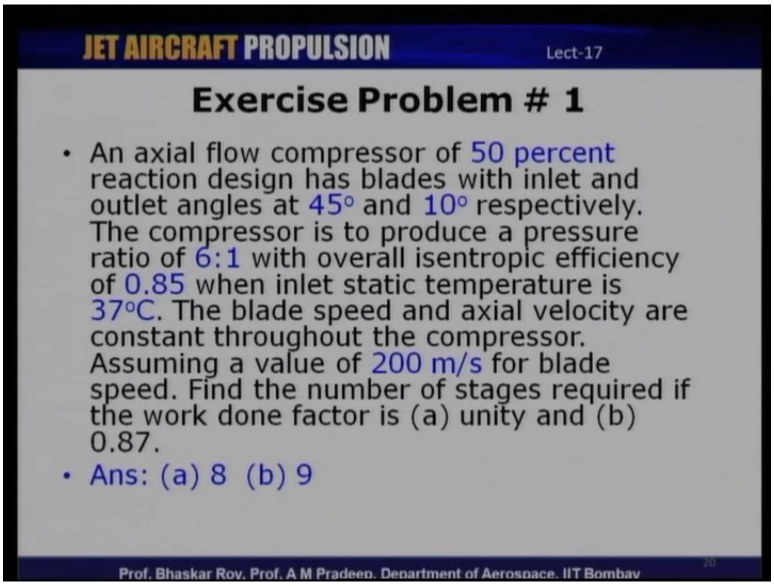
So, we have mass flow rate, which is specified as 25 kg’s per second work done. We have already calculated 7534.8 joules per kilogram, and this divided by mechanical efficiency would give us the power required, and we are dividing by mechanical efficiency. Because mechanical efficiency represents the loss of power, which is occurred while the power was been transferred to the compressor; that means the compressor will require so much power extra to overcome the mechanical losses and therefore, mechanical efficiency indicates losses in transmission from the turbine to the compressor so the, which is why the power required has been divided by mechanical efficiency.

So, this is 204.75 kilo watts, so this is the power required to drive this compressor. So, we have now solved three different problems pertaining to axial compressors and in all the cases we have seen the basic problem solving should begin with the velocity triangle; and once the velocity triangle is specified and we mark the different components, which are known and specified in the problem that makes the problem solving much more easier and chances of making mistakes are less likely.

And, so we have in all the three problems we have seen that given a certain given certain geometric parameters and angles pertaining to a stage of the compressor we can make use of the velocity triangles to solve the velocity triangle as well as to calculate, and determine the other parameters like work done factor work done per stage or power required or the other dimensions like the hub to tip diameter ratio and so on.

So, there are three problems that we have solved in today's lecture. So far, and I have a few exercise problems that problems for you, which you can attempt to solve based on what we have discussed. In today's lecture as well as our discussion during the earlier lectures, so let us take a look at the first exercise problem that I have for you.

(Refer Slide Time: 47:41)



## Exercise Problem # 1

- An axial flow compressor of 50 percent reaction design has blades with inlet and outlet angles at 45° and 10° respectively. The compressor is to produce a pressure ratio of 6:1 with overall isentropic efficiency of 0.85 when inlet static temperature is 37°C. The blade speed and axial velocity are constant throughout the compressor. Assuming a value of 200 m/s for blade speed. Find the number of stages required if the work done factor is (a) unity and (b) 0.87.
- Ans: (a) 8 (b) 9

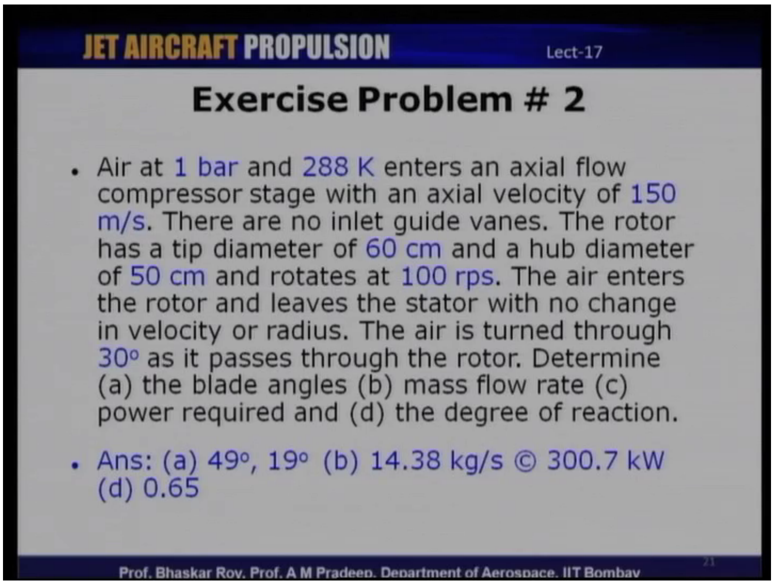
So, the first exercise problem in for today is an axial compressor with 50 percent reaction design has blades with an inlet and outlet angles of forty five degrees and ten degrees respectively.

The compressor is to produce a pressure ratio of 6 is to 1 with an isentropic efficiency of 0.85, when the inlet static temperature is 37 degree celsius the blade speed and the axial velocity are constant throughout the compressor assuming a value of 200 per second for the blade speed find the number of stages required if the work done factor is part a that is unity and part b that is 0.87.

So, in this question we have a rotor, which has a degree of reaction of 50 percent, which means that the velocity triangles are symmetric and mirror images 2 of the angles have been specified. So, all the angles of the velocity triangle are known and you have been given the blade speed and overall pressure ratio and based on this. We need to find out the number of stages required in two cases; that is if the work done factor was unity and second case was if work done factor was 0.87.

So, answer to this question is if it if the work done factor is unity; the number of stages required is 8 and if you take a work done factor of .87; the work done factor comes out to be 9. And so you can see that there is an increase in the number of stages as the as we start assuming values of work done factor. Because that reduces the per stage temperature raise, which means that the number of stages required for generating the pressure ratio which is given as 6 is to 1. in this case would be higher as work done factor comes into play.

(Refer Slide Time: 49:42)



JET AIRCRAFT PROPULSION Lect-17

## Exercise Problem # 2

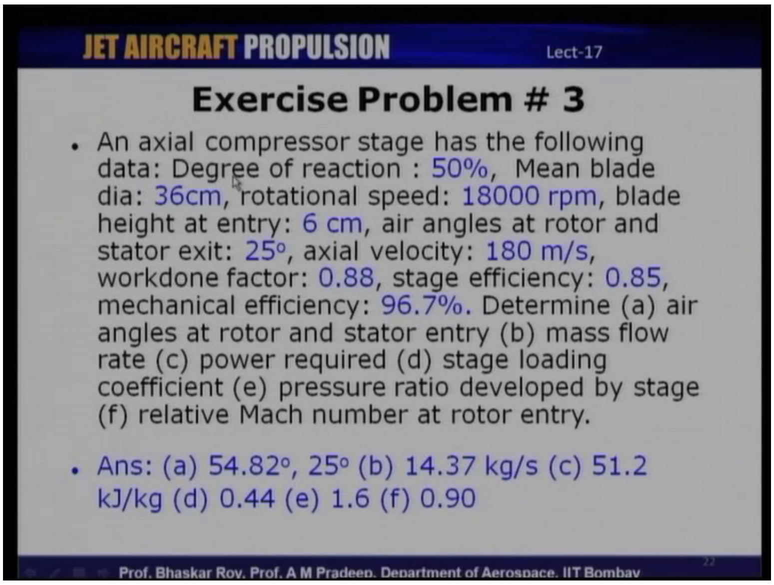
- Air at 1 bar and 288 K enters an axial flow compressor stage with an axial velocity of 150 m/s. There are no inlet guide vanes. The rotor has a tip diameter of 60 cm and a hub diameter of 50 cm and rotates at 100 rps. The air enters the rotor and leaves the stator with no change in velocity or radius. The air is turned through 30° as it passes through the rotor. Determine (a) the blade angles (b) mass flow rate (c) power required and (d) the degree of reaction.
- Ans: (a) 49°, 19° (b) 14.38 kg/s © 300.7 kW (d) 0.65

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

The second exercise problem is stated as follows air at 1 bar and two 88 kelvin enters an axial flow compressor with an axial velocity of 150 meters per second. And there are no inlet guide vanes specified the rotor has a tip diameter of 60 centimeter and hub diameter 50 centimeter and rotate rotates at 100 revolutions per second. The air enters and the rotor and leaves the stator with no change in velocity or radius the air is turned through 30 degrees as it passes through the rotor determine part a the blade angles part b the mass flow rate part c the power required and part d the degree of reaction.

So, this is a problem, which is very similar to the problem that we solved in the first problem that we solved today so in this case the blade angles come out to be 49 degrees and 19 degrees mass flow rate as 14.3 kilo grams per second; part c as 3300; that is power required as 300.7 kilo watts, and degree of reaction as 0.65.

(Refer Slide Time: 50:56)



JET AIRCRAFT PROPULSION Lect-17

## Exercise Problem # 3

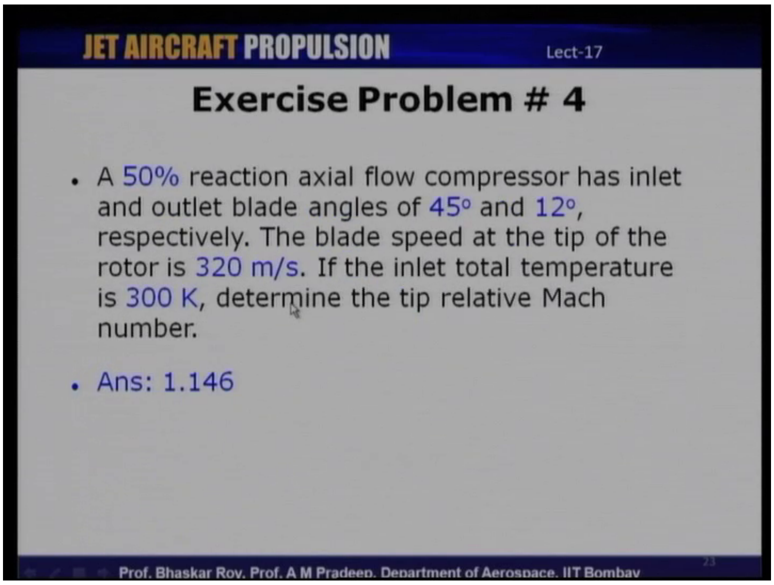
- An axial compressor stage has the following data: Degree of reaction : 50%, Mean blade dia: 36cm, rotational speed: 18000 rpm, blade height at entry: 6 cm, air angles at rotor and stator exit: 25°, axial velocity: 180 m/s, workdone factor: 0.88, stage efficiency: 0.85, mechanical efficiency: 96.7%. Determine (a) air angles at rotor and stator entry (b) mass flow rate (c) power required (d) stage loading coefficient (e) pressure ratio developed by stage (f) relative Mach number at rotor entry.
- Ans: (a) 54.82°, 25° (b) 14.37 kg/s (c) 51.2 kJ/kg (d) 0.44 (e) 1.6 (f) 0.90

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 22

Problem number 3 is axial flow compressor stage has the following data degree of reaction is 50 percent; the mean blade diameter is 36 centimeter rotational speed is 18000 r p m blade height at entry is 6 centimeters air angles at the rotor and stator exit are 25 degrees axial velocity is 180 meters per second work done factor has .88 stage efficiency is 0.85 mechanical efficiency is 96.7 percent determine part a; the air angles at the rotor ,and stator entry; part b mass flow rate, part c power required, part d the stage loading coefficient, part e pressure ratio developed by the stage, and part c part f that is relative mach number at the rotor entry.

So, the answer to this question that part a the angles are 54.82 degrees, and 25 degrees. The mass flow rate is 14.37 kilo grams per second the power required is 51.2 kilo joules per kilo gram; and the stage loading is 0.44 pressure ratio developed is 1.6 and mach number relative mach number at rotor entry is 0.9.

(Refer Slide Time: 52:17)



JET AIRCRAFT PROPULSION Lect-17

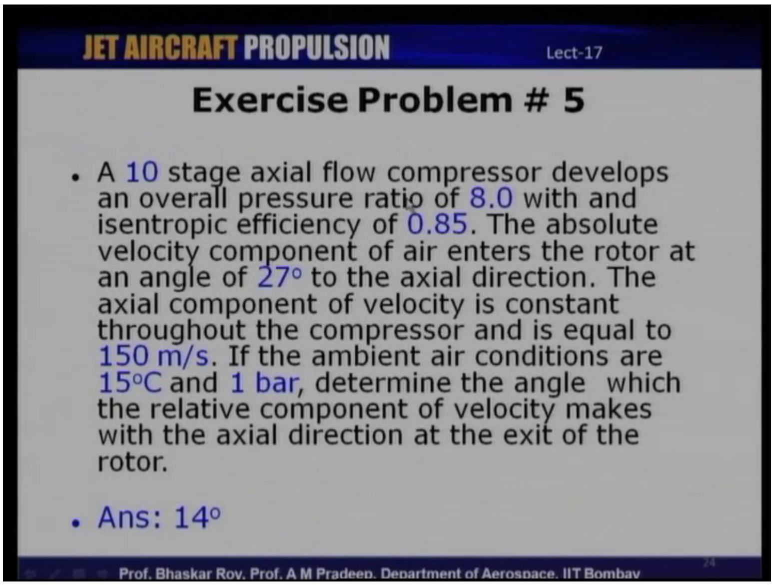
## Exercise Problem # 4

- A 50% reaction axial flow compressor has inlet and outlet blade angles of 45° and 12°, respectively. The blade speed at the tip of the rotor is 320 m/s. If the inlet total temperature is 300 K, determine the tip relative Mach number.
- Ans: 1.146

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 23

Question number four, exercise problem number 5 is a 50 percent reaction axial flow compressor has inlet and outlet blade angles of 45 degrees and 12 degrees respectively. The blade speed at the tip of the rotor is 320 meters per second, if the inlet total temperature is 300 kelvin, determine the tip relative Mach number.

(Refer Slide Time: 52:48)



JET AIRCRAFT PROPULSION Lect-17

## Exercise Problem # 5

- A 10 stage axial flow compressor develops an overall pressure ratio of 8.0 with and isentropic efficiency of 0.85. The absolute velocity component of air enters the rotor at an angle of 27° to the axial direction. The axial component of velocity is constant throughout the compressor and is equal to 150 m/s. If the ambient air conditions are 15°C and 1 bar, determine the angle which the relative component of velocity makes with the axial direction at the exit of the rotor.
- Ans: 14°

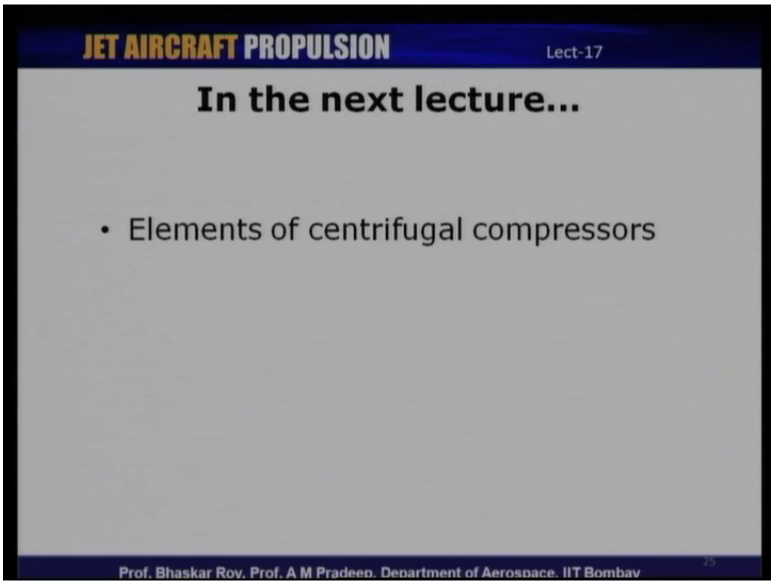
Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay 24

So, in this case the tip relative mach number is 1.146 six and the last problem for you to solve is a 10 stage axial flow compressor develops an overall pressure ratio of 8 with an isentropic efficiency of 0.85. The absolute velocity of component of air enters the rotor at angle of 27 degrees to the axial direction the axial component of velocity is constant throughout the compressor, and is equal to 150 meters per second, if the ambient air conditions are 15 degree Celsius in 1 bar determine the angle, which the relative component of velocity makes with the axial direction at the exit of the rotor.

So, this case the angle comes so that is beta 2 is 14 degrees. So, in today's lecture so there are 5 exercise problems for you to solve and the key to solving all these problem is to get the velocity triangles right, because angles are specified in many of them. And so you should begin with trying to get the velocity triangles for this these problems. And then proceed towards solving these problems. So, I hope you would be able to solve these problems based on, what we have discussed in today's lecture as well as, what we have discussed in the last few lectures on theory, and working of a the axial flow compressors.

So, that brings us to the end of today's lecture, which is, which was basically a tutorial session in the next lecture. We shall begin the next chapter, and we shall be discussing about centrifugal compressors.

(Refer Slide Time: 54:11)



JET AIRCRAFT PROPULSION Lect-17

**In the next lecture...**

- Elements of centrifugal compressors

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

25

So, we will begin with the discussion on elements of centrifugal compressors in the next lectures, and we will take up centrifugal compressors in the next couple of lectures as well; so that brings us to the end of a today's tutorial session.