

**Jet Aircraft Propulsion**  
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**Lecture No. # 16**  
**Free Vortex Theory; Single – Multi-Stage Characteristics**

Hello and welcome to lecture number sixteen of this lecture series on jet aircraft propulsion. We have been talking about axial compressors over the last few lectures, and we are going to continue our discussion on axial flow compressors today. And this would be the last lecture on the theory of axial flow compressors, and subsequently in the next lecture we are going to discuss about we will basically have a tutorial on axial compressors.

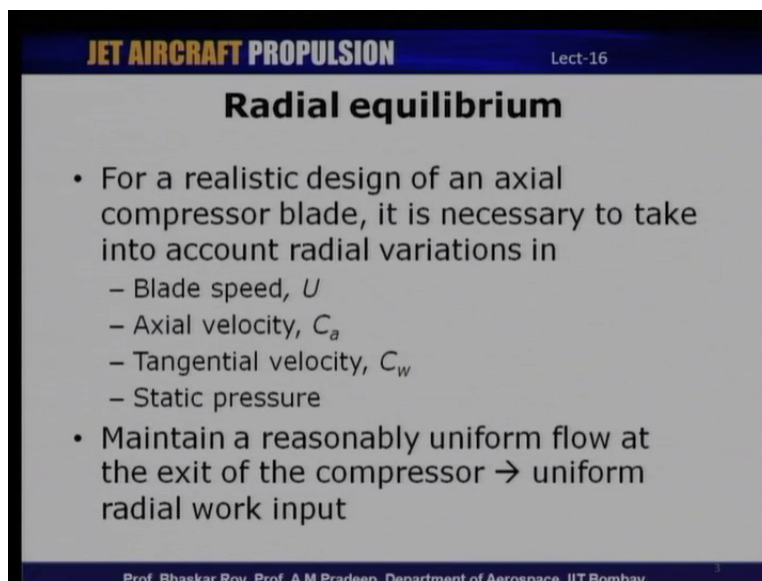
Now, what in today's lecture we are going to cover two very important aspects of axial flow compressors. One of them is to do with a very fundamental form of design of the axial flow compressor blades, and the that is what we will initiate our discussion on and subsequently, we are going to discuss about the characteristics performance characteristics of single and multi stage axial flow compressors. And the performance characteristics is a very important part of axial flow compressor design, and also it plays a very significant role in the performance of the engine as a whole, because the compressor is to be matched with the turbine, and matching of the compressor and turbine is in some way related to the performance characteristics.

And that also reflects the performance of the engine as a whole in the sense that performance characteristics would tell us that what is the band of operation for this particular compressor where in the operation is safe. And what happens, if we exceed these bands of operation obviously, the performance is going to degrade. And in some cases as we shall see today.

The compressor might enter into might enter into certain unstable modes of operation which can effect, which can drastically effect the performance, and working of an engine as a whole. So, in this context it is very important for us to understand the significance of performance characteristics. So, but before that let us discuss about a very fundamental, form of or method of design of the axial compressors in what is known as a free vortex design, and we will see what is the principle behind a free vortex design, and why we need to consider such a design methodology.

Subsequently, we will discuss about single and multi stage compressors, and we will also towards the end of the lecture discuss in brief, about two instability modes of operation of an axial compressor. These are known as rotating stall and surge, so that we will take up towards the end of the lecture. So, we are going to discuss begin the lecture today with discussion on what is known as free vortex design. And in subsequently we will take up the performance characteristics.

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### Radial equilibrium

- For a realistic design of an axial compressor blade, it is necessary to take into account radial variations in
  - Blade speed,  $U$
  - Axial velocity,  $C_a$
  - Tangential velocity,  $C_w$
  - Static pressure
- Maintain a reasonably uniform flow at the exit of the compressor → uniform radial work input

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So, let us discuss about what is meant by free vortex design, but before that let me give you a background about what is meant by a radial equilibrium basically radial equilibrium is one of the conditions that needs to be satisfied in the sense that all the flow parameters in terms of velocity pressure etcetera, need in the radial direction also needs to be considered. In whatever design methodology or the velocity triangle etcetera, which we have discussed over the last few lectures, we have not really considered variations of properties in the radial direction.

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We have assumed that in a particular cross section the blade speed is a constant, the tangential velocity is constant and many other properties remain a constant. But in an actual blade you know that the blade speed is going to vary, all the way from the hub to the tip. And so how do you consider or account for this in a design methodology. So, radial variations of all these properties needs to be considered and factored into when we take up a serious design of an

axial compressor blade. And that is one of the principles that will be made use of in, what we will be discussing about in free vortex design. So, for a realistic design, what we need to do is to consider radial variations of the following one is of course the blade speed, which we know will vary any way, from hub to tip because it is a direct function of the radius. The other parameter that we will need to be considered is the axial velocity. Then the radial variation in tangential velocity and static pressure.

So, if we have to maintain a reasonably uniform flow at the compressor exit. And, why do we need to maintain that we need to maintain a uniform flow at the compressor exit. Because the next stage or hum that is going to follow a particular stage, will depend upon, what is coming into the stage from the previous one. And so, if there is a uniform flow that exits one stage of an axial compressor. That is also beneficial for these succeeding stages.

So, one of the ways of ensuring that we can have a fairly uniform flow exiting the compressor, is to ensure that there is a uniform distribution of specific work input at each of the cross sections that is starting from the hub all the way to the tip. If we can maintain, or at least try to maintain, relatively uniform distribution of specific work in the radial direction, then it possible that we should be able to get a fairly uniform flow at the compressor outlet, which is obviously good thing for the succeeding stages.

Now, in order to ensure that we have uniform specific radial work distribution. Now, we know that the enthalpy difference across a stage  $\Delta h$  or even a compressor  $\Delta h_{not}$ , is equal to enthalpy thrice across a compressor stage. That is equal to the product of the blade speed that is  $U$  multiplied by  $\Delta C W$  right that is something that we had discussed in one of the earlier lectures that the ah

Enthalpy rise, specific enthalpy rise will be equal to the product of blade speed times the difference in the tangential velocity between the inlet and exit. So,  $\Delta C W$  is basically the difference in the tangential velocity. So, if we equate this now we are trying to maintain relatively uniform specific work input. So, we should be able to get some hints from this as to how we can do that.

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### Free vortex design

- Since  $\Delta h_0 = U \Delta C_w = \Omega r \Delta C_\theta$
- This means that for a given rotational speed,  $r \Delta C_\theta$  must be a constant.
- One such configuration that satisfies the above is the Free Vortex Design.
- In this approach, the product  $r C_\theta$  is held a constant across the exit of each blade row.
- From the axial velocity and the blade speeds at various radial sections, the velocity triangle can be completed.

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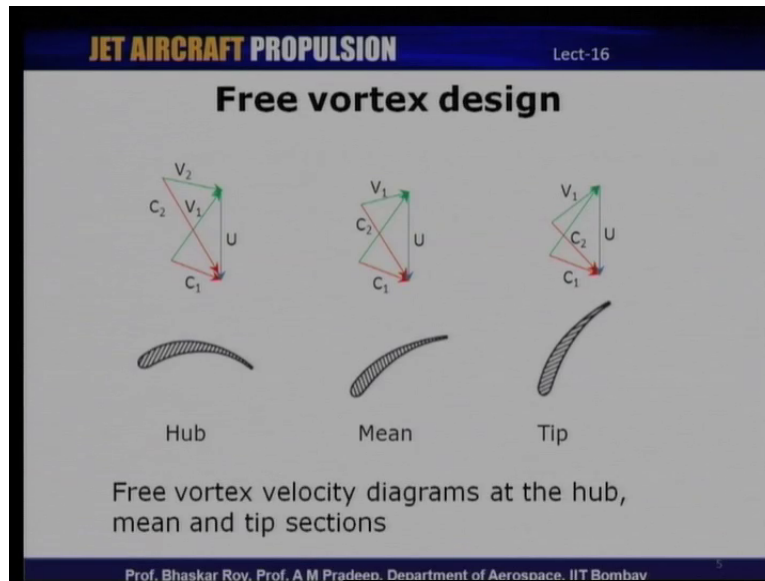
So, we know that  $\Delta h_0$  which is specific enthalpy change across a stage is equal to blade speed times  $\Delta C_w$  or  $C_\theta$ . And, which is equal to  $\Omega r$ , which basically the blade speed is the product of the angular velocity times the radius this multiplied by  $\Delta C_w$  is basically equal to the specific work input.

So, what does mean is that for a given rotational speed, that is if we fix  $\Omega$  then  $r$  times  $\Delta C_w$  must be a constant. So, the product of the radius, and the tangential velocity must be a constant. Now, this can ensure that we can have for a given rotational speed; we can ensure that the specific work is a constant. So one configuration or one design methodology, which can ensure that this is satisfied is known as the free vortex design.

And in a free vortex design, basically we keep the product  $r$  times  $C_w$  it is kept a constant that they exit for each of these blade rows. And so, given the axial velocity and the blade speed at different cross section from the hub to tip we have axial velocity and so on. And so, from hub to tip we know all these parameters; and if we to if we were to ensure that the product  $r$  times  $C_w$  is kept a constant. We can actually solve the velocity triangle; all the way from hub to tip.

So, in vortex design methodology that is one of the basic principles trying to keep the product  $r$  times  $C_w$  a constant, which will ensure that the specific work input, required for this particular blade would be a constant. So, if that is the case we can ensure that  $r$  times  $C_w$  is kept a constant.

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Now, for example let us, take a look at one example where in we can see what has happens, if we were to maintain the velocity triangles if we were to solve the velocity triangles, keeping  $r$  times  $C W$  a constant. If we were to do that; so let me repeat what I was saying so, basically if we have  $r$  times  $\Delta C W$ , which we are trying to maintain a constant for different cross sections all the way from hub to the tip. Then from axial velocity, which is known at the inlet and blade speed we can solve the velocity triangle.

So, let us take one example of, what would happen if we were to take to do this, what is shown here are three cross sections at the hub the mean and the tip. And their corresponding velocity triangles and if we were to use the free vortex method for designing such a blade. This is how the velocity triangles are likely to look like.

This is just one example now let us, take a look at the hub section, where we have the velocity triangle at the inlet. And the exit of the blade; so this particular velocity triangle, which is shown here.  $C_1$  is absolute velocity entering the blade at the leading edge at the hub  $v_1$  is the corresponding relative velocity.  $U$  is the blade speed; and at the exit we have  $C_2$ , which is the **velocity** absolute velocity exiting the blade and  $V_2$ , which is the relative velocity exiting the blade.

And if you try to maintain  $r$  times  $C W$  constant; we can see that the blade cross section well the orientation of the blade itself changes drastically as we move from the hub towards the tip, which means that and we can see velocity triangles at the mean; and also at the tip.

So, what you notice is that if you were to is were to stag these different cross sections from the hub all the way to the tip. You can clearly see that the blade is going to be twisted it is no longer going to be a straight blade as we have been discussing so far, where we can at least the velocity triangles we have discussed in the last few lectures we were seeing that the blades were little bit straight. Now, here with the free vortex design methodology we can see that the blade is no longer going to remain straight.

It is going to have a fairly significant twist. The other significant thing that you can try and notice is that during our discussion on degree of reaction. We have seen that when degree of reaction is close to 0.5 or is equal to 0.5. The velocity triangles, become symmetric. So, that is something that you can probably try to see here at the mean cross section of the blade. We can see that the velocity triangles are more or less symmetric. And this is indication that the degree of reaction here is likely to be close to 0.5, whereas at the hub and the tip. It is quite different.

And, what comes out is that in most of the design a processed that one would encounter. The degree of reaction approaches zero close to the hub; it becomes very low. And towards the tip it becomes much higher than 0.5 it might approach one, and even though we are maintaining a degree of reaction of around 0.5 at the main section.

So, this is just an example of one case of how we can use free vortex design methodology to design a blade cross section, and there of course, the other variants of this methodology, which we shall discuss now. And by using certain modifications to the free vortex design methodology we can of course, arrive at other methods of designing a blade.

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### Free vortex design

- The conditions of radial equilibrium, i.e., the satisfaction of the three-dimensional equations of fluid motion must be considered.
- Possible distributions that satisfy the constant specific radial work criterion are:
  - Free vortex,  $rC_\theta = a$
  - Forced vortex,  $rC_\theta = ar^2$
  - Exponential,  $rC_\theta = ar + b$
  - Constant reaction,  $rC_\theta = ar^2 + b$

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Now, what is important in all these methods is that we need to satisfy the conditions of radial equilibrium. That is the equations of motion the three-dimensional equations of motion needs to be satisfied.

That is one fundamental requirement in all these design methodologies. So based on this besides free vortex design, where in we have  $r$  times  $C W$  is equal to constant there are other methods like forced vortex, which is  $r$  times  $C W$  is equal to a  $r$  square exponential method which could be  $r$  times  $C W$  is equal to a  $r$  plus  $b$ . And constant reaction, which is  $r C W$  is a  $r$  square plus  $b$ , where  $a$  and  $b$  are of course constants, and needs to be fixed (()).

So, these are other variants of the methods, where in we can try to ensure a constant specific radial work. And that is required, because we would like to maintain a fairly uniform set of properties in the exit of a particular rotor blade. So, free vortex design basically is one of the attempts or one of the methods by, which we can try to ensure a constant specific radial work criteria.

So, what we have discussed in brief for now is one of the ways of one of the methods or popular methods of design of an axial compressor blade. And what we shall discuss next would be as I discussed had initially. We should be talking about the different performance characteristics of single and multi stage axial compressors.

So we will begin our discussion, with consideration of a single stage axial flow compressor. We will first discuss about performance characteristics of single stage compressor, followed by this we will be talking about the multi stage characteristics. Now stage of an axial

compressor as you probably know by now compressors comprises of a rotor and a stator. So let us, take up a characteristic rotor and the stator and then we will discuss about, how we can characterize this particular rotor and see what happens as you change different properties.

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**Single stage performance characteristics**

- Let us consider a typical axial compressor stage comprising of a set of rotor blades followed by a set of stator blades.

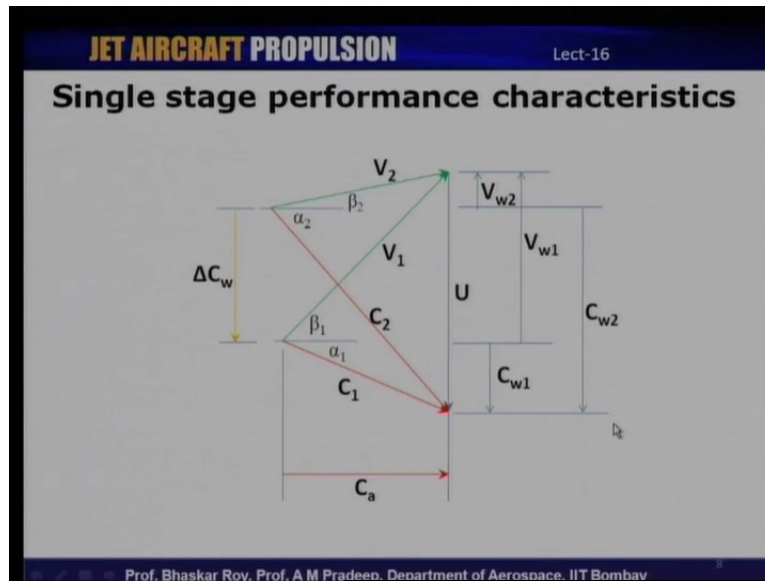
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Now, we have a typical axial compressor stage here, which consists of a set rotor blades, followed by a set of stator blades. We have already seen this velocity triangle (( )) on. At the rotor inlet we have absolute velocity entering at  $C_1$  relative velocity at  $V_1$  blade speed is zero. And at the exit of the rotor we have, absolute velocity is equal to  $C_2$  and relative velocity is  $V_2$ . And blade speed of course, remains the same its  $U$ . And this velocity triangle is, what is basically goes into the stator basically the absolute velocity that enters the stator. And exits at velocity of  $C_3$  living at angle  $\alpha_3$ .

So, here we can see as the as the flow enters the rotor and exits the rotor. There is a deceleration in terms of the relative velocity and that is, what leads to the diffusion, and similarly, as the flow enters the stator. The absolute velocity decelerates from  $C_2$  to  $C_3$  leading to diffusion.



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So, the combined velocity triangle is something, which we have discussed earlier as well. If we were to overlap the two velocity triangles at the inlet and exit of the rotor then we get a combined velocity triangle with all the angles indicated here as well. Let me, just quickly go through these velocity triangles we have, absolute velocity at an angle  $\alpha_1$  at the inlet, of the rotor and relative velocity  $V_1$  at an angle  $\beta_1$  at the inlet, at the exit we have the absolute velocity leaving the rotor at an angle  $\alpha_2$ . And relative velocity leaving the rotor at angle  $\beta_2$ .

And  $C_{w1}$  corresponds to the tangential component of the absolute velocity  $C_{w2}$  corresponds to the tangential component of the absolute velocity at the exit of the rotor. And similarly,  $V_{w1}$  and  $V_{w2}$  are the corresponding relative velocity components.  $\Delta C_w$  is the difference between  $C_{w2}$  and  $C_{w1}$ ; and  $C_a$  is that axial component of the absolute velocity.

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**Single stage performance characteristics**

- From the above velocity triangles,

$$C_{w2} = U - C_a \tan \beta_2 \quad \text{and} \quad C_{w1} = C_a \tan \alpha$$

Since,  $\Delta h_0 = U \Delta C_w$

$$\Delta h_0 = U [U - C_a (\tan \alpha_1 + \tan \beta_2)]$$

or,  $\frac{\Delta C_w}{U} = \frac{\Delta h_0}{U^2} = 1 - \frac{C_a}{U} (\tan \alpha_1 + \tan \beta_2)$

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So, with this in mind from the velocity triangles we can infer that  $C_{w2}$  is equal to  $U$  minus  $C_a \tan \beta_2$ . So, let see where that comes from  $C_{w2}$  is this component this should be equal to  $U$ , which is the blade speed minus the this particular component, which is what is given by  $C_a \tan \beta_2$ . So  $C_a$  is this component;  $C_a \tan \beta_2$  is this component.

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So,  $U$  minus  $C_a \tan \beta_2$  is  $C_{w2}$  similarly,  $C_{w1}$  is  $C_a \tan \alpha$ ,  $C_a C_{w1}$  is this so,  $C_a \tan \alpha_1$  is this component. Now we also know that  $\Delta h_0$  that is the specific enthalpy rise across a stage should be equal to  $U$  times  $\Delta C_w$ . And  $\Delta C_w$  is  $C_{w2}$  minus  $C_{w1}$ .

So, if you go to substitute for  $C_{w2}$  and  $C_{w1}$  here, we get  $\Delta h_0$  is equal to  $U$  multiplied by  $U$  minus  $C_a \tan \alpha_1$  plus  $\tan \beta_2$  or  $\Delta C_w$  by  $U$  is equal to  $\Delta h_0$  divided by  $U^2$ , which is  $1$  minus  $C_a$  divided by  $U$  into  $\tan \alpha_1$  plus  $\tan \beta_2$ . So, what we have here is the specific work ratio or the stage loading, which is expressed in terms of two important parameters. One is the axial velocity; and the blade speed and of course the angles the blade the inlet angle  $\alpha_1$  and the blade outlet angle. So, the specific work ratio or the blade loading has expressed in terms of two distinct parameters.

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**Single stage performance characteristics**

- Change in the design mass flow rate affects  $C_a$ , change in rotor speed affects  $U$ .
- Change of either  $C_a$  or  $U$  changes the inlet angle  $\beta_1$  at which the flow approaches the rotor.
- The above equation shows that the blade performance depends upon the ratio  $C_a/U$ .

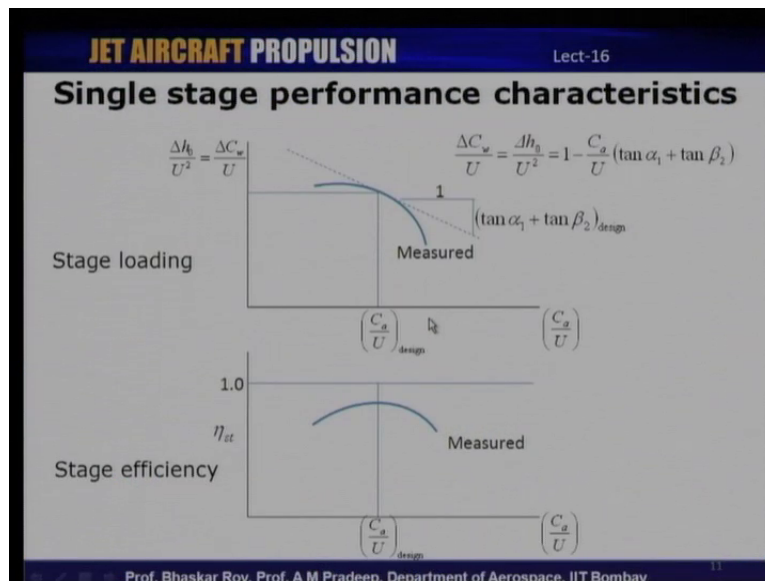
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So, what this means is that if we go to change the design mass flow rate; mass flow rate is directly proportional to the axial velocity  $C_a$ . And so, if we change design mass flow rate or the blade speed either of them is going to effect the loading characteristics. Because  $\Delta C_w$  by  $U$  is equal to  $1 - C_a/U \tan \alpha_1 + \tan \beta_2$ . So the stage loading is directly a function of either the mass flow or the blade speed, besides of course, the angle so if we were to assume that the inlet angle angles are fixed.

This means that the performance of the stage will directly depend upon the mass flow, which is in turn equal to the axial velocity or the blade speed  $U$ . That is this ratio  $C_a/U$  plays a significant role in the stage performance characteristic. So the blade performance or stage performance is a direct function of the ratio  $C_a/U$ . And so, we have already defined  $c_a/U$  if you recall earlier on we have defined  $C_a/U$  as the blade loading in terms of  $C_a/U$  as we have defined in the last lecture.

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So, let us look at what happens as you change  $C_a$  by  $U$ . That is basically the flow coefficient thus defined last slide. So, what we have plotted here are two parameters one is the stage loading. In terms of what we just now derived  $\Delta h_0$  not by  $U$  square, which is  $\Delta C_w$  by  $U$ . And these stage efficiency; so,  $\Delta C_w$  by  $U$  is equal to  $\Delta h_0$  not by  $U$  square that is  $1 - C_a/U \tan \alpha_1 + \tan \beta_2$ . So, if you were to plot this for different values of the flow coefficient  $C_a$  by  $U$ . As we keep changing the flow coefficient what happens to this loading characteristics.

So, what we will see is the one we indicated by this thick blue line refers to the measured loading characteristics as  $C_a$  by  $U$  is changed, which could be either by changing the mass flow or by the speed. So, we observe a characteristic as shown here, which means that as we reduce the mass flow the pressure rise or the stage loading increases up to a certain point. And then subsequently if we continue to reduce the mass flow it will decrease. And this has certain implications, which we will discuss during the multi stage performance characteristic as well. And so we see that at this point that is indicated by  $C_a$  by  $U$  which is corresponding to the design condition. We have this particular characteristic and if we were to look at the corresponding stage efficiency. We get the maximum efficiency around that point. That is corresponding to  $C_a$  by  $U$  design, which is of course less than a hundred percent of there is

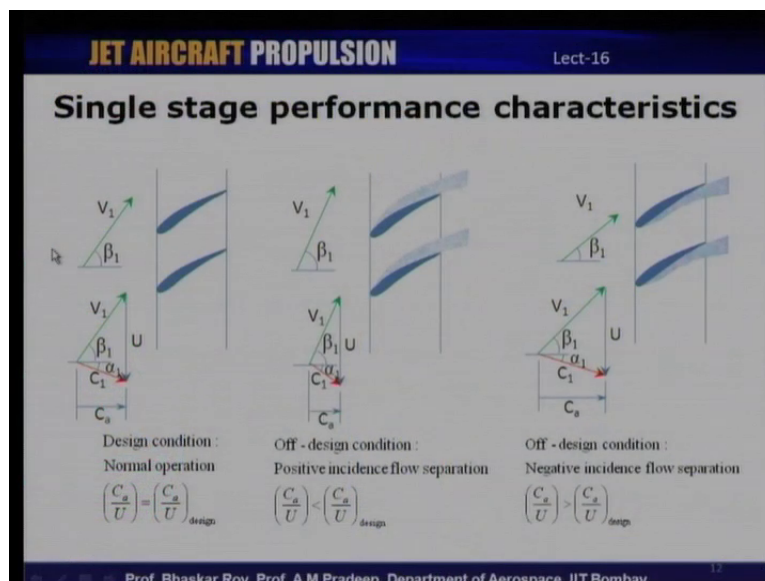
a certain efficiency associated with the stage. And if we were to draw a tangent at this point where  $C_a$  by  $U$  is given by this point if you draw a tangent at that point.

The slope of that is basically given by  $\tan \alpha_1 + \tan \beta_2$  corresponding to the design condition. So, this the slope of this line basically is, what is given here by  $\tan \alpha_1 + \tan \beta_2$  corresponding to the design condition. So, what is shown here is basically, how a particular stage of an axial compressor is going to behave as we change the mass flow or the speed.

So, in terms of either of them we have represented them by the flow coefficient that is  $C_a$  by  $U$ . So as  $c_a$  by  $U$  changes how is it that the stage performance changes in terms of the loading of the stage as well as the efficiency of the stage. So what we see here is that basically the measured performance and, how it performs in terms of the efficiency corresponding to  $C_a$  by  $U$  design. So, what this means is that the as one deviates from  $C_a$  by  $U$  design. The stage is not going to perform as it is supposed to be for the design conditions.

So, what we will see next is for off design conditions, where in  $C_a$  by  $U$  is either greater than  $C_a$  by  $U$  design or  $C_a$  by  $U$  is less than  $C_a$  by  $U$  design. How the performance of the stage would get effected. So, let us take a look at what happens in terms of the velocity triangles as well as what happens on the blade as we change the flow coefficient from its design value.

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So, as  $C_a$  by  $U$  let us take a look at three different cases here; the first case is, when  $C_a$  by  $U$  is actually equal to the design condition. Then we have the stage performing normally that is normal operation under design condition of this particular stage. So the velocity triangle with the relative velocity at entering the stage at an angle  $\beta_1$  and absolute velocity at  $C_1$  entering at  $\alpha_1$ .

Now, let us consider the first off design condition, that is if  $C_a$  by  $U$  is less than  $C_a$  by  $U$  design. And if you assume that this is only by reducing the axial velocity or changing the mass flow alone keeping the blade speed same. That means if we fix  $U$  and just change the mass flow it means that for a lower mass flow, how the stage is going to perform. So, as  $C_a$  is less than  $C_a$  by  $C_a$  design. This means the axial velocity is now less than the design axial velocity. As we reduce the axial velocity what, happens is we see that the velocity triangle getting altered.

So, this was the original velocity triangle; and as we reduce the axial velocity the velocity triangle has got altered. Because now,  $C_a$  by  $U$  is now less than  $C_a$  design, which means that the relative velocity will now enter the rotor at angle, which now greater than what it was at the design condition. So, here  $\beta_1$  is actually greater than the  $\beta_1$  at the design.

So, therefore, it means that the flow approaches the stage at a very high angle of incidence, which is basically a positive incidence here. There is a chance that the flow would separate from the suction surface of this rotor.

So, there is flow separation occurring here on the suction surface of this blade. Therefore, this is basically a positive incidence flow separation. The other extreme of this is  $C_a$  by  $U$  is greater than  $C_a$  by  $U$  design, which means  $C_a$  is greater than  $C_a$  design. If  $U$  were to be fixed the blade  $(C)$  is fixed then there is a possibility that the incidence is now negative. And there could be flow separation from the pressure surface and that basically means a negative incidence flow separation flow might separate from the pressure surface of the blade.

So, these are two extreme cases of operation of the stage when the flow coefficient is different from the design flow coefficient. It is either less than design coefficient or greater than if it is less than if the blade fixed then  $C_a$  is less than the design. Axial velocity and the flow separates from the suction surface might separate from the suction surface leading to positive incidence separation. And if  $C_a$  is greater than  $C_a$  of design then it could lead to negative incidence separation.

So, having understood some of the aspects of a single stage performance characteristics of an axial compressor. Let's now move on to multi stage axial compressor. That is if there multiple number of these stages one after another how does the performance, how can we evaluate the performance of such an axial compressor. Now, to do that we need to understand, what are the different parameters or variants or variables based on which we can evaluate this performance.

So, at the outlet of the compressor we are interested in two things one is the pressure ratio; stagnation pressure ratio across the compressor. And the other parameter is the efficiency of this compressor. So, these are two parameters, which we will be interested in. And so, we will need to see what are the other variables on which these two parameters depend that is efficiency and pressure ratio of the compressor expressed in terms of other variables. So, that we can then go ahead and then look at the performance characteristics.

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**Multi-stage performance characteristics**

- Let us know consider a multi-stage compressor. Inlet station is denoted by 1 and exit of the compressor by 2.
- Therefore the overall pressure ratio of the compressor is  $P_{02}/P_{01}$ .
- The compressor outlet pressure,  $P_{02}$ , and the isentropic efficiency,  $\eta_C$ , depend upon several physical variables

$$P_{02}, \eta_C = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

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So, for a multi stage compressor now we are going to denote the inlet of the multi stage compressor by station one. And exit by of the compressor by two that means the overall pressure ratio of the compressor is  $P_{02}$  by  $P_{01}$ . So, the compressor outlet pressure and efficiency isentropic efficiency is a function of several properties several variables. So it could be mass flow rate, the inlet stagnation pressure, inlet temperature, the rotational speed, the ratio of specific heats gas, constant for the working fluid, viscosity of the working fluid,

the design of the blades themselves and the diameter  $d$ . So, these are the different parameters on, which the outlet pressure outlet stagnation pressure and outlet efficiency depend upon.

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**Multi-stage performance characteristics**

$$P_{02}, \eta_c = f(\dot{m}, P_{01}, T_{01}, \Omega, \gamma, R, \nu, \text{design}, D)$$

In terms of non - dimensionless parameters,

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\gamma RT_{01}}}{P_{01}D^2}, \frac{\Omega D}{\sqrt{\gamma RT_{01}}}, \frac{\Omega D^2}{\nu}, \gamma, \text{design}\right)$$

For a given design, we can assume that  $\gamma$  and  $\nu$  do not affect the performance significantly. Also,  $D$  and  $R$  are fixed. Therefore the above reduces to

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$$

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So, if you were to express this in terms of non dimensional parameters, so if we carry out dimensional analysis of these terms that we have listed here. Then non dimensional parameters come out to be on the left hand side we have the stagnation pressure ratio of the compressor,  $P_{02}$  by  $P_{01}$  and the efficiency.

This is a function of mass flow rate multiplied by square root of  $\gamma R T_{01}$  divided by  $P_{01} D^2$ . The other parameter is  $\Omega D$  by square root of  $\gamma R T_{01}$ ; then we have the  $\Omega D^2$  by viscosity  $\mu$ . Then, the ratio of specific heats and the design. Now, for a particular design suppose we have frozen a particular design and we can assume that  $\gamma$ , which is of this particular fluid and the viscosity do not really affect the performance significantly or do not change much and for a given diameter of the engine.

Because, the diameter has been frozen the diameter has also been frozen; and the gas constant fixed. So this particular set of non-dimensional parameters will now reduce to  $P_{02}$  by  $P_{01}$ . And the efficiency as a function of  $\dot{m} \sqrt{T_{01}}$  by  $P_{01}$ ; and  $n$  times  $T_{01}$  square root of  $T_{01}$ , where  $n$  is the rotational speed.

So, the pressure ratio and the efficiency of the compressor depends up on basically two parameters here. One is corresponding to mass flow rate and the other corresponding to the



rotational speed. And so, let us further reduce it in terms of standard conditions, which we will see that is known as corrected mass flow and the corrected speed. So, pressure ratio and efficiency as a function of mass flow rate times the root of inlet stagnation temperature divided by  $P_{01}$ , and the speed that is  $n$  times square root of  $T_{01}$ .

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**Multi-stage performance characteristics**

Usually, this is further processed in terms of the standard day pressure and temperature.

$$\frac{P_{02}}{P_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\theta}}{\delta}, \frac{N}{\sqrt{\theta}}\right)$$

Where,  $\theta = \frac{T_{01}}{(T_{01})_{\text{Std. day}}}$  and  $\delta = \frac{P_{01}}{(P_{01})_{\text{Std. day}}}$

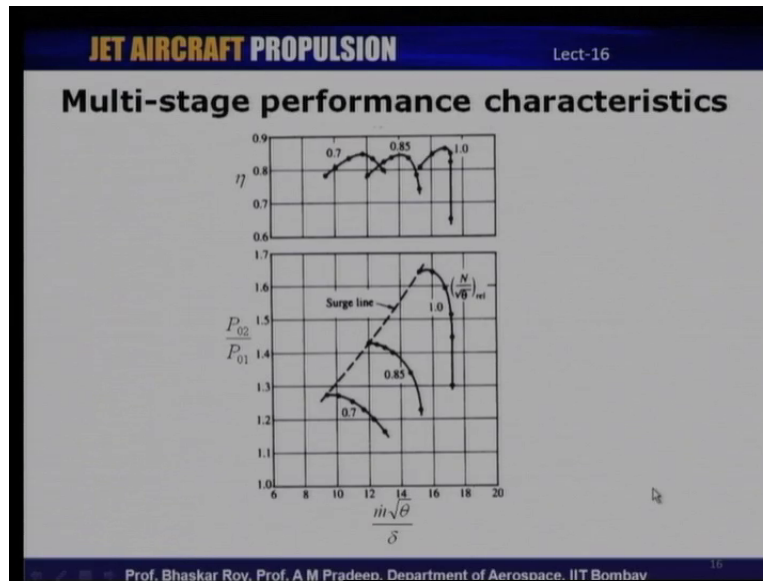
$(T_{01})_{\text{Std. day}} = 288.15 \text{ K}$  and  $(P_{01})_{\text{Std. day}} = 101.325 \text{ kPa}$

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So, we will further express them in terms of standard conditions. That is standard ambient pressure and temperature; so, that this performance characteristics can be used in any other ambient conditions. So, that once it is standardized and corrected it can be used under other conditions as well. So, if you process this further in terms of standard day pressure and temperature. When we have  $P_{02}$  by  $P_{01}$ , and the efficiency are functions of  $\dot{m}$  square root of  $\theta$  divided by  $\delta$  and  $N$  divided by square root of  $\theta$ .

Here  $\theta$  is equal to  $T_{01}$  divided by  $T_{01}$  standard day. And  $\delta$  is equal to  $P_{01}$  divided by  $P_{01}$  standard day, where  $T_{01}$  standard day is 288.15 kelvin. And  $P_{01}$  standard day is 101.32 kilo pascal.

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So, if you go to express these performance parameters in terms of the characteristic of a compressor, multi stage compressor. So, what I have shown here is one typical performance characteristics of a particular axial compressor.

We have on the Y axis the pressure ratio  $P_{02}$  by  $P_{01}$  and expressed in terms of the X axis that is mass flow rate  $m \cdot \sqrt{\theta}$  divided by  $\delta$ . We also have the efficiency; as a function of the mass flow rate  $m \cdot \sqrt{\theta}$  by  $\delta$ . So let us, look at the pressure ratio characteristics first. So, typical compressor multi stage axial compressor characteristic would comprise of pressure ratio versus mass flow at different speeds. So, these lines that are shown here are for different speed ratios that is  $N$  by square root of  $\theta$ .

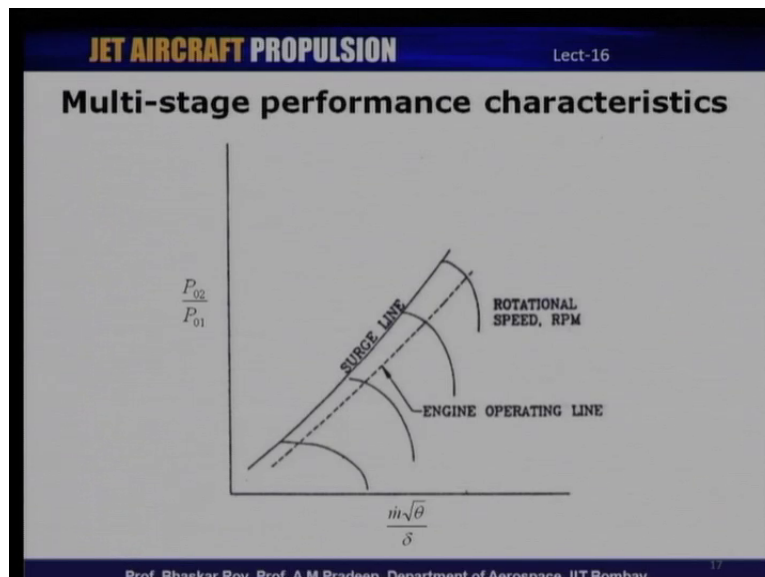
That is for different speeds of rotation how the characteristic change. Let us, take up one particular speed let us say .7. So, what does this line mean is that as we change the mass flow rate? As the mass flow reduces the pressure ratio increases. And it reaches a particular peak beyond, which the pressure ratio, which is not show here. It will droop or it will fall drastically. The reason by I not showing ah the points on the left hand side of the line, which is indicated as the surge line is because after this point the compressor operation is unstable. Because of what is known as surge, which I will discuss shortly what is meant by surge.

So, there is an instability in the compressor performance, which prohibits the compressor operation on the left hand side of this line. So, compressor operation is possible; stable compressor operation is possible; only on towards the right hand side of the compressor of

this particular line. So the surge line dictates one of the limits of the compressor operation. And so, what performance characteristic here shows how the performance in terms of pressure ratio varies as we change the mass flow.

We also have the efficiency characteristics shown here how the efficiency changes as we change the mass flow. So, we have peak efficiency corresponding to the mass flow, which is shown here, so far a particular mass flow at 0.85 speeds. We have the peak efficiency of occurring at a mass flow somewhere around here. And that is true for other mass flows as well.

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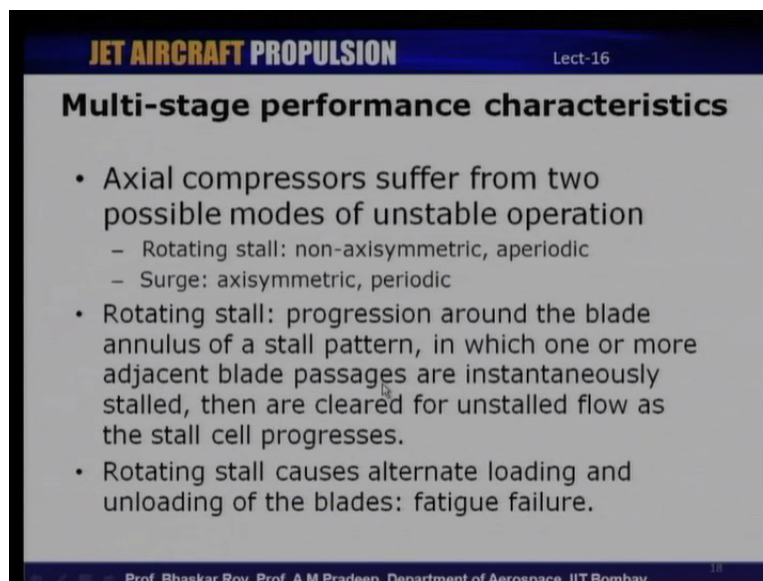
So, mass flow versus efficiency, mass flow versus total pressure ratio both of these put together. Define the performance of a multi stage axial flow compressor. Now, let me take a closer look at the pressure ratio versus the mass flow characteristic. So, you might have noticed here that the peak efficiency is occurring at a point, which is slightly away from the surge line. And at the same time I mentioned that operation on the left hand side of the surge line is not possible, because of the presence of instabilities.

So, we would like the compressor to be operating at point at a safe operating point, which is away from the surge line. So, what is shown here is in terms of this dotted line corresponds to

the actual operating line of an engine, which means we are really not operating very (( )) we would not want to operate very close to the surge line. Because there is always a risk that the compressor might go into surge. If there is a certain fluctuation in the mass flow rate.

So, the engine the compressor is usually operated or designed for operation along an operating line, which is away from the surge line. And the difference between these two points is basically referred to as surge margin. So, there is always a certain margin provided between the engine operation and the surge line, which indicates or denotes that there is a certain margin provided for a safe operation of the compressor. So, surge lines is one of the extremes of operations of the compressor. That the engine actually (( )) slightly away from the surge line, which is known as the engine operation line. So, the multi stage performance characteristic as defined here is in terms of the efficiency as well as the pressure ratio. So, in some books and literature you might also find that the efficiency plots are shown in the same pressure ratio plot as well so you will also find contours of constant efficiency plotted on the same graph.

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### Multi-stage performance characteristics

- Axial compressors suffer from two possible modes of unstable operation
  - Rotating stall: non-axisymmetric, aperiodic
  - Surge: axisymmetric, periodic
- Rotating stall: progression around the blade annulus of a stall pattern, in which one or more adjacent blade passages are instantaneously stalled, then are cleared for unstalled flow as the stall cell progresses.
- Rotating stall causes alternate loading and unloading of the blades: fatigue failure.

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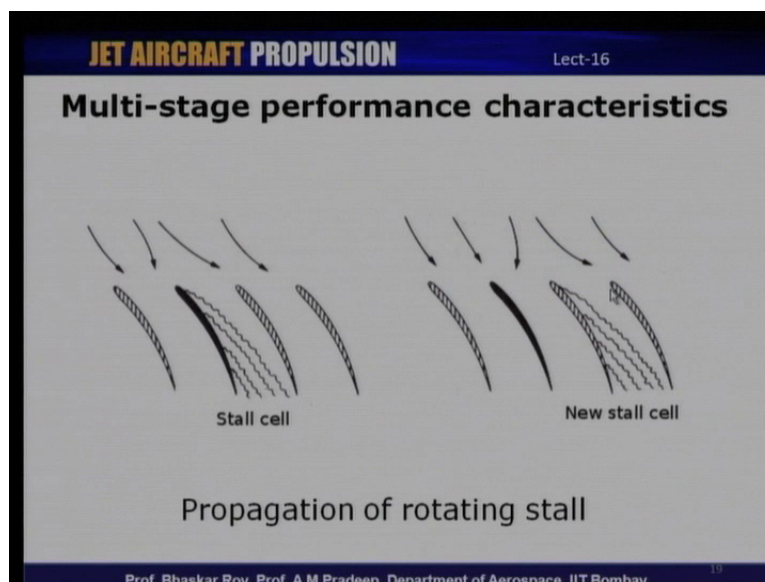
Now, I mentioned that the axial compressor performance is limited by surge on one of the sides. And that there is one of the instability modes of operation of the compressor. And so before I discuss surge let me talk about yet another mode of instability called, which is known as the rotating stall.

So, axial compressor performance is hindered by two instability modes there of course other modes as well. But primarily two modes of instability one is known as the rotating stall, and the other is known as surge.

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Now rotating stall is a non axis symmetric mode of instability. And on the other hand surge is the axis symmetric mode of instability and it is periodic, where as rotating stalled is not necessarily periodic it is a periodic in nature. But both of these modes of operation eventually can hamper the performance of a compressor drastically. So rotating stalled basically involves progression around the blade unless of a stall pattern. That is the stall pattern progressively moves around the annulus of the axial compressor, where in one or more adjacent blade passages are instantaneously stalled. And then the stall cell continuously moves or progresses around the annulus of the compressor. So, this means that the rotating stall can lead to alternate loading, and unloading of the of the blade, which means that there is a possibility of fatigue failure eventually if rotating stall mode was to be present continuously.

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So, let us see what actually happens during rotating stall. Now, during rotating stall let us say there is a certain pocket of non uniformity in the incoming flow. And this pocket of non uniformity may cause a certain one either one blade or a set of adjacent blades to operate

under stall condition. So here we have a non uniform flow; that means ah there is a flow entering the compressor at a very high incidence than the design condition.

So, high non uniformity entering into this particular set of blades causes this blade let us say to get to be stalling, which means that as this blade stalls. It also causes the flow entering the second the adjacent blade to enter at a higher incidence, because the blades are actually rotating. So, there is also a relative motion here the blades rotate.

And, because of this the non uniformity now progresses, and move towards the adjacent blade. Causing the adjacent blade, also to now stall you has this blade stalls it unstalls this particular, which is earlier under stall. Because the flow is now deflected in this fashion here. And also because of the rotation of the blades themselves.

So, this stall blade, which was undergoing a stall. And this stall cell as you see here moves towards the adjacent blades and the new stall is now on this particular blade. And eventually it unloads the previous blade, which was undergoing stall. So, this continuously happens and this stall cell moves progressively moves from one end to another around the annulus.


And, what you can immediately see here is that the direction of the rotation of the stall cell is opposite to that of the rotation of the blades themselves. So rotating stalls cells ah rotate in a direction, which is opposite to that of the rotor rotation. And you can see the rotor rotation obviously for this compressor is in the direction from right to left. Whereas, the stall cell moves from left to right. So similarly on a annulus you can imagine that the rotating stall cells will actually move in a direction opposite to that of the rotor rotation.

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### Multi-stage performance characteristics

- Rotating stall often precedes surge.
- The stall patterns move in a direction opposite to that of the rotor revolution.
- The stall frequency can be as high as 50% of the rotor frequency.
- Rotating stall may be initiated by an incoming flow non-uniformity.



The diagrams show three stages of stall progression in a compressor annulus, represented as a ring between an inner hub and an outer casing. 1. Part span stall: Shaded regions are located at the top and bottom of the annulus, representing stalls affecting only a portion of the span. 2. Full span stall: The shaded regions have expanded to cover the entire width of the annulus from the hub to the casing. 3. Surge: The entire annulus is filled with diagonal hatching, indicating a complete flow reversal or surge.

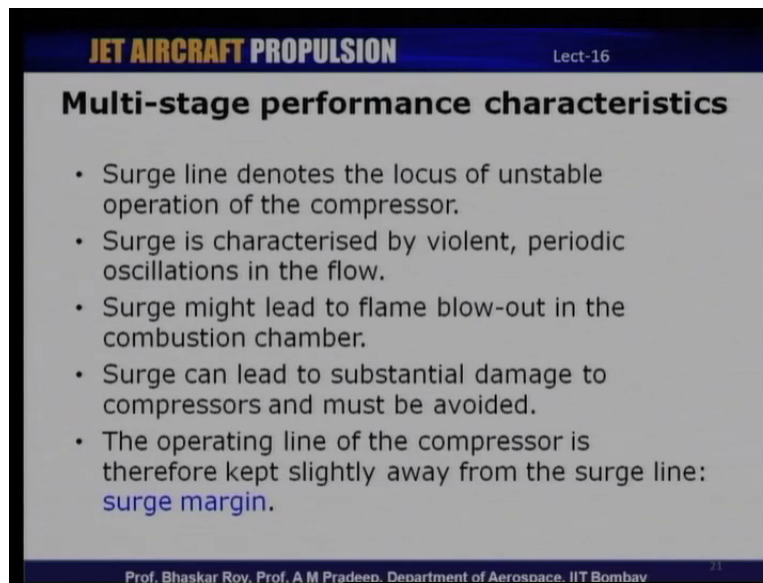
Part span stall      Full span stall      Surge

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So the rotating stall is an instability, which often precedes surge. And we will see that a little later; and the stall patterns will obviously move in a direction, which is opposite to that of rotor revolution. And the frequency of the rotation of a stall cell is of course, not fixed it can keep changing, but it can be as high as about fifty percent of rotor frequency.

And rotating stall may usually be initiated by the presence of a non uniform flow (( )) rotor blades. So, what I have shown here are three different modes of operation of the stall. One is that rotating stall may be affecting only a part of span. This is the annulus of the compressor; this is the hub, and this is the casing. So, rotating stall may be affect part of the span or it may be a full span stall. That it affects all the way from the hub to the tip. And eventually it may lead to surge, where in the entire annulus undergoes a stall. So, this means that the rotating stall is one of the precursors is likely to be one of the precursors of the surge. That is one of the rotating stall were to continue and progress leading to full annulus stall. It may eventually lead to surge.

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**JET AIRCRAFT PROPULSION** Lect-16

### Multi-stage performance characteristics

- Surge line denotes the locus of unstable operation of the compressor.
- Surge is characterised by violent, periodic oscillations in the flow.
- Surge might lead to flame blow-out in the combustion chamber.
- Surge can lead to substantial damage to compressors and must be avoided.
- The operating line of the compressor is therefore kept slightly away from the surge line: surge margin.

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So, which brings us to what is meant by what is actually meant by surge, we have already seen surge line is seen in compressor characteristic, which was on the left hand side. On one of the limits of compressor operation and I mentioned that you cannot really operate the compressor beyond the surge line. Because of the occurrence of surge itself. So, let us understand what we mean by surge. Now, surge is surge line is obtained as by joining all the points on the speed lines of compressor, where surge is likely to occur. So, surge line is the locus of all the points of a multi stage axial compressor.

Surge basically, involves fluctuation of fluid back and forth along the annulus. Axial fluctuation of the fluid of the working fluid. And the entire annulus of the compressor is affected by surge, which is why I mentioned that surge is axis symmetric. That is the entire annulus of the compressor is affected by a separating flow. And it is violent oscillation of the fluid back and forth. So, the onset of surge can lead to very drastic effects on the compressor, and as the engine as a whole. Because compressor is feeding in to the compressor chain bar (( )) and the turbine and so on, which means that if there is something drastic happening compressor. It will immediately affect the combustion chamber and therefore, the turbine which means the whole engine gets affected as a result of surge.



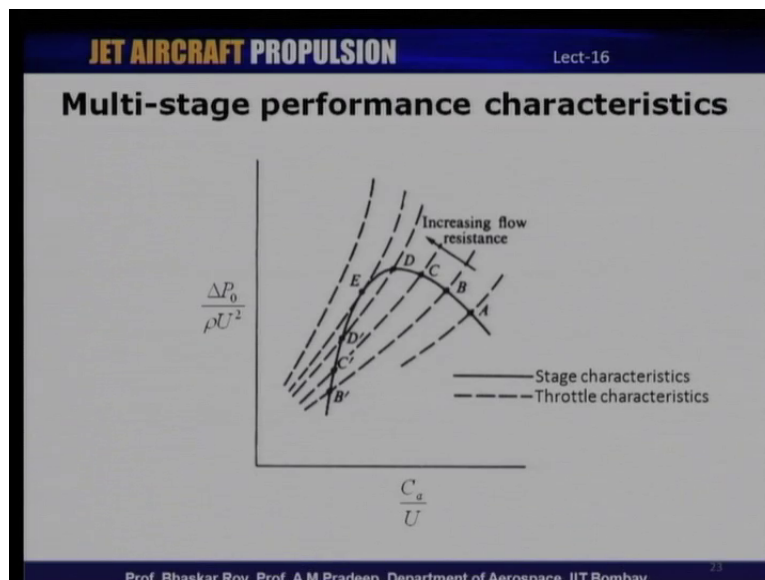
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So surge is characterized by violent and periodic oscillations in the flow. It might lead to flame blow out in the combustion chamber. Because if there is a back and forth movement of fluid in the combustion in the compressor.

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It can lead to a flame out a flame blow out in the combustion chamber. And obviously it can lead to substantial damage of compressors, which is why surge is something that always needs to be avoided. Designers always try to keep a safe margin between the surge line and the operating line, which is basically known as the surge margin.

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I mentioned that there is a certain margin provided between the surge line and the engine operating line. And this is basically referring to this surge margin. And so designers would always like to keep a certain margin there. So now, let us try to understand what is actually meant by a surge; and let me take up the performance characteristics once again we have the mass flow in terms of the flow coefficient I have taken at only one stage of the compressor here.

We have flow coefficient and the pressure coefficient, what happens to the stage performance as we keep changing the mass flow. Now, let us say we have the stage characteristics, which have been plotted here and in solid line. And the throttle characteristics

in terms of these dashed lines, which means that as we throttle the compressor as we reduce the mass flow.

The throttle characteristics keeps changing. And this particular stage let us say operating at point a. As we throttle the compressor will continuously move towards the left. And it keeps moving along these dotted lines. A to B and to C D and so on, but beyond a certain point what we see what are indicated by the points here with dash. That is D dash C dash and B dash are those points, which corresponds to unstable operation of the compressor

And why is it exactly unstable, because what happens here is that as we reduce mass flow in on the right hand side of the particular line. Let us say, after on the right hand side of E. As we reduce the mass flow it is accompanied by an increase in the pressure ratio. Whereas, on the left hand side we have as we reduce the mass flow rate. It also result in reduction in pressure ratio which means as the pressure ratio reduces the pressure downstream is lower than the pressure in the upstream.

And so, that can lead to sudden fluctuation in the mass flow rate. And that is, why operation on the left hand side of what is given here as the throttle characteristic can lead to an unstable operation of the compressor. So, what happens really here is that beyond this point, which is given here as the tangent of the stage characteristic any point on the left hand side of this, where this stage characteristic will have a positive slope. We mean that reduction in mass flow will be accompanied by ah reverse flow, which can fluctuate back and forth. Because momentarily would have a lower pressure of upstream than the downstream, which means that there would be a reverse flow.

And as the flow moves from the downstream stages to the upstream stages. There is a decrease in pressure downstream as compared to upstream and then this continues in a cycle leading to rapid fluctuations in the flow back and forth. So, this is what leads to what is meant by surge, where in there is violent oscillation of mass flow back and forth. And something that can be explained using the stage characteristic and the throttle characteristic.

So, as we change the throttle characteristic from extreme left hand side right hand side as shown here. As we keep throttling, which means it indicates some increase in flow resistance. We obtain different performance characteristic as denoted here by these points. But all these points as you can see have a negative slope. Whereas, as the stage characteristic reaches a point, where in the slope is positive.

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That indicates that these are the points where basically the operation of the compressor is unstable. And ah this is what is reflected in the multi stage characteristic as well. So, what I had shown was one particular line here. And beyond this as you can see the slope will have will be basically positive. Whereas, on this side will be negative. So, operation of the compressor is safe on all these lines and all these points, where the slope is positive.

And the locus of all these points together constitute what is known as the surge line. So, operation of the compressor is unstable on the left hand side of the surge line. Whereas, it is safe to operate the compressor on the right hand side ah of this surge line. So, surge is basically the one of the modes of instability of an axial compressor, which is something that the designer would always want to eliminate and avoid.

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And rotating stall is one of the possible one of possibly one of the precursors of surge. And that is something that would eventually lead to the occurrence of surge. So, let me wind up today's lecture, where in we were discussing the about two important aspects. One was the stage characteristics, the single stage characteristic as well as the multi stage performance characteristics of the axial compressor. And the other aspect that we were discussing was the free vortex theory we started the lecture today with some preliminary discussion about the free vortex theory.

And how it can be used for in preliminary design of the axial compressors, subsequently, we started discussing about single stage characteristic. And how we can ah how we can basically evaluate the performance of a single stage by relating the flow coefficient to the pressure rise or the stage loading.

And subsequently, we discussed about multi stage performance characteristic in brief. And then we also discussed about two modes of instabilities. One is known as the rotating stall, where in the stall cell progressively moves around the annulus. We then saw that rotating stall can eventually or if allowed to continue may lead to occurrence of surge as well, where in the entire annulus of the compressor undergoes flow separation or flow reversal.

And there could be rapid oscillations or fluctuations of the flow, which could also lead to extensive damage to the engine as a whole. So, single stage multi stage performance characteristic the significance of that and also the free vortex theory. These were some of the topics we had discussed in today's lecture. And what we will take up for tomorrow's well for the next lecture would be basically a tutorial.

On axial compressors and we will wind up our discussion on axial compressor with the next lecture, which will basically be a tutorial. We will try to solve some problems pertaining to axial compressors. And also we will have a few exercise problems, which we can solve based on the discussion in the lecture. So, let us take up these topics for discussion in the next lecture.