Jet Aircraft Propulsion

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Lecture No. #14

Axial Compressors: Two Dimensional Analytical Model

Hello and welcome to lecture number fourteen of this lecture series on jet propulsion. In the last several lectures, you have been exposed to some of the basic aspects the thermodynamic aspects of the jet engines. We have analyzed the ideal cycles, the component performance parameters like the intake, and the compressors turbines combustion chambers, and nozzle and so on. And how we can incorporate these component performance parameters into a real cycle analysis?

So, during the component performance analysis, we just assigned some performance parameters like isentropic efficiency or pressure laws associated with various components, without really going into the working the basic aspects of working of these components. So, what we will begin to do in the next several lectures is to take up these components and understand the working of these components in greater detail. You already understood the thermo dynamics behind or thermo dynamic operation of compressors, and turbines.

So in today's lecture, we will begin to understand we will we will start our understanding of the working of axial flow compressors in little more detail. So, what we will be talking about in today's lecture will be on axial compressors; let us take a look at those topics that we shall be discussing in today's lecture.

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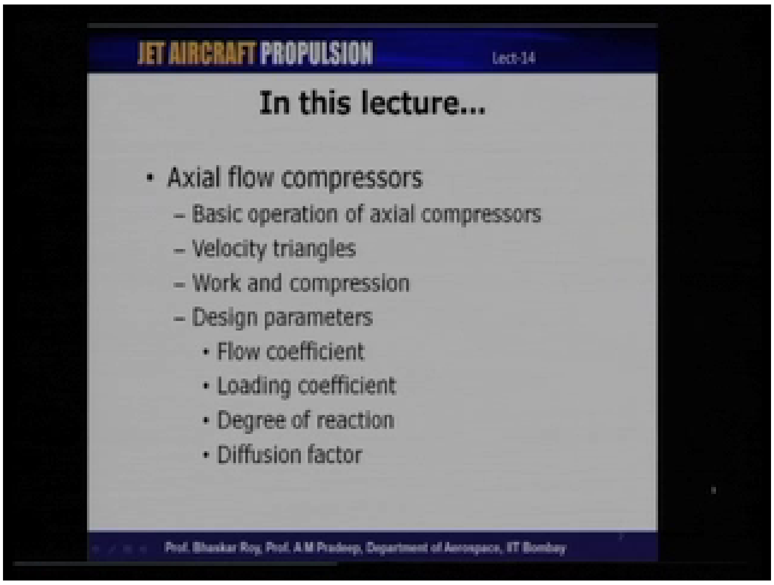

So, we will start off our discussion today with some brief description of the basic operation of axial compressors you have already been exposed to some of these aspects in some of the earlier lectures; then we will also understand about the construction of velocity triangles of axial compressors. We will derive some expressions for the work, and the pressure ration of axial compressors and how they are related.

And then we will also discuss about some of the design parameters which are involved in design of axial compressors. We will only list a few of them this some of them which are important in the design of axial compressors these are to do with flow coefficient the loading coefficient degree of reaction and diffusion factor. So, we will have some very quick understanding, and discussion about some of these different terminologies, which are used in design of axial compressors. So, we will begin with our discussion the basic operation of axial compressors.

Now, axial compressors as your already your familiar with their basic function is to increase the pressure of the incoming flow which is coming from the intake, and then deliver a high pressure, and also high temperature relatively higher temperature air to the combustion chamber, where fuel is added, and the actual heat addition, and combustion takes place in the combustion chamber.

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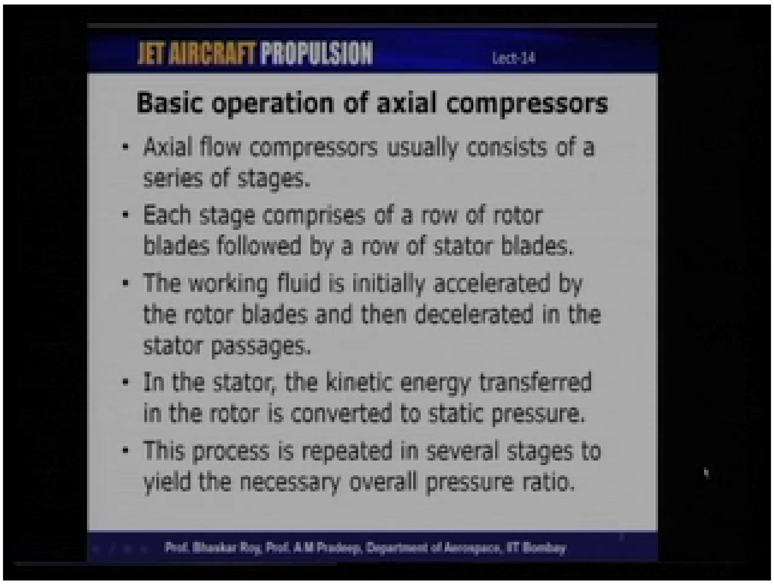

So, axial compressors form one of the basic components or aspects of a gas turbine engine, and in axial compressors as you might have already noticed that they usually consist of a series of stages, and each stage basically comprises or consists of a row of rotor blades followed by a row of stator blades.

So, a combination of a rotor and the stator constitute or comprise a stage of an axial compressor, and the typical modern day axial compressors have several stages, and several stages of rotors, and stators, and what basically happens in these stages is that the working fluid is initially accelerated by the rotor blades. We will see how this is done later, and subsequently it is decelerated in the stator passages.

And what happens in the stator is that the kinetic energy which was transferred to the fluid in the rotor gets converted to increase in static pressure, and so what the rotor basically does is that it derives its energy form the turbine; the turbine basically drives the compressors. So, the energy that the rotor derives from the turbine is transferred to the working fluid in the rotor.

And then in the stator, stator has two functions basically one is of course, that it will increase the static pressure by converting that kinetic energy into pressure energy, the other function of course, is that at the exit of the rotor the fluid also has a tangential component of velocity. So, the stator acts as a sort of a guide where in some sense that it turns the flow back to its original direction partially. So that the next stage of rotor can operate efficiently. So, stator has these two functions that it not only increases the pressure but, also deflects the fluid in a particular desired direction.

So, and this process of the rotor transferring energy to the working fluid; and then the stator doing its function is repeated in several stages. So a multi stage axial compressor will have this process getting repeated over several stages, which could be in a modern day compressor it could be as high as ten to twelve stage or even more than that where in this process is repeated.

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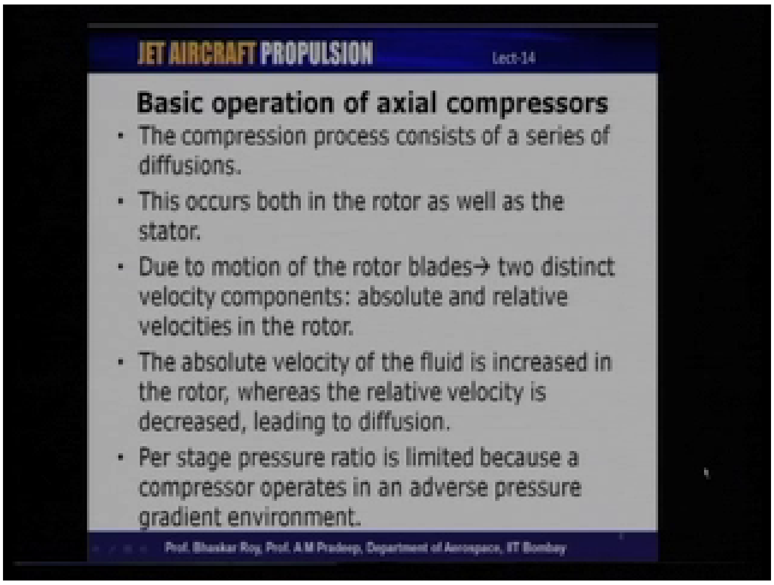

So, in an axial compressor the compression as I mentioned consists of a series of diffusions that is diffusion occurs both in the rotor as well as in the stator now in the case of rotor because the rotor itself has a rotary velocity that is there is a tangential velocity associated with the rotor blade itself, one can identify two distinct velocity components; the absolute, and the relative velocities that is if we were to observe the fluid particle as an observer from outside the compression system that is the stationary frame of reference then we see one particular velocity, and if the observer is actually seated on the blade itself that relative frame of reference then we see another component of velocity.

And therefore, rotation of the blade itself leads to two distinct components of velocity the absolute, and relative components, and these constitute basically the velocity triangle which we shall discuss shortly.

So, what happens in the rotor is that the absolute velocity is increased as a result of the momentum or the energy that is transferred by the rotor on the working fluid, where as there is a decrease in the relative velocity leading to diffusion, which is why I mentioned that there is diffusion in both the rotor as well as the stator.

Now one of the factors that limits the amount of you may wonder why is that we need to have so many stages of an axial compressor why cannot we have a limited number of stages of axial compressor, the basic reason is that the per stage pressure ratio of pressure raise in an axial compressor is limited, because an axial compressor as you might have already guessed operates in an adverse pressure gradient environment that is the pressure downstream is always higher than the upstream pressure, which means that the fluid has to overcome this pressure gradient as it passes through an axial compressor stage therefore, if we try to carry out too much of diffusion in one stage the adverse pressure gradient may be so high that the flow will not be able to withstand these pressure gradients, and there could be stalling of the flow from the blades.

And then that is one of the reasons that limits the per stage pressure ratio or per stage pressure raise that can be achieved in an axial compressor, which is why we need to have multiple stages of axial compressor each of them increasing the pressure by a certain amount.

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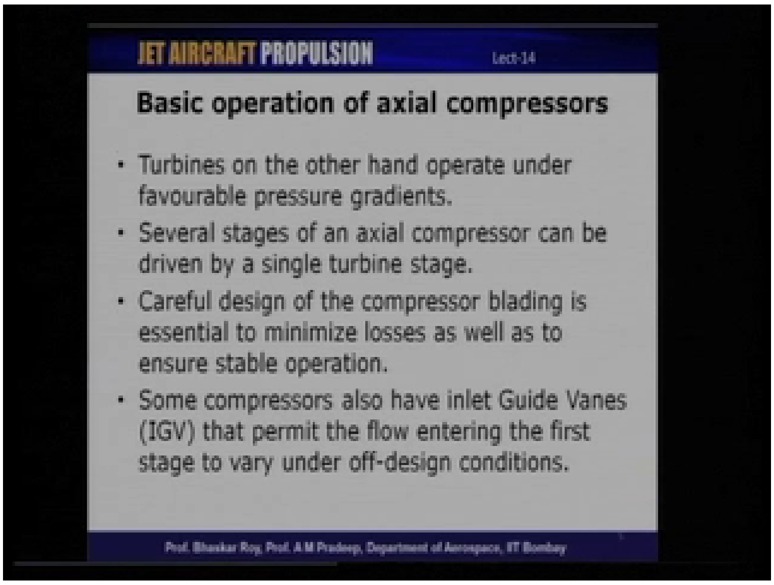

So, on the other hand turbines do not really have this problem, turbines always operate in a favorable pressure gradient mode and therefore, it is possible to have a much larger pressure drop across a turbine stage as compared to the pressure raise that we get from one stage of an axial compressor.

This means that in one stage of a turbine it is usually possible to achieve a substantial pressure drop which can be achieved only in several stages of an axial compressor and this is another reason if you next time when you take a look at a cutout diagram of a jet engine you can notice that you will see several stages of axial compressors but, only very few stages of turbine. So, typically a few stages of a in fact one stage of an axial turbine can drive several stages of an axial compressor, and the reason for that is basically that the per stage pressure ratio of compressors are limited due to adverse pressure gradient. But, that problem is not really there for a turbine, because they always operate in a favorable pressure gradient mode, and so you can actually expand in one stage much more than what you would be able to compress in one stage of an axial compressor.

And so in a jet engine it is usually something you would notice that a few stages of turbines would be driving several stages of a compressor, and that is one of the aspects that you need to keep in mind. And in the case of compressor of course, which means that compressor design in some sense would probably be little more challenging at least aerodynamically as compared to a turbine.

Now, so a careful design of a blade is very much essential, because as I mentioned that you can increase the pressure ratio per stage by only a limited amount and therefore, if you would like to minimize losses at the same time try to maximize the pressure ratio that you can get from one stage of a compressor that definitely calls for a very careful design of the compressor blades.

And in some cases you might notice in some engines that the compressor begins with one set of a stationary blades, which are known as in that guide range usual compressors usually compressors or actual compressors stage consists of a rotor followed by a stator that is rotor imparts the kinetic energy to the fluid, and then stator increases the pressure by converting that energy to pressure. But, in some engines the first stage of the compressor is preceded by a row of stationary vanes which are known as inlet guide vanes.

And inlet guide vanes basically have a function in the sense that they permit the flow to enter at a desired angle; especially, when the engine is operating at off design conditions that is as the engine operates over several operating points, which may not necessarily be the design operating point, then if the incoming flow angles are different then that can affect the performance of the compressor, and therefore, the engine.

So, inlet guide vanes tend to deflect the flow at a certain desired angle. So, as so that the off design performance of a the engines can be better than what it would normally be. So, in an axial compressor the basic understanding of the working of an axial compressor begins with understanding of what are known as the velocity triangles.

So, that is what we will be doing in the next few slides how to we construct a velocity triangle or basically what do we mean by a velocity triangle. So, I think I already mentioned that in a rotor because of the presence of the peripheral velocity of the blade speed because of by virtue of its rotational speed one can identify to distinct velocity components one is to do with the stationary frame of reference, and the other is the rotating frame of reference, and so if you were to combine and all the three that is the blades speed the relative velocity, and the absolute velocity or related to each other vectorially.

And so it is possible to construct a triangle, which relates which shows the relationship between the blade speed the absolute velocity, and the relative velocity. And of course, as we construct the velocity triangle it will also involve certain angles these are to do with the inlet flow angles, and the blade angles which we will discuss shortly.

And so these angles also will be part of the so called velocity triangle. So, what we will do in the next few slides is to see how we can construct given a certain rotor, and stator combination how is it that. We can go about constructing a velocity triangle, because that is where the basic design an understanding of the working of an axial compressor begins.

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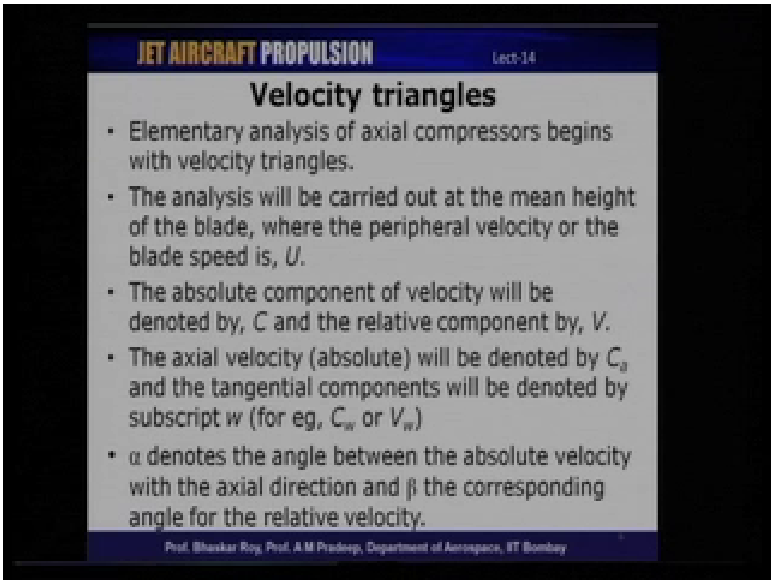

So, what we do is that we will carry out or the analysis that we are going to do is limited to a certain height of the blade. We will take the main height of the blade, and we will assign the blade speed or the peripheral velocity to a symbol U. So U denotes the peripheral velocity or the blade speed at that particular station, at which the analysis is carried out. And we are going to carry out this analysis for the main blade height.

So, we will denote the absolute component of velocity by symbol C, and relative velocity by V. So, absolute components of velocity will be denoted by C and the relative component by V, and axial velocity the absolute component of that is usually denoted by symbol C, subscript a whereas, a is for the axial, and the tangential components will be denoted by a subscript w that means if we are denoting the tangential component of the absolute velocity then it would be denoted by C subscript w; and the tangential component of the relative velocity will be denoted by V subscript w.

And we will as I mentioned there are also certain angles which we will need to be familiar with we will denote alpha for the angle between the absolute velocity and the axial direction. So, the angle which the absolute velocity makes with the axial direction will be denoted by alpha, and beta will be the angle corresponding to the relative velocity that is the direction that is angle between the relative velocity, and the axial direction. So, alpha is for the absolute velocity, and beta is for the relative velocity and absolute velocity is to be denoted by C relative velocity or relative component by V.

So, what we will take up next is we will take up one row of a rotor blades, and followed by a row of stator blades, and then we will see how we can think of beginning to understand, and develop the velocity triangle for this kind of a configuration. So, what we will do is to express the rotor blades on a two dimensional plane, you know that rotor blades are arranged on an axis.

So, let us assume that the rotor blades are at arranged in a linear fashion which is what I will show in the next slide, and that is basically for simplicity, and that is particularly true for a particular cross section. In this case, we will assume that it is a the main blade height section.

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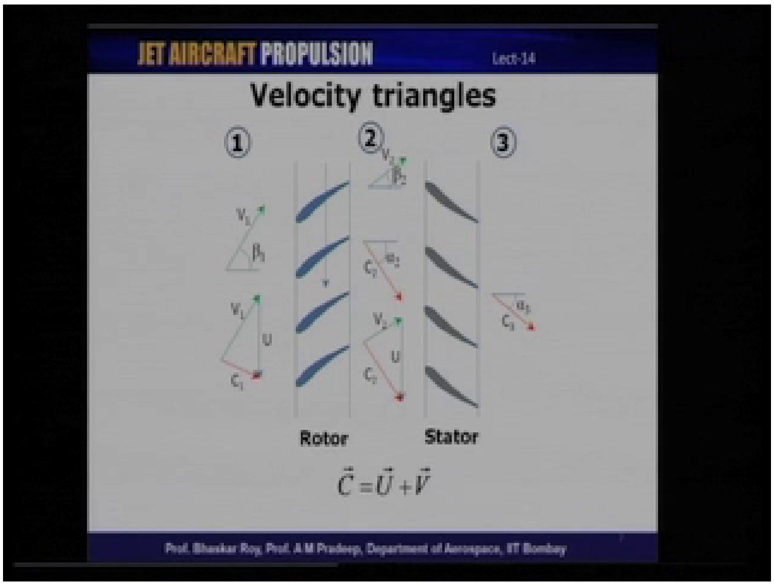

So, if we consider a row of rotor blades, and stator blades so what is shown here by the these set of blades that are shown here are the rotor blades let us say, and set of stator blades following them. And we will denote these stations just upstream of the rotor by 1 station 1 and just downstream of the rotor is 2 which is also the inlet for the stator 3 denotes the exit of the stator.

So, all these velocity triangles which we will construct is just at the leading edge of the rotor. So, which is denoted by symbol or 1 or station 1 trailing edge of the rotor is you know denoted by station 2 which is also the leading edge of the stator, and exit of the stator is by a 3 station number 3. Now, this is the rotor, and stator configuration that is a stage that we have, and so this rotor has a rotational direction as indicated here, and so this is how the rotor would of a compressor would rotate.

And because this is imparting energy to the fluid that is a rotor blades do work on the fluid and therefore, if the work has to be done on the fluid this obviously has to be the direction in which the rotor blades rotate. In the case of turbines which we will see latter on it would be the other way round because in turbines work is extracted from the fluid and therefore, the direction of rotational velocity is opposite.

So, now we will begin with the inlet of the rotor and as I mentioned the angel between the relative velocity and the axial direction is beta and since this is at the leading edge of the rotor beta is have has a subscript of one. So, if beta one corresponds to the angle between the relative velocity and the axial direction.

And so the relative velocity in this case is obviously V 1 because one corresponds to the leading edge of the rotor. Now, why is it that we have indicated a direction of V 1 in this fashion. Now, you might notice that the blades both the rotor as well as stator blades have already been set at a particular angle, and so the angle which is shown here corresponds to the angle at which the relative velocity of the velocity of this particular arrangement enters the blade in a tangential direction that is if you were to draw the camber at of this particular rotor, and draw a tangent at the leading edge of the camber line that would have an angle of beta one with the axial direction. Of course, this is assuming that there is no incidents and things like that, which we will discuss in later lectures.

Now, for simplicity we are assume we are assumed that the blade angle is represented by beta 1. So, relative velocity is the angle which is basically the angle of at the leading edge of the if you draw if you want to draw a tangent at the leading edge on the camber of the blade.

Now, we already know the direction of the blade speed which is already shown here by this blue arrow, and so how are these three components the relative velocity the absolute velocity, and the blade speed related. So, vectorially they are related like this C which is the absolute velocity is equal to the vector sum of U which is the blade speed, and the relative velocity V.

So, with that in mind let us construct the velocity triangle for the leading edge or the inlet of the rotor so at the inlet of the rotor we now have all the 3 components of velocity. We have the relative velocity which is V 1 and if we add vectorially add V 1 and U we get the absolute velocity which is indicated here by the red arrow which is C one.

So, this represents the velocity triangle at the inlet of the rotor or at the leading edge of the rotor. So, similarly we can now construct the velocity triangles at the trailing edge as well. Now, before that let me also mention that C 1 has an angle of alpha 1 with reference to the axis. So alpha 1 would be this angle that is the angle which C 1 makes with the axial direction whereas, beta 1 is angle which the relative velocity makes with the axis.

Now, at the trailing edge the same argument is valid that V 2 which is the relative velocity exciting the rotor that is at the trailing edge it exceeds the trailing edge at an angle beta 2 here again as I mentioned for simplicity we assume that the blade angle at the trailing edge is what is shown here as beta 2 there is no deflection incidental deflection we will discuss possible in the next class or little later.

So, V 2 is the relative velocity exciting the blade at the trailing edge and U remains the same that is blade speed at a given cross section remains unchanged, and if that is so we can now construct the velocity triangle at the trailing edge the same way, we constructed it for the leading edge.

And So, alpha 2 here corresponds to the angle which is the angle which the absolute velocity makes with the axial direction. So, this is how the velocity triangle at the trailing edge now looks; so, trailing edge velocity triangle where in V 2 is added vectorially U 2 which U 2 and the U 1 are of course, the same gives us the absolute velocity, which is C 2 C 2 making and angle of alpha 2 with the axial direction.

Now, unlike a rotor we which which had a blade speed stator blades are stationary so there is no blade speed associated with stator blades, which means that stator blades simply deflect the incoming flow at a desired angle depending upon how the blades are set.

So, C 2 is the angle or the velocity which enters the stator, and it leaves the stator another velocity which obviously would be lower than C 2 because stator blades defuse the flow, and therefore, C 3 will be less than C 2 and it leaves the stator blades at an angle alpha 3.

And what we will see little later on is that by design normally we would like to keep alpha 3 equal to alpha 1, which means that another set of rotor blades can now follow this stator blades, and this can be repeated over several stages, and that is how a multi stage axial compressor is developed.

So, let me just quickly go through what we had done in this particular construction of velocity triangle. Now, we have rotor blades set at a certain angle we know the direction of rotation of the rotor blades, and so depending upon the blade angle at the leading edge, which in this case is beta 1. We can determine the direction of the relative velocity, and blade speed is already known and therefore, the absolute velocity can be determined, because these three components are related to 1 another vectorially

The same thing is followed for the trailing edge, where we know the trailing edge the angle blade angle at the trailing edge, and assuming that the blade leaves the trailing edge in the same direction we get the direction of the relative velocity blade speed is known and therefore, the absolute velocity that is C 2. In the case of stator, there is no more blade speed and therefore, there is no relative velocity in a stator, and so C 2 is the velocity which enters the stator blade at its leading edge, and C 3 is the velocity living the blade stator blades at an angle of alpha 3.

So, in this particular slide we were trying to discuss about how given a certain configuration of rotor, and blade stator combination one can go about trying to make or construct the velocity triangle, because that is perhaps one of the starting points of the design; where one would like to analyze a given configuration of blade rotor, and stator blades, and so velocity triangles help us in trying to design or begin the design of an axial compressor stage.

So, we will take a closer look at the velocity triangles you already constructed velocity triangles at the leading edge and trailing edge. We can now super impose these two velocity triangles, because one of the components of velocity is common for both that is the blade speed U for leading edge and trailing edge both of them are same, which means that with common U. We can now, supper impose these two velocity triangles and then try to infer a few more aspects of axial compressor working from the supper imposed velocity triangle.

So, if you take a look at the velocity triangle the combined velocity triangle that has been obtained by supper imposing the inlet velocity triangle with that of the outlet keeping the blade speed that is peripheral velocity the same. So, what we see here is that in the case of the velocity triangle of the leading edge, and as we moved from the leading edge to the trailing edge. We see an increase in the absolute velocity.

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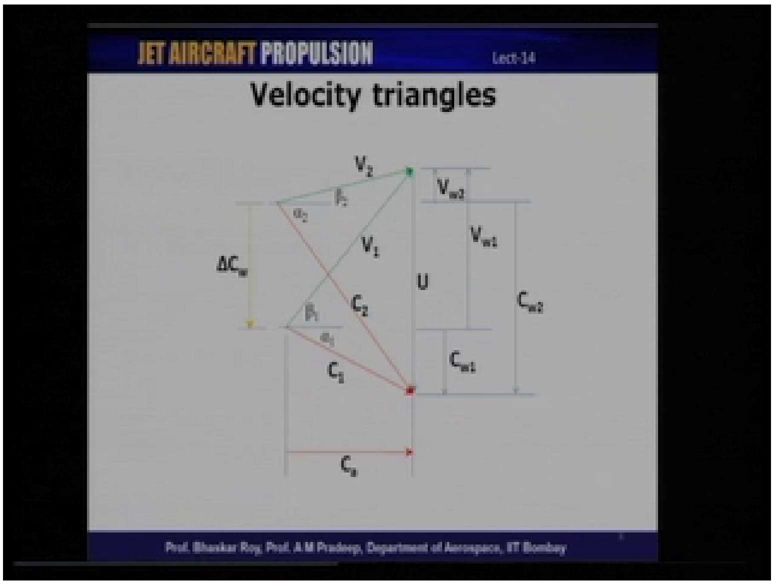

And so, there is an increase in absolute velocity as you might have noticed at the inlet this is the velocity triangle consisting of C 1, which is absolute velocity at leading edge at the inlet of the rotor b 1 is the relative velocity, and U is the blade speed. Alpha 1 corresponds to the angle of, which the absolute velocity makes with the axis beta 1 is the relative velocity angle between relative velocity and the axis.

Similarly, at the exit we have C 2 which is the absolute velocity at the exit V 2 which is the relative velocity at the exit, and correspondingly there angles which are alpha 2 and beta 2. Now, I have indicated a few more of these components or if you were to take component of these velocities in different directions. So, the axial velocity component is as I mentioned denoted by C a, which is the velocity component of the absolute velocity in the axial direction that is C a.

And we also have these tangential components for example, the tangential component of a V 1 which is the relative velocity at the inlet is denoted by V w 1 which is basically this component of velocity which is this, and V w 2 corresponds to the tangential component of the relative velocity. So, relative velocity component in the tangential direction which is U is denoted by V w 2 and this is the direction of these two velocity components. Similarly, we have the corresponding components of the absolute velocity C w 1 is the tangential component of the absolute velocity at inlet C w 2 is the tangential component of the absolute velocity at the exit of the rotor.

And the difference between C w 2 and C w 1 is denoted by delta C w, and that we shall see little later is delta C w does play a significant role in the performance analysis of these compressor. So, delta C w is basically the difference between C w 2 and C w 1 which are basically the tangential component of the absolute velocities, and taken in the tangential direction.

So, this particular slide gives us an idea of how these velocity triangles are related in the sense that, we can immediately see from these two velocity triangles that between C 1 and C 2; C 2 obviously is higher than C 1 that means that there is an increase in absolute velocity. But, at the same time we can see that relative velocity has reduced or decreased so, V 1 is greater than V 2. So, there is a deceleration in terms of the relative velocity and that is why we see an increase in pressure even in the rotor, because the relative velocity has decreased. So, there is a diffusion of occurring in the rotor and obviously there is also a diffusion occurring in the stator, because a we have seen in the stator C 3 is less than C 2, and there is no longer any more relative velocity components there.

So, this is one of the configurations of velocity triangles of course, there depending upon the blade design you might see different types of velocity triangles some of them we will discuss a little later. On how the velocity triangles can look like if certain design parameters are changed, so that we will discuss a little later.

So, having understood some basic aspects of velocity triangle, and how we can construct velocity triangle we have now understood that in the rotor basically there is an increase in the absolute velocity. But at the same time there is a deceleration in terms of their relative velocity which is by we get an increase in the pressure even at the rotor and the diffusion process is split between the rotor and the stator.

So, what we will see next is to just get some idea of how the properties vary as we move from rotor to stator that is in a stage of an axial compressor how different properties is like enthalpy or pressure varies, as we move from inlet of the rotor towards the exit of the stator.

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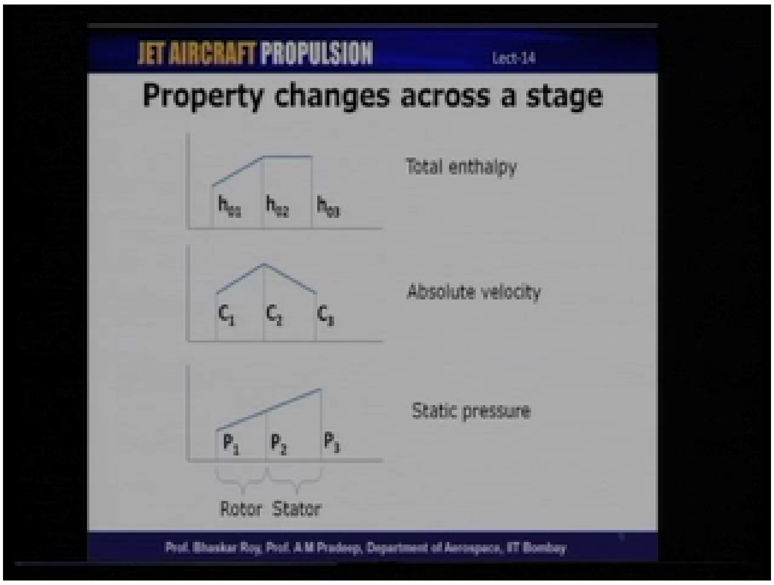

So, in a stage what basically happens is that the properties obviously undergo a certain change. For example, if you were to look at the enthalpy b the stagnation enthalpy or total enthalpy what is indicated here just an illustration that is shown here. We have the rotor and the stator both of them put together constitute a stage of an axial compressor. So, if you were to express the variation of total enthalpy across a stage what we see is that in the rotor there is an energy addition occurring that is energy is added on the fluid rotor does work on the fluid, and so there obviously is an increase in the total enthalpy or stagnational enthalpy. So, x 0 2 is greater than x 0 1.

On the other hand, in the case of stator there is no more energy addition taking place, and therefore, if you have to assume that the flow passing through the stator is adiabatic in which case this stagnation enthalpy does not change. So, stagnation enthalpy remains fixed between stations 2 and 3 which is the inlet of stator 2 inlet of the exit of the stator.

On the other hand, we look at the absolute velocity in the rotor there is an increase in absolute velocity. So, C 1 is less than C 2 that is there is an increase in absolute velocity from the inlet of the rotor to the exit of the rotor and in the case of stator there is a deceleration occurring that is what we had observed earlier as well that C 3 is less than C 2.

And static pressure on the other hand, static pressure as I mentioned the diffusion is split between the rotor, and stator diffusion begins in the rotor and continues in the stator. So, there is a continuous increase in static pressure all the way from inlet of the rotor till the exit of the stator.

Similarly, we can also map out other properties like the relative velocity relative velocity know we know is yeah decreases in the rotor and there is no relative velocity as such in the stator. Total pressure also can be expressed in a similar way like total enthalpy, because if you were to assume that there are no frictional losses in the stator total pressure in the stator does not change. So, total pressure increase basically occurs in the rotor, because that is where you are adding energy.

So, total in the same way as total enthalpy total pressure basically raises in the rotor and remains constant at least in the ideal case in the stator. So, this is just to give an illustration or of how these properties vary and in a stage of an axial compressor, and so what we have discussed is to one of the basic ways of beginning design of axial compressors basically starting from the velocity triangles.

So, what we will do next is that given this understanding of the velocity triangle and there construction we will try to now derive an expression or relate the pressure rise in an axial compressor to some of the other parameters, and see on what parameters does pressure rise across a stage of an axial compressor would depend upon.

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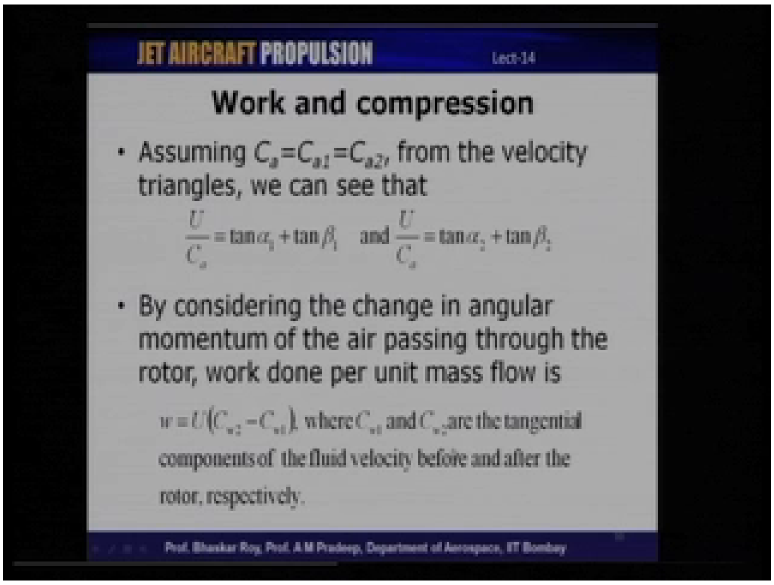

So, basically what we are going to do is to assume that the axial velocity will remain the same. So, we will assume that the axial velocity C a 1, and C a 2 is basically equal to C a that is axial velocity remains unchanged. So, if that is so from the velocity triangles we will let me go back to the velocity triangle so from the both these velocity triangles let's take up the inlet velocity triangle which has C 1 and B 1 and U.

So, if we were to take this ratio U, U divided by C a that is basically equal to the tan of alpha 1 plus tan of beta 1. So, tan beta 1 is basically this component of U this part of U divided by C a and tan of alpha 1 is the remaining component of U divided by C a, which means U by C a should be equal to tan alpha 1 plus tan beta 1.

Similarly, for the second velocity triangle that is the exit velocity triangle we have tan of alpha 2 plus tan of beta 2 is equal to U by C a. So, U by C a, we have expressed in terms of the angles tan alpha 1 plus tan beta 1 similarly, it is also equal to tan alpha 2 plus tan beta 2.

Now, if you look at the change in angular momentum of the air that is passing through the rotor. We know that as the fluid passes through the rotor there is a change in the velocity components, and you have also already identified that the component of the absolute velocity in the tangential direction are denoted by C w 1 and C w 2.

So, as the rotor does work on the fluid its basically the tangential component of the velocity that plays a significant role here, because it is the tangential motion of the rotor which basically does work, and so we can basically express the work done per unit mass flow as a product of the blade velocity, and the difference between the tangential component of these velocities between the inlet of the rotor, and exit of the rotor; that means work done per unit mass would be basically the product of the blade speed the peripheral velocity and the difference in the tangential component of these absolute velocities.

So, U multiplied by C w 2 minus C w 1 basically gives us tells us how much work is done on the fluid per unit mass. So, work done per unit mass of the air passing through the compressor can basically be expressed in terms of product of U times C w 2 minus C w 1, which basically U times delta C w, and here C w 1 and C w 2 are the tangential component of velocities of the fluid before and after the rotor respectively.

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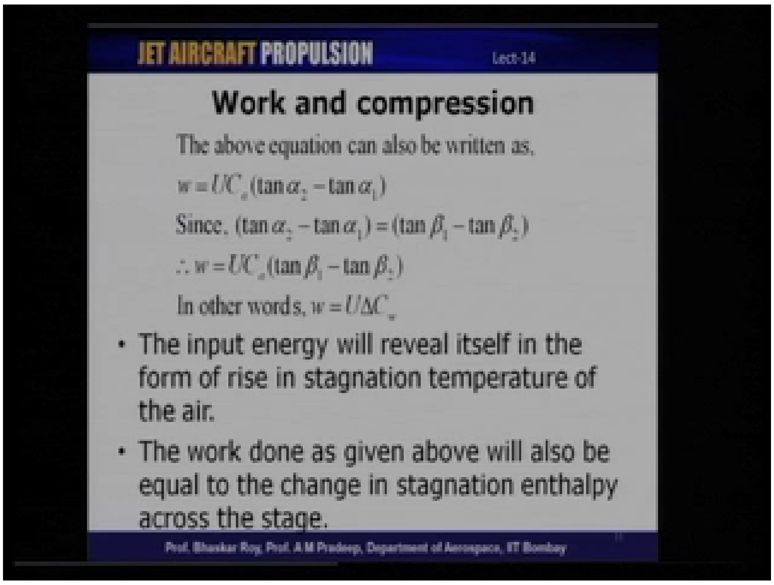

So, if you were to look at the first equation we wrote it was U by C a is equal to tan alpha 1 plus tan beta 1 and U by C a is also equal to tan alpha 2 plus tan beta 2. We combine that with this worked on equation, then we get work done per unit mass is equal to U times C a into tan alpha 2 minus tan alpha 1.

Now, tan alpha 2 minus tan alpha 1 is also equal to tan beta 1 minus tan beta 2, which you can let's go back to the velocity triangle again. So, the difference between tan alpha 2 and tan alpha 1 is also equal to the difference between tan beta 1 and tan beta 2.

So, work done per unit mass can also be expressed as U time C a into tan beta 1 minus tan beta 2. So, you could use either of these expressions to determine how much work is done per unit mass which means that one of the advantages of having the velocity triangle is that you can calculate how much work is done per unit mass if we know the angles and obviously the peripheral loss T would be known because the speed at which the rotor rotates is kind of a design parameter. So, you would know a clearly how much is the speed of rotation of the rotor.

And knowing that and the axial velocity one can express these components, that is the blade speed, and axial velocity and the blade angles in terms of the work done per unit mass. And what it means is that work done per unit mass is basically a function of the blade speed, and delta C w. And so how will it affect the performance so, basically this work done per unit mass will manifest itself in the form of an increase in stagnation temperature because you are adding energy to the flow. We have seen that stagnation enthalpy changes that h 0 2 is greater than h 0 1, because we are adding energy to the flow therefore, whatever work is being done on the flow that is U times delta C w is basically equal to or will manifest itself as increase in stagnation enthalpy, which is basically increase in stagnation temperature.

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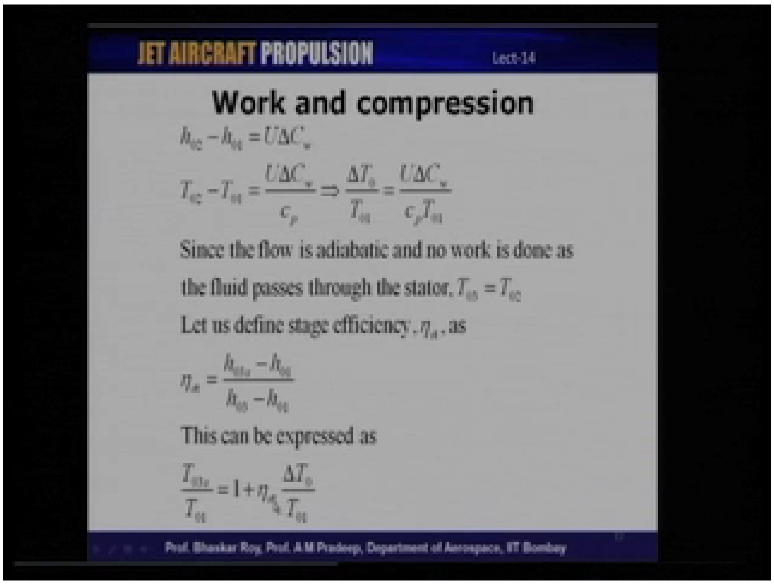

That means U times delta C w should be equal to h 0 2 minus h 0 1. So, this is per unit mass so work done have so here as we have calculated U times delta C w should be equal to h 0 2 minus h 0 1. So, if that is the case then we can simplify this further that is multiplying this by C P and T in delta temperature so T 0 2 minus T 0 1 is equal to U times delta C w by C P, which again can be simplified as delta T not divided by T 0 1 is U times delta C w by C P T 0 one.

Now, if we assume that the flow is adiabatic and since there is no work done in the fluid as it passes through the stator it means that T 0 3 is equal to T 0 2, and so let us now define a stage efficiency we have already seen what is meant by isentropic efficiency that is defined for the whole compressor at that time I have also mentioned that in addition to isentropic efficiency, and polytrophic efficiency we can also define stage efficiency; which is basically the isentropic efficiency applied for one stage. So, stage efficiency is h 0 3 s minus h 0 1 divided by h 0 3 minus h 0 1 which can be simplified as T 0 3 s divided by T 0 1 is equal to 1 plus stage efficiency times delta T not by T 0 1.

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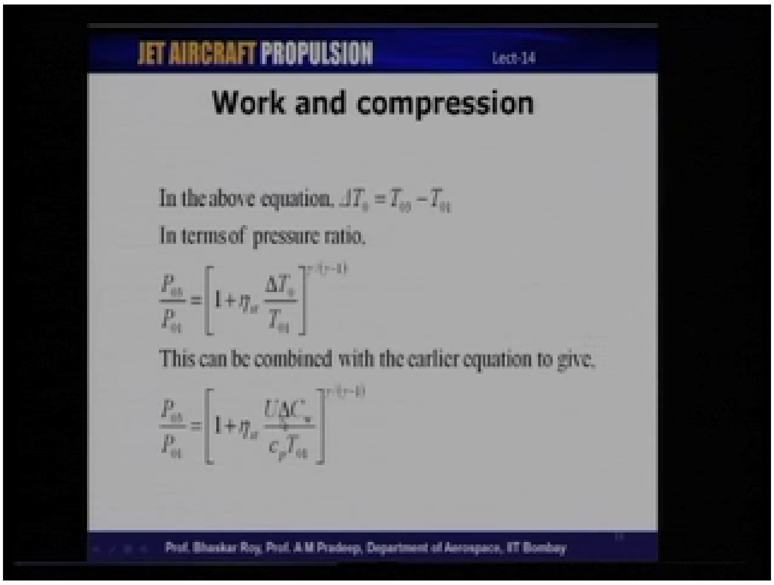

So, in the above equation we can we can basically substitute for T 0 1 delta T not as T 0 3 minus T 0 1, because there is no change in stagnation temperature in the stator. So, in terms of pressure ratio we now have P 0 3 divided by P 0 1, which is because we have this temperature ratio which is isentropic T 0 3 s by T 0 1 can be equated to pressure ratios rise to the gamma minus 1 by gamma.

So, P 0 3 by P 0 1 that is pressure ratio per stage will be equal to 1 plus stagnation stage efficiency into delta T not by T 0 1 raise to gamma by gamma minus 1. We have already calculated what is delta T not by T 0 1 this is basically equal to U times delta C w divided by C P T 0 1. Therefore, pressure ratio per stage is equal to 1 plus stage efficiency into U times delta C w divided by C P T 0 1 rise to gamma by gamma minus 1.

So, here we have an expression which is basically the pressure ratio the stagnation pressure ratio across a stage being related to a few other parameters like the temperature raise or in terms of the velocity triangle components like the blade speed delta C w, and of course, the stage efficiency. So, once we know once we can determine the velocity triangle for particular stage, and of course, if we know the stage efficiency we can actually calculate how much pressure ratio this particular stage would be able to develop.

So, pressure ratio per stage as you can see depends upon a few parameters one of them is the blade speed that means higher the blade speed better is the velocity pressure ratio per stage. And of course, there is other parameter that is delta C w, and delta C w is basically related to the blade angles beta 1, and beta 2 that is deflection of the fluid also determines or increases the pressure ratio per stage. So increasing the blade speed increasing the deflection these are different ways of improving the pressure ratio per stage but, that is not always a possible as we will see little later.

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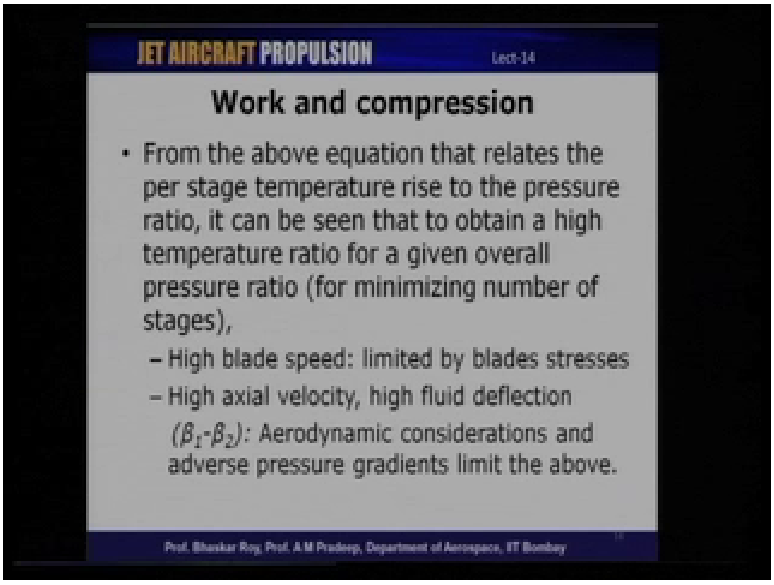

So, what we can see that the it basically the equation relates the temperature raise per stage to the pressure ratio, and so to obtain a high temperature ratio for a given pressure ratio that is to basically minimize the number of stages. We could either increase the blade speed or axial velocity or fluid deflection but, increasing the blade speed has certain limitations that is if we if we were to rotate the blades at very high speeds it can obviously lead to very large amounts of stress.

So, blade stresses will limit the blade speed high axial velocity, and fluid deflection also are limited because of the increase in adverse pressure gradients which may lead to these stalling of these blades. So, there are certain limitations because of which we cannot increase these parameters, and therefore, as we had discussed in the initial part of the lecture that per stage pressure ratio is very much limited. So, we have now identified what are the parameters or based on which the pressure ratio can be increased or improved and we have also seen what limits these parameters.

Let us now take a look at some of the important design parameters which are often used in design of axial compressor, and then we will we will take a closer look at some of these parameters like degree of reaction and the diffusion factor.

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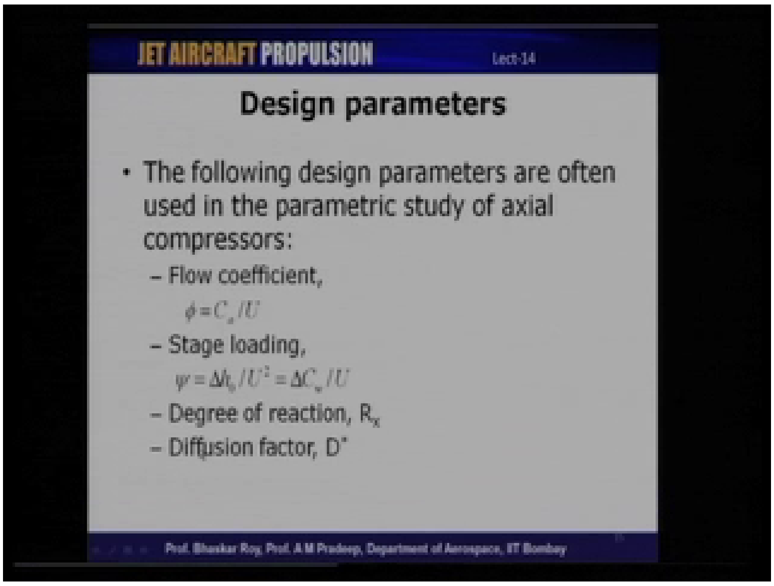

So, there are I have listed here four important design parameters one of them is the flow coefficient; flow coefficient is usually denoted by five, and this is a equal to the ratio of the axial velocity to the blade speed. So, C a by U is basically the flow coefficient besides that often one would use the stage loading coefficient, which is denoted by psi, and that is basically equal to the enthalpy rise across the stage divided by U square. So, delta h not by U square which is in turn equal to delta C w divided by U. So, this gives us an idea of how much work is done per stage.

The other parameter which is often used in fact always used in the design of a these compressors is the degree of reaction, which is denoted by R subscript x, and also the diffusion factor which is denoted by d star.

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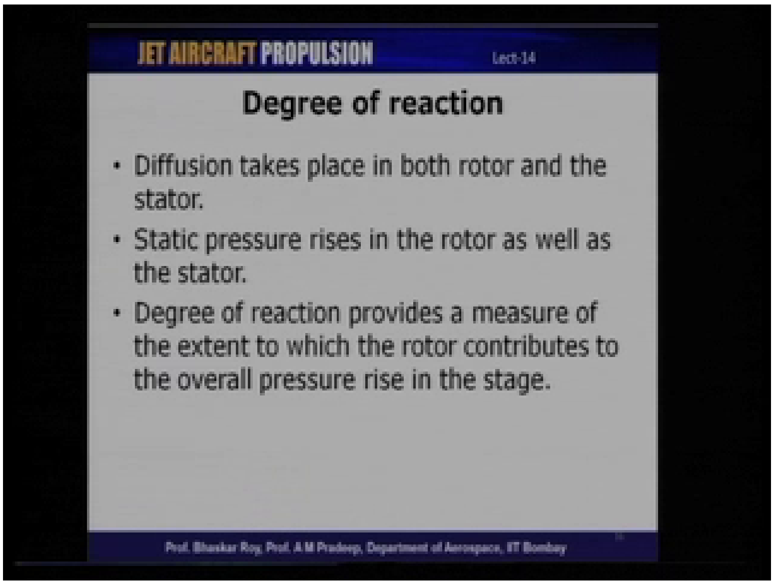

So, we will take a closer look at what is meant by degree of reaction; and the diffusion factor. Now, degree of reaction is basically an indication of the split of work or pressure raise between the rotor and the stator as I mentioned that pressure rise is shared by both the rotor, and the stator. So, how much of this total pressure rise that is done by the stage is being done by the rotor is what degree of reaction indicates it indicates how much of a pressure rise of course, in the rotor as compared to the whole stage.

So, degree of reaction gives us an indication of what is the ratio of the pressure rise in the rotor to the pressure rise across the stage. So, it basically provides a measure of the extent to which the rotor assists or contributes towards the overall pressure ratio in this stage.

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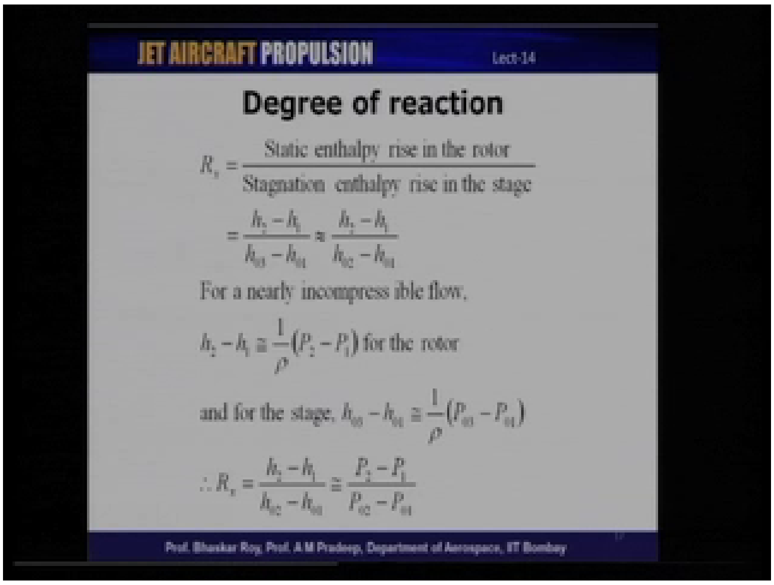

So, diffusion factor is a defined as the static enthalpy rise in the rotor to the stagnation enthalpy rise in the stage, which is basically h 2 minus h 1 divided by h 0 3 minus h 0 1; which we can also approximate as h 2 minus h 1 divided by h 0 2 minus h 0 1, because h 0 2 is equal to h 0 3. So for so we are going to simplify this for a relatively in compressible for nearly incompressible flow, we can approximate h 2 minus h 1 from the energy equation as 1 by row into P 2 minus P 1, and for the stage h 0 3 minus h 0 1 as 1 by row into P 0 3 minus P 0 1, if that was the case then the diffusion degree of reaction would be h 2 minus h 1 divided by h 0 2 minus h 0 1 which is approximated as P 2 minus P 1 by P 0 2 minus P 0 1.

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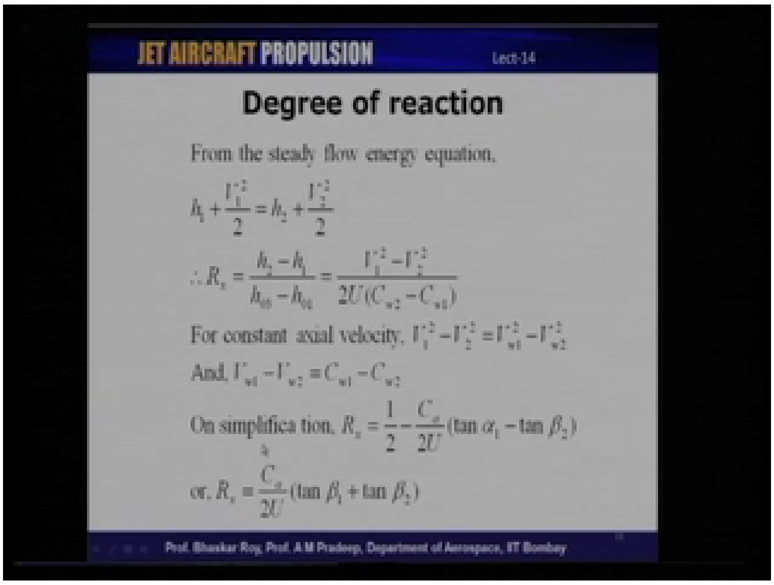

Now, from the energy equation if we were to look at the apply the energy equation in the relative frame of reference, then we have in the rotor h 1 plus V 1 square by 2 is equal to h 2 plus V 2 square by 2, which is that V 1 and V 2 are the relative velocities at the inlet, and exit of the rotor. So, we can express the degree of reaction as V 1 square by 2 V 1 square minus V 2 square divided by 2 U into C w 2 minus C w 1, because delta h not is a expressed in terms of U times delta C w.

So, for a particular stage if you were to assume axial velocity to be constant then we can express these velocity components as V 1 square minus V 2 square is equal to V w 1 square minus V w 2 square and V w 1 minus V w 2 is C w 1 minus C w 2. So, this we can derive as we look at the velocity triangle and if you assume constant axial velocity. So, if we were to assume these, and we substitute this in the degree of reaction on simplification we get degree of reaction as 1 by 2 minus C a by 2 U into tan alpha 1 minus tan beta 2 or from the velocity triangle degree of reaction is also equal to C a by 2 U into tan beta 1 plus tan beta 2.

And of course, there are many other expressions for degree of reaction based on simplification, and the understanding of the velocity triangle; so degree of reaction as we can see here can be expressed in terms of the axial velocity the blade speed, and the angles as we get it from the velocity triangles. So, it basically tells us what is the amount of work sharing between the rotor as compared to the whole stage.

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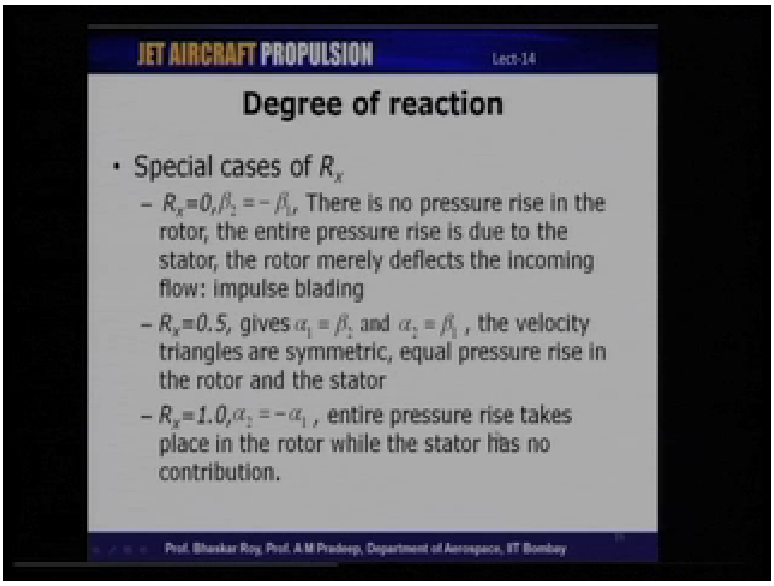

So, let us take a look at a few special cases of the degree of reaction degree of reaction if it were to be 0 if we if we put R as 0 here, then we get beta 2 is equal to minus beta 1 this means that there is no pressure rise occurring in the rotor the entire pressure raise is due to the stator of the rotor basically deflects the flow incoming flow.

So, this is something like an impulse bleeding which we will discuss a little later when we take up the turbines; degree of reaction is equal to 0.5 gives us alpha 1 equal to beta 2, and alpha 2 equal to beta 1 which means the velocity triangles are symmetric or mirror images, and there would be equal pressure rise taken place in the rotor and the stator. And degree of reaction is equal to 1 means that alpha 2 is equal to minus alpha 1, and the entire pressure rise takes place in the rotor where as the stator does not contribute to the pressure rise.

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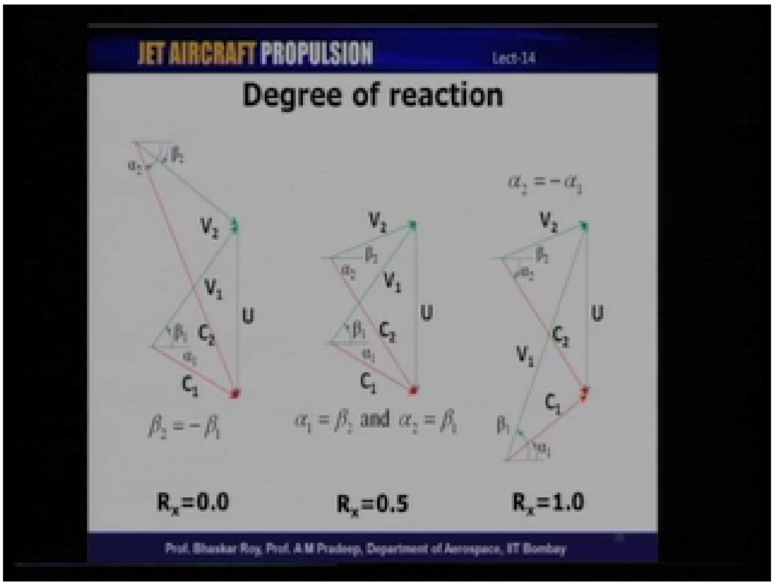

So, the velocity triangles would look something like this for all these three cases if degree of reaction was 0 then we get beta 2 is minus beta 1. So, this is beta 2 here this is exactly opposite equal and exactly opposite to that of beta 1; this is how it would be if we have an impulse balding for an axial compressor degree of reaction equal to 0.5 means alpha 1 is equal to beta 2, and alpha 2 is equal to beta 1, so that is the velocity triangles are mirror images of one and other.

So, this the inlet velocity triangle if you take a mirror image of that we get the exit velocity triangle. So, this is how it would be if degree of reaction was 0.5 for degree of reaction of one, we get alpha 2 is equal to minus alpha 1. So, here we have alpha 2 which is equal, and opposite to that of alpha 1.

So, here there is the pressure rise is occurring only because of the rotor. Now, let us now look at the last parameter that we will discussed today that is called the diffusion factor, and we have already seen that pressure ratio per stage pressure ratio is significantly dependent on the fluid deflection that is difference beta 1 minus beta 2 that is a strong parameter that influences the pressure ratio.

But, we also seen that there are limitations as you keep increasing the deflation angle that is deflection beta 1 minus beta 2, then there is an increasing adverse pressure gradient, and there is a threat of flow separation or blade stalling that can influence the performance of the compressor.

So, we will now define a parameter which will tell us how close you is how close is the rotor towards what are the chances that the flow will separate on the section surface of the blade.

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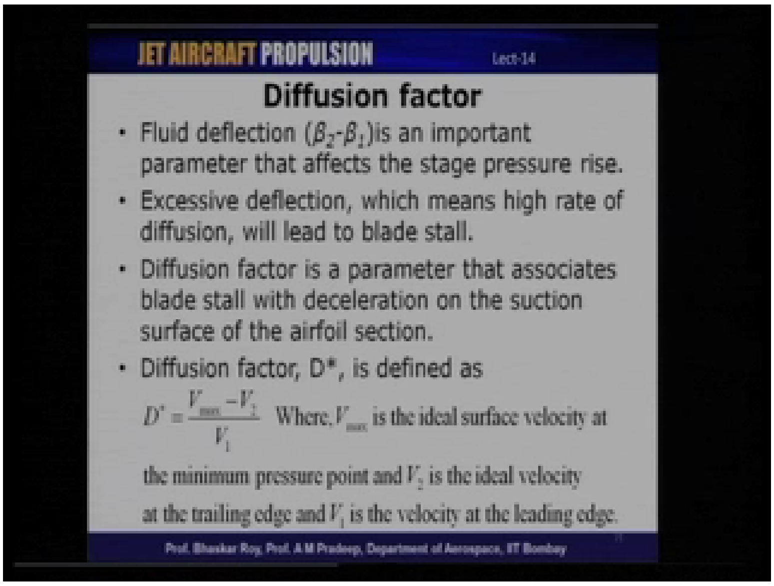

So, fluid deflection is a basically what decides the pressure ratio one of the parameters which decides the pressure ratio but, there are certain limitations to that. So, diffusion factor is a parameter that associates the blade stall with deceleration on the section surface of the airflow. So, it’s basically defined as the difference between the maximum velocity, and the minimum velocity that is at the trailing edge to the inlet velocity at the leading edge. So V max minus V 2 by V 1 basically defines or tells us, what is the chance that there could be a blade stall occurring on the section surface?

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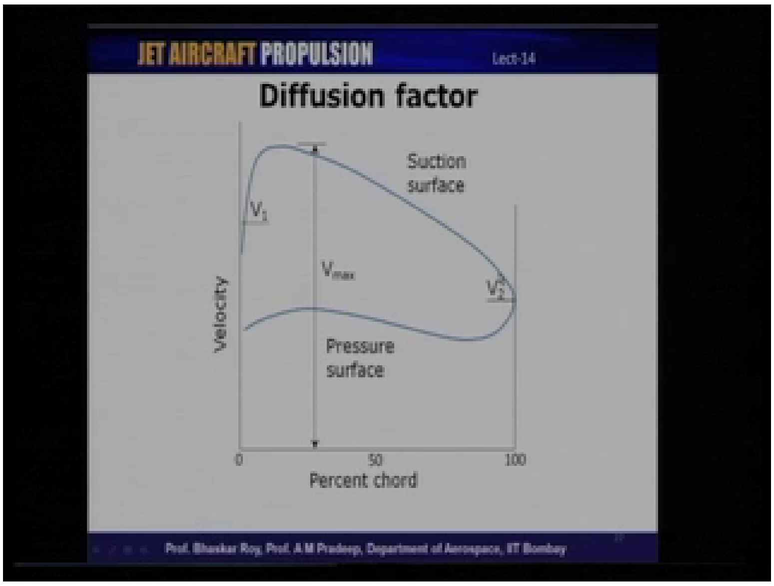

So, let us understand how this V max minus V 2 by V 1 come from so here we have the velocity distribution on the blade surface. So, this is the pressure surface shown here and this is the section surface of the blade the velocity on the section surface. So, we know that at from the leading edge to the max pressure minimum pressure point the flow accelerates velocity reaches its speed, and that is where we get V max V 2 on the other hand is a velocity at the trailing edge.

And so that is the minimum velocity that basically is that velocity at the trailing edge, and we have this pressure surface velocity distribution. So, the higher we keep V max by V 2 which is also true as we keep increasing beta 1 minus beta 2 this difference also increases beyond a certain level we are trying to decelerate the flow too much and the obviously the flow will not be able to sustain itself, and with stand the pressure gradient, and it may separate so diffusion factor relates this V max to V 2, and so that is why we get the diffusion factor V max minus V 2 by 1.

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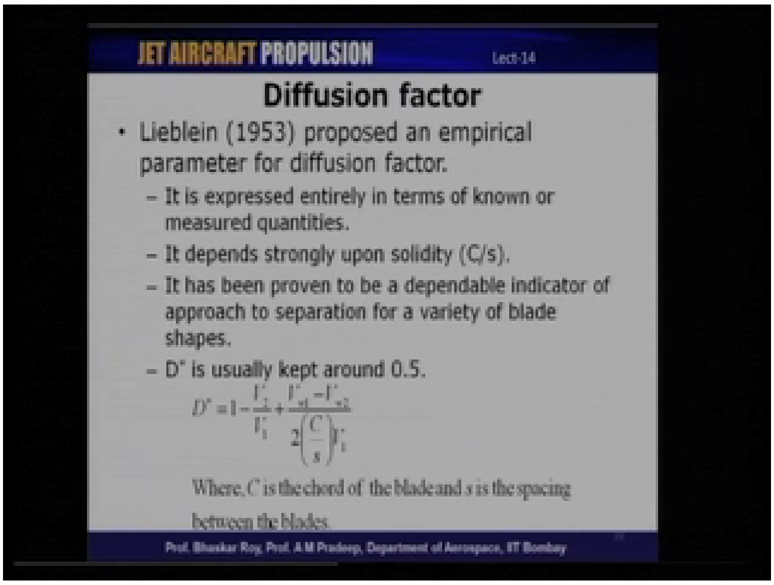

And so in the earlier days when this was sort of identified of course, it was in go back in 1950's Lieblein basically propose an empirical parameter, because V max minus V to by V 1 is something that is in those days it was difficult to calculate and measure from experiments, and they were not have computational tools to measure to estimate these numbers.

So, he came with an empirical parameter for diffusion factor basically tried to express diffusion factor in terms of parameters, which can be measured or it can be known and as we can see as we will see it depends strongly upon the solidity which is basically the cord to the spacing ratio, and over the years it has been proven to be a sort of a dependable parameter indicator of the approach to separation for a variety of blade shades.

So, diffusion factor according to Lieblein is defined as 1 minus V 2 by V 1 plus V w 1 minus V w 2 divided by 2 C by s into V 1. So, here we can see that all these parameters can be known based on velocity triangles the velocity components can be known and therefore, it is possible for us to calculate the diffusion factor given velocity triangles, and if the cord and is spacing are known.

So, this is much more handier expression to calculate the diffusion factor, and also estimate how much it is for a given design, and so what has been observed is that diffusion factor around 0.5 is considered to be a safe number, and as we exceed diffusion factor of 0.5 there is a I mean a threat of a the chance of blade stall that might occur. But, of course, modern day design or tools like computational tools etcetera have helped us in trying to exceed these limits to the extent possible.

So, diffusion factors is we parameter that we have seen is one of the other design parameters, which will tell us how close our design is or what are the chances that the blade might stall given a certain design conditions.

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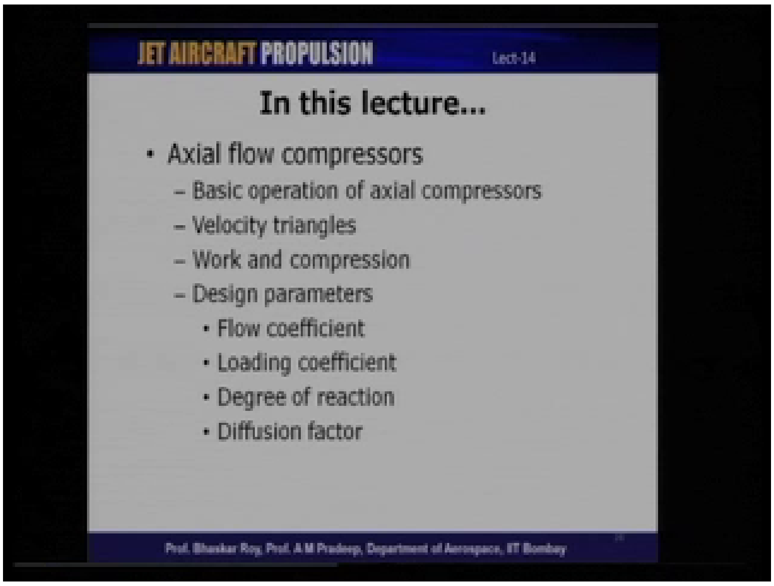

So, let us take a relook at what we had discussed in today's class; today's class was basically an introductory class on lecture on axial flow compressor basic operation of axial compressors. We have discussed about velocity triangles, and how we can construct a velocity triangle for a given configuration. And then we have also derived expression for work done on the fluid by the compressor and pressure ratio as related to the geometric parameters, and like the plate speed, and delta C w and so on.

And so towards the end of the lecture we have also discussed about a few design parameters like the flow coefficient the loading coefficient the degree of reaction, and the diffusion factor which are basically parameters which are used during the initial stages of design of axial compressors.

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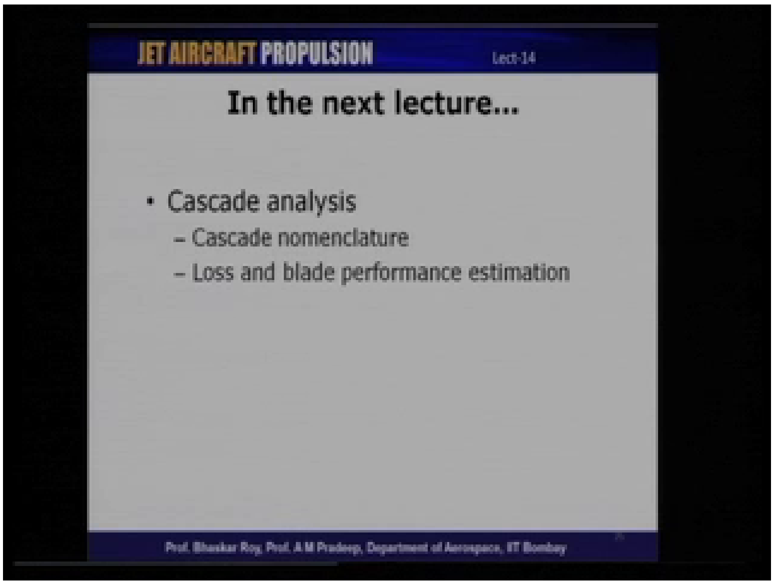

So, we will continue discussion of our study of axial compressors in the next lecture as well, where we will basically we will be talking about what is meant be a cascade, and cascade analysis which will involve a nomenclature of different parameters of cascade. We will also be talking about certain loss parameters which are used in performance analysis, and we will take up some of these topics for discussion during the next lecture; subsequently we will also take up discussion of performance of single stage, multi stage characteristics in later lectures. So, let us discuss these topics during the next lecture.