Jet Aircraft Propulsion

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Lecture No. # 13

Thermodynamics of Turbines

We are talking about thermodynamics of compressors and turbines. One of the things that we talked about in the last class is that compressor basically is a aero dynamic machine, so is turbine. However, their performance is to a large extent decided by certain thermodynamic aspects in so far as both the compressor and the turbine are part of a heat engine, the jet engine. And as a result of this, it is necessary that we always keep one eye on the thermodynamic aspects of the functioning of compressor and turbine. And because of that, we decided to take a little closer look at some of the thermodynamic aspects of the compressor and the turbine. You are doing the whole thermodynamics of cycle analysis involving the entire engine including compressors and turbines.

In these two lectures, we just have a little closer look at some of the aspects of thermodynamics of compressors and turbines. And later on of course, we will be doing a little more details of the aerodynamics of compressors and turbines in the following lectures. So in the last class, we did a little bit of thermodynamics of compressors under various kinds of fluid flow in so far as the compressors, turbine in jet engine involving flow in fluid at fairly high speeds actually. And hence we looked at some of those aspects, we looked at the thermodynamics of compressors from static parameter change point of view as well as from stagnation parameter change point of view and we could see that there are certain differences in the way the thermodynamics may need to be handled or calculations, preceded taking into account that some of the changes are in terms of the stagnation parameters. In which case, if you want to make a little more rigorous analysis of compressors turbines, and other such components, you may like to adopt those methods, those are more involved methods, those would indeed take more time for computation. And as a result of which for simple handy calculations you may not like to them, but you want to do more rigorous calculations, you may like to adopt those methods.

Today, we will look at thermodynamics of turbines. In the same way in that we did in the last class, we look at the turbine flow as it is as part of a jet aircraft engine. And we see how thermodynamics affects certain aspects of the turbine functioning, and we also know that thermodynamic processes can be categorized under various process titles; and we will take a look at how if these processes are invoked for turbine operation, how the turbine operation may behave slightly differently from each other. So let us take a look at thermodynamics of turbines.

Now, in this particular lecture, where we are talking about thermodynamics turbines, we are not explicitly talking about actual turbines or radial turbines the difference of those things are probably known to you, and we will be doing the details of those things in the following chapters. So today, we will take a look at simply what we call a turbine without bothering about whether it is a actual turbine or a radial turbine, whatever we are doing is overall in a generic sense valid for both kinds of turbines. In some specific cases, you might see more resemblance to an actual turbine, which is the more used kind of turbine in various jet engines, and the that will be obvious to you also however, we will keep an eye on all aspects of functioning of turbine.

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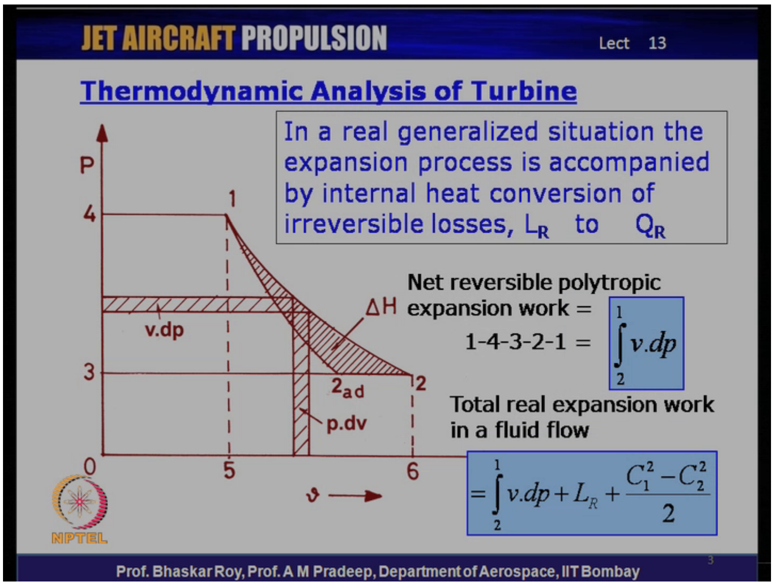

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Thermodynamic Analysis of Turbine

In a real generalized situation the expansion process is accompanied by internal heat conversion of irreversible losses, L_R to Q_R

Net reversible polytropic expansion work = 1-4-3-2-1 = $\int_2^1 v.dp$

Total real expansion work in a fluid flow

$$= \int_2^1 v.dp + L_R + \frac{C_1^2 - C_2^2}{2}$$

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If we look at how the turbine functions through to begin with let us say a P V diagram, something which you are already very well familiar with through your cycle analysis lectures. And if we take a case, let us say in which the flow is to begin with adiabatic let us say, which is 1-2 adiabatic. And then compared to that we take a let us a more realistic case, which is a generalized situation, and which we normally call a poly tropic case, which is from 1-2.

So, the different between these two is that extra work delta H that would be required to be spend in extracting the work, which means the when the work is extracted, so much less work would be available for any other kind of work or functioning. Now, which is essentially means that if you are going from station 1 to station 2, this is the kind of extra work that will have to be spent in extraction of the work.

Now, the net reversible poly tropic expansion work, we can say is given by 1-4-3-2-1 that is, 1-4-3-2-1. So this is the area that you would normally be using for figuring out from the P v diagram what could possibly be the reversible poly tropic expansion work, and this we have seen before it is it is normally given in terms of 1-2 v dp and correspondingly the real expansion work of a fluid flow can be written down in terms 1-2 v dp plus L R, which is the loss normally occurring, and the differential of the kinetic energy at the entry and exit of the turbine.

So those are the terminologies that come when you have a flowing fluid through the turbine as we normally have in jet engines. And those parameters then come into the picture and loss of occur is what occurs during the process of the fluid flow through the turbines. So this is how we can start making a thermodynamic analysis and to begin with we are using the P v diagram.

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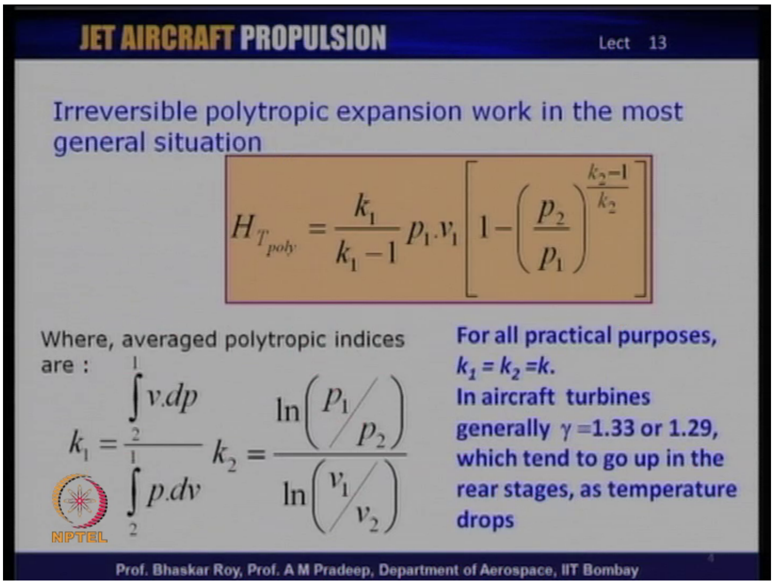

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Irreversible polytropic expansion work in the most general situation

$$H_{T_{poly}} = \frac{k_1}{k_1 - 1} p_1 \cdot v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{k_2 - 1}{k_2}} \right]$$

Where, averaged polytropic indices are :

$$k_1 = \frac{\int_2^1 v.dp}{\int_2^1 p.dv} \qquad k_2 = \frac{\ln\left(p_1/p_2\right)}{\ln\left(v_1/v_2\right)}$$

For all practical purposes, $k_1 = k_2 = k$.
In aircraft turbines generally γ =1.33 or 1.29, which tend to go up in the rear stages, as temperature drops

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If we write down the irreversible poly tropic expansion the earlier one we wrote down was reversible poly tropic expansion in the most general case. We can say that H T poly tropic, and that will be given by k 1 divided by k 1 minus 1 whole thing to the power p 1.v 1, and then whole thing multiplied by 1- p 2 divided by p 1 to the power k 2 minus 1 divided by k 2. Now, we we see here there are two case one is k 1, which we had earlier called in the earlier lecture in the compressors the first mean index, and the k 2, which is called the second mean index, and the 2 hard, 2 slightly different definitions, which we had discussed in the last lecture in connection with the compressor so same thing is valid over here, very vigorously.

If you really wish to do that, you could have possibly two poly tropic indices to handle the situation of a poly tropic process; however, for all practical purposes, k 2 is equal to k 1 is equal to k. Now, in aircraft gas turbines generally the value is 1.33, and it could go down to 1.29 depending on the temperature.

Now, we know that as the temperature operating temperature through the turbines go down. The value of the gamma also tend to go up; when the temperatures are very high that is in the HP turbines the value of gamma is likely to be lower value that is 1.29, but a later on the value of gamma could go up to 1.33 in the LP turbines. So, those are some of the values you may like to use if you want to do a more rigorous analysis; because gamma as well as c p c v, as you knows are dependent on the operating temperature, at that particular location.

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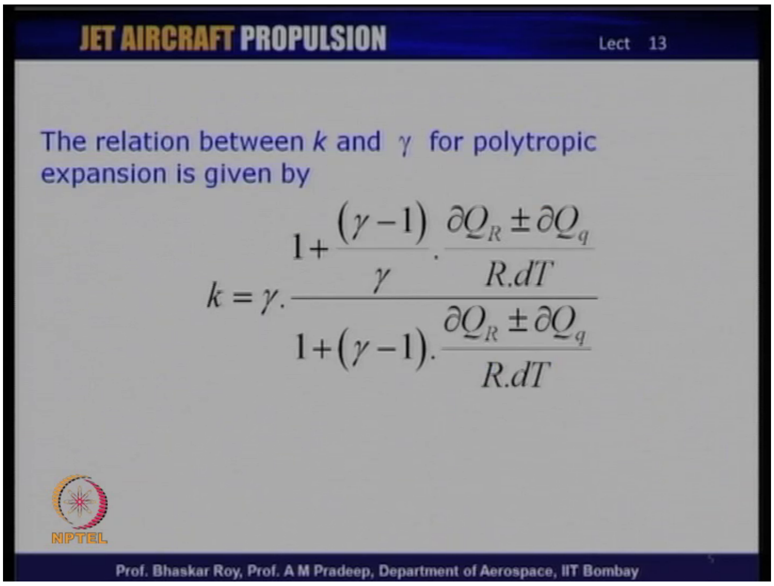

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The relation between k and γ for polytropic expansion is given by

$$k=\gamma.\frac{1+\dfrac{(\gamma-1)}{\gamma}.\dfrac{\partial Q_R \pm \partial Q_q}{R.dT}}{1+(\gamma-1).\dfrac{\partial Q_R \pm \partial Q_q}{R.dT}}$$

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So, the relation between k and gamma for a polytropic process is given more rigorously assuming that k 1 is equal to k 2, and we have one value of k, and that is given in terms of gamma. The heat transfer that is taking place, this could be the internal heat transfer this could be the external transfer coming in or going out T is change of temperature through the process R is the gas constant and gamma of course, is the ideal specific heat ratio. So, if you use this relation, you would get a k a poly tropic index for that particular operating condition involving all the heat transfers that may be taking place internally or externally.

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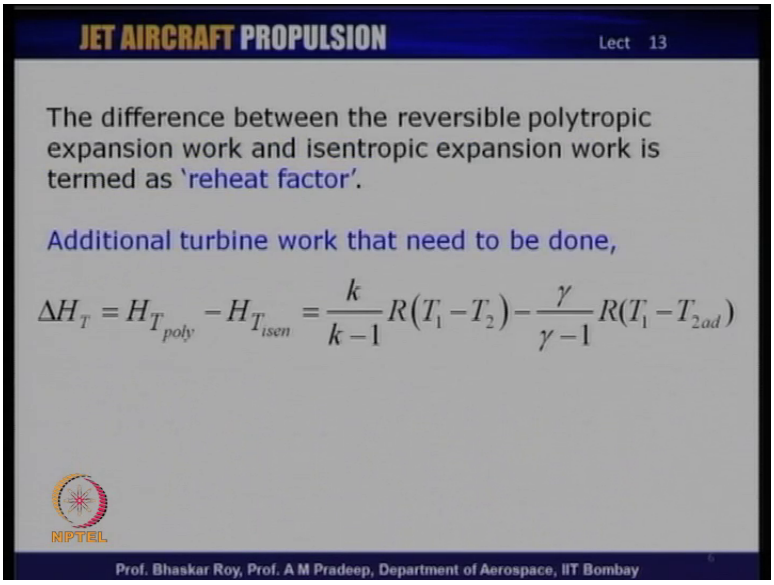

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The difference between the reversible polytropic expansion work and isentropic expansion work is termed as 'reheat factor'.

Additional turbine work that need to be done,

$$\Delta H_T = H_{T_{poly}} - H_{T_{isen}} = \frac{k}{k-1} R(T_1 - T_2) - \frac{\gamma}{\gamma - 1} R(T_1 - T_{2ad})$$

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Now, the difference between the reversible poly tropic expansion work, and the isentropic expansion work is often termed the re-heat factor. Now, this means that if you are to do same amount of actually expansion work, the additional turbine work that need to be done is given in terms of delta H T and that H T polytrophic minus H T isentropic and that is equal to k by k minus 1 multiplied by R multiplied by T 1 minus T 2 minus gamma by gamma minus 1 R T 1 minus T 2 adiabatic. So, the difference of the two can be the extra work that the turbine needs to do to extract the work from the high energy gases, which means of course that that much less work would be available on the shaft for supply to the compressor or to any other functionary in the jet engine. We will have a look at this reheat factor again in a few minutes.

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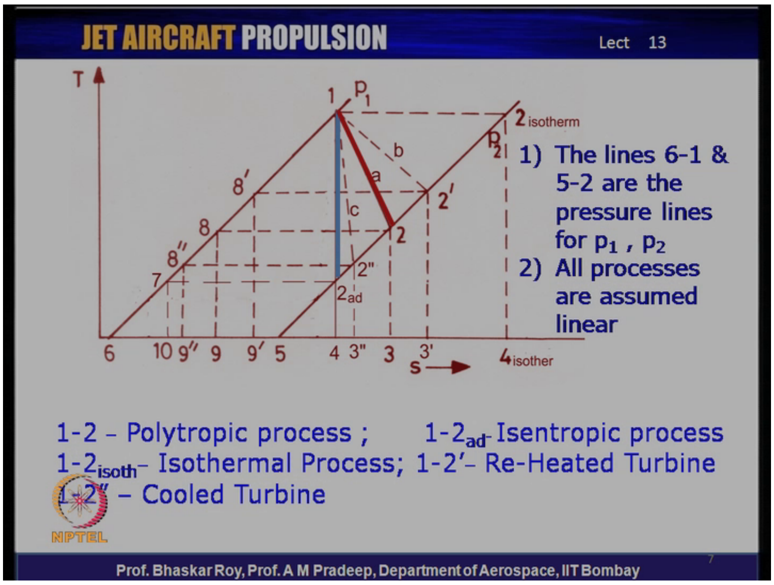

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1) The lines 6-1 & 5-2 are the pressure lines for p_1 , p_2

2) All processes are assumed linear

1-2 – Polytropic process ; 1-2_{ad}- Isentropic process

1-2_{isoth}– Isothermal Process; 1-2′– Re-Heated Turbine

1-2″ – Cooled Turbine

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If you now look at the T S diagram typically of a turbine, and let us say it is operating from 1 to 2. So it is operating between pressure P1 and P2, the two pressure lines within which this turbine is said to be operating. We also assume that the processes that are being shown here all linear path they follow a linear path, it is entirely possible that they may not follow linear path in fact many of the poly tropic processes do actually follow long linear slightly curvilinear path also the pressure lines that are shown here are actually shown as linear lines indeed in a T S diagram, the pressure lines may not be linear, also they may not be parallel they are slightly curvilinear and they are slightly diverging, now for the sake of our simple analysis here.

We assume that all of them are linear and the path followed by the turbine is also following a linear path from 1-2. So, 1-2 is a realistic poly tropic process what we call thermodynamically a poly tropic process on the other hand 1-2 adiabatic is the so called isentropic process, which is vertical line from 1-2 adiabatic there are slightly bluish line.

And then the 1-2 isotherms the horizontal line the dotted line is the isothermal process, and then we have 1-2, which is a re-heated turbine process. And then we have 1-2, which is one may call for a absence of any other terminology 1-2 as a cool turbine. As you will probably learn very soon that quite often many of the turbines are indeed subjected to cooling for (( )) purposes, and hence those also have certain thermodynamic ramifications. So, these are the various possibilities obvious possibilities that could occur in a actual gas turbine of jet engine. And we will see how these things actually affect the operation of the turbine. Now let us take a look at the situation let us say 1-2, which is a poly tropic process, what we call a realistic poly tropic process the case a.

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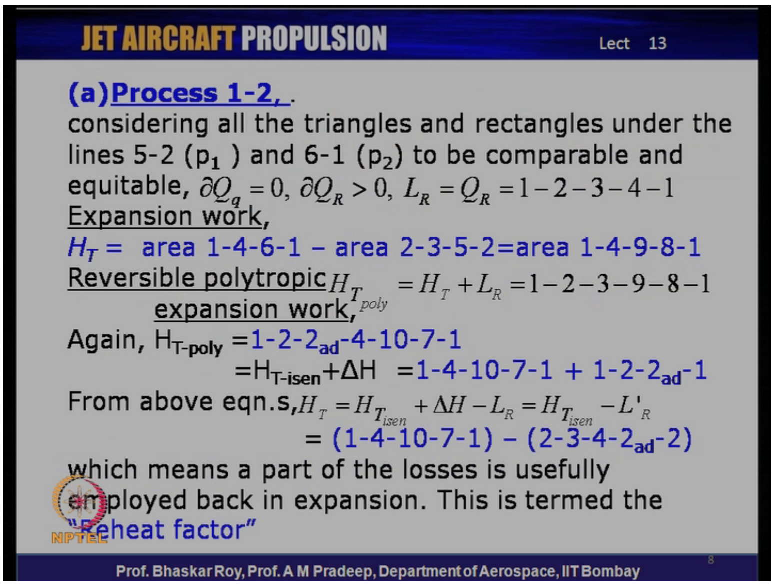

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(a)Process 1-2, .

considering all the triangles and rectangles under the lines 5-2 (p_1) and 6-1 (p_2) to be comparable and equitable, $\partial Q_q = 0,\ \partial Q_R > 0,\ L_R = Q_R = 1-2-3-4-1$

Expansion work,

$H_T =$ area 1-4-6-1 – area 2-3-5-2=area 1-4-9-8-1

Reversible polytropic expansion work, $H_{T_{poly}} = H_T + L_R = 1-2-3-9-8-1$

Again, $H_{T\text{-}poly}$ =1-2-2_{ad}-4-10-7-1

=$H_{T\text{-}isen}+\Delta H$ =1-4-10-7-1 + 1-2-2_{ad}-1

From above eqn.s, $H_T = H_{T_{isen}} + \Delta H - L_R = H_{T_{isen}} - L'_R$

= (1-4-10-7-1) – (2-3-4-2_{ad}-2)

which means a part of the losses is usefully employed back in expansion. This is termed the "Reheat factor"

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Now, if you look at the case a, and we have already assumed that all the paths are actually linear. So, we have triangles and rectangles to be considered; and we say that the heat transfer from external sources is 0. However there is a internal heat transfer, which is normally in terms of conversion of one kind of energy to heat principally the kinetic energy may get converted to heat due to friction and other reasons, and that is normally given in terms of L R equal to Q R and that is as we have said non-zero and that is given in terms of 1-2-3-4-1.

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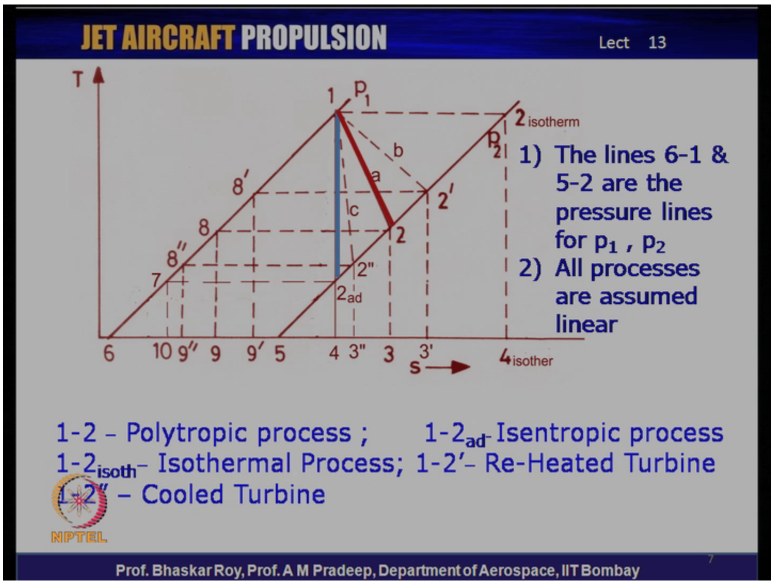

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1-2 – Polytropic process ; $1\text{-}2_{ad}$- Isentropic process
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So, let us look take a look at that so 1-2-3-4-1 so that is the area, which is typically one may say the losses that are occurring and that is a amount we say that it is internally converted to heat within the fluid flow system. The expansion work H T for this particular process may be given in terms of area 1-4-6-1. So, if we look at that area that is 1 all the way down 4-6, and then one so that is a triangle, which is the first area minus the area 2-3-5-2 and that is 2-3-2 is the end of the process so all the way down 3, and then 5-2 another triangle, so this triangle minus this triangle is the work that is expansion work that is being hope to be extracted, and hence this would be 1-4-9-8-1.

So that is 1-4-9-8-1 that means this triangle 8-9-6-8 is considered now equal to 2-3-5-2. So that assumption that all the things being linear and parallel to each other we can arrive at that and as a result, which the expansion works is 1-4-9-8-1. Now the reversible polytrophic expansion works H T poly tropic is H T plus let us say the losses, and that is 1-2-3-9-8-1. If we go back 1-2-3-9-8-1 that is the total amount of work, that is being performed including the losses which as I said earlier is 1-2-3-4-1. And as a result of which, these rectangles and triangles give you the reversible poly tropic expansion work. Now the H T poly tropic can also be written in terms of 1-2 adiabatic -4-10-7-1.

So 1-2adiabatic-4 and then 10-7-1 so, if you go along this red line 1-2, then 2-2adiabatic and 2adiabatic down to 4 and then to 10 and to 7-1.

This also of course, from thermodynamic depictions gives you the poly tropic work. So from the above you know relations and the rectangles triangles that we have considered we can write that the H T the work that is extracted is H T isentropic plus delta H minus the losses, losses of course, are not available to for functioning of anything, and which would be then H T isentropic minus the losses, which we can now write in terms of 1-4-10-7-1, which is 1-4-10-7-1 that we have shown earlier.

And then that minus 2-3-4-2 adiabatic to 2, so 2-3-4-2 adiabatic to 2; so this is the rectangle which gets subtracted from the earlier rectangle and that is your work done. So which means a part of these losses is somehow getting usefully employed back in the process of expansion, and this is what is often termed a reheat factor, which we mentioned a little earlier that some of the losses that occur in a turbine do kind of transform themselves in the form of heat internally, and if that happens then that heat is available for expansion purpose and one can say that it is gainfully employed for the purpose of expansion. So that is what that may happen in a poly tropic process.

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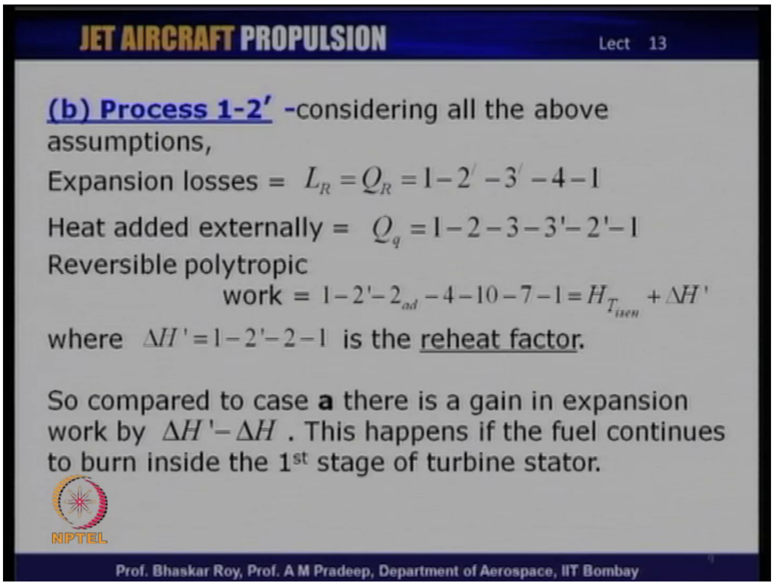

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(b) Process 1-2′ -considering all the above assumptions,

Expansion losses = $L_R = Q_R = 1-2'-3'-4-1$

Heat added externally = $Q_q = 1-2-3-3'-2'-1$

Reversible polytropic work = $1-2'-2_{ad}-4-10-7-1 = H_{T_{isen}} + \Delta H'$

where $\Delta H' = 1-2'-2-1$ is the reheat factor.

So compared to case **a** there is a gain in expansion work by $\Delta H' - \Delta H$. This happens if the fuel continues to burn inside the 1st stage of turbine stator.

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Now let us take a look at the processes b which is a 1-2’. Now we continue with the assumption that we had made earlier, and we can say that the expansion losses are now we call expansion as loss L capital L, and that is also equal to Q R that is internal heat that is added to the system, and this would be 1-2’-3’-4-1. So if we go back to the diagram so it is 1-2’-3’-4-1. So, as you can see this area is going to be larger than the earlier area 1-2-3-4-1.

And this is what will impact our calculations. And the heat let us say now being added externally is 1-2-3-3’-2’-1 now this is the heat that is being added externally, and this is a process 1-2’ in which heat is been added externally that is 1-2-3-3’-2’-1. So, 1-2-3-3’-2’-1 so this if you take this area that is this rectangle plus this rectangle plus this triangle on top, So that is the additional work or heat that is being added to the system, because of the external influence. And that is what is appearing as a additional area and hence the reversible poly tropic work can now be written in terms of 1-2’-2adiabatic-4-10-7-1 as we had done before and that will be H T isentropic plus delta H’, so 1-2’-2adiabatic-4-10-1

As we had done before 1-2’-2 adiabatic -4-10-7-1 so same things, we have done for the case one case a. And as a result of which we can see that reversible polytrophic work will be somewhat different now. And this delta H’ 1-2’-2-1 is the reheat factor for this particular case in which certain amount of extra heat has been influenced from external source. So compared to case a there is a likely hood of gain in expansion work by delta H’ minus delta H.

Now this happens if the fuel continues to burn inside the nozzle of the turbine in case of turbine as you know the nozzle of the stator comes before the rotor. So if there is a certain amount of heating going on inside the nozzle this would add to the reheat factor and as a result, an additional expansion possibility would come up. This also opens up the possibility reheating between the turbines or inter heating between the turbine stages after the H P as you know it loses a lot of its energy in the process of doing work and there is a provision now that if you manage to do a little bit of reheating between the L P and the H P so before it heats L P if you do that reheating. This clearly shows that you can now expect to get more work extraction from the L P than before and this opens up the possibility of reheat inter reheat between the H P and the L P.

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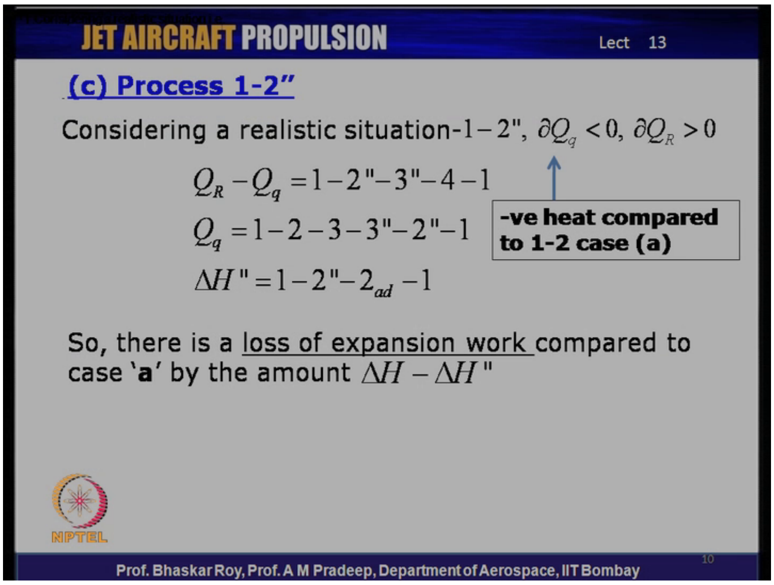

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(c) Process 1-2”

Considering a realistic situation-$1-2''$, $\partial Q_q < 0$, $\partial Q_R > 0$

$$Q_R - Q_q = 1-2''-3''-4-1$$

$$Q_q = 1-2-3-3''-2''-1$$

-ve heat compared to 1-2 case (a)

$$\Delta H'' = 1-2''-2_{ad}-1$$

So, there is a loss of expansion work compared to case ‘**a**’ by the amount $\Delta H - \Delta H''$

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Let us take a look at the process c which is processed 1-2’’. Now this is a realistic situation in most of the of the modern jet engines because, most of the modern jet engines are indeed slightly cooled essentially to increase the life of the blades. And in this process as we can see a negative heat is you know applied and that is delta Qq is now < 0 however, there is always the possibility that delta Q R that is the internal heat heating due to friction and other things would always be > 0 so this different between these 2 can be given in terms of 1-2’’-3’’-4-1 so, let us take a look at the picture. So 1-2’’ that is a dotted line down here and 3’’-4 and then 1 so that is the net heating that may be going on different between the heat that is taken out by cooling and the heat that is added by may be friction internally.

Now the result is that we can write down the delta H’ the Q heat that is been taken out is the negative heat is can be written down also from 1-2-3-3’’-2’’-1 that is shown in the diagram. And we will go back to the diagram with this delta H’’ which is now 1-2’’-2adiabatic-1.

Now this means that 1-2’’-2adiabatic-1, so this is the amount of work. Let us say that is finally, actually not being available to the turbine for work. That the work has to be done or expanded the energy has to be expanded. So, there is a loss of expansion work compared to case a, that is the different between delta H and delta H’’ so, when you are applying cooling for improving the life of a turbine, you are making a small sacrifice that the amount of work that would be available poly tropically, now would be slightly on the lower side, so, this is to be done in a knowledgeable manner and this allows you to make a reasonable calculation and a priory knowledge of how much you might be sacrificing in the process of applying cooling to the turbine blades.

So those are the various obvious processes that occur in a typical jet engine gas turbine and we should, we would now like to look at a realistic slightly more realistic gas turbine now. You see we were looking at a turbine going from process 1 to process 2 the point is no where there we talked about flowing fluid. The process in actual turbine is a flowing fluid it has very high kinetic energy with, which it goes in and it is even higher kinetic energy with which it flows out.

And the net kinetic energy with which it flows through the turbine is often quite high. So, some whole other that kinetic energy component needs to be brought in to the picture. The simplistic thermodynamic analysis we have just done does not take that into account it simply goes from process 1-2 without really taking into account the kinetic energy components at station 1 and at station 1 or what is happening in between.

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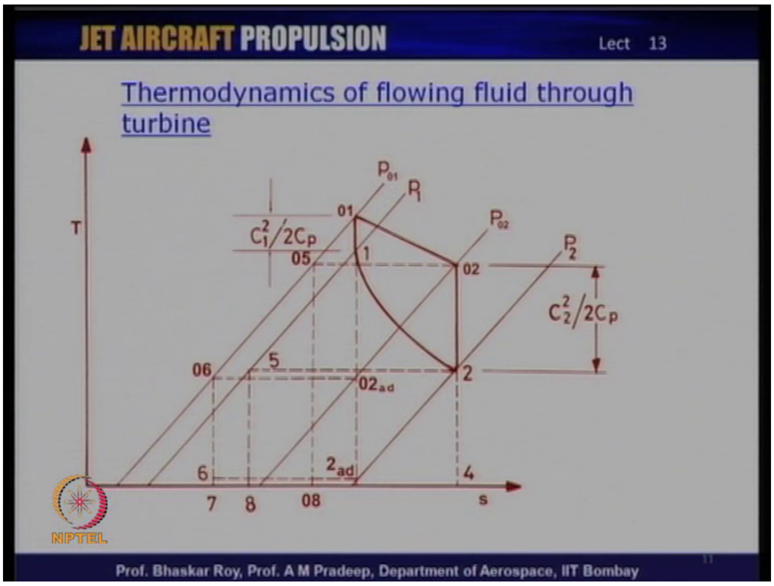

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Thermodynamics of flowing fluid through turbine

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Let us take a look at that now including the fluid flow effects if you look at this again a T S diagram, now we have a turbine in which its flowing from 1 to 2, but at station 1 it has a kinetic energy head of the order of C 1 square by 2Cp. This is something you have done in your cycle analysis quite a lot, so we will just use that straight away, and say that its going from station 1 to station 1.

Now between station 1 and 2 as you have done in your cycle analysis, what happens is it flows from point 1 to point 2 and at point 2 it has a higher kinetic energy, which is given by let us say C 2 square by 2Cp a much larger kinetic energy head actually. And at this station you have value 02, which is the total head so the total process has moved from P01 to P02 that is a pressure drop across the turbine, and that is what turbine actually does. And in the process the static pressure is dropped from P1 to P2. So that is, what is overall happening across a turbine that is going into the turbine with a kinetic energy.

And it has static values and stagnation values we can talk in terms of temperatures also. And then of course, it is it is going to station 2 that is the exit of the turbine, where it has a static value and a stagnation value. And of course, it has the kinetic head, corresponding adiabatic process would be you know 0 to adiabatic on the line P02 and if you take two adiabatic on the line P2 that will be right down here. So, straight down along the dotted line would be the adiabatic process or the isentropic if it is a reversible process.

So, those are the issues that you would probably looking at if you have a flowing fluid through the turbine taking into account, the entry and exit kinetic heads. Now what happens is the flow through the turbine is indeed a little more a little more complex. A turbine stage typically has a stator and a rotor, the flow first goes through the stator with a large enhancement in kinetic energy conversion of potential energy to kinetic energy. And then of course, it goes to the rotor in the process of doing a lot of work it gives up lot of energy to the turbine for doing a lot of work. Now, we should try to bring that in to the thermodynamic diagram; so let us take a look at that which is a step forward from that we have just now.

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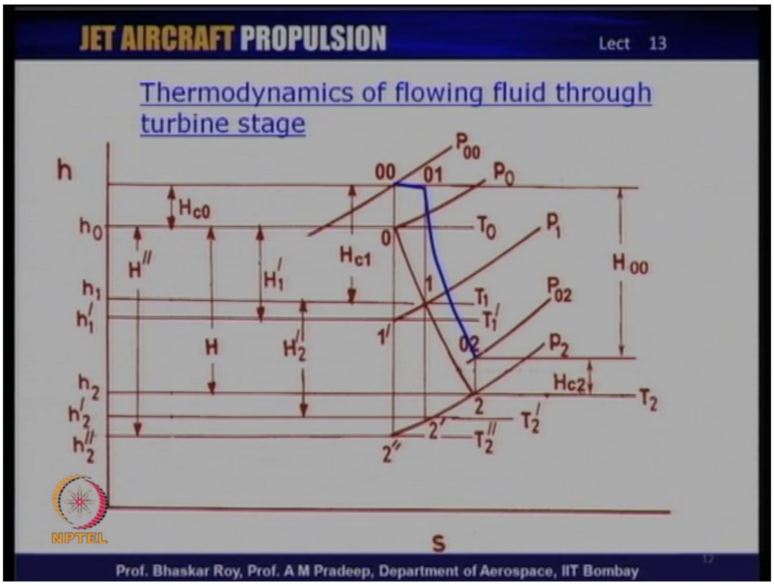

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Thermodynamics of flowing fluid through turbine stage

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So, this actually shows the flowing fluid through the whole turbine stage, and it shows that the flow now we have three stations let us say station 00, then station 01 and station 02, 00 is where the flow is entering the turbine, and it has a kinetic energy head given by Hc0, let us say and then across the stator it acquires a large kinetic head given by Hc1 and so, 01 is over here and 1 is down here. So, it is moved from 0 to 1 on the static map, so 012 is your static head h s diagram.

We are now showing h s diagram, which as we have discussed before is similar to T s diagram, and so we have moved from 0 to 1 to 2 on the static parameter on the total parameter 00 to 01 that simply depicts that no work has been done. And hence the total head parameters remain conserved from 00 to 01, so we are bringing in the things that you have done in your cycle analysis.

To look at the thermodynamic diagram in connection with what we have just talked about. Now we have from 01 to 02 that is where the work is being done, that is the flow through the rotor. And as a result of it, at the end of it, the kinetic energy head goes down from Hc1 to Hc2 much lower. So it gives up a lot of energy including, lot of kinetic energy, it gives up a lot of pressure, it indeed gives up a lot of enthalpy as you can see here, one can talk in terms of temperature. So, it falls from this energy level to this energy level and this kinetic energy level Hc1 to kinetic energy level Hc2 in all of it should be possible to be shown on this thermodynamic diagram.

So, if you can show all of it, then you have covered all the ground that is happening thermodynamically inside a turbine. Now, we say that H00 is the total head work done from station 00 to 02, so that is the work that can be said to be done on the basis of stagnation parameter change on the other hand H is the work that is done from 0 to 2 based on the static parameter change from 0 to 2 the 2 may not be equal to each other.

If Hc0 is not equal to Hc1, if they are equal to each other, then this H the static work would be equal to the total work H00. So, that is what the thermodynamic tells us some are on the left you can a H’’ that is nothing but the isentropic work done from the station 0 to station 2’’ that is vertically dropped from 0 to 2’’, and that is the thermodynamic ideal work done in isentropic process.

So as you can see here, now we it is possible for us to show more or less everything that is happening inside a gas turbine engine, and that you should be able to show everything both in terms of thermodynamic process as well as in terms of gas dynamic process. So the aerodynamics or gas dynamics that is happening is also quite clearly shows able over here in terms of thermodynamic parameters.

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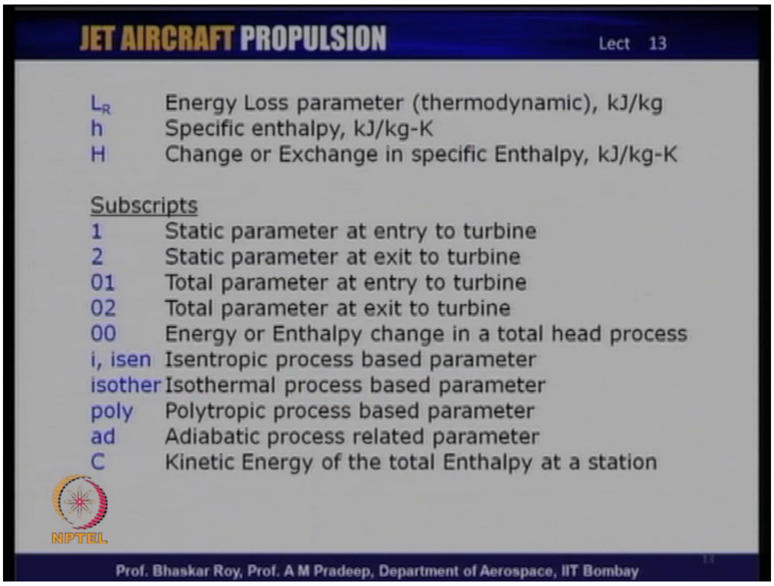

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L_R	Energy Loss parameter (thermodynamic), kJ/kg
h	Specific enthalpy, kJ/kg-K
H	Change or Exchange in specific Enthalpy, kJ/kg-K

Subscripts

1	Static parameter at entry to turbine
2	Static parameter at exit to turbine
01	Total parameter at entry to turbine
02	Total parameter at exit to turbine
00	Energy or Enthalpy change in a total head process
i, isen	Isentropic process based parameter
isother	Isothermal process based parameter
poly	Polytropic process based parameter
ad	Adiabatic process related parameter
C	Kinetic Energy of the total Enthalpy at a station

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The parameters that we talked about are listed over here, so we have talked about energy loss parameter, which we had talked said as L R, h is a specific enthalpy and H is the change in specific enthalpy across one station to another and these are the subscripts 1 is the np, 2 is the exit and 01 is the total parameter at np 02 is the total parameter to exit.

00 is the normally the enthalpy or energy change in the total head process of course, the isentropic process isothermal is isothermal process poly is a poly tropic process and ad is the adiabatic process quite often we talked about adiabatic reversible process, and C is the kinetic energy of the total enthalpy at the station.

As we have talked about Hc0, Hc1, Hc2 etc. In this diagram you can also see here H1' H1 H2' which are the adiabatic work from 0 to 1 and then from 1 to 2. So, if you separate out the two components the rotor and stator the adiabatic work across the stator is 0 to 1 and then across the rotor is from 1 to 2', so the two can be also separated out if you wish to. So we have gone through the whole system of turbines looking at the rotor stators and what is happening across the entire turbine.

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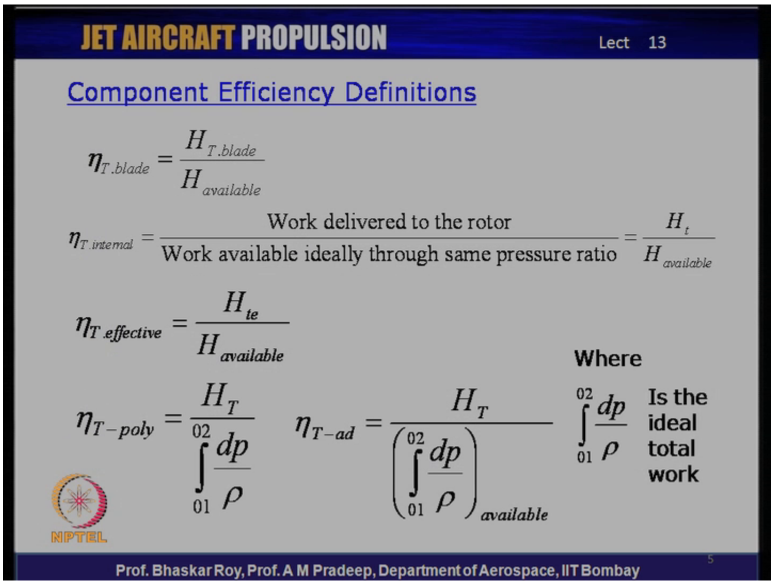

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Component Efficiency Definitions

$$\eta_{T.blade} = \frac{H_{T.blade}}{H_{available}}$$

$$\eta_{T.internal} = \frac{\text{Work delivered to the rotor}}{\text{Work available ideally through same pressure ratio}} = \frac{H_t}{H_{available}}$$

$$\eta_{T.effective} = \frac{H_{te}}{H_{available}}$$

$$\eta_{T-poly} = \frac{H_T}{\int_{01}^{02}\frac{dp}{\rho}} \qquad \eta_{T-ad} = \frac{H_T}{\left(\int_{01}^{02}\frac{dp}{\rho}\right)_{available}}$$

Where $\int_{01}^{02}\frac{dp}{\rho}$ Is the ideal total work

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We can now summarize by simply writing down the component efficiencies of the various components; the blade efficiency can be simply written in terms of the H T blade by the Ht H that is available, as we have seen lot of losses may occur across the blade and all of it may be not be available, and that may be simply given as blade efficiency. One can actually even write down a sectional efficiency of a particular section of a blade through this efficiency definition. The eta T internal of course, is the work delivered to the rotor or the rotor shaft as opposed to the work availability ideally through the same pressure ratio, now in this is given as H available, and this is of course, the H that is available to the rotor. The effective efficiency is the H T effective turbine work that is effective with reference to H available, and this includes the some of the losses that we have talked about and a slightly different efficiency can be configured out of those values if they are available.

The poly tropic efficiency that we have talked about, and we have talked about a lot of it in your cycle analysis can be also written down in terms of H T divided by integral of 01 to 02 dp by rho, and corresponding adiabatic efficiency can be written down in terms of H T divided by integral 01 to 02 dp by rho as the work available.

Now 01 to 02 dp by rho of course, is the ideal total work based on the total heads 01 to 02 that is ideal work that may be available to the turbine for doing its work. So, these are the fundamental thermodynamic deficiencies from thermodynamic point of view. One can make use of them as you wish to for configuring the various processes that are occurring inside a turbine.

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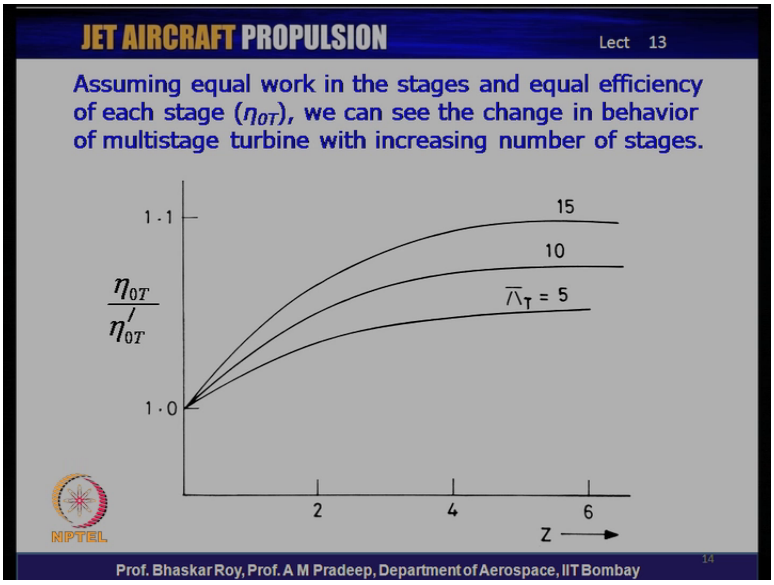

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Assuming equal work in the stages and equal efficiency of each stage (η_{0T}), we can see the change in behavior of multistage turbine with increasing number of stages.

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If you now take into account the fact that we have equal work in other stages and equal efficiency of each stage of a multi-stage turbine as you know most of the turbines in a jet engine are multi-stage turbines, with we can see that with the increasing number of stages the efficiency that you get it from corresponding to the ideal efficiency.

Actually, you start from one as you keep on increasing the number of stages; actually the efficiency goes up and with a increase of pressure ratio across the turbine the efficiency actually goes up. So, if you have a large number of stages or multi-staging and correspondingly, you are able to get more and more pressure drop the efficiency of the turbine system actually goes up. So instead of trying to get a lot of pressure drop across one single stage; if it is possible to spread it across number of stages, you get more efficiency advantage, however that needs to be practically balanced against the weight of the engine and space of the turbine that it takes, and quite often this kind of optimization indeed needs to be done to get the final turbine configuration. So, this what we are showing is purely from thermodynamic point of view.

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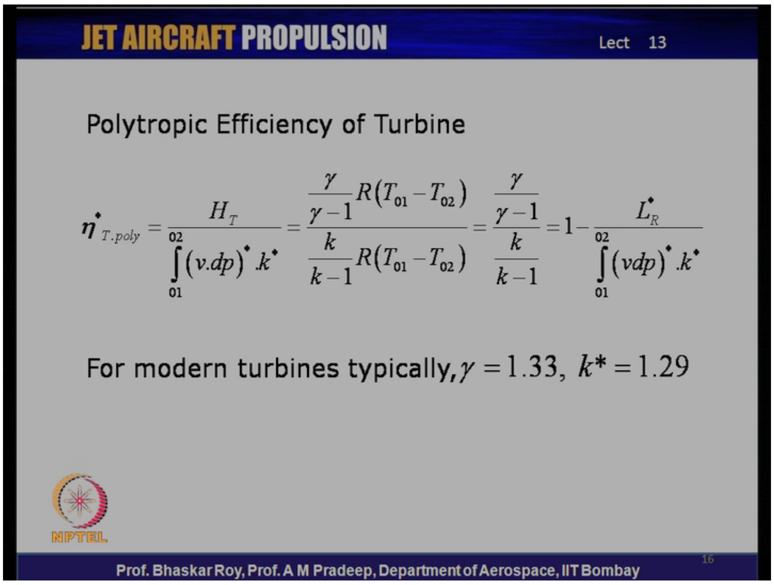

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Polytropic Efficiency of Turbine

$$\eta^*_{T.poly} = \frac{H_T}{\int_{01}^{02}(v.dp)^* .k^*} = \frac{\frac{\gamma}{\gamma-1}R(T_{01}-T_{02})}{\frac{k}{k-1}R(T_{01}-T_{02})} = \frac{\frac{\gamma}{\gamma-1}}{\frac{k}{k-1}} = 1-\frac{L_R^*}{\int_{01}^{02}(vdp)^* .k^*}$$

For modern turbines typically, $\gamma = 1.33,\ k^* = 1.29$

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So, the poly tropic efficiency as we have seen before can also be written down in terms of the work that is being done v dp, and here we are showing in terms of star, now eta T star is the design poly tropic efficiency, as we have seen turbine, compressor and many of the other components are designed at a particular operating condition of the jet engine. And this is been shown in terms of those particular operating condition of T 01 minus T 02 that is the operating condition which for which this turbine is being designed, and we are looking at the design efficiency of the turbine, in terms of the operating condition as specified by the designer. And in this case, the value of k design k star is given as 1.29 typically in most of the modern turbine, which actually works at reasonably high temperatures.

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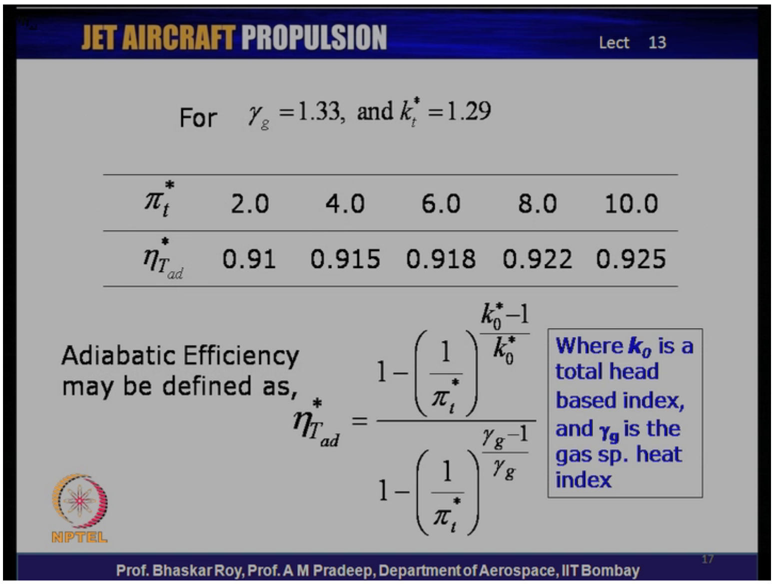

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For $\gamma_g = 1.33$, and $k_t^* = 1.29$

π_t^*	2.0	4.0	6.0	8.0	10.0
$\eta^*_{T_{ad}}$	0.91	0.915	0.918	0.922	0.925

Adiabatic Efficiency may be defined as,

$$\eta^*_{T_{ad}} = \frac{1-\left(\frac{1}{\pi_t^*}\right)^{\frac{k_0^*-1}{k_0^*}}}{1-\left(\frac{1}{\pi_t^*}\right)^{\frac{\gamma_g-1}{\gamma_g}}}$$

Where k_0 is a total head based index, and γ_g is the gas sp. heat index

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So, we have gone through a number of issues and at the end, we can see that if you increase the turbine pressure ratio, there is a efficiency advantage that keeps on happening as I just mentioned that whether or not you need to do with more number of stages, you have to balance against the aircraft requirement in terms of weight and space of the required by the turbines inside the engine.

The adiabatic efficiency may be also written down the design value that we talked about in terms of K0, which we have talked about in detail in terms of the compressor, when we discusses the compressor in the last lecture, where K0 is the total head based index and of course, gamma g is the gas specific heat index ideal value. So if you invoke those values, then you can arrive at adiabatic efficiency definition based on K0 star and gamma g ideal value and of course, pi T star, which is the turbine design pressure ratio as defined by the designer. So, these are the rigorous things you may like to do, if you want to do it in a very rigorous fashion.

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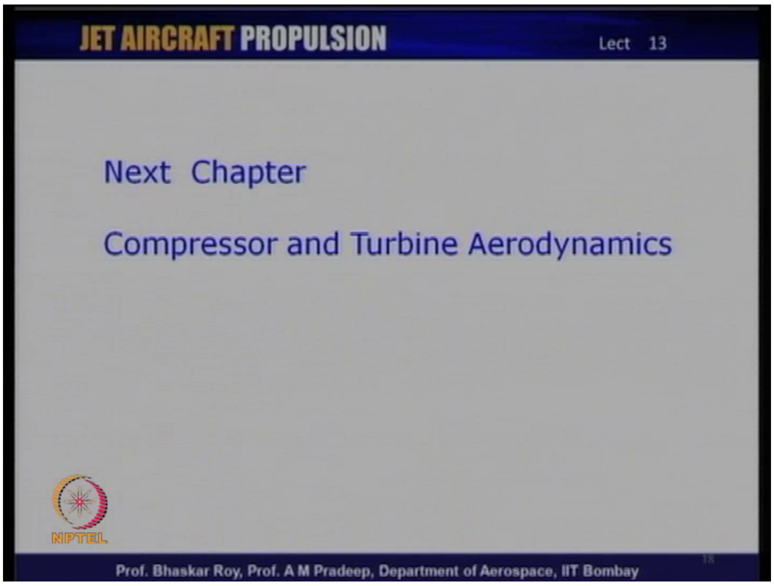

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Lect 13

Next Chapter

Compressor and Turbine Aerodynamics

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We have covered the ground with respect to compressors and turbines; and looked at thermodynamics, you also had a very good look at various aspects of cycle analysis. We had a closer look at the compressors and turbines. And if you wished to go into more rigorous thermodynamic analysis, we had a look at what can be done obviously your computational time and computational procedure will be much longer, if you want to get more and more accurate and rigorous analysis done through these thermodynamic methods.

In the coming lectures, we will be looking now at the various aero dynamic and gas dynamic aspects of compressors, and turbines we will be looking at actual compressors. We will be looking at centrifugal compressors, and then we will be looking at actual turbines and centrifugal turbines, and this time we will be looking at them more with reference to the dynamics, and gas dynamics of the flow through these components of jet aircraft engine.