Jet Aircraft Propulsion

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Lecture No. # 12
Thermodynamics of Compressors

In the last few lectures, you have been exposed to cycle analysis, and the associated aerothermodynamics of flow through the jet engines; various kinds of jet engines, which are used on aircraft for flying. These aero thermodynamics cycle analysis must have been exposed to you all kinds of component analysis, engine analysis as a whole; and this has been covered by professor Pradeep.

In today's lecture, we will take a look at one of the components and that is compressors, and try to look at it from thermodynamic point of view; one of the reasons is that compressor is a working machine, it actually absorbs work supplied by their turbine, and then converts that work to pressurization of air. Now, that is one of the intentions of the compressor inside the gas turbine engine. The other purpose is that the compressor as a working medium is also part of the jet engine, which incidentally is also heat engine. So all affairs connected to any kind of heat engine needs to be analyzed in the overall matrix of thermodynamics. So, any component, which is there inside a jet engine must necessarily be subject to a scrutiny using fundamental thermodynamic laws. And this is what to some extent has been done in great detail through the cycle analysis. And today we will look at thermodynamics of compressors; and in the next lecture we will look at thermodynamics of turbines.

So let us take a look at thermodynamics of compressors. The compressor thermodynamics essentially comes from the fact that you have a flow which is in our case essentially air which is performing work, and this work is available or made available by running the turbine, and the depiction of this work the expression of this work can be put down in form of thermodynamic expressions, which means the work that is transferred from turbine to compressor, and the work that is done by the compressor in the form of compression of pressurization can be expressed in terms of thermodynamic parameters. And this is what we intend to look at a closely today through various methods of fundamental thermodynamic relations.

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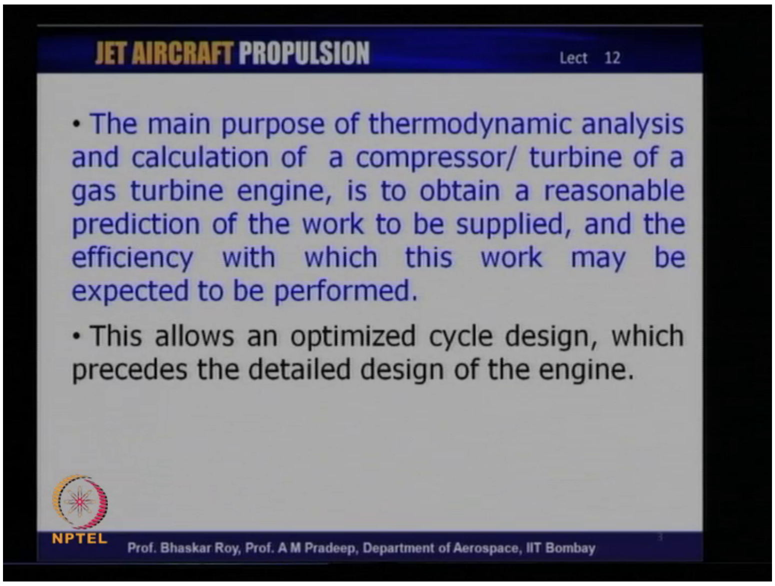

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- The main purpose of thermodynamic analysis and calculation of a compressor/ turbine of a gas turbine engine, is to obtain a reasonable prediction of the work to be supplied, and the efficiency with which this work may be expected to be performed.
- This allows an optimized cycle design, which precedes the detailed design of the engine.

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Let us take a look at fundamental thermodynamics of compressors, one of the reasons I said that we would need to analyze the compressors and of course, later on turbines through thermodynamic means is because it is part of the heat engine and they need to be subject to the scrutiny under thermodynamic matrix, and if we can show that they are amenable to fundamental thermodynamic rules and laws then it is possible to provide a prediction mechanism of the amount of what they are doing the amount of work that is transferred between turbine and compressor, and the efficiency with which this work is being transferred or this work is being done.

Now this is the prediction that is required by the engine designer because, he needs to know how each of these components is going to perform, and then overall how the engine is going to perform. So, this prediction mechanism that thermodynamics can provide is an important aspect of the development and creation of the engine. So this is one of the reasons why we need to have a good look at the thermodynamics of compressors and turbines; this also allows another important aspect of creation of the engine, and that is it allows for optimization of the cycle design, you have done all the cycle analysis over the last few lectures much of this cycle cycle analysis would be subjected to an engine design effort and during that process is inevitable.

Now, a days that an optimization would need to be arrived at so the final design is invariably an optimized cycle design which is the fundamental basis of the development of the engine.

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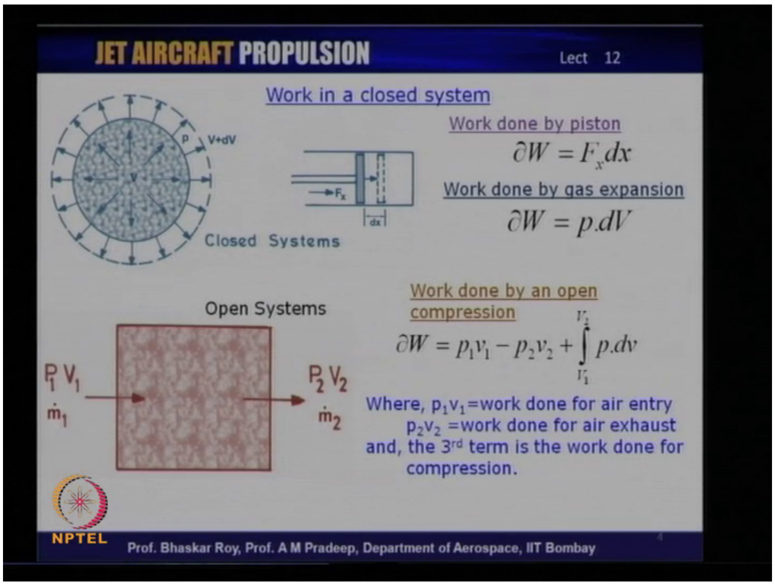

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Work in a closed system

Work done by piston

$$\partial W = F_x dx$$

Work done by gas expansion

$$\partial W = p.dV$$

Closed Systems

Open Systems

Work done by an open compression

$$\partial W = p_1 v_1 - p_2 v_2 + \int_{v_1}^{v_2} p.dv$$

Where, p_1v_1=work done for air entry
p_2v_2 =work done for air exhaust
and, the 3rd term is the work done for compression.

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So this optimization is another possibility that thermodynamic analysis throws up let's take a look at some of the fundamental issues, if we have a simple compressor in which you have let us say to begin with a closed system something which many people are familiar with you have a closed system in which work is expected to be done, either by let us say a movement of a piston or by expansion of gases.

It could be negative expansion and the work done can be expressed in terms of the motion of the piston or the change of volume of the enclosed gas. Now, those expressions are written down there, and it it shows that for a closed system it is it is comparatively simple probably to write down the work that needs to be performed to do the process of compression, now in open systems the work done by an open system or a open compression is a little more involved to begin with the flow is coming into a system which is let us say the square red system that we have and its coming in with a pressure p 1 volume v 1 and it has a mass flow rate m.1, and it is exiting the system with pressure p 2 volume v 2 and a mass flow rate m.2.

It is possible under many systems m1 would be equal to m 2 but that is a separate issue the work done by this kind of an open system in which it is flowing the fluid is flowing its coming in and going out on a continuous basis. The work done is expressed in terms of p 1 v

1 minus p 2 v 2 plus integral of v 1 to v 2 of p dv. Now, the third term is the work done for the compression process the first term is the work done for the air entry; and the second term is the work done for air exiting the system.

So in a open system it enters the system with a certain amount of work, it exits a system with a certain amount of work, and during the process of its resident time within the system a certain amount of work is done, and in our case for the purpose of compression which means in the process of the flow going from 1 to 2 it experiences compression or pressurization. So this is a simple way of writing down how the work can be expressed for a compression process.

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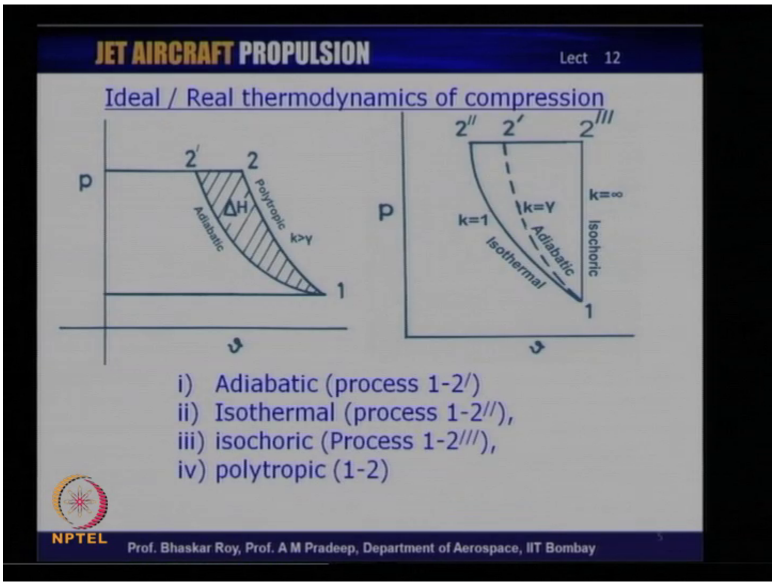
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Ideal / Real thermodynamics of compression

i) Adiabatic (process $1-2'$)
ii) Isothermal (process $1-2''$),
iii) isochoric (Process $1-2'''$),
iv) polytropic (1-2)

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If we show the thermodynamic process of compression in p v diagram something you have already done before in your cycle analysis, let's take a quick look at the same thing may be with a different idea in our mind, the process from 1 to 2 in the p v diagram is typically a polytrophic process a real process the corresponding ideal process is 1 to 2 prime and that is often called the adiabatic process. If it is a reversible adiabatic we would probably call it isentropic process, now these are the two processes to which with which we could start off and the difference between the two is delta h that is the work extra work that is needed to be done.

To accomplish the pressurization from pressure 1 to pressure 2 from line 1 to line 2; and this process of pressurization requires that much of extra work, if the work done is not ideal that is adiabatic but polytrophic now on the right hand side you would see a number of other possibilities in a p v diagram you could have a process which is isothermal in between we have the process which is adiabatic the so called ideal process, and the third process which is a theoretical possibility is an isochoric process, where the specific volume remains constant and the pressurization goes from 1 to 2 triple point. So 1 to 2 double prime is the isothermal process, where the temperature remains constant and1 to 2 triple prime is the isochoric process, where the specific volume remains constant now as you can see.

The index which is normally used for signifying the process, in case of adiabatic process normally this index is gamma, in case of other processes we often might like to call it some k in case of a general polytrophic process non adiabatic process k is likely to be greater than gamma, in case on isothermal process k would be equal to 1 and in case of isochoric process the k would be infinity.

So, these are the theoretical possibilities in which the polytrophic process is a realistic process, which does not confound to any of these adiabatic isothermal or isochoric theoretical possibilities but a general polytrophic process.

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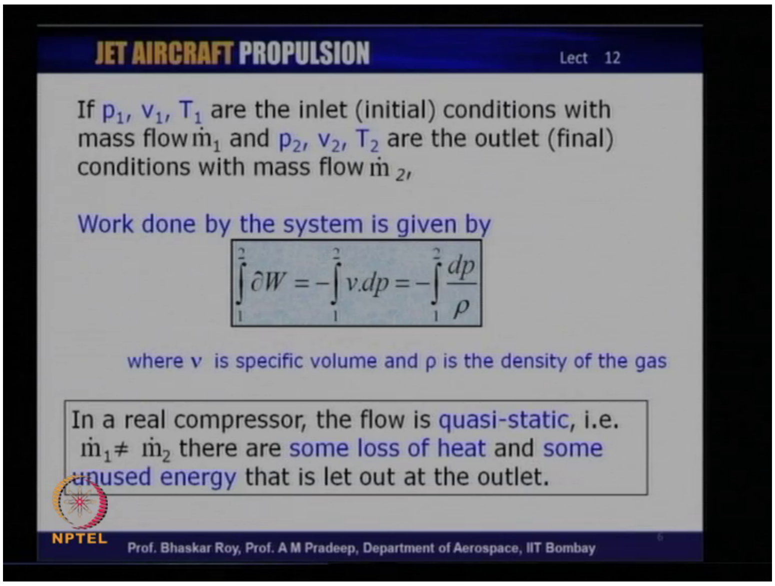

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If p_1, v_1, T_1 are the inlet (initial) conditions with mass flow $\dot{m}_1$ and p_2, v_2, T_2 are the outlet (final) conditions with mass flow $\dot{m}_2$,

Work done by the system is given by

$$\int_1^2 \partial W = -\int_1^2 v.dp = -\int_1^2 \frac{dp}{\rho}$$

where v is specific volume and ρ is the density of the gas

In a real compressor, the flow is quasi-static, i.e. $\dot{m}_1 \neq \dot{m}_2$ there are some loss of heat and some unused energy that is let out at the outlet.

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If we say that p 1, v 1, T 1 are the inlet or the initial conditions with which the mass flow of m.1 is coming in, and P 2, v 2 and T 2 are the exit or final conditions with a mass flow of m 2 the work done by the system is given by integral 1 to 2 dou w that is the work done would be equal to minus of integral 1 to of v dp, which we can also write down as minus of integral 1 to 2 of dp by row row of course, is the density of the gas or air.

And V of course, is the specific volume of gas or air, now in a real compressor as I just mentioned a little while ago the flow is likely to be quasi static, which means m.1 may not be equal to m2. And there is a possibility that the flow may not be adiabatic that means it could have some loss of heat and some unused energy that is let out at the outlet that means certain amount of energy that was used for the intended purpose of compression.

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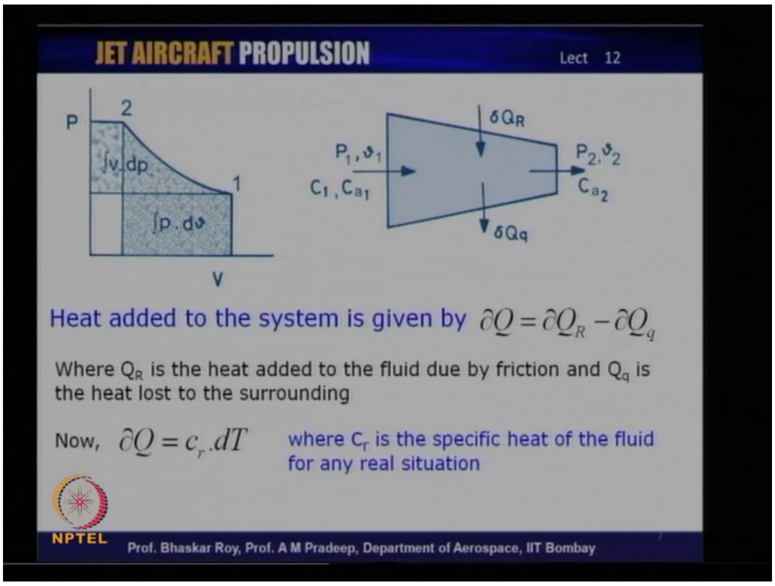

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Heat added to the system is given by $\partial Q = \partial Q_R - \partial Q_q$

Where Q_R is the heat added to the fluid due by friction and Q_q is the heat lost to the surrounding

Now, $\partial Q = c_r.dT$ where C_r is the specific heat of the fluid for any real situation

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Is now available as a exhaust energy and goes out with the outgoing flow, so that is what is likely to happen in a real compressor which is we are generally calling a polytrophic process. If you look at the p v diagram and on the side you have a system in which the flow is coming in with P 1 V 1, and let us say a velocity C 1. In which case it could be an actual velocity C a1 coming straight into let us say a ducting system, and then we say that this process now has a possible heat addition or rejection.

Now this heat addition is shown as delta Q R and the heat rejection is shown as delta Q Q subscript small q, so the heat net heat added to the system is then given by delta Q is equal to delta Q R minus delta Q subscript small q, and this Q R that is said to be added to the fluid that is flowing in inside the system may be assumed to have been added by the fluid friction, so that is one of the possibilities and Q subscript small q is the heat loss to the surrounding which often happens due to the normal heat transfer to the surroundings another result of this we can write down from thermodynamics that delta Q the net heat that is added to the system is C r into dT, where C r is the specific of the particular fluid gas or air.

For this particular real situation we know that specific heat for a constant process pressure is known as c p constant volume, it would be called c v this is a process which is neither constant pressure nor constant volume some real process, and we simply call it we are simply calling it c r for the time being.

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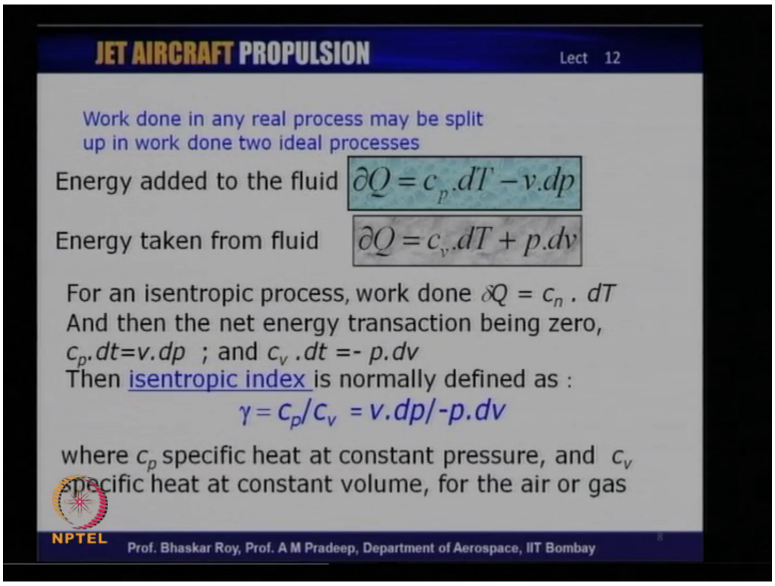

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Work done in any real process may be split up in work done two ideal processes

Energy added to the fluid $\boxed{\partial Q = c_p.dT - v.dp}$

Energy taken from fluid $\boxed{\partial Q = c_v.dT + p.dv}$

For an isentropic process, work done $\delta Q = c_n \,.\, dT$
And then the net energy transaction being zero,
$c_p.dt=v.dp$; and $c_v.dt =- p.dv$
Then isentropic index is normally defined as :

$$\gamma = c_p/c_v = v.dp/-p.dv$$

where c_p specific heat at constant pressure, and c_v specific heat at constant volume, for the air or gas

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So, the process that we are seeing here is a little more realistic process subjected to a compression, now if we proceed along this we can say that the work done in any real process may be split up in work done for two ideal processes one is the energy added to the fluid that is by delta Q is equal to C p dT minus V dp. And we we showed the V dp in the earlier diagram this is your v dp and this is of course, delta p dv.

So, if we add the those work done to c p dt which is the energy added to the fluid and energy taken away from the fluid can be put down in terms of c v dt plus p dv, then for an isentropic process the work done can be written down simply as delta q is equal to c n dt where c n is the specific heat of the isentropic process.

Let us say, let us put it that way and then the net energy transaction in an isentropic process is typically 0 because, it is an adiabatic process, so delta q is actually the heat added or subtracted from the system is likely to be 0 in which case c p dt would be equal to v dp and c v dt is would be equal to minus p dv from the above equations.

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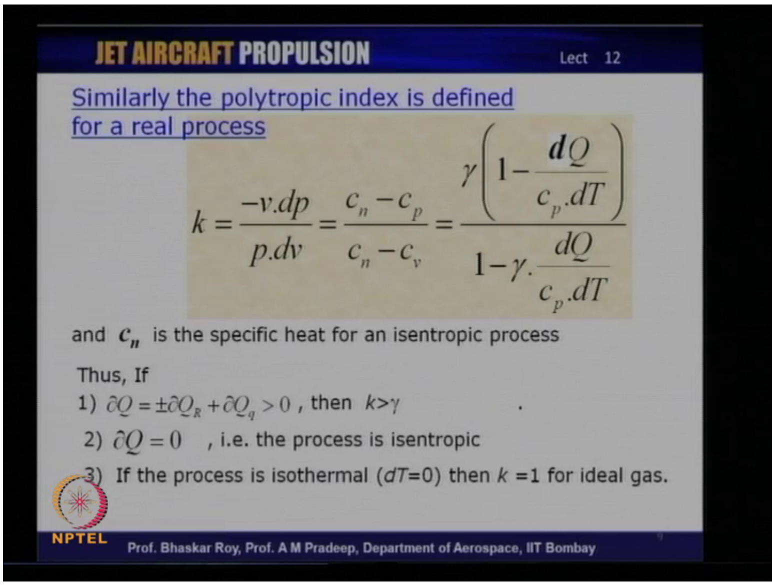

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Similarly the polytropic index is defined for a real process

$$k = \frac{-v.dp}{p.dv} = \frac{c_n - c_p}{c_n - c_v} = \frac{\gamma\left(1 - \frac{dQ}{c_p.dT}\right)}{1 - \gamma.\frac{dQ}{c_p.dT}}$$

and c_n is the specific heat for an isentropic process

Thus, If

1) $\partial Q = \pm\partial Q_R + \partial Q_q > 0$, then $k > \gamma$

2) $\partial Q = 0$, i.e. the process is isentropic

3) If the process is isothermal ($dT=0$) then $k=1$ for ideal gas.

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If we do that and then use the concept of isentropic index, which you have used in your cycle analysis this is normally defined as gamma is equal to C p by C v. And now, we can see that it can be written down in terms of v dp divided by minus p dv, where C p is the specific heat at constant pressure process and C v is the specific heat.

At the constant volume for the air or gas it it depends on whether it is air or what kind of gas, so the value of C p and C v would depend on the values of properties of the air or gas they also depend on the conditions of the air or gas notably the temperature of the air or gas.

In if we do that the concept of specific heat ratio can now be extended, and we can write down that the polytrophic index for a real process can also be defined for a real process as k just like gamma and the it is also a ratio of V dp by P dv, so that ratio remains as the fundamental definition of the specific heat ratio, and now we are saying it is a real process a polytrophic process and we say.

We designate it by k small k, and this can now be written down from the earlier equations as C n minus C p, where C n is the specific heat for a isentropic process, C p is that for a constant pressure process and that is divided by C n minus C v and that would be equal to from the earlier equations 1 can derive gamma multiplied by 1 minus d Q by c p dT and the whole thing divided by 1 minus gamma into dQ by c p dT.

So that is in a net shell the fundamental definition or concept of polytrophic index similar to the isentropic index gamma, which is used in cycle analysis specially in ideal cycle analysis thus. We say that if delta q is equal to plus or minus delta Q R plus delta Q q and if that is greater than 0, that means that the net that is going into the system is greater than 0 then the polytrophic index k would be greater than gamma.

If the delta q is 0 then it is an isentropic process and k would be equal to gamma, if the process is isothermal then the value of k is 1 for the ideal gases, and this is what we saw in 1 of the earlier p v diagrams, so this is what comes out from the definition of polytrophic index as its used for the gases ideal or real used in various cycle analysis of the engines.

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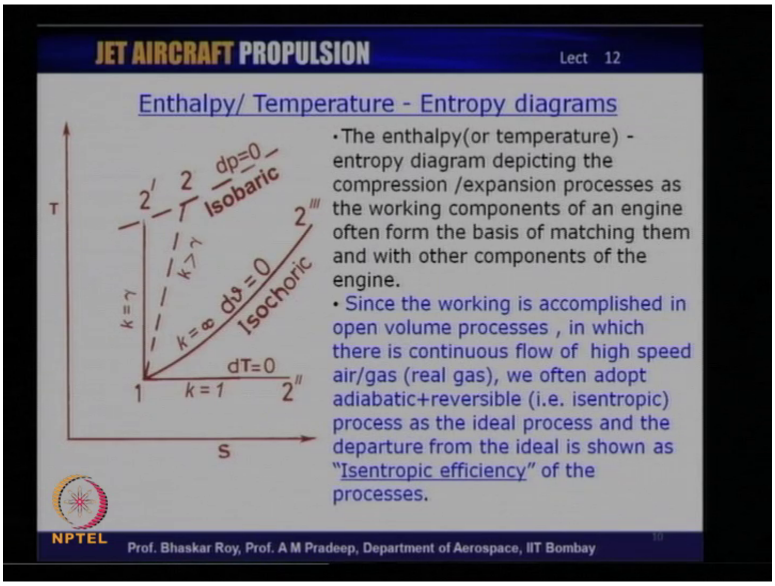

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Enthalpy/ Temperature - Entropy diagrams

- The enthalpy(or temperature) - entropy diagram depicting the compression /expansion processes as the working components of an engine often form the basis of matching them and with other components of the engine.
- Since the working is accomplished in open volume processes , in which there is continuous flow of high speed air/gas (real gas), we often adopt adiabatic+reversible (i.e. isentropic) process as the ideal process and the departure from the ideal is shown as "Isentropic efficiency" of the processes.

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Now, you we look at the t s diagram or more appropriately referred to as temperature entropy diagram, which can also be referred to as enthalpy entropy diagram in the sense the enthalpy entropy. And the temperature entropy diagrams looks exactly same except that in one case the y axis is enthalpy, and the other case the y axis is temperature in both the cases the x axis is entropy.

Now, if we put the various compression processes we were talking about earlier, and we can see here that the isentropic process, where k is equal to gamma goes straight up from 1 to 2 prime and then we have the polytrophic process, where k is greater than gamma and that is from 1 to 2 that is a real process, and then we have k equal to 1 which is the isothermal process that is 1 to 2 double prime over here.

And then k equal to infinity where change of specific volume that is a is 0 that is isochoric process; and it goes from 1 to 2 triple prime, so all the processes we had talked about earlier are now shown here on the t s diagram and those are the typical lines or the path through which the compression process would proceed from 1 to 2 2 being the end of the compression process.

Now in case of compressor that we are talking about in in typical jet engines the working of compression is accomplished in a open volume; and there is a continuous flow of high speed air or gas in case of compressor it is air, and we often adopt the so called isentropic process as a reference or a bench mark to categorize or find out the goodness or badness of a real compression process.

This comparison between the ideal compression process or isentropic compression process, and the real compression process is referred to as the isentropic efficiency of the process, so isentropic efficiency of the process is essentially a comparison between real process and isentropic process.

And hence we need to know a priory what is the isentropic work done before we know what is a real work that needs to be done because, that then we will know what the isentropic efficiency is and we will know how close we are to an ideal isentropic process and in a sense how close we are to the original joule bration cycle which is the fundamental matrix of jet engines.

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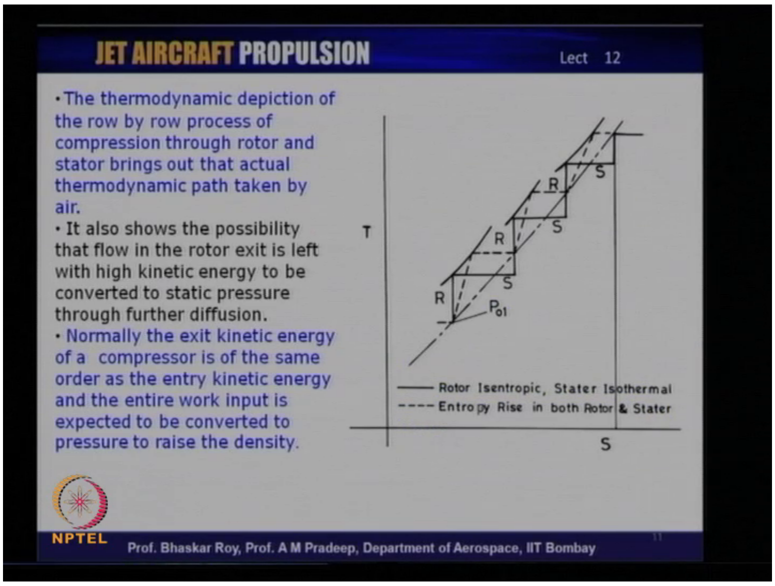

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- The thermodynamic depiction of the row by row process of compression through rotor and stator brings out that actual thermodynamic path taken by air.
- It also shows the possibility that flow in the rotor exit is left with high kinetic energy to be converted to static pressure through further diffusion.
- Normally the exit kinetic energy of a compressor is of the same order as the entry kinetic energy and the entire work input is expected to be converted to pressure to raise the density.

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If we look at the thermodynamic depiction of row by row process of a compression through router and stator, we know the rotor does work, and stator does not do any work really speaking, we will be talking about those things in great detail in the chapters to come the thermodynamic paths taken by the air through these compressors through router and stator can also be shown in terms of the T S diagram.

The path through the router is actually the work done, so it in isentropic work is is the vertical work along the vertical line, and then the stator what is a horizon to line where no work is being done and the work is essentially isothermal. So, this is the isothermal work that we were talking about and this is the isentropic work that we were talking about earlier.

So along this k is equal to gamma along this k is actually equal to 1, so it is a combination of these two that make up for the entire compression process through router and stator, so that is how the work through the compression process actually proceed it is k equal to gamma, and then k equal to 1 then again k equal to gamma. And then again k equal to 1 so it is the step by step process by which the compression actually proceeds through the thermodynamically in the typical T S diagram, if one does a rigorous analysis what of course, happens is that the exit kinetic energy of the compressors is often of the same order as the inlet kinetic energy, and as a result of which its expected that the entire work that is supplied would be converted to pressure.

As a result of which what is often done is through this steps an average line is drawn, and that average line is what we see in our cycle diagram representing the entire compression process actually proceeds along the line that we have shown, if it is real process it proceeds along the dotted lines. And then it reaches the end of compression what we show in a cycle diagram is we simply connect the entry point, and the exit point through an average curve and that is depicted as a compression process in a T S diagram.

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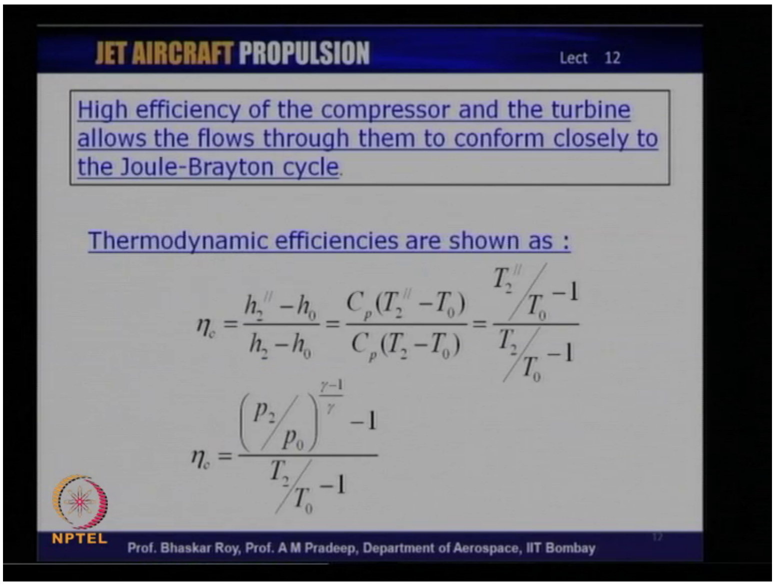

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High efficiency of the compressor and the turbine allows the flows through them to conform closely to the Joule-Brayton cycle.

Thermodynamic efficiencies are shown as :

$$\eta_c = \frac{h_2'' - h_0}{h_2 - h_0} = \frac{C_p(T_2'' - T_0)}{C_p(T_2 - T_0)} = \frac{T_2''/T_0 - 1}{T_2/T_0 - 1}$$

$$\eta_c = \frac{\left(p_2/p_0\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_2/T_0 - 1}$$

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So rigorous depiction of the compression process would have to be progressed in a step by step manner, let us take a look at how you show the efficiencies of the compression, the high efficiency of the compressor and the turbine that we would like to achieve actually takes the cycle pretty close to the joule brayton ideal cycle closer, we are to the ideal cycle closer we can say that at the option of the brayton cycle as the basis of a jet engine is fundamentally viable preposition.

Further we are away from the brayton cycle in actual engine operation adoption of the brayton cycle is then not a acceptable or it become remains of questionable matrix of analysis of jet engine. So, only way we can make sure that the jet engine is pretty close to a brayton cycle is by ensuring that the compressors. And the turbines another components of the jet engine do have very high efficiency operation, and they are pretty close to the isentropic process; and this efficiency that we are talking about is indeed the isentropic efficiency which I introduced just a few minutes back this thermodynamic efficiency or what we call the isentropic efficiency can be shown in terms of the work that is being done ideally the work that is being done really. And the real and the ideal work can be put together in one relation; and that relation captures the essence of the work that is going on through the compression process.

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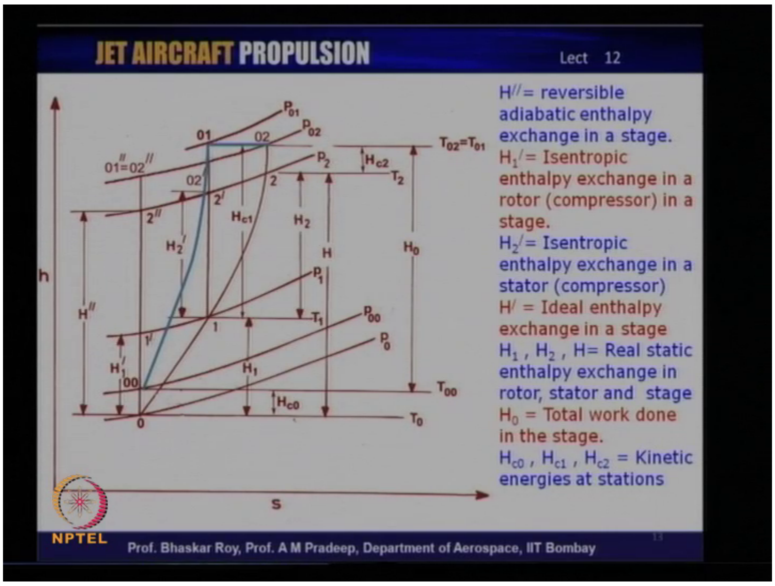

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H''= reversible adiabatic enthalpy exchange in a stage.
H_1'= Isentropic enthalpy exchange in a rotor (compressor) in a stage.
H_2'= Isentropic enthalpy exchange in a stator (compressor)
H' = Ideal enthalpy exchange in a stage
H_1 , H_2 , H= Real static enthalpy exchange in rotor, stator and stage
H_0 = Total work done in the stage.
H_{c0} , H_{c1} , H_{c2} = Kinetic energies at stations

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So, let's take a look at this thermodynamic efficiency definition now this definition to understand this definition a littlie, let us take a look at the thermodynamic depiction of the compression process and this time I have put a h s diagram which is the enthalpy entropy diagram very similar to the t s diagrams that we have looked at and what you shown here of course, are the various work that is being done. And as you can see here this is the work ideal work that is being done ideal adiabatic work in a stage, and then you have the work split up in rotors and rotor and stator, and then of course you have the real work done over here in h that is a real work done through the stage again that can be split up in rotor and stator that is H 1 and H 2. And then of course, H subscript 0 is the total work done in term of the stagnation parameters through the particular stage.

So, all the various kinds of work that this particular compressor stage is doing is being shown here, the blue line that is going from 00 to 01 and then to 02 is the total change parameter that is stagnation change parameter. And hence can be called the stagnation work done from 00 to 01, and then from 01 to 02 which is of course, the stator where no work is being done, so the actual work being done is form 00 to 01.

The corresponding static work is from 0 to 1, and then from 1 to 2, so these works are all shown here, and they are written down over here, the last three are the kinetic energies at the various stations shown as H c 0 and H c 2 and H c 1, so these are the three kinetic energies that the flow carries through the compressor at stations 01 and 2.

Now, one the basis of this depiction of the work done through the compressor stage the definition of the efficiencies can now be looked at again this is something very similar to what you must have already done in your cycle analysis. Let us take a look at it and understand how we can make use of this in our thermodynamic analysis of compressors. Now, what is shown here is the efficiency of compressor in terms of the work done, this is the ideal work done, this is the real work done and work at station 0 this is work at station 2, and the numerator is the ideal work and denominator is the real work done, and the ratio of the 2 is the efficiency which we have called isentropic efficiency.

The work done as you know can be written down in terms in terms of C p into temperature differential, we assume that the c p of real and ideal processes are same, and that is a an assumption for the time being the temperature differential or there ideal temperature differential. And the real temperature differentials come into the picture, and the C p gets cancelled out on what we have is the temperature ratios the ideal temperature ratio to the real temperature ratio across the compression process the ideal temperature ratio can also be shown in terms of the pressure ratio across the compression process. Now, we can see that the compression ratio that we intend to achieve comes into the picture over here, and if this is the compression ratio we intend to achieve.

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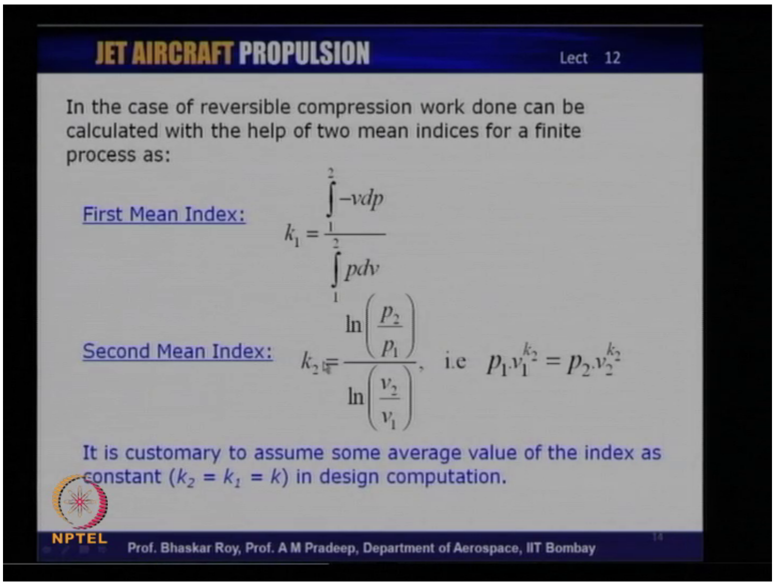

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Lect 12

In the case of reversible compression work done can be calculated with the help of two mean indices for a finite process as:

First Mean Index:

$$k_1 = \frac{\int_1^2 -v dp}{\int_1^2 p dv}$$

Second Mean Index:

$$k_2 = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\ln\left(\frac{v_2}{v_1}\right)}, \quad \text{i.e} \quad p_1 . v_1^{k_2} = p_2 . v_2^{k_2}$$

It is customary to assume some average value of the index as constant ($k_2 = k_1 = k$) in design computation.

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This can be put here and would give us the ideal work that is necessary the denominator, of course is the real work that is actually being consumed and that ratio of the two of course, is the overall isentropic efficiency of the compression process. Now, this compression work can be shown in terms of two mean indices specially, if they are finite processes as it is normally in a stage, if we consider a stage which is a small change in pressure.

We may consider it a finite process, and then the indices that we have talked about k can be shown in two different ways, the first one is often called thermodynamic the first mean index k 1, and that is something which we have defined before and that is integral 1 to 2 minus v dp divided by integral 1 to 2 p dv of the finite compression process, there is an another definition. And that is the second mean index often referred to as k 2, and that is defined as ln that is natural logarithm p 2 by p 1 divided by ln of v 2 by v 1 v 2 of course, v of course, is the specific volume.

And this can be written down in terms of what is known as the polytrophic relation that is p 1 v 1 to the power k 2 is equal to p 2 v 2 to the power k 2, which means we can write for a compression process p v to the power k is equal to constant. So that is another way of showing the polytrophic process in case of isentropic process it could be p v to the power gamma equal to constant in our analysis of jet engines without getting into very minute thermodynamic analysis. It is often customary to assume that the average value of the index as shown above the two of them are equal to each other, and hence k 2 is equal to k 1 is equal to k in all designed computations of a typical jet engine, so the two mean indices their definitions are slightly different from each other can be considered to be one and the same.

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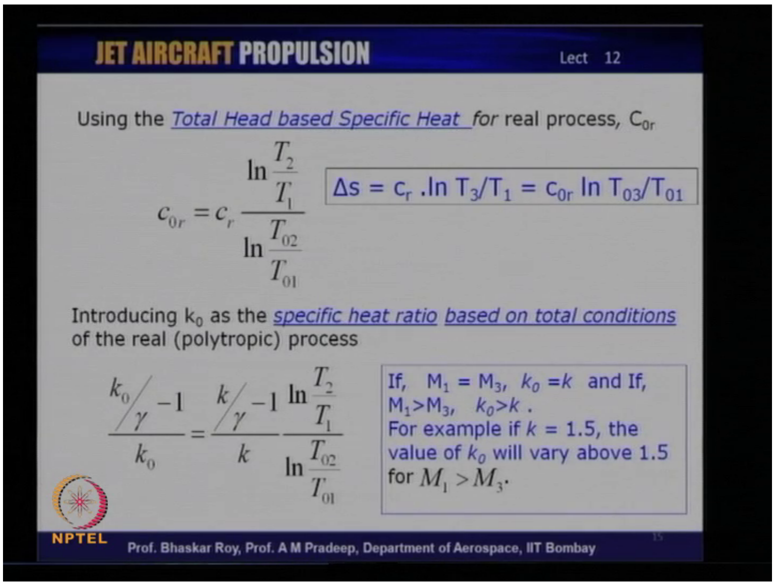

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Lect 12

Using the *Total Head based Specific Heat* for real process, C_{0r}

$$c_{0r} = c_r \frac{\ln \frac{T_2}{T_1}}{\ln \frac{T_{02}}{T_{01}}}$$

$\Delta s = c_r . \ln T_3/T_1 = c_{0r} \ln T_{03}/T_{01}$

Introducing k_0 as the *specific heat ratio based on total conditions* of the real (polytropic) process

$$\frac{k_0/\gamma - 1}{k_0} = \frac{k/\gamma - 1}{k} \frac{\ln \frac{T_2}{T_1}}{\ln \frac{T_{02}}{T_{01}}}$$

If, $M_1 = M_3$, $k_0 = k$ and If, $M_1 > M_3$, $k_0 > k$. For example if $k = 1.5$, the value of k_0 will vary above 1.5 for $M_1 > M_3$.

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So, if we can get a value of the index through one of the mechanisms, we can use it in the other relation that is p v to the power k and carry on our analysis thermodynamic analysis of compression process. Now, this total head based compression process that we have talked about let us have a quick look at that H S diagram that we had a look at.

You see the blue line shows the compression process where the changes are occurring in stagnation parameters form 00 to 01 and from 01 to 02 which means there is a total temperature change from T 00 to T 01. Similarly, there is a total pressure change from P 0 to P 01; and then to P 02 and there is a enthalpy change form H 0 to H 01 and H 02.

Those are the stagnation properties stagnation temperature stagnation pressure stagnation enthalpy, now those stagnations changes are what is shown here through this blue line, if we show a thermodynamic process in our cycle through the stagnation property chain then it stands to reason that we should be using a total head based specific heat which means instead of C r that we have written down earlier for v l process. We should be writing C r, now this C or for our total head bases specific heat definition can now be written down as C r to the power ln T 2 by T 1 divided by ln T 02 by T 01, such that delta S S of course, is the entropy is equal to C r into ln T 3 by T 1 across the entire compression process and that would be equal to C 0r into ln T 03 by T 01.

T 03 being the exit condition, T 01 being the entry condition, so it is possible to actually come out with a specific heat based on total temperature based; and total pressure based process in which specific heat is now being shown as C 0r as oppose to C r as we have done before. Now that brings us to the point that if we do that then the specific heat ratio that we have talked about k for a polytrophic process also should be shown in terms of the total change of parameters, and we can say it is depicted as k 0 for a real polytrophic process this k 0 can be written down in terms of k 0 by gamma minus divided by k 0; and that would be equal to k by gamma. Minus 1 divided by k into ln T 2 by T 1 divided by ln T 02 by T 01.

Now, these are all derivable definitions form the earlier thermodynamic definitions that we has just introduced what happens is if you do that; if you introduce this specific heat ratio as a total parameter, then if the entry and the exit mach numbers through the compression process are equal.

As we have said it sometimes could be equal then for all practical purposes k 0 is equal to k, and we do not have to bother about k 0; however if m 1 is greater than m 3 then k 0 is likely to be greater than k 1, and in many practical compressors quiet often it may well be, so if for example, if k is equal to 1.5 and the value of k 0 will vary quiet a lot above 1.5 if m 1 is greater than m 3. And this variation along with the mach number or velocity variation across the compressor then would have to be factored into the rigorous cycle analysis, that needs to be done to get a more realistic appraisal of the work done and the efficiency of the compression process.

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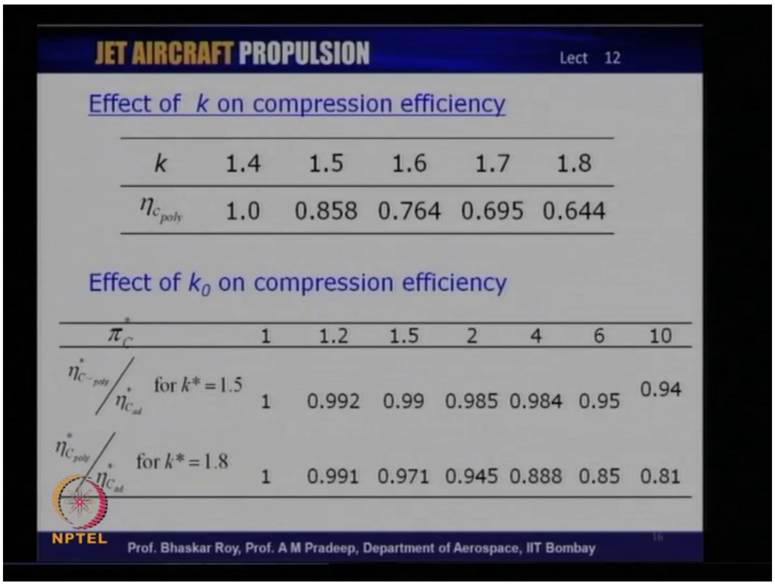

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Effect of k on compression efficiency

k	1.4	1.5	1.6	1.7	1.8
$\eta_{c_{poly}}$	1.0	0.858	0.764	0.695	0.644

Effect of k_0 on compression efficiency

π^*_C	1	1.2	1.5	2	4	6	10
$\eta^*_{C-poly} / \eta^*_{C_{ad}}$ for $k^* = 1.5$	1	0.992	0.99	0.985	0.984	0.95	0.94
$\eta^*_{C_{poly}} / \eta^*_{C_{ad}}$ for $k^* = 1.8$	1	0.991	0.971	0.945	0.888	0.85	0.81

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Let us quantify the effect of these changes, if we have a change of k as we have seen k varies away from the ideal value which is 1. 4 and if for example, we have a compression process where it is ideally 1.4 that is air we say that the compression has an efficiency of one, now if the value of k is 1.5 the effeminacy starts falling. And if it goes all the way up to 1.8 the efficiency of the same compression process would fall all the way to point 644 which is only 64. 4 percent, which is rather low efficiency.

So, we can see that the compression efficiency would be dependent quietly a lot on the value of k, and the value of k of course, depends on the real polytrophic process that the compression is executing, if we look at k 0. And its effect on the compression efficiency we see that has the value of k 0 changes from k to 1 value to another, and as the compression ratio changes the value of efficiency changes substantially.

If we have a compression ratio which is very let us say one ideal that means no compression at all we say that the efficiency is one, and when the value of k 0 changes it remains 1, but once the compression ratio increases from 1.2, 1.5 which are pressure ratios, and then multistage pressure ratios of 2 4 6 and 10 for a value of k which is now let us say 1.5.

The or k 0 for example, if the value is 1.5 the efficiencies won't start falling and we can see that the efficiency falls all the way up to 94 percent, if the efficiency ratio that are looking at if the value of k now changes to 1.8 the efficiency would start falling substantially, and this efficiency ratio that we are looking at falls from 1 to 0.81.

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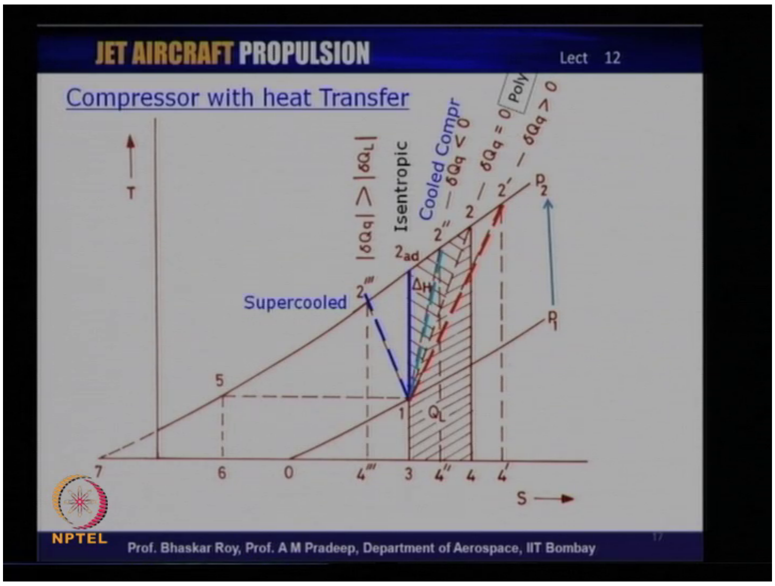

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Compressor with heat Transfer

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So these are the various effects of the value of k and k 0 if we use k 0 for total based cycle analysis, and we see that they have substantial effect on the efficiency computation that one may like to do for cycle analysis of jet engines. Now, we take a look at what happens? If we have a compression process with a certain amount of heat transfer what does happen is that when the process of heat transfer is going on the compression is affected, now the blue line one to two adiabatic is indeed the ideal process or we may call isentropic process.

On the other hand the red line 1 to 2 is the polytrophic process, where heat transfer across the process is still 0, so it is a polytrophic process where there is no heat transfer isentropic process of course, has no heat transfer anyway, and then we consider two processes one in which the heat transfer is greater than 0 another in which the heat transfer is less than 0.

So in one case the compression process or compressor is subjected to to certain amount of heating, and in the other case the compressor is subjected to certain amount of cooling. Now, we can see that the area under the line 1 to 2 is indeed the extra work that is done to overcome the losses of a real process or polytrophic process, and that is the extra work that one has to do to overcome this path 1 to 2 for the time being we are considering this to be a linear path 1 to 2, we have seen before that path could be curved for a long compression process.

Let us consider a small compression process in which this path is let us say a linear path, and then we can say each of these can be considered as rectangles or triangles an as a result of this. Ee have a heat transfer now being introduced into the compression process in which extra work is necessary to overcome the work done in a polytrophic process

As we can see now, if you introduce heating you would actually need to do more work because, the entropy rise or the loss in such a process be even higher and you would actually need to do more work, because the path from 1 to 2 prime. Now is longer to accomplish the same pressurization from P 1 to P 2, see in all these processes the pressurization is same from P 1 to P 2; this path is longer which means you would need to do more work the temperature rise from T 1 to T two prime is higher, so you would actually need to do more work.

On the other hand, if you have a cooled compression process the area under that is smaller than the earlier polytrophic process, and as a result of which you have a lesser work to be done for the compression process, and this means that if you have a certain amount of cooling introduced in the compression process you can actually get away with doing less of compression work. And this is the thermodynamic concept which has given rise to various kinds of cooled compression process or in case of jet engines or various kinds of gas turbine engines.

The concept of inter cooling or even simple cooling has been used for many many years, wherever there is a lot of the working condition is very hot as it is in places like India you often refer to cooling of the flow that is going into the compressor. And this cooling as we can see now form pure thermodynamics introduces a simple thermodynamic basis, which allows the compressor to do the same amount of compression with lees work necessary to be supplied by the turbine; and this is what thermodynamics tells us that if you have some method by by which you can accomplish a little bit of cooling actual work of compression can be done with less of work supplied.

There is a process here which is which shows that if you do a lot of cooling which can one call as super cooling you can get away with doing a lot less work to accomplish the same pressurization from 1 to 2 triple prime, so the pressurization would indeed be achieved, but its possible that you would be arriving at a temperature of a lower value at the end of the compression process even lower than the ideal in which case for a jet engine purpose that may not be a very desirable situation. Because the flow from the compressor is going to combustion chamber, and a super cooled flow going into the combustion chamber would require additional heating in the combustion chamber for the flow to be taken to high temperature, so super cooling may not be a desirable phenomenon in a typical jet engine operation.

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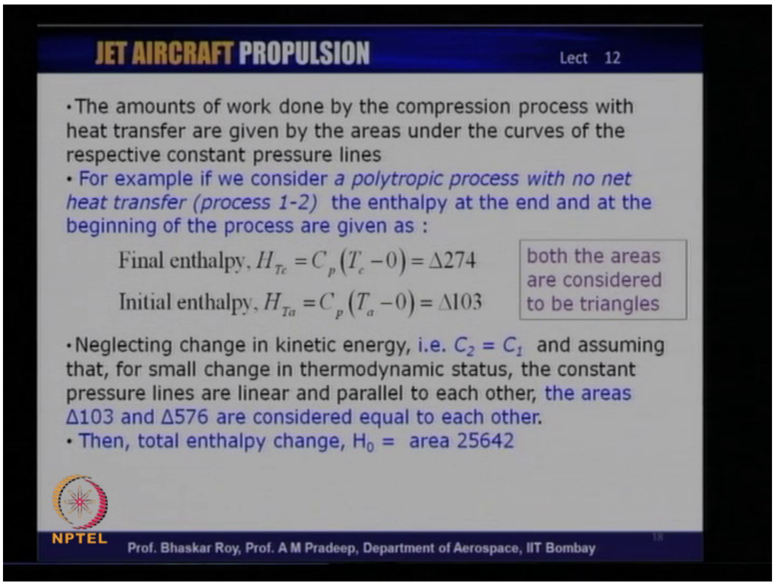

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- The amounts of work done by the compression process with heat transfer are given by the areas under the curves of the respective constant pressure lines
- For example if we consider *a polytropic process with no net heat transfer (process 1-2)* the enthalpy at the end and at the beginning of the process are given as :

$$\text{Final enthalpy, } H_{Tc} = C_p\left(T_c - 0\right) = \Delta 274$$

$$\text{Initial enthalpy, } H_{Ta} = C_p\left(T_a - 0\right) = \Delta 103$$

both the areas are considered to be triangles

- Neglecting change in kinetic energy, i.e. $C_2 = C_1$ and assuming that, for small change in thermodynamic status, the constant pressure lines are linear and parallel to each other, the areas Δ103 and Δ576 are considered equal to each other.
- Then, total enthalpy change, H_0 = area 25642

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But a certain amount of cooling, as we can see is certainly a desirable phenomenon for more efficient compression process, we can summarize all this by simply saying that the as we have said those diagrams are in terms of linear diagrams. And hence the final enthalpy can be written down in terms of the triangle 274 which is shown here in terms of 2 and 7 and 4, and that is the triangle we are approximating it as a triangle, and that is the final enthalpy the initial enthalpy is the triangle 103 and that is shown here in terms of 03 and 1 and that is the triangle with which the flow is entering the system.

So, it enters the system through one goes out through the system from two, so this is your energy at the entry point, and this is the energy at the exit point, so these are the various phenomena that is going on in a typical compression process as shown in a little simplified thermodynamic diagram where the processes are considered to be linear. So that some of those can be simply referred to as triangles or rectangles, and as a result of which those some of those total enthalpy change can now be shown in terms of the area 2 5 6 4 2 and that is 2 5 6 4 2, and that is a area which is the total enthalpy change in terms of total parameter H 0.

So, this is what we learnt from simple thermodynamic analysis that you can use a thermodynamics as a matrix try to figure out what is going on it terms of compression process, and then see whether you can get your compression process done a little more efficiently a little more effectively. So, that it serves the purpose of the jet engine, and brings the jet engine closer to the ideal brayton cycle which is the matrix on which the jet engine is functioning.

So, this is what we have done in today's class the thermodynamics of compression process, and we can see that the cooled thermodynamic compression can give us some benefits. In the next class what we will be doing is thermodynamics of turbines similar thermodynamic analysis. We will then refer to in thermodynamic diagrams and see whether we can learn something from a fundamental thermodynamic understanding, and make the turbine do work in a more efficient manner to conform to our ideal cycles that we have studied over the last few lectures. So, in the next class we will look at turbines.