Jet Aircraft Propulsion

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Lecture No. # 11

Tutorial - 2

Hello and welcome to lecture number eleven of this lecture series on jet propulsion. We have been discussing about the various aspects of jet engine cycles; we have discussed about the ideal cycles, the real cycles, and also the component performance parameters based on which we were able to derive the real cycles. So, based on our discussion during the last several lectures, you must have had some idea of what is involved in carrying out a cycle analysis. And as I have been discussing earlier, it is also being assumed that you have undergone some sort of training in the form of the ideal cycle analysis; specially, if you have undergone the previous course on introduction to aerospace propulsion.

So, in today's class as promised in the last class we shall be discussing a few problems we will basically be having a tutorial session, we will solve a few problems in today's class. And we will see how we can carry out a real cycle analysis for a few configurations of the jet engine. So, we will basically be having a problem on a simple turbo jet. And then I have configured a problem on a turbo jet the same problem with after burning.

We will subsequently solve a problem on turbo fan, one of the configurations of a turbo fan. And then I also have a problem for you on turbo prop engines. And towards the end of the class, I will also give you a few exercise problems, which you can try and solve based on our discussion in the previous classes as well as based on the tutorial session that we have today.

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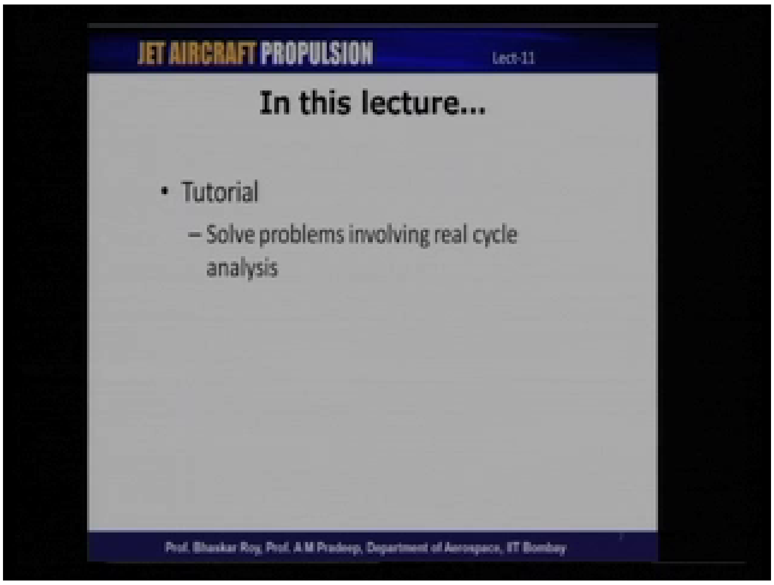

So, today's class is basically a tutorial session, we will solve a few problems from the jet engine cycle we will basically be solving real cycle problems; and not the ideal cycles. So, the first problem that we have for today's discussion is a simple turbo jet. So, let us take a look at the problem statement.

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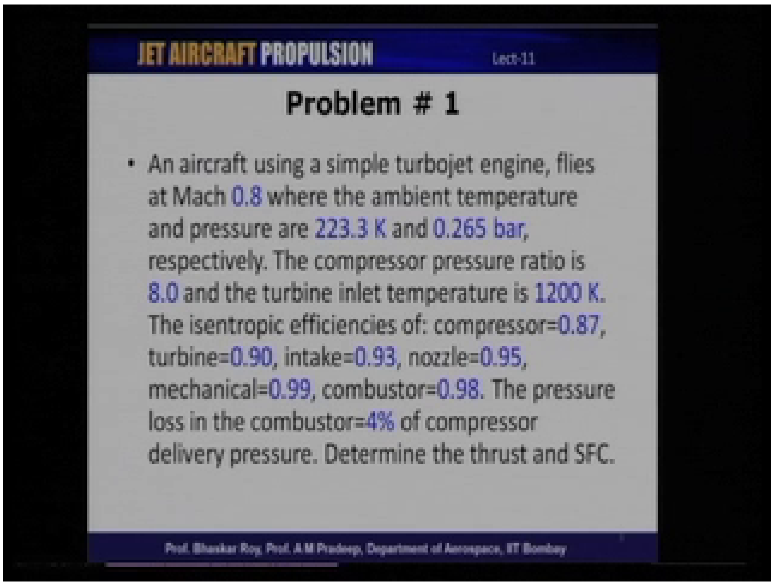

So, problem number one states that an aircraft using a simple turbo jet engine flies at mach 0.8, where the ambient temperature, and pressure are 223.3 Kelvin, and 0.265 bar respectively the compressor pressure ratio is 8 and the turbine inlet temperature is 1200

Kelvin the isentropic efficiencies of the compressor are 0.87 the turbine is 0.90 the intake is 0.93.

The nozzle is 0.95 the mechanical efficiency is 0.99 the combustor efficiency is 0.98. The pressure loss in the combustor is 4 percent of the compressor delivery pressure determines the thrust; and specific fuel consumption so, this is the first problem that we shall be attempting to solve today.

It is basically on a simple turbo jet engine a simple turbo jet as you know is a turbo engine, which does not have any form of after burning. So, which means that there is no reheating in this particular cycle so, we have been given a lot of data corresponding to this particular turbo jet engine. We have the compressor pressure ratio the turbine inlet temperature. And all the efficiencies of the components like the compressor the turbine the fuser nozzle. And so, one we also have been given the ambient conditions; and the flight mach number so.

Based on this data's we should be able to carry out a cycle analysis; and determine what is the thrust; and specific fuel consumption of such an engine so, given this particular problem statement, where do we first begin to solve this problem so, as you must have already realized by now the first step towards solving such a problem is to get the cycle diagram first.

That is on either on a temperature entropy plot or on a pressure volume plot so, once we get the cycle diagram right. And also we mark the points, where we have the data with us. And we would therefore, be able to determine, which are those points, where we need to find the properties of temperature, and pressure, and therefore will help us in determining the exhaust velocity, and hence the thrust, and fuel consumption.

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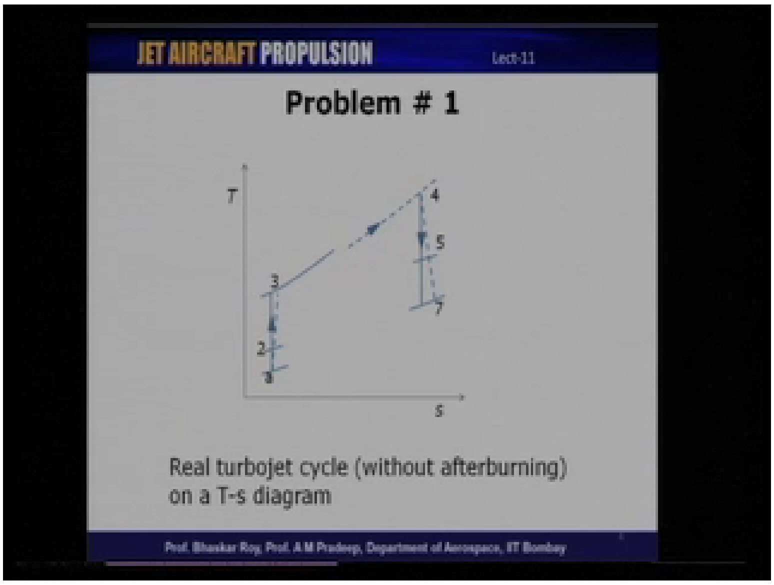

So, the first step towards solving this problem is to draw the cycle diagram. So, this is a turbo jet cycle without after burning. And this problem does not have this particular jet engine that we have does not have any after burning. So, this is the real turbo jet cycle that we had discussed in the last class on a temperature entropy plot. So, I will just quickly review the different processes involved in this particular turbo jet cycle, the ambient conditions are denoted by a subscript a station two corresponds to the intake exit or the diffuser exit, which is also the compressor inlet.

Station 3 is the compressor outlet station 4 is the turbine outlet 5 is turbine outlet; and 7 is the nozzle exit so, process between a; and 2 is the compression in the intake of the diffuser; and the line shown by this dotted line is the real process or the actual process; and because it is an actual process. And because they have been given diffuser efficiency it means that that process is not isentropic.

So, process a to two is non isentropic the compression begins at from station between 2; and 3 is the compressor again this is also a non isentropic process, which is why we have this dotted line which indicates the compression process between process states 3, and 4 we have the combustion chamber or the heat addition.

And we have been given the combustion efficiency as well as the pressure loss in the combustion chamber so, this process is no longer a constant pressure process so, there is a pressure loss occurring in the heat addition process or the combustion process between stations 4, and 5 is the turbine. And the turbine process expansion process is non isentropic; and that is why the expansion process is indicated by this dotted line.

Between stations 5, and 7 is the nozzle, and this again is a non isentropic process, and basically we get the exhaust dust as a result of expansion through the nozzle so, these are the various processes that are that basically constitute the turbo jet cycle, and what we shall try to do now is that we will identify those points where we have been given the data. And then we will get some idea of which are those points where we need to find out. Temperature, and pressure, and that is of course, that is what will be required for calculating the exhaust velocity so, we have been given the ambient velocity ambient temperature, and pressure, and the mach number, and then the diffuser efficiency is given the compressor pressure ratio is given then we have the turbine inlet temperature.

So, these are the different property or parameters which have been specified in the problem besides of course, the efficiencies of all the components so, based on the ambient temperature, and pressure, and the mach number we can calculate the ambient stagnation conditions, and also because intake efficiency is given we can calculate the intake exit conditions like its stagnation temperature and pressure.

And then we have the compressor pressure ratio, therefore compressor inlet conditions being known we can find out the compressor exit conditions from the pressure ratio. And the efficiency then we arrive at the combustion chamber where again we have been given the efficiency the pressure loss; and the exit conditions of the combustor that is the turbine inlet temperature.

So, from the combustor analysis we can find out the fuel to air ratio; and then we have the turbine inlet condition is known. And so, we can find out by equating the turbine work to the compressor work the turbine exit conditions. And also the exhaust nozzle which will be solved so, these are the steps that will be that are involved in solving this particular problem.

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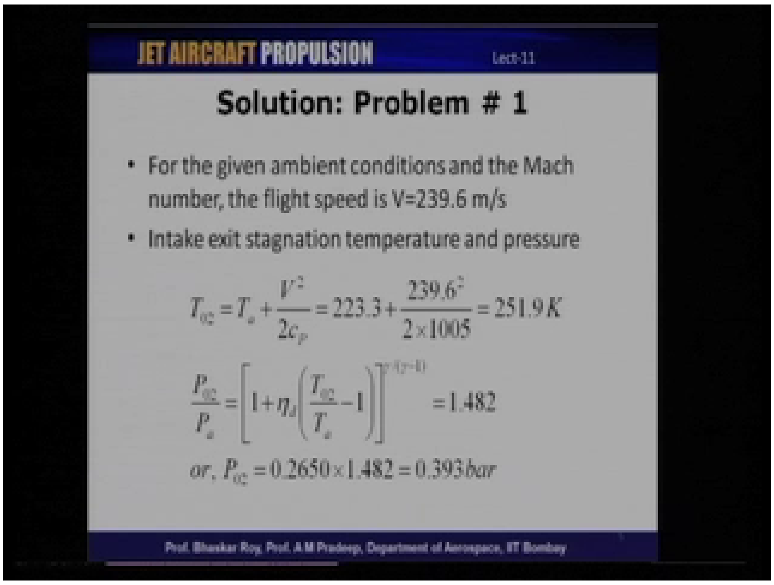

So, let us begin with the intake, where we have the intake inlet conditions that is we have the ambient temperature pressure Mach number; and the intake efficiency so, for the given ambient conditions that is the temperature, and pressure, and the mach number we can calculate the flight speed, because flight speed will be equal to Mach number into the speed of sound that is square root of gamma RT.

So, from that we can calculate flight speed which comes out to be 239.6 meters per second so, this is calculated, because the Mach number is known; and the ambient temperate is known; so we can calculate the speed of sound, which is square root of gamma RT that multiplied by the Mach number gives us this flight speed.

And so, once we know the flight speed or the Mach number we can calculate the intake exit stagnation conditions - that is the stagnant temperature, which is the ambient temperature plus V square by 2C p. And so, we the ambient temperature is known the velocity is known; and C p for air we will be assuming as 1.005 kilo joules per kilo grams per Kelvin; and for the exhaust products or the combustion products we will assume it to be 1.47 kilo joules per kilo grams Kelvin.

So, based on these when we substitute these values; and simplify we get the stagnation condition as 251.9 Kelvin; and now similarly, the stagnation pressure is given by this stagnation is related to the stagnation temperature to be isentropic efficiency of the diffuser so, P 0 2 by P a is equal to one plus diffuser efficiency multiplied by T 0 2 by T a minus one raised to gamma by gamma minus 1.

So, diffuser efficiency is given to us it is 0.93 stagnation temperature we have just now calculated ambient temperature is known; and gamma for air is 1.4, so if we substitute these values we get P 0 2 by P a is equal to 1.482. And since ambient pressure static pressure is known stagnation pressure can be calculated by P a multiplied by 1.482 that is 0.265 into 1.482; that is 0. 393 bars so, what we have calculated now are the exit conditions of the intake which will basically be the inlet conditions of the compressor so, based on these conditions that are correspond to the compressor inlet we can now proceed towards calculating the compressor exit conditions. And how do we calculate that compressor exit pressure is straight forward.

Because the pressure ratio is specified so, given the pressure ratio we can calculate the compressor exit stagnation pressure which is basically the stagnation pressure multiplied at the inlet of the compressor multiplied by the pressure ratio so, P 0 3 will be equal to pi c which is the compressor pressure ratio multiplied by P 0 2. And how do you calculate the temperature is basically calculated using the compressor isentropic efficiency definition, which is eta d is equal to T 0 3 minus T 0 2 T 0 3 s by T 0 2 divided by T 0 3 minus T 0 2. And if we simplify this we get an expression in terms of the pressure ratio; and therefore, we can calculate T 0 3, we have discussed that during the last lecture, when we took up the cycle analysis of the turbo jet.

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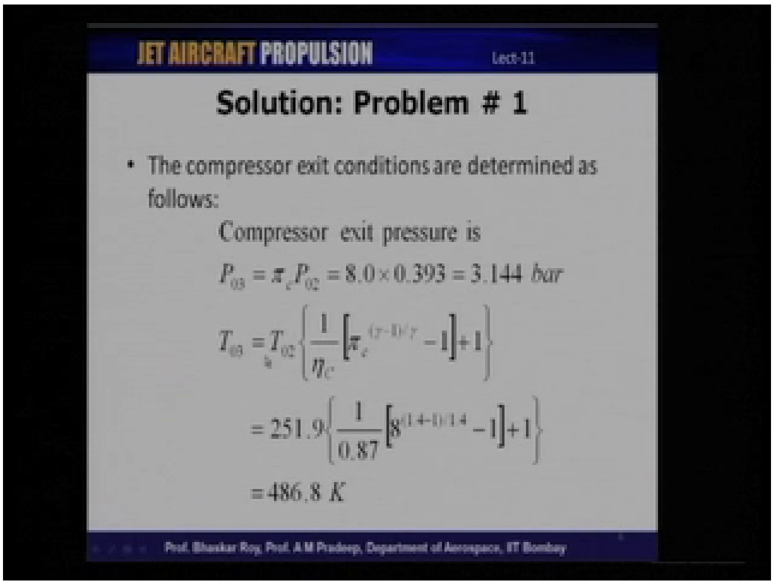

So, compressor exit pressure is basically just the product of the pressure ratio; and inlet stagnation pressure, which is already calculated as 0. 393 bar. And so, that multiplied by the exit by the pressure ratio that is 8.0 gives us the exit stagnation pressure. So, compressor exit stagnation pressure is 3.144 bar, and exit stagnation temperature is from the efficiency definition.

T 0 3 is equal to T 0 2 multiplied by one by eta c which is the isentropic efficiency of the compressor multiplied by pi c raised to gamma minus 1 by gamma minus 1 plus 1 so, this basically gives us the stagnation temperature at the compressor exit so, all these parameters on the right hand side are known to us, now T 0 2 we have calculated in the previous expression in the previous slide. Eta c is specified c is the pressure ratio gamma is 1.4 so, we substitute for all these values in this expression on the right hand side we should be able to get an expression for we should be able to find out the compressor exit stagnation temperature so, substituting for all these values we get T 0 3 is equal to 486.8 Kelvin

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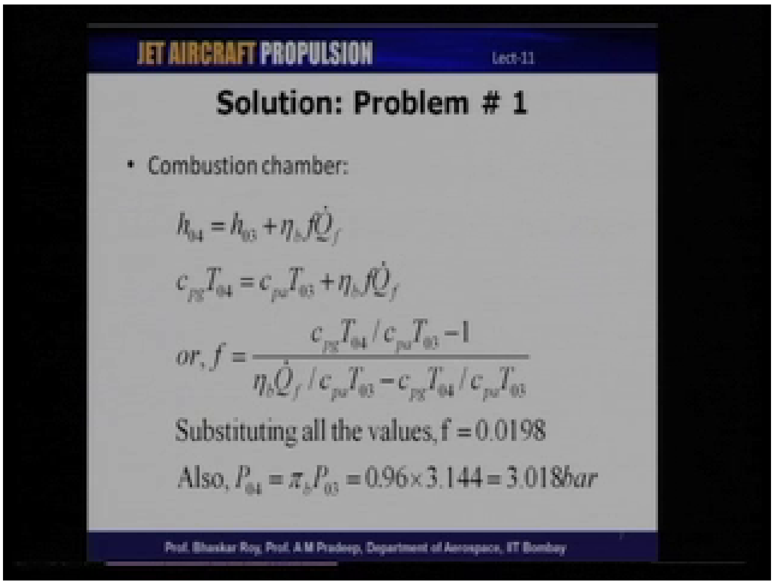

So, this is the stagnation temperature at the compressor exit so, compressor exit stagnation temperature we have related that to the pressure ratio; and the efficiency based on which we can calculate the compressor exit conditions so, the next component is we have now called or carried out the cycle analysis up to the compressor exit. And so, we now have data up to the compressor exit in terms of the stagnation pressure; and the stagnation temperature we also have which is basically the combustion chamber inlet conditions.

And for the combustion chamber we have the combustion efficiency, which has been specified; and the pressure loss taking place in the combustion chamber. And what else we have we also have the combustor exit temperature stagnation temperature, which is the turbine in the temperature so, based on this much data that we have we should be able to calculate the fuel to air ratio. And how do we calculate that if you recall in the last lecture we had discussed to calculate the fuel to air ratio, which is carry out energy balance from between the inlet; and exit. So, energy balance will be able to help us in calculating the fuel to air ratio so, enthalpy at inlet which is the enthalpy of the air coming in from the compressor plus the enthalpy or heat of reaction of the fuel.

Is equal to enthalpy at the exit of the combustion chamber so, we solve this; and we should be able to get the fuel to air ratio. So, h 0 4 which is the enthalpy at the exit of the combustion chamber, which is the turbine inlet is equal to h 0 3, which is the compressor exit plus the burner efficiency or the combustion chamber efficiency.

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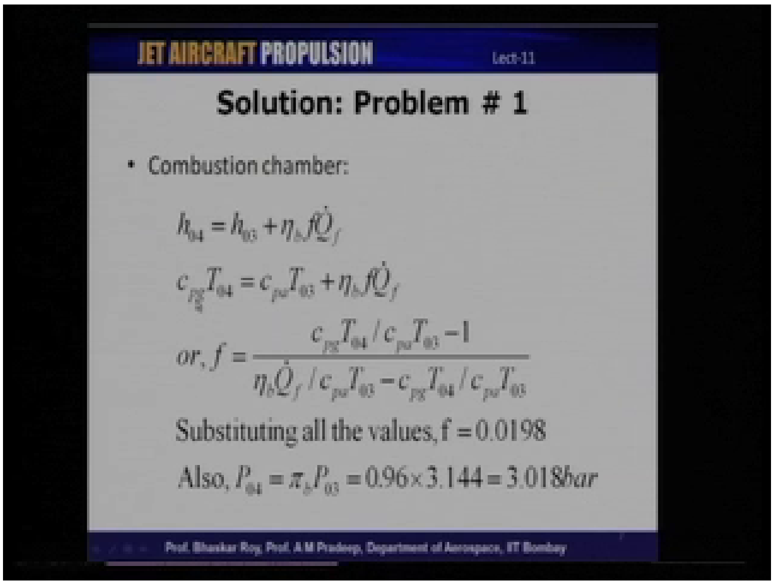

Multiplied by the fuel to air ratio; and the heat of reaction of the fuel we should be able to solve this; and get the fuel to air ratio so, C p times T 0 4 is equal to C p a times T 0 3 eta b into f times Q dot f so, C p g here corresponds to the specific heat of the combustion product C p a corresponds to the specific heat of air. So, all these numbers are known to us now so, f we can simplify. And that would be equal to C p g by into T 0 4 divided by C p a into T 0 3 minus one divided by burner efficiency or combustion efficiency into Q dot f into C p a T 0 3 minus C p g T 0 4 by C p a T 0 3. Now we will assume a value of q dot f, because it is not explicitly stated in the problem usually for jet engines the fuel that is used is known as aviation turbine fuel. And that is a fuel which is similar to kerosene.

And for that particular fuel the Q dot f or heat of reaction is about forty four mega joules per kilo gram so, we will assume that value here; and rest of the values are already known to us so, we substitute for all these values; and if we substitute for all of them T 0 4 is basically the turbine inlet temperature which is given T 0 3 is compressor exit stagnation temperature, which we have calculated, and C p a, and C p g we have known eta b burner efficiency is also given so, f is equal to if we simplify substitute for all these values, and simplify we get 0.0198.

Now so, this is the fuel to air ratio what else is required to be calculated we need to know to calculate the combustion chamber exit stagnation pressure exit stagnation temperature is known. That is the turbine inlet temperature exit stagnation pressure is equal to the pressure loss in the combustion chamber multiplied by the inlet time machine pressure it is given in the problem that 4 percent loss occurring in the combustion chamber so, pi b is basically equal to 0.96 so, P 0 4 will be pi b multiplied by P 0 3 that is 0.96 into 3.144 so, that is 3.018.

So, this is how we have estimated; and calculated the properties all that way from the inlet then proceeding towards the compressor. And the combustion chamber the next component is the turbine, we have the turbine inlet stagnation temperature, and pressure, and we also have the mass flow rate in the sense that we have mass flow rate of air plus mass flow rate of fuel we have just calculated the fuel to air ratio.

So, we should now be able to calculate the fractional mass flow rate that is actually going into the turbine so, all these conditions are known for the turbine, now what we need to calculate are the exit conditions of the turbine. And how do we calculate that remember in a simple turbo jet engine the basic function of a turbine is only to drive a compressor. It means the turbine is generating just enough work to drive the compressor, so turbine work the work output of the turbine will be equal to the work input of the compressor. And of course, there is there is a mechanical efficiency involved that means the work output of the turbine is getting diminished by a certain fraction which is equal to the mechanical efficiency.

So, multiplied by the turbine work will be equal to the compressor work this if we simplify which should be able to calculate the compressor turbine exit stagnation temperature. And then from the efficiency definition we can calculate the stagnation pressure. And so that is how we will be proceeding towards calculating turbine exit conditions.

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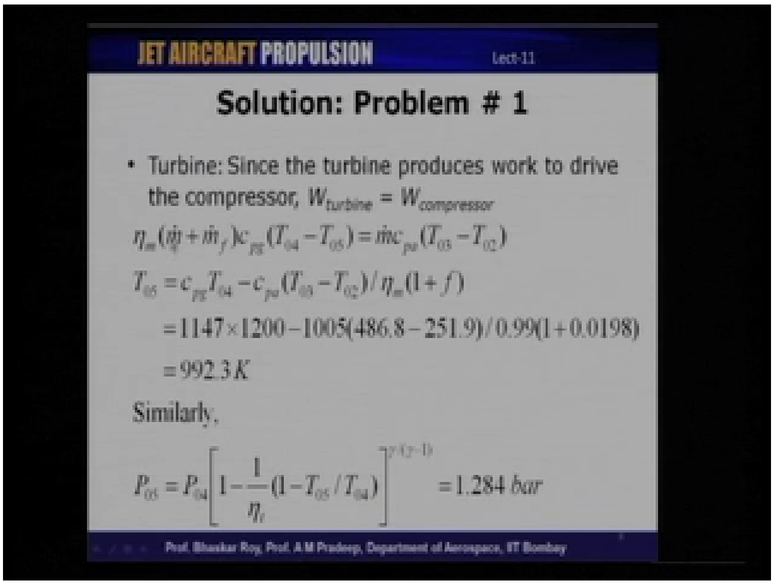

So, let us do that; and see what the turbine exit conditions come out to be now since we know that turbine exit is work done by the turbine is equal to that of the compressor, we know that the mechanical efficiency multiplied by m dot, which m dot mass flow rate of air plus mass flow rate of fuel this sum of this multiplied by C p of gases into the temperature difference across the turbine that is T 0 4 minus T 0 5 is equal to m dot, which is mass flow rate of air into C p of air into T 0 3 into T 0 2 which is the differential temperature across the compressor.

So, left hand side is the work done by the turbine right hand side is the word done by the required by the compressor. So, let us simplify this once you simplify this in this equation the only unknown is T 0 5. So, if you simplify this; and we get an expression in terms of T 0 5; so, T 0 5 on simplification is equal to C p into well C p g into T 0 4 minus C p a into T 0 3 minus T 0 2 divided by mechanical efficiency into one plus f, because we cancel out m dot, so we get 1 plus f, so the entire all the terms which are involved on the right hand side are known from our cycle analysis so far. So we have C p g, which is 1147 into T 0 4 which is 1200 minus C p a, which is 1005 multiplied by T 0 3 that is 486.8 minus 251.9 divided by mechanical efficiency that is 0.99 into 1 plus f that is 1 plus 0.189, if we calculate this we will get the exit temperature as 998.3 Kelvin, so this is the stagnation temperature at the turbine exit now, how do we calculate the stagnation pressure at the exit? We will now make use of the efficiency definition of the turbine well efficiency of the turbine isentropic efficiency of the turbine is defined as eta t is equal to the inlet temperature stagnation temperature T 0 4 minus T 0 5, which is the actual temperature divided by T 0 4 minus T 0 5 isentropic. Now if we divide this and convert them into temperature ratios in the denominator, we have the isentropic temperature ratio T 0 5 s divided by T 0 4, which is equal to P 0 5 by P 0 4 raised to the gamma minus one by gamma from this implication, we should get an expression for T 0 5, which is the turbine exit stagnation pressure.

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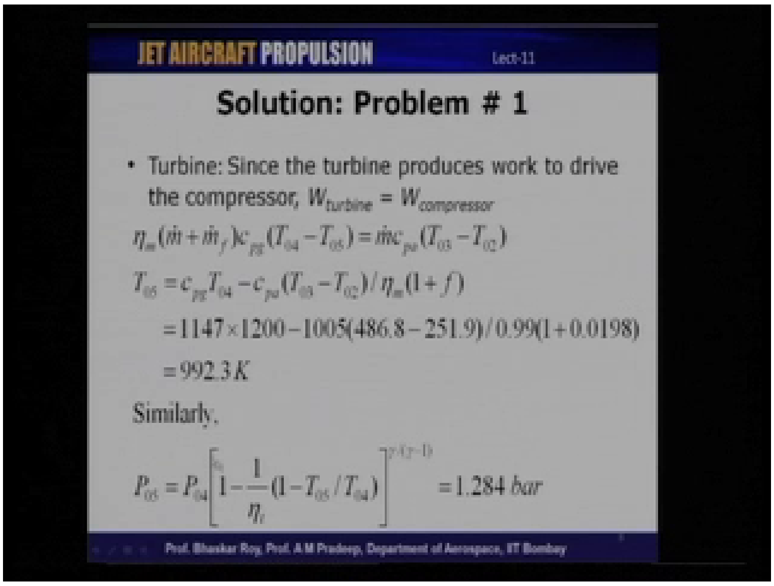

We already derived that in the last lecture; and so T 0 5 can be expressed in terms of the efficiency; and the temperature ratio, so if we do that we get P 0 5 is equal to P 0 4 by four multiplied by one minus one by eta t the turbine efficiency into one minus T 0 4 this raised to gamma by gamma minus 1. So, all these values are known to us T 0 4 is known turbine efficiency is known. And these two temperatures are also known; and remember we are going to use gamma here as one 0.33; and not 1.4, because we are not dealing with the combustion products; and so gamma is we are going to assume an average gamma for all the combustion products; and that is going to be assumed as 1.33; and for the average gamma for air is 1.4.

So, if we substitute for all these values here; and simplify we get 305, which is the turbine exit stagnation pressure which his equal to 1.284 bar, so this is the pressure of the of the exit of the turbine. So we have data now all the way up to turbine exit, we also have now the turbine exit stagnation pressure.

And the temperature; and in this particular engine configuration there is no after burner; and since there is no mention of any other losses after the turbine except the nozzle efficiency we can assume that all these parameters, which we have calculated for the turbine exit is valid for the nozzle inlet also. That means T 0 5; and P 0 5 will be the same as the nozzle entry, so we will use these same values at the nozzle entry to calculate the nozzle exit condition. And therefore, the velocity of the jet, but before that we have to make one check, which is basically to see this nozzle is operating under choked conditions or not. Because if it is choking then the exit conditions get fixed by the critical parameters that is the exit temperature will be the critical temperature exit pressure will be the critical pressure, and density will be critical density and so on.

So based on which we need to know calculate the other conditions; if it is un chocked, then the nozzle exit conditions are to be calculated using the enthalpy drop across the nozzle something, you have done in the last class during the cycle analysis discussion.

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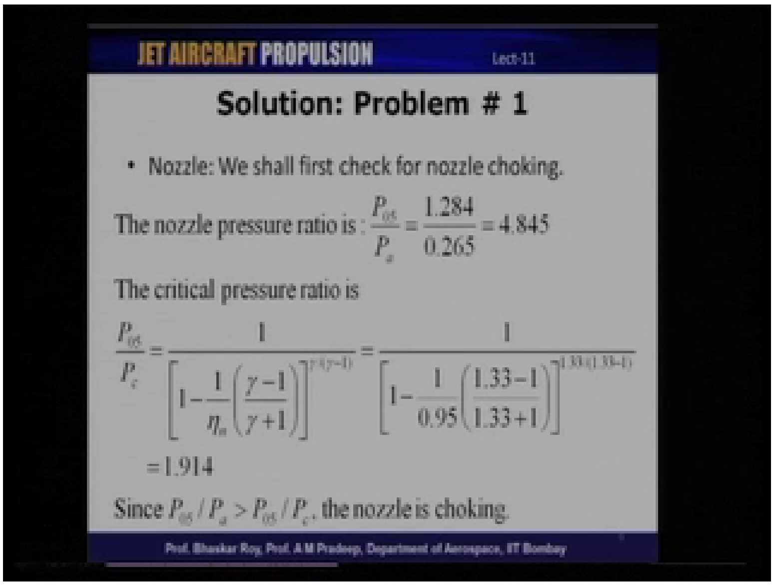

So, let us check for the chocking of the nozzle, so we first check for nozzle chocking. And how do we do that if you are familiar with the ideal cycle analysis, you must have had some experience of calculating the nozzle pressure ratio. And to check whether its indeed chocking the nozzle pressure ratio is basically, P 0 5 by P a P 0 5 is known from our previous calculation.

That is 1.284; and P a is given as 0.265 bar so, P 0 5 by P a is equal to 4.845; and that is the nozzle pressure ratio; and what is the critical pressure ratio that is P 0 5 by P c how do we calculate this critical pressure ratio? This is basically calculated by equating mach number is equal to one. And so, we can from isentropic relations we should be able to calculate the critical pressure ratio, but in that case we have been given a nozzle efficiency as well as, so there is a nozzle efficiency that also comes into picture here, so this basically can be derived when we either from the nozzle efficiency definition or from the pressure ratio definition, so critical pressure ratio is 1 by 1 minus 1 by eta n into gamma minus 1 by gamma plus 1 this raised to gamma by gamma minus 1.

So, if you substitute for all these values, we have the efficiency; and gamma which is 1. 33, then we get a critical pressure ratio 1.914 so, here we have now two pressure ratios 1 is the actual nozzle pressure ratio which is 4.845; and the critical pressure ratio which we have calculated as 1.914. So what we see here is that the nozzle pressure ratio is greater than the critical pressure ratio; and what that means that means that is the nozzle pressure ratio being greater than nozzle the critical pressure means that the nozzle is chocking. And why should that be, because from the pressure ratio that we have it means that the nozzle entry pressure is much higher than the critical pressure that is it will expand only up to the critical pressure ratio, if we have a pressure ratio higher than that it will still mean that the exit conditions of the nozzle are choked or fixed.

Because it is a convergent model, so because it happens to be a convergent nozzle then the exit conditions are fixed that is Mach number will become one mass flow rate will be maximum mass flow rate. And therefore, it is choking; and so, if it is chocking then the nozzle exit conditions are fixed; and they fixed, because those conditions will be equal to the critical parameters that is the critical temperature; and critical pressure; and the density.

That will also fix the exhaust velocity, because since Mach number is equal to one the exhaust velocity will now be equal to square root of gamma r t which is speed of sound based on the critical temperature so, in this problem we now have a scenario where the nozzle is chocking. And therefore, we now will calculate the nozzle exit conditions based on the critical parameters. So, let us see how we can calculate the critical parameters so, we will calculate the critical temperature pressure. And density so, nozzle exit conditions will get fixed by critical parameters.

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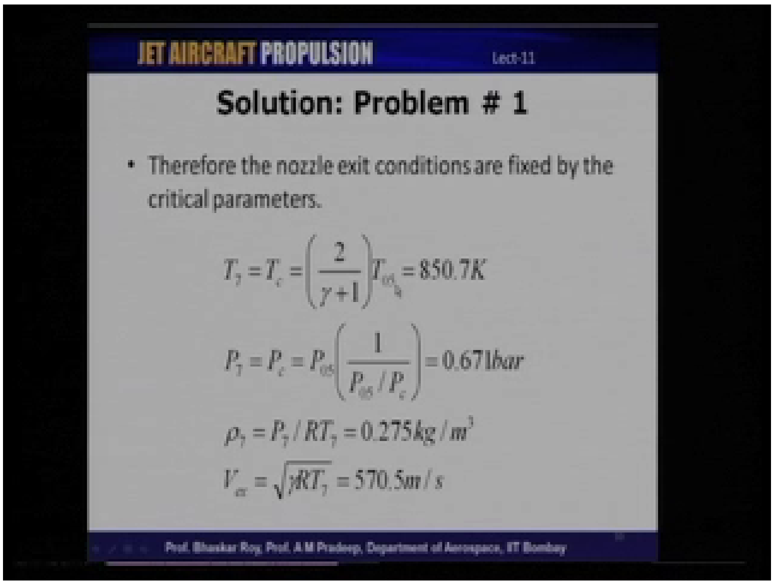

So, the critical temperature that is static temperature T 7 will be equal to critical temperature T c which is 2 by gamma plus 1 into T 0 5. And it becomes 2 by gamma plus 1, because of the temperature ratios that is T 0 5 by T 0 c is 1 plus gamma minus 1 by 2 m square; and if we put m is equal to 1 we get this expression so, T c is 2 plus gamma into T 0 5 so, if we substitute for all these values 850.7 Kelvin. Similarly, P 7 or P c is equal to T 0 5 multiplied by 1 by P 0 5 by P c, therefore this comes out to be 0.671 bar.

The density rho seven is equal to P 7 by RT 7 this on simplification we get 0.275 kilograms per meter cube, so we have temperature pressure; and density at the exit, so exhaust velocity v exit is equal to square root of gamma RT 7. And we substitute, and we find out that the exhaust velocity is 570.5 meters per second, so this is the exit velocity.

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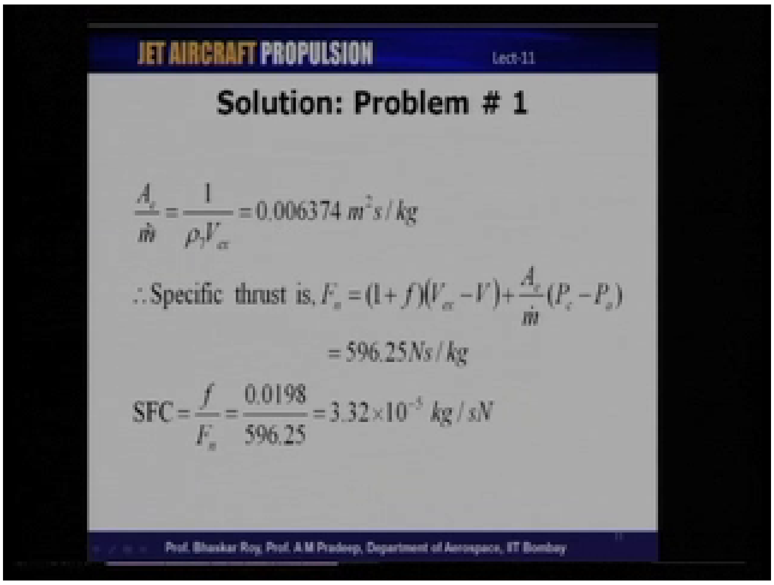

We can also calculate from what the data that is known to us this particular parameter that that is area to mass flow rate ratio, because this will be required for calculating thrust so, exit area by mass flow rate is equal to one by rho seven into V exit, because mass flow rate is rho into area into velocity. So, this parameter that is area divided by mass flow rate is 0.006374 zero meters square second per kilogram therefore, we now have the exit velocity the flight mach number; and the pressures that is P c minus P f critical parameter pressure is known that is exit pressure is known that is the ambient pressure is also known; and they are not equal.

And so, if we substitute for all these values the specific thrust we can calculate as 1 plus f into V m exit minus V plus A e by m dot into p c minus p a so, if we substitute all the values we get 596.25 Newton second per kilogram so, this will be the thrust specific thrust which is Newton per that is thrust per unit mass flow that is 596.25 Newton second per kilogram so, we have calculated the thrust what is the next parameter to be calculated that is fuel consumption or specific fuel consumption.

So, specific thrust we have calculated. And that give that is basically 596.25 Newton second per second; and next parameter to be calculated is the fuel consumption the specific fuel consumption we have already calculated he fuel to air ratio. So that divided by the specific thrust will give us the specific fuel consumption, so 0.0198 minus the fuel to air ratio divided by 596.25. So, specific fuel consumption comes out to be 3.32 into ten raised to minus 5 kilo grams per Newton second. And sometimes this is also converted to kilograms per Newton hour that is multiplied by 3600. And but basically I have I have expressed it the SI units here.

So, we have now completed cycle analysis for this first problem which was basically a simple turbo jet engine without any after burning. And given a certain amount of data, we were able to calculate the different temperatures; and pressure across various components finally, leading to the nozzle exit conditions; and the nozzle exhaust velocity; and so using that we were able to calculate the specific thrust.

And also the specific fuel consumption so, this is how you would be carrying out real cycle analysis based on the data that is provided to us of course, with certain assumptions like we assumed a constant specific heat for air all the way up to compressor exit. And we have also assumed an average specific heat for the combustion products; and similarly, the pressure ratio is heat and so on.

And so, the next problem that we will be take up that is basically on the same turbo jet, but if the turbo jet were to be operating in an after burning mode that is if this particular turbo jet is to operate with an after burner, then what would be the corresponding thrust, and fuel consumption.

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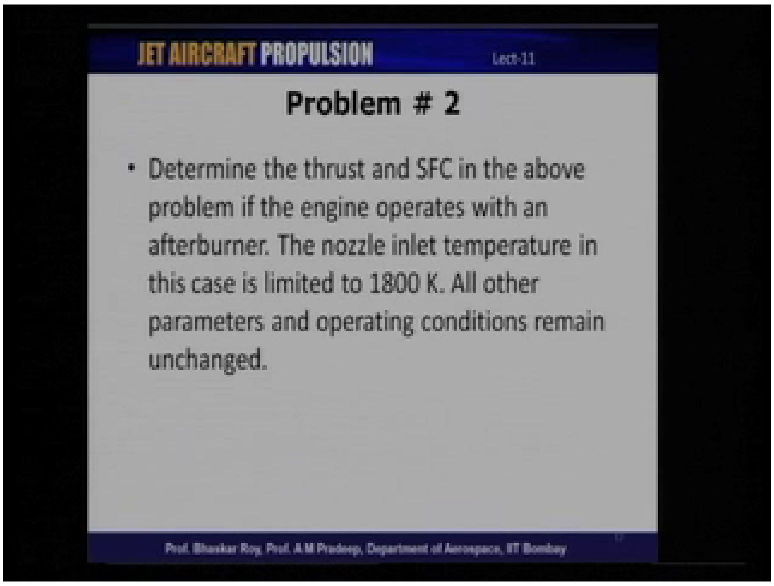

So, the problem statement for the second problem statement is determined the thrust and specific fuel consumption in the above problem if the engine operates with an after burner.

The nozzle inlet temperature in this case is limited to 1800 Kelvin all other parameters; and operating conditions remain unchanged that is if we assume that other parameters can be fixed as it is in the previous case, the only change is that the after the turbine exit we have now the after burner that is additional fuel is added in the after burner taking the temperature to 1800 Kelvin in which case what will be the new value of the thrust as well as the fuel consumption. So, before we start solving this problem, we will first take a look at the cycle diagram like we did in the previous case so, we look at a turbo jet with after burning. And then we shall attempt to solve this problem. And find out the thrust or the increase in the thrust, and fuel consumption.

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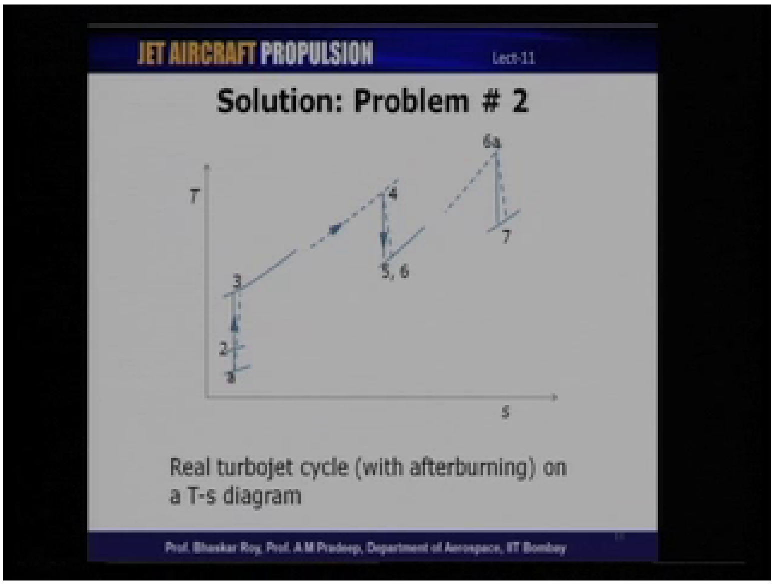

So, for an after burning turbo jet real turbo jet cycle with after burning the cycle diagram up to five is the same as what we discussed in the previous problem, because it is the same turbo jet engine now at the exit of the turbine instead of a nozzle now we have an after burner after burner will increase the temperature. And so, the temperature which was at T 0 5 at the exit of the turbine will now increase; and reach T 0 6.

That is temperature after the exit of the after burner or the nozzle entry; and after the after burner we have the nozzle which expands the combustion product; and generated the thrust so, we can see that there is a substantial increase in the temperature that is expected in the case of after burning turbo jet. The turbine in that temperature was 1200 Kelvin where as the temperature at the exit of the after burner is specified as 1800 Kelvin so, we can afford to have higher temperatures in the after burner. And therefore, we should be able to get substantial increase in the thrust.

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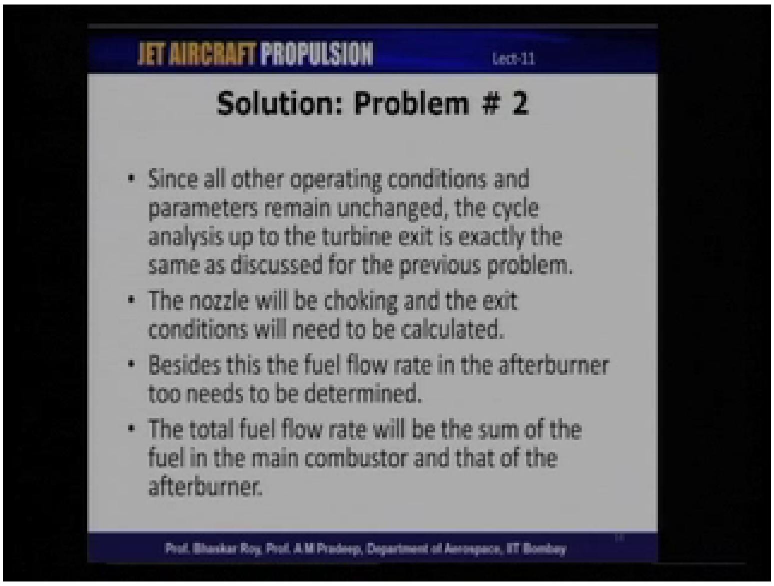

So, let us see how much increase in thrust we should be able to achieve. So, all other it is also mentioned that the operating conditions or operating parameters remain unchanged up to the turbine exit, which means that the cycle analysis. Of up to the turbine exit is exactly the same. And so, we will not repeat the same calculations here. And so, we will now need to see if the increase in temperature what kind of how would it affect s the performance, so the nozzle will continue to be chocked, because we have calculated the nozzle pressure ratio. And so, we have seen in the previous case that the nozzle was chocking, which means that even if we use an after burner the nozzle will continue to be chocked. But we will have an increase in the stagnation temperature, but we will see if it makes an difference in the thrust, so what else we will change, we will have an additional fuel flow rate in the after burner, because we have to add additional fuel in the after burner.

Which is what will lead to the increase in the temperature, therefore the fuel consumption is going to change, and the total fuel flow rate will now be equal to the fuel you added in the combustion chamber that is main combustor plus the fuel added in the after burner so, this will directly impact the fuel consumption, but besides this there will also be a change in the thrust.

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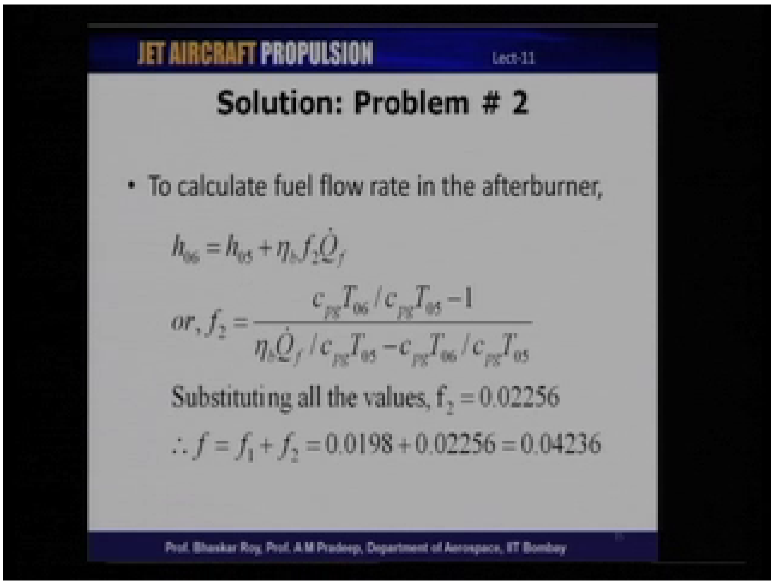

So, to calculate the fuel flow rate we calculated first in the after burner, because the rest of the parameters up to the turbine exit remains unchanged. So if we will use the same principle if we did for the main combustor that is basically an energy balance across the after burner, so enthalpy of the after burner exit is h 0 6 which is equal to h 0 5 plus the efficiency or burner efficiency into fuel flow rate multiplied by the heat of reaction or the calorific value of the fuel.

So, this on simplification we get f 2, which is the fuel added in the after burner is equal to C p into T 0 6 by C p g into T 0 5 minus 1 divided by the burner efficiency into calorific value divided by C p into T 0 5 minus C p T 0 6 by T 0 5. So, all these numbers are known to us T 0 6 is specified as 1800 Kelvin other parameters are known from our previous cycle analysis. So if we substitute all these values we get f 2, which is fuel flow rate in the after burner is equal to 0.02256. And therefore, the total fuel flow rate is f is equal f 1 plus f 2, where f 1 is the fuel plus air ratio in the main combustor that plus f 2 in the after burner, so the total fuel flow rate is 0.04236, so that is as you can see more than 50 percent there is more than 100 percent increase in the fuel to air ratio, in this the turbo jet with after burning.

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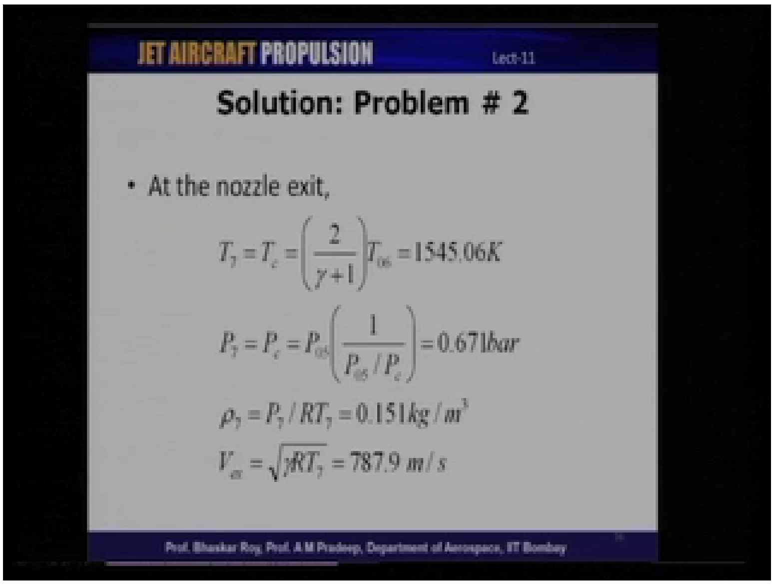

The nozzle exit conditions, we will now calculate we have already calculated. And seen that he nozzle is operating under chocked condition. So, the nozzle exit temperature critical temperature will now be equal to 2 by gamma plus 1 into T 0 6 that T 0 6 is given as 1800 Kelvin, so this is 1545.06 Kelvin. Similarly, the exit pressure which is the critical pressure is equal to P 0 5 by 1 by P 0 5 by P c which is 0.671 bar which is same as what we calculated in the last problem.

The density will now change, because the temperature has changed that is P 7 by RT 7 that is 0.151 kilograms per meter cube therefore, the exhaust velocity will be equal to square root of gamma RT 7, where T 7 is something which we have already calculated so, this is equal to 787.9 meters per second. So, at the nozzle exit we now have the exit velocity; and also the critical parameters.

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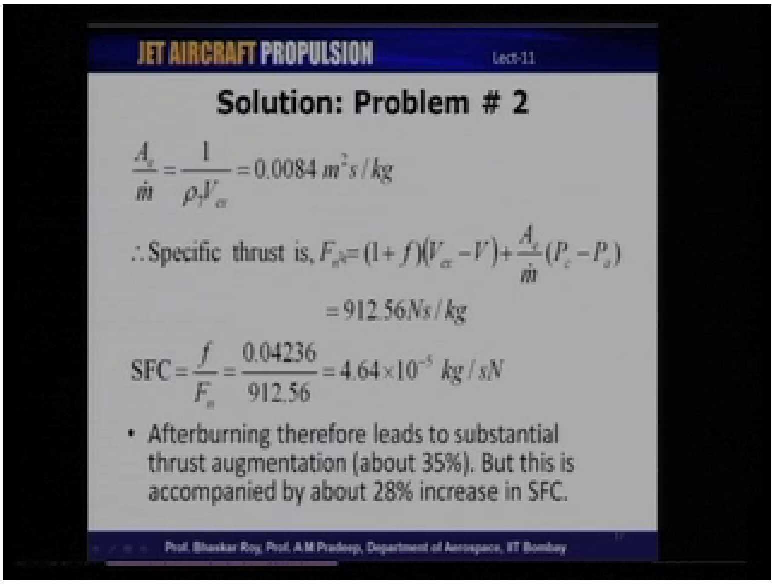

So, we now calculate the specific thrust in the same way as we calculated in the previous case, we have the specific thrust is equal to one plus f into V exit minus b plus a m dot into P c minus P a here, f is different from what we calculated in the previous problem; and so, is the exhaust velocity. So, if we substitute these values we get the specific thrust as 912.56 Newton second per kilogram. Similarly, the specific fuel consumption is f divided by specific thrust this comes out to be 4.64 into 10 raised to minus 5 kilograms per second Newton second.

So, if we compare these values with what we had calculated in the first problem we will see that after burning it leads to a very substantial increase in thrust we get about 35 percent increase in the thrust. And this is also accompanied by a corresponding increase in the fuel consumption it is with a cost of 28 percent in the fuel consumption. So, this is just to highlight that after burning is something that is used to increase the thrust or to achieve momentary increase in thrust or so, to sustain high mach numbers in supersonic flights. And so, after burning turbo jets are usually used in these applications where there is need to increase thrust momentarily. And so, since we have seen that after burning also leads to an increase in fuel consumption.

It is not something used in civil aviation turbo fan normally do not operate in after burning so, after burning is usually used or limited to military engines. So, we have now solved two problems related to turbo turbo jet engines one was a simple turbo jet engine without any after burning; and the second problem was an extension of the first problem with after burning; and we have seen how we can calculate; and carry out cycle analysis.

Realistic cycle analysis, because we have efficiencies of all the components; and how the cycle analysis can be carried out in a systematic manner so, we will now take up an another version of jet engine we will now discuss about the turbo fan engine we will solve one problem on turbo fan engine followed by another problem on turbo prop engine.

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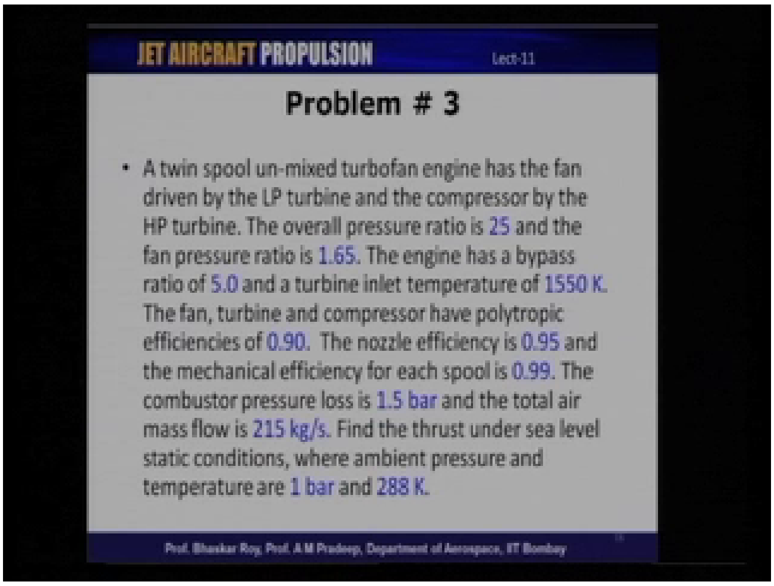

So, let us take a look at the third problem, problem number 3 states that a twin spool unmixed turbo fan engine has the fan driven by the LP turbine; and the compressor driven by the HP turbine the overall pressure ratio is 25; and the fan pressure ratio is given as 1.65 the engine has a bypass ratio of 5; and the turbine in the temperature of 1550 Kelvin. The fan turbine and the temperature have polytrophic efficiencies of 0.9 the nozzle efficiency is 0.95; and the mechanical efficiency of each spool is 0.99 the combustor pressure loss is 1.5 bar; and the total air mass flow rate is 215 kilograms per second.

Find the thrust under the sea level static conditions, where ambient temperature; and pressure are 1 bar; and 288 Kelvin, so in this problem to do with an unmixed turbo fan, which means that there are two separate nozzles that are used as cold nozzle or the bypass nozzle or the secondary nozzle. And the primary nozzle which is like that used in a turbo jet engine.

So, we will solve these two streams separately. And then the total thrust will be equal to sum of the thrust generated by the primary nozzle; and the thrust developed by the secondary nozzle so, the steps that are going to be followed are very similar to what we had done in the case of a turbo jet engine. So, I will I will go through this problem little more quickly a little faster than what we had discussed for the turbo jet, because the steps are identical.

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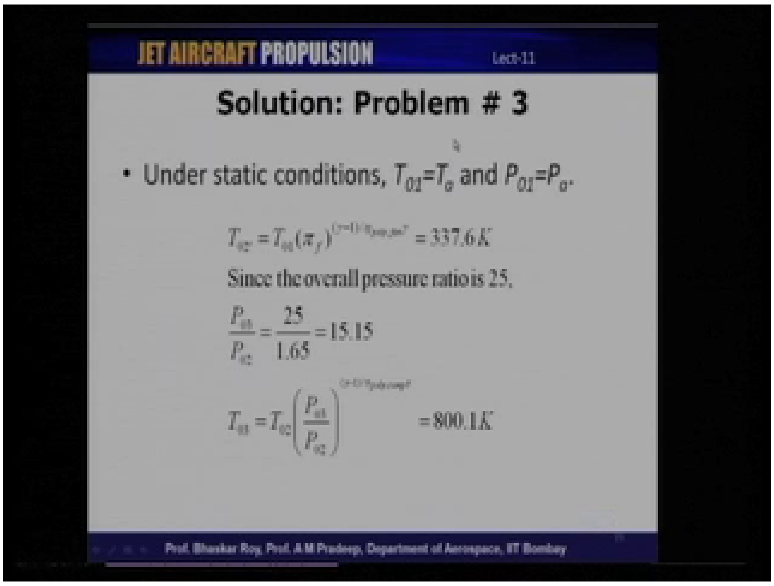

So, we will begin with the fan; and it is mentioned in the problem that is we are required to calculate the thrust under static conditions, which means that the engine is mach number or flight speed is 0 therefore, T 0 1 will be equal to ambient temperature P 0 1 will be equal to ambient pressure. And therefore, the fan exit temperature T 0 2 prime is equal to the inlet temperature.

Multiplied by the fan pressure ratio into gamma minus 1 divided by in this problem we have been given the polytrophic efficiency of the fan. And if you recall during our discussion little earlier when we were talking about the component performance, we had discussed about polytrophic efficiency of the compressor.

And we have seen that we can relate the polytrophic efficiency to the isentropic efficiency; and through the ratio of specific heat. So we are going to use the polytrophic efficiency here so, we have pi f which is fan pressure ratio raised to gamma minus 1 divided by the polytrophic efficiency of the fan multiplied by gamma.

So, this comes out to be 330.6 Kelvin now it is given that the overall pressure ratio is 25; and the fan pressure ratio is known which is 1.65, therefore the compressor pressure ratio will be equal to overall pressure ratio divided by the fan pressure ratio, so we have 25 divided by 1.65. That is 15.15 so, the compressor exit stagnation temperature T 0 3 will be equal to T 0 2 into the fan the compressor pressure ratio that is pi c raised to gamma minus 1 by polytrophic efficiency of the compressor into gamma so, this temperature comes out to be 800.1 Kelvin.

So, we will first calculate the secondary nozzle properties we will calculate the thrust developed by the secondary nozzle. And then we will proceed to the primary nozzle; and calculate the thrust developed by that; and then add up the two to get the final thrust generated by the turbo fan.

Now in the case of nozzle as we have seen, we will first need to determine whether the nozzle is chocking or not. So, we will calculate the nozzle pressure ratio; and the critical pressure ratio; and see if check, if the nozzle is indeed chocking.

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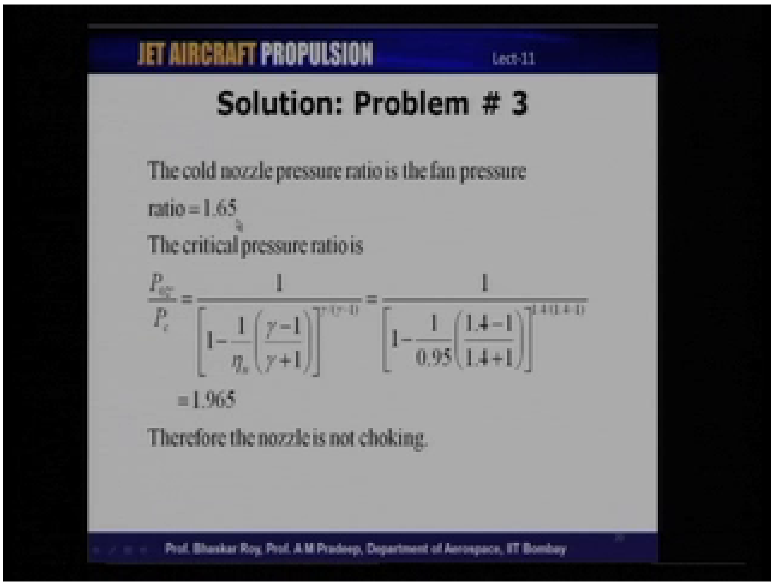

The cold nozzle pressure ratio is the fan pressure ratio, because that is what is available at the inlet so, it is 1.65. And critical pressure ratio is P 0 2 prime by P c which is one by 1 minus 1 eta n into gamma minus 1 raised to gamma by gamma plus 1 so, for the cold nozzle we will take gamma as 1.4 where as for the second nozzle we will take gamma as 1.33. So, if we substitute the nozzle efficiency; and the value of gamma we get the critical pressure ratio as one point nine six five now since the pressure ratio of the nozzle is less than the critical pressure ratio it means that the nozzle is not chocking.

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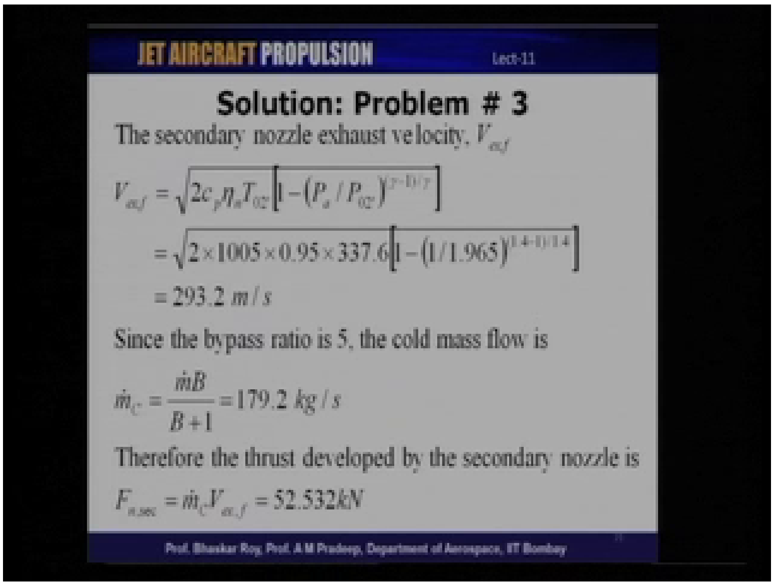

So, if the nozzle is not chocking then we have the nozzle exhaust velocity which will be equal to square root of 2C p into nozzle efficiency into the temperature T 0 2 prime into 1 minus P a by P 0 2 prime raised to gamma minus 1 by gamma. So, how we have derived his expression we have discussed in earlier lecture on cycle analysis this basically comes from the enthalpy drop across the nozzle; and if we simplify we get the exhaust velocity.

So, this is equal to two into the C p which is 1005 nozzle efficiency temperature; and the pressure ratio so, we get the exhaust velocity at the fan at the secondary nozzle at 293.2 meters per second now it is given that the bypass ratio is 5. Therefore the cold mass flow m dot c is equal to mass total mass flow into bypass ratio divided by b plus 1 that is bypass ratio plus 1, which we get as 179.2 kgs per second therefore, the thrust developed by the secondary nozzle mass flow rate into exhaust velocity that is 52.532 kilo newtons.

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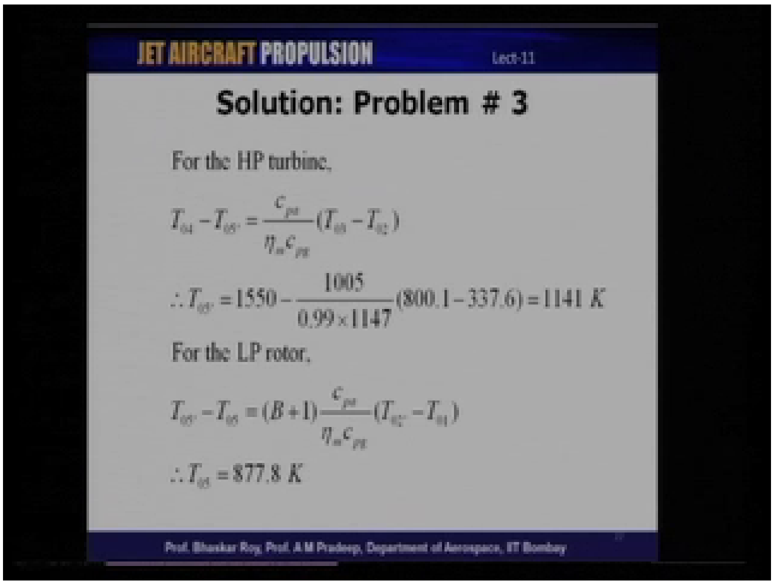

Now we now move on to the core nozzle or the core engine so, for the HP turbine we have the work done work done by turbine to be equated by the work required by the compressor. So, HP turbine drives the compressor so, the temperature across the HP turbine T 0 4 minus T 0 5 prime is C p a divided by mechanical efficiency into C p g into T 0 3 minus T 0 2. So, we simplify this, and we get the HP turbine exit stagnation temperature which is 1141 Kelvin.

Similarly we can calculate the LP turbine exit conditions, because LP turbine drives the fan; and the fan mass flow rate as we can see here is different from the core mass flow, because the fan drives a larger amount of mass flow, so T 0 5 that is LP turbine exit temperature is 877.8 Kelvin.

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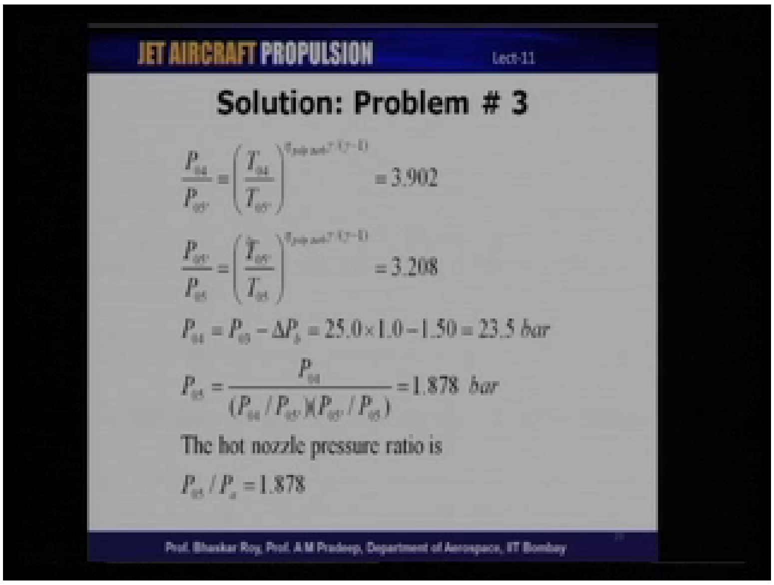

Similarly, we proceed towards finding the exit pressures, we have these pressure ratios P 0 4 by P 0 5 prime, which is temperature ratio raised to gamma minus 1 by gamma, which is 3.902. Similarly, to the LP turbine; and the turbine inlet pressure is equal to compressor exit pressure minus delta p pressure loss in the burner. So, that is 25 minus 1.0 minus 23.5, so P 0 5 is basically P 0 4 divided by these pressure ratios. And so, we get 1.878 bar, and that the hot. Therefore the hot pressure ratio of pressure ratio of the hot nozzle is 1.878.

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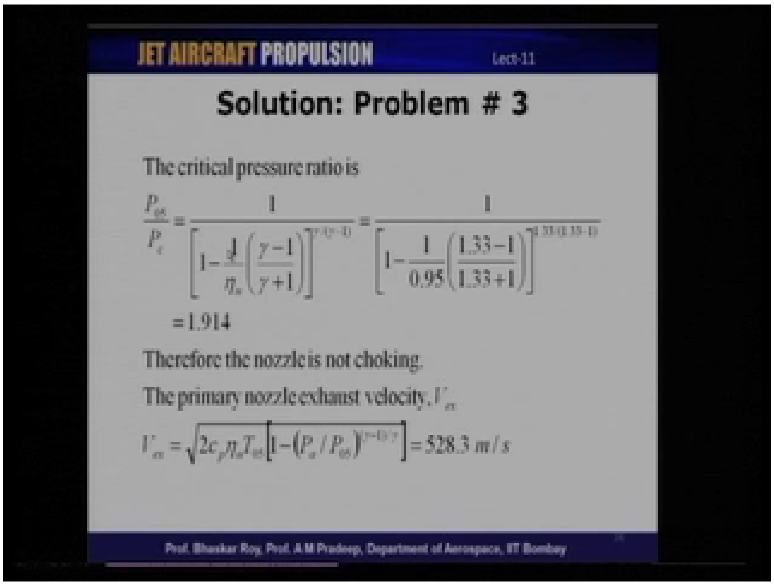

The critical ratio pressure ratio for this nozzle is different nozzle is primary nozzle from that of the secondary nozzle; because gamma is different here gamma is 1.33. Critical pressure ratio is actually 1.914 one, so we find that since the pressure ratio is less than the critical pressure ratio this nozzle is also not chocking. So we proceed to find out the exit velocity exhaust velocity here in the same manner as we calculated for the secondary nozzle; and that we get 528.3 five meters per second.

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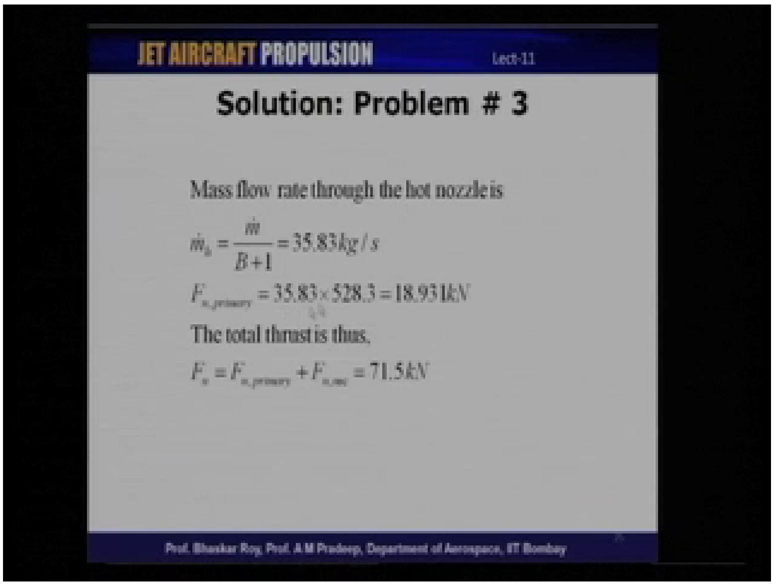

And the mass flow rate through the hot nozzle is 35.83. Therefore, thrust developed is 35.83 times the exhaust velocity which is 18.931. Therefore the total thrust is equal to primary thrust, which is developed by the primary nozzle plus that of the secondary nozzle that is 71.5 kilo Newton's. So, this is a problem, which involved a turbo engine; and unmixed turbo fan engine which was consisting of two separate streams each of them generating a thrust; and contributing towards he total thrust.

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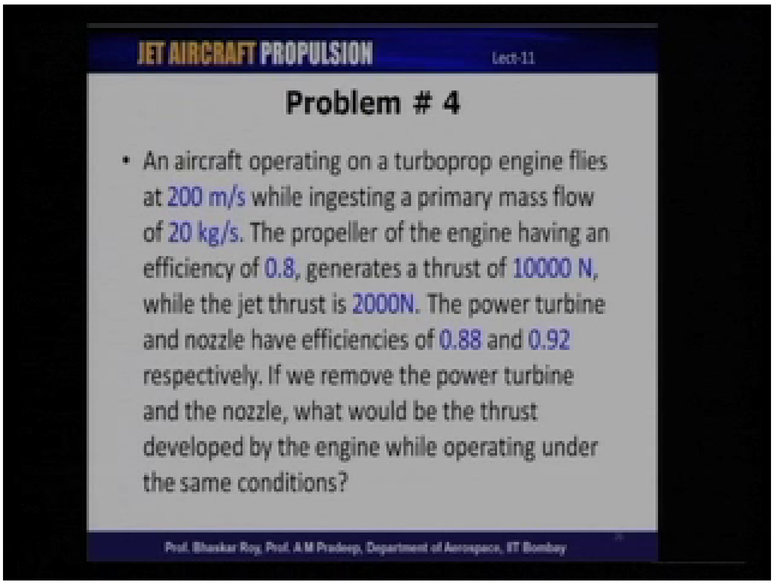

The last problem that we will solve today corresponds to that of a turbo prop engine, where in an aircraft operating on a turbo prop engine flies at 200 meters per second. While ingesting a primary mass flow of 20 kgs per second the propeller of the engine having an efficiency of 0.8 generates a thrust of 10000 Newton, while the jet thrust is 2000 Newton the power turbine. And the nozzle have efficiencies of 0.88, and 0.92 if we remove the power turbine; and nozzle what would be the thrust developed by the engine while operating.

Under the same conditions so, here we have a turbo prop; and we have been given all the efficiencies of the propeller the turbine; and the nozzle so, we have required to find out that if we remove the power turbine; and the nozzle what will be the thrust developed by the same engine operating under the same conditions.

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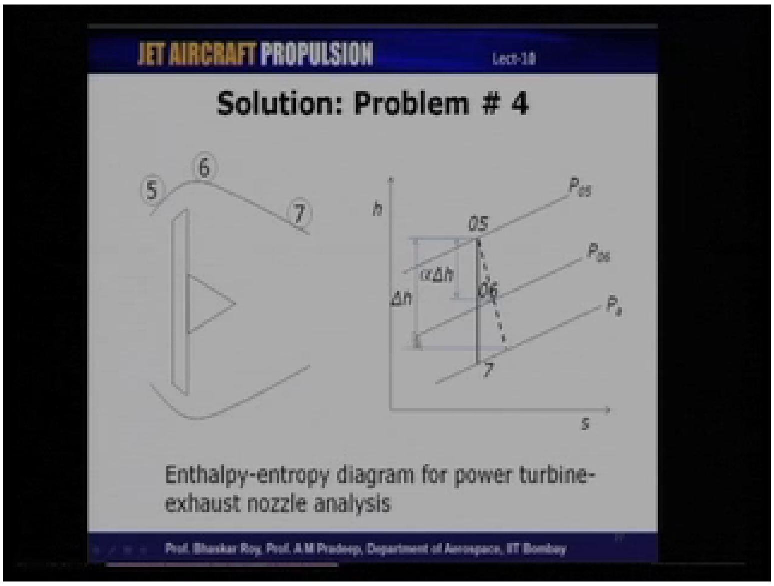

So, in this case we will basically be calculating the enthalpy drop across the power turbine plus the nozzle combination. And then we see that if we remove the power turbine then the entire enthalpy drop is available for exhaust through the nozzle.

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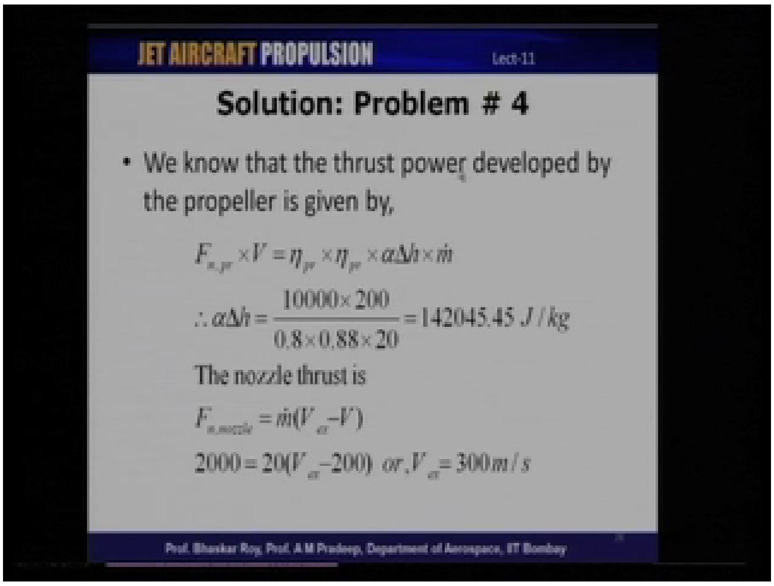

And that is how we will be able to calculate the thrust developed by this particular engine. So, thrust power that is developed by the propeller we have already discussed this is our last class thrust power by the propeller is basically equal to the thrust developed by the propeller into the velocity this is equal to the propeller efficiency into the power turbine efficiency into alpha delta h into m dot. Therefore, if we simplify this we get alpha delta h as the thrust by the propeller, which is given as 10000 Newton into velocity 200 divided by propeller efficiency 0.8 or turbine efficiency 0.8 into m dot which is 20. So, alpha delta h is 142045 joules per kilo gram; and the nozzle thrust is also given it is 2000 Newton. So, nozzle plus 2000 is equal to m dot into exit velocity minus V, therefore we get the exit velocity as 300 meters square per second.

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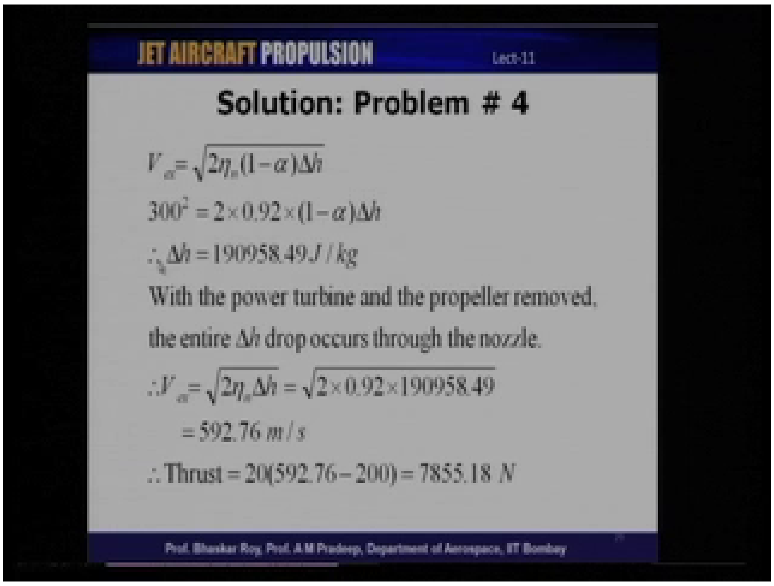

We also know that the exit velocity is equal to square root of two into nozzle efficiency into 1 minus alpha into delta h so, from this we can calculate what is delta h, because alpha delta h we have already calculated so, delta h comes out to be 190508.49 joules per kilogram. Now it is mentioned that, now if you remove the power turbine; and propeller what will be the thrust which means that this entire delta h will now be available for expansion through the nozzle; therefore, the exit velocity now will change it was earlier three hundred now the exit velocity is not equal to one minus alpha delta h, but it is just equal to delta h so, we get exit velocity is equal to square root of 2 into efficiency of the nozzle into delta h which is equal to 592.76 meters per second.

Therefore, the thrust the new thrust will now be equal to 20 that is mass flow rate into V exit minus V that is 592.76 minus 200 that is 7855.19 Newton, so this is the thrust that will be developed by the nozzle if we were to remove the power turbine; and the propeller.

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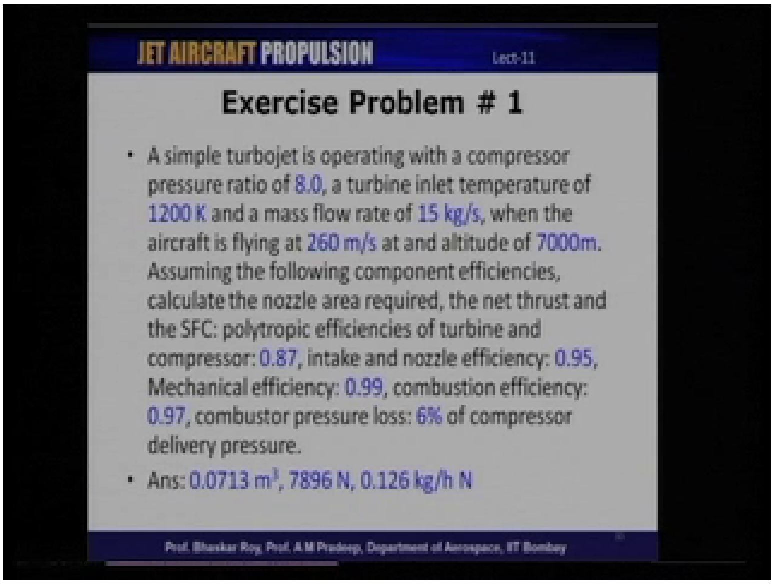

So, we have today discussed four problems we have solved four problems two of them related to turbo jet one of them to turbo fan; and one with turbo prop so, I have now four exercise problems for you. And the first one is to do with the turbo jet engine which is operating with a compressor pressure ratio of eight turbine inlet temperature as twelve hundred; and mass flow rate of 15 kgs per second.

The altitude of the aircraft; and the speed is given so, assuming these efficiencies that is polytrophic efficiencies of turbine; and compressor as 0.87 intake; and nozzle efficiency as 0.95 mechanical efficiency as 0.99 combustion efficiency 0.97 pressure loss as six percent of compressor delivery. We need to calculate the net thrust area the nozzle area required the thrust; and specific fuel consumption so, the answer to the 0.0713 meter cube meters square 0.7896 Newton which is the thrust; and fuel consumption is 0.26 kilograms per Newton hour.

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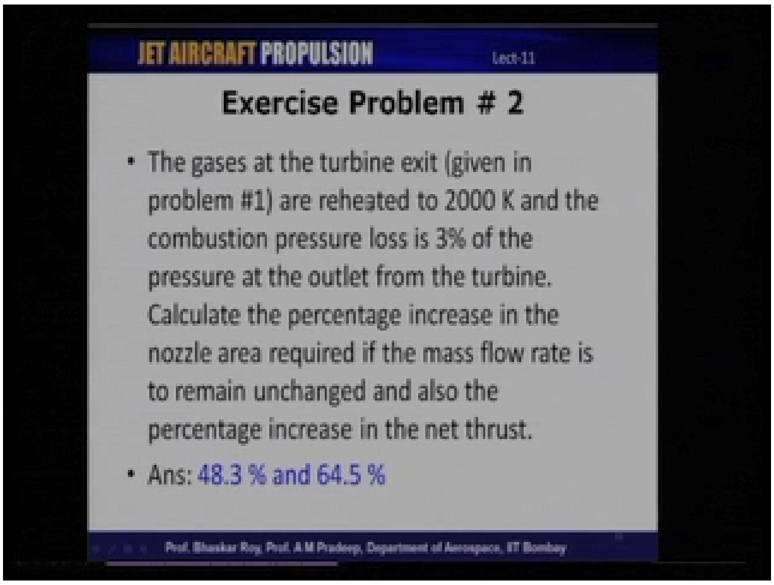

The second problem is the same problem with turbo with after burning so, at the turbine exit the gases are reheated to two thousand Kelvin; and pressure loss is 3 percent calculate the percentage increase in nozzle area required if the mass flow rate is to remain unchanged. And also the percentage increase in net thrust that means we need to calculate how much area increase of the nozzle is required if we have to operate the engine keeping the same mass flow.

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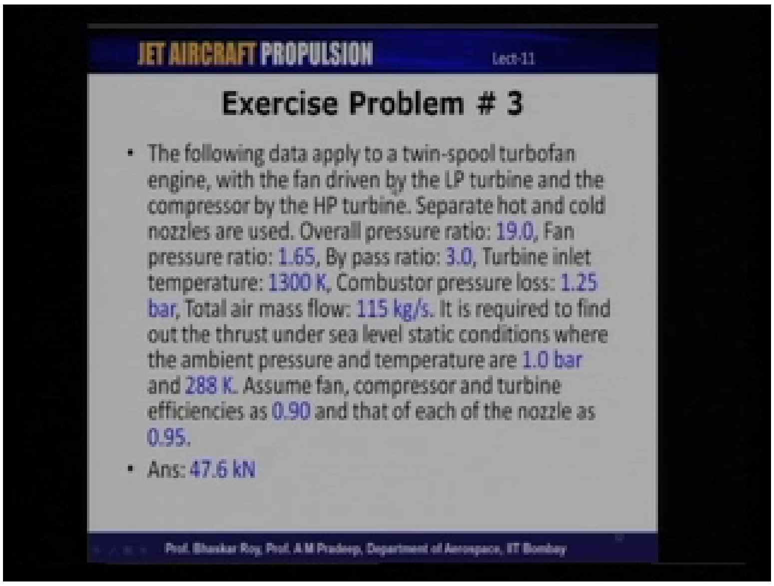

So, the answer to that is 48.3 percent increase in area; and 64.5 percent increase in net thrust third problem is to do with a turbo fan engine, a twin spool turbo fan engine, its again unmixed above fan the overall pressure ratio. The fan pressure ratio bypass ratio is given as three turbine in the temperature is 1300 power loss is given as 0.25 in the combustor. Total air mass flow is 115 kgs per second we need to find out the thrust, when the ambient pressure and temperature are 1 bar; and 288 Kelvin the fan; and compressor turbine efficiencies are 0.9 nozzle has an efficiency both the nozzles have an efficiency is 0.95 so, thrust in this case is 47.6 kilo Newton.

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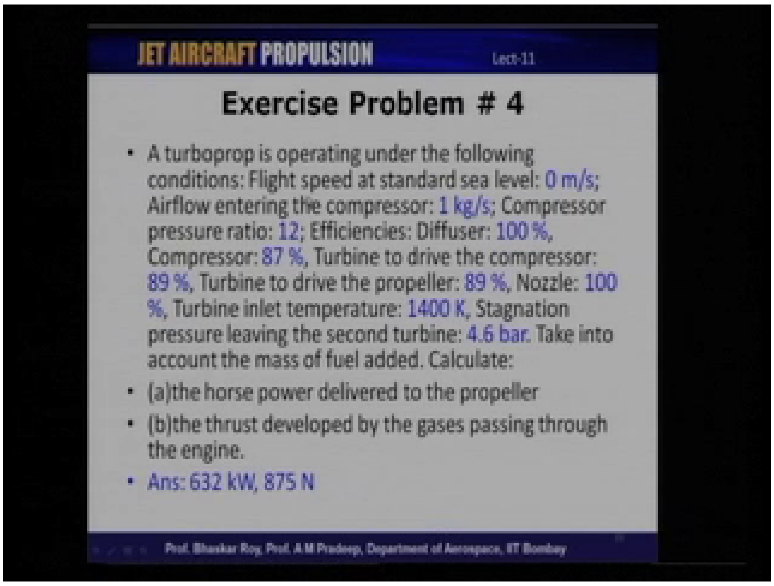

And the last problem is to do with the turbo prop turbo prop operating under sea level conditions with flight speed as 0 air flow entering the compressor is 1 kgs per second compressor pressure ratio is 12, all the efficiencies are given turbine in the temperature is 1400 Kelvin stagnation pressure leaving the second turbine is 4.6 bar.

And we need to calculate the horse power delivered to the propeller that is by the power turbine; and the thrust developed by the gases passing through the engine so, the power developed by the propeller is 632 kilowatts; and the thrust developed by the nozzle is 875 Newton. So, these are four different problems that I have sorted out for you basically to do with turbo two of them to do with turbo jet one to do with the turbo fan. And another one with turbo prop engine all these problems are exactly in line with what we have solved with slight differences here and there. So, I am sure you would be able to solve these problems based on what we discussed during today’s tutorial session, so that brings us to the end of today’s tutorial session. In the next class, we will be taking up analysis of compressors axial flow compressors; we will begin a simplistic analysis of axial flow compressors.