

Jet Aircraft Propulsion
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Module No. # 01
Lecture No. # 10
Analysis of Engine Real Cycles

Hello and welcome to lecture number 10 of this lecture series on Jet Propulsion. In the last few lectures, we have been discussing about the Brayton cycle, the ideal and the real Brayton cycles, and how we can use the Brayton cycle, the basic thermodynamic cycle for application in **in** the form of jet engines. We have also discussed the ideal cycles for jet engines, the different forms of jet engines like the turbojet, the turbofan, turbo shaft, turboprops and the ram jets.

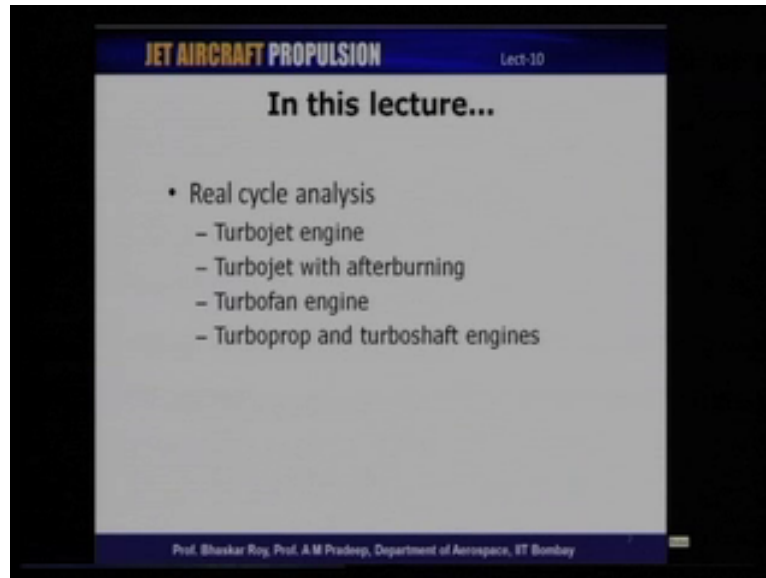
Subsequently, we have also discussed about how we can account for some of the efficiencies or the loss in performance occurring in many of these components like in the diffuser, the compressors, the combustion chamber, turbine and the nozzle. So, we have associated all these components with certain performance parameters like efficiencies and pressure drop occurring in these components.

So, in today's lecture what we are going to do is, to see how these efficiencies affect the performance of the ideal cycles, how we can take into account the loss of performance in all these components and how we can modify the cycle analysis, the ideal cycle analysis taking into account all these efficiency parameters. So, today we will be discussing about the real cycle analysis of jet engines. So, we will be discussing about the real cycle analysis for turbojet engines, with and without afterburning and then we will take up the turbofan engines and subsequently the turboprop and turbo shaft engines.

So, we will be taking a quick look at the real cycle analysis of some of these jet engines. And of course, I am assuming that you have already undergone the ideal cycle analysis in either the thermodynamics course or in the precursor to this course, the introduction to Aerospace

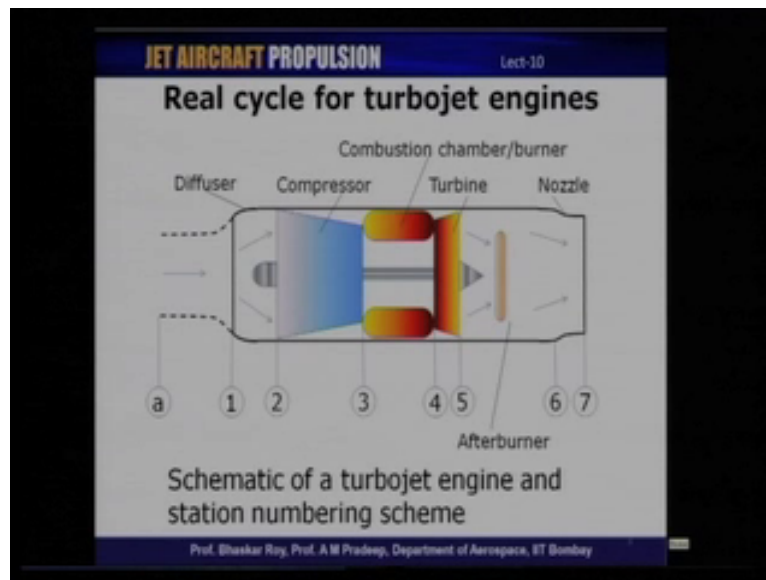
propulsion. So, ideal cycle analysis, of course we had some brief discussion of this when we were discussing about Brayton cycle as applied to jet engines and detailed ideal cycle analysis would have been covered in your thermodynamics course.

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So, in today's lecture, we will be discussing about real cycle analysis as applied to turbojet engines, to begin with that is simple turbojet engine without afterburning; we will then take up turbojet with afterburning, subsequently we will discuss about turbofan engine and then turboprop and turbo shaft engines.

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So, let us begin with the turbojet engines. So, this is schematic of a turbojet engine which I had also shown a couple of lectures earlier, showing all the components which constitute a turbojet engine and most of these components will also be present in the other forms of these jet engines like the turbofan or turboprop engines.

So, one of the first components that we can see in a turbojet engine is the diffuser and then the compression process begins with the diffuser and continues in the compressor. And subsequent to the compressor, we have the combustion chamber where heat is added that is fuel is burned in the combustion chamber and the temperature reaches its maximum temperature in this entire cycle that is in the combustion chamber.

So, these (O) products at high pressure and temperature are expanded through the turbine which generates a work output. So, the work output of the turbine is primarily used for driving the compressor. So, after the turbine, we may have an afterburner present, where in additional fuel can be added and the temperature cycle temperature can further be taken up by reheating.

So, the combustion products are then expanded through the nozzle which generates the thrust. So, this is just a schematic of a turbojet engine, I have also indicated certain numbers beneath each of these components which are basically the station numbers which we will use in our cycle analysis. For example, the number that is shown here that is a, which primarily denotes

the free stream, 1 is for the diffuser entry, 2 is for the diffuser exit which is also the compressor entry. station number 3 is for the compressor exit which also happens to be the combustion chamber inlet. 4 is the combustion chamber outlet and the turbine inlet. 5 is the turbine outlet, and 6 is the afterburner exit or the nozzle entry, and 7 is the nozzle exit. So, we are going to use this numberings scheme, not just for this turbojet analysis that we are going to initiate right away, but also for the turbofan and turboprop analysis which we will take up subsequently.

Now, in a turbojet engine as we have already discussed, the various processes involved are can be basically map to Brayton cycle. The cycle begins with compression in the case of turbojet engine we have compression beginning in the diffuser and then it continues in the **in the** compressor. And after compression process, we have the heat addition which is what it is in Brayton cycle; in the case of a jet engine we have combustion taking place in the combustion chamber. And then at the end of the combustion process or the heat addition process, the temperatures reaches the maximum in **in** this cycle and the combustion products are then expanded through the turbine and the expansion continues in the nozzle.

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JET AIRCRAFT PROPULSION Lect-10

Real cycle for turbojet engines

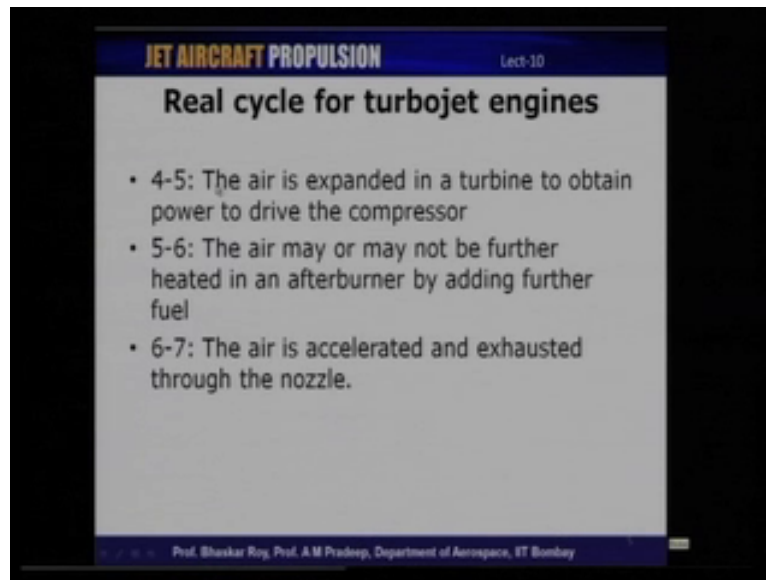
- The different processes in a turbojet cycle are the following:
- a-1: Air from far upstream is brought to the air intake (diffuser) with some acceleration/deceleration
- 1-2: Air is decelerated as it passes through the diffuser
- 2-3: Air is compressed in a compressor (axial or centrifugal)
- 3-4 The air is heated using a combustion chamber/burner

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And so, if you look at the different processes, process from a to 1 is that the air is compressed in the free stream itself which we will again discuss later on when we discuss about diffuser that, how compression can actually take place before even the diffuser begins. And then between 1 and 2, there is decelerated as it passes through the diffuser, between 2 and 3 we

have the compressor where air is compressed and it could be either an axial compressor or centrifugal compressor, we will discuss that in detail in later lectures. 3 to 4 is that air is heated using a combustion chamber or a burner.

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4 to 5 is expansion through a turbine which generates the work output to drive the compressor, 5 to 6 is the afterburner which may or may not be present, and 6 to 7 is the air is accelerated and then exhausted through the nozzle. So, basically the expansion is completed when it passes through the nozzle.

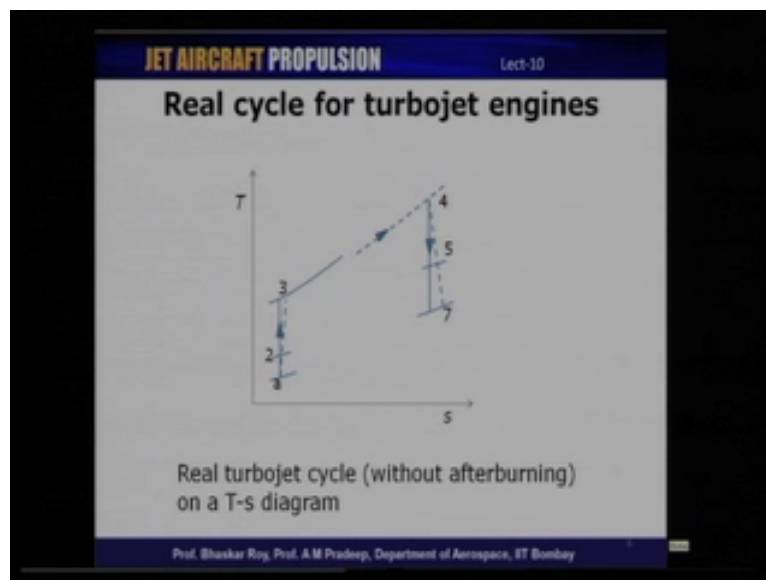
So, all these processes we can now represent on a cycle diagram, we have already seen the ideal cycle of a turbojet engine, where in the first process was an isentropic compression and then we had a constant pressure heat addition in the combustion chamber, subsequently we had the isentropic expansion process. Now, in a real cycle as we have just discussed that all these processes are not going to be reversible or ideal anymore. And so, that is basically because of the presence of irreversibility in all this components.

For example, the compression process will no longer be isentropic and so, there would be some change in entropy taking place across the diffuser as well as the compressor. And then in the heat addition process that is the combustion chamber **the** the, it would no longer be a constant pressure process, there would be some loss of pressure due to viscous effects present

in the combustion chamber, so there would be pressure loss occurring through the combustion chamber.

Besides this, there would also be some efficiency associated with combustion itself that is the combustion may not be complete combustion. So, there could be some loss of efficiency in the combustion process itself. Then in the turbine again, we no longer have an isentropic expansion taking place, there could be some losses associated with the expansion in the turbine and also the nozzle. So, if we were to incorporate all the different losses into a turbojet cycle, the basic thermodynamic cycle, the ideal cycle will get modified slightly. So, let us take a look at how the modified thermodynamic cycle of a turbojet would look like.

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So, what I have shown here are both the ideal as well as the real processes, those shown by the dotted lines are the real processes, the ones in solid line is what we had seen earlier the ideal process. So, the cycle begins at station a which is the far off streamer, free stream and then it goes on, the compression goes on all the way upto 3 and in the ideal process, it would have been isentropic. And so, we have a vertical line which represents the isentropic process, but the actual process is not isentropic. And it, so we can see a deviation of the process from the ideal cycle and so, the dotted line represents the actual process.

So, the actual process is non isentropic and therefore, it is no longer vertical. So, process from a to 3 is non isentropic and that is represented by this dotted line. At the end of the

compression process which is at point 3, we have the heat addition in the combustion chamber and the ideal cycle, this would have been a constant pressure line and in an actual cycle or real cycle, it is no longer a constant pressure. So, there is loss of pressure occurring in the combustion chamber, which is why we have two different pressure lines shown here.

At the end of combustion we reach the point 4, which is the turbine inlet and the expansion process again is non isentropic and so, we have two different lines here. Turbine expansion which is between 4 and 5 is non isentropic, **the** the isentropic process is shown by the solid line which is vertical and the expansion process being non isentropic, it is represented by the dotted line between stations 4 and 7.

So, this is a turbojet cycle, a real turbojet cycle without afterburning, we will also see the cycle diagram for the turbojet with afterburning which is basically a reheat after the turbine expansion. Now, to carry out a cycle analysis of such a system, what we basically do is that, there are certain parameters which are fixed or which are designed parameters. Using some of these known parameters, **we will** we would now calculate and determine the other parameters which are required for calculating the thrust and efficiency developed by the, by this particular engine.

So, some of the parameters which will be known are the ambient pressure and temperature which is basically from the altitude at which the engine has been designed for operation, we would also be knowing the flight mach number, and then the compressor pressure ratio, the turbine inlet temperature, and the properties of the fuel which is used in the combustion chamber. So, these are some of the parameters which would be known a priori, which are basically design parameters; besides this, we would also be assuming some values of efficiencies for the different components like the diffuser, the compressor, combustion chamber, turbine and the nozzle.

So, these are also efficient, **there** there is certain efficiency ranges which I had displayed during last lecture where we were discussing about performance penalties associated with each of these components. So, these efficiencies are also basically in our cycle analysis, we are going to assume that these efficiencies are known. So, given these efficiencies, how can we carry out a cycle analysis for to begin with a turbojet engine?

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Real cycle for turbojet engines

- For cycle analysis we shall take up each component and determine the exit conditions based on known inlet parameters.
- Intake: Ambient pressure, temperature and Mach number are known, P_a , T_a and M

$$T_{02} = T_a \left(1 + \frac{\gamma-1}{2} M^2 \right)$$

$$\eta_d = \frac{T_{02} - T_a}{T_{02s} - T_a} = \frac{T_{02}/T_a - 1}{T_{02s}/T_a - 1} = \frac{T_{02}/T_a - 1}{(P_{02}/P_a)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$\therefore P_{02} = P_a \left(1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right) \right)^{\frac{\gamma}{\gamma-1}}$$

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So, we will take up each of these components one by one and based on the parameters which are known **for the** for this particular component, we can determine the exit conditions of each of these components.

So, the first component that we shall take up is the intake or the diffuser and for the diffuser, the ambient pressure and temperature as well as the Mach number would be known. Because if the flight Mach number and the altitude are known, we can calculate the mach ambient pressure and temperature, this means that the ambient pressure temperature and the Mach number are known. Now, since the ambient pressure is known, temperature is known we can calculate the corresponding stagnation temperature.

Now, T_{02} will be equal to T_{0a} that is the ambient stagnation temperature, because there is no loss in stagnation temperature in the intake or the diffuser, because there is no heat addition or heat rejection, we are assuming that this process is adiabatic, there is no heat addition or heat rejection talking place in the intake which is by T_{02} is basically equal to T_{0a} , which can be calculated from the isentropic relations.

So, T_{02} will be equal to T_a multiplied by $1 + \frac{\gamma-1}{2} M^2$, so this is following directly from the isentropic relations. Now, to calculate this stagnation pressure, we would need to know the diffuser efficiency; now if **if** the diffuser efficiency is known, we can easily calculate the diffuser exit stagnation pressure. So, how do we that, now

the diffuser efficiency we have already defined in the last lecture, it is basically from the stagnation temperature differences for the actual process and the ideal process. So, diffuser efficiency is defined as $T_{02} - T_a$ divided by $T_{02s} - T_a$, where the T_{02s} is the stagnation temperature corresponding to the isentropic process.

So, if you take the ratio of these temperature we get T_{02} divided by T_a minus 1 by the corresponding isentropic temperature ratio minus 1. So, this denominator, the temperature ratio in the denominator can be converted to the corresponding pressure ratios, which is P_{02} divided by P_a raised to $\gamma - 1$ by γ which is from the isentropic relation.

So, we now have a relation which basically gives us a relation between the efficiency, diffuser efficiency, the stagnation temperature and the stagnation pressure. So, from this if we simplify, we get P_{02} is equal to P_a multiplied by $1 + \eta_d$ which is the diffuser efficiency into T_{02} by T_a minus 1, this whole thing raised to γ by $\gamma - 1$. T_{02} is known from our previous equation, diffuser efficiency if known we can now calculate the stagnation pressure at the exit of the intake. So, we now have this stagnation pressure and the temperature at the exit of the diffuser based on our knowledge of the Mach number and the ambient conditions as well as the diffuser efficiency.

Now, once we calculate the diffuser efficiency and therefore, the stagnation pressure and temperature at the exit of the diffuser, we can now proceed towards the compressor. Now, the compressor pressure ratio as I mentioned would usually be known, it **it** would be a fixed parameter, a design parameter, let us denote that by π_c . Now, if that is a fixed parameter, we can calculate the compressor exit pressure directly, P_{03} will be equal to π_c times P_{02} .

Now, we now need to calculate the compressor exit stagnation temperature, now to do that we will need to use the efficiency definition once again. The compressor efficiency, isentropic efficiency of the compressor **is** is defined as the temperature difference for the isentropic process divided by the temperature difference for the actual process. So, $T_{03s} - T_{02}$ divided by $T_{03} - T_{02}$, we again take the temperature ratio like we did for the diffuser, in which case the numerator we have the temperature ratio, the isentropic temperature ratio which is basically the compressor pressure ratio π_c raised to $\gamma - 1$ by γ minus 1 divided by T_{03} by T_{02} minus 1.

So, if we simplify this, we will now be able to calculate, we will get an expression for the stagnation temperature in terms of the compressor pressure ratio and the efficiency. So, if we simplify this, we get T_{03} is equal to T_{02} multiplied by $1 + \frac{\gamma - 1}{\eta_c} \left(\frac{p_{03}}{p_{02}} \right)^{\frac{\gamma - 1}{\gamma}}$ which is the compressor efficiency into $\frac{\gamma - 1}{\eta_c}$ raised to $\frac{\gamma - 1}{\gamma}$ by $\frac{\gamma - 1}{\gamma}$ plus 1. So, this basically gives us an expression for calculating the stagnation temperature at the exit of the compressor, once we know the stagnation temperature the inlet of the compressor, the compressor pressure ratio and the efficiency. So, having known these three parameters, we can calculate the compressor exit pressure that is stagnation pressure as well as the temperature.

So, we now have the pressure and temperature, all the way up to the combustion chamber inlet which is basically the compressor exit. So, the next component which we will take up for analysis is the combustion chamber. And in the combustion chamber, we have two performance parameters, one is the combustion efficiency and the second is the pressure loss occurring across the combustion chamber. So, we will need to make use of both of these efficiency parameters to determine, what happens across the combustion chamber.

So, the first thing that we will do is to carry out an energy balance across the combustion chamber, we know the inlet stagnation pressure in temperature, we know that there is certain amount of fuel that is added and fuel properties are known, and we also know the turbine inlet temperature, that is the combustion chamber exit temperature is again fixed. So, based on this, we can carry out an energy balance and therefore, we can calculate how much fuel is to be added in the combustion chamber if these parameters are fixed.

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JET AIRCRAFT PROPULSION Lect-10

Real cycle for turbojet engines

- Combustion chamber: From energy balance,

$$h_{04} = h_{03} + \eta_b f \dot{Q}_f$$

$$c_{pg} T_{04} = c_{pa} T_{03} + \eta_b f \dot{Q}_f$$

$$\text{or, } f = \frac{c_{pg} T_{04} / c_{pa} T_{03} - 1}{\eta_b \dot{Q}_f / c_{pa} T_{03} - c_{pg} T_{04} / c_{pa} T_{03}}$$

Also, $P_{04} = \pi_b P_{03}$

- Hence, we can determine the fuel to air ratio and the combustion chamber exit total pressure.

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So, from the energy balance we get h_{04} , which is the combustion chamber exit stagnation enthalpy is equal to h_{03} , which is the inlet stagnation enthalpy plus η_b which is the burner efficiency or combustion chamber efficiency multiplied by f that is the fuel to air ratio and the fuel property that is \dot{Q}_f that is the calorific value of the fuel.

So, this stagnation enthalpy will now be converted to the corresponding C_p times the temperature that is stagnation temperature. So, we have C_{pg} which is representing the specific heat constant pressure for the combustion products multiplied by T_{04} is equal to C_{pa} , which is specific heat constant pressure for air multiplied by T_{03} plus the efficiency times fuel to air ratio times the heat of reaction or calorific value. So, if you simplify this, we can find out, what is the fuel to air ratio. So, fuel to air ratio f is equal to C_{pg} into T_{04} divided by C_{pa} into T_{03} minus 1 divided by the combustion efficiency multiplied by \dot{Q}_f divided by C_{pa} T_{03} minus C_{pg} T_{04} by C_{pa} T_{03} .

So, this you might notice that if you recall from our, if **if** you have, if you can recall what you have done in ideal cycle analysis, we normally assume that C_p is a constant, we assume that combustion products have the same properties as that of air and therefore, C_p would have gotten cancelled. And so, in **in** the case of real cycle analysis, of course we cannot assume that the C_p is the same before and after combustion. And therefore, we have an average value of C_p after combustion and an average value of C_p before combustion. So, fuel to air ratio can be determined from the energy balance.

And the next parameter that will affect the combustion chamber performance is the pressure loss occurring across the combustion chamber and pressure loss again, if **if** we know the amount of pressure loss occurring across the combustion chamber, the stagnation pressure at the exit of the combustion chamber $P_0 4$ will be less than the stagnation pressure at the inlet of the combustion chamber. And that difference is basically coming from the pressure loss occurring in the combustion chamber, so $P_0 4$ will be equal to $p_i b$ which is the pressure loss in the combustion chamber multiplied by $P_0 3$.

So, from this analysis, we can find out the fuel to air ratio as well as the combustion chamber exit total pressure which is basically the turbine inlet total pressure. So, having determined the properties all the way up to combustion chamber exit, we now move towards this the next component which is the turbine. So, we have the temperature and pressure at the combustion chamber exit which is the turbine inlet. Based on these temperature and pressure values which are known to us, we can now carry out the cycle analysis for the turbine that the component analysis for the turbine to determine, what is the amount of pressure, what is the pressure across the turbine, and what is the temperature at the exit of the turbine. You will again make use of the fact that turbine is generating work output which is basically driving the compressor. So, if we equate the work developed by the compressor to that of the turbine, we should be able to find out the temperature at the exit of the turbine and from the turbine efficiency, we can find out what is the pressure at the exit of the turbine.

So, from these two equalities, that is one is work done by the compressor is equal to that of the turbine and from the efficiency which is basically relating the ideal and the actual temperature difference across the turbine, we can also find out the pressure, the total pressure at the exit of the turbine.

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JET AIRCRAFT PROPULSION Lect-10

Real cycle for turbojet engines

- Turbine: Since the turbine produces work to drive the compressor, $W_{\text{turbine}} = W_{\text{compressor}}$

$$\eta_m (\dot{m} + \dot{m}_f) c_{ps} (T_{04} - T_{05}) = \dot{m} c_{pa} (T_{03} - T_{02})$$

$$T_{05} = c_{ps} T_{04} - c_{pa} (T_{03} - T_{02}) / \eta_m (1 + f)$$

$$\eta_t = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}} = \frac{1 - T_{05}/T_{04}}{1 - T_{05s}/T_{04}} = \frac{1 - T_{05}/T_{04}}{1 - (P_{05}/P_{04})^{(\gamma-1)/\gamma}}$$

Simplifying,

$$P_{05} = P_{04} \left[1 - \frac{1 - T_{05}/T_{04}}{\eta_t} \right]^{\gamma/(\gamma-1)}$$

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So, let us do that and we will see that, for the turbine we will first do the turbine work is equal to compressor work. Of course, we will also include the mechanical efficiency here, because mechanical efficiency is the amount of work that the turbine produces is actually reduced by the amount of the mechanical efficiency.

So, turbine work will get reduced by the amount given by the mechanical efficiency. So, mechanical efficiency multiplied by mass flow rate in the turbine which is mass flow rate of air which is \dot{m} plus mass flow rate of fuel multiplied by C_p of the combustion product C_{ps} multiplied by the temperature difference T_{04} minus T_{05} is equal to \dot{m} which is mass flow rate of air into C_{pa} into T_{03} minus T_{02} . So, if you simplify this, we can determine the temperatures, stagnation temperature and the exit of the turbine.

So, T_{05} is equal to C_{ps} into T_{04} minus C_{pa} into T_{03} minus T_{02} divided by mechanical efficiency multiplied by $1 + f$, where f is equal to \dot{m}_f / \dot{m} . So, this will help us in finding out the stagnation temperature at the turbine exit, the next task is to find out the stagnation pressure at the turbine exit, for that we will use the efficiency definition, turbine efficiency is equal to T_{04} minus T_{05} divided by T_{04} minus T_{05s} where T_{05s} is the stagnation temperature isentropic at the exit of the turbine.

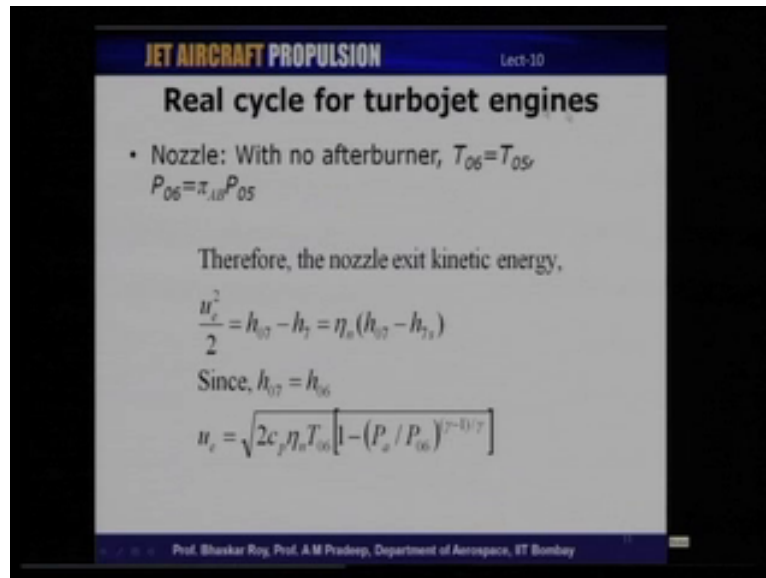
You take the temperature ratios now and so, we have here the isentropic temperature ratio which is equal to the corresponding pressure ratios. And so, we have efficiency being related

to the temperature ratio as well as the pressure ratio. So, from simplification of this, we can calculate $P_0 5$ is equal to $P_0 4$ multiplied by $1 - \eta_t$ into $1 - \frac{T_0 5}{T_0 4}$, the whole thing raise to γ by $\gamma - 1$. So, for the turbine now, we have the **the** pressure as well as the temperature, temperature we got from equating the work done by the turbine to that of the compressors.

So, we get the temperature at the exit of the turbine, for determining the pressure we use the efficiency definition, we can also find the stagnation pressure at the exit of the turbine. So, for a non afterburning turbojet, the only component that is left is the nozzle. So, in the nozzle again we will use the nozzle efficiency definition to determine, what is the exit velocity, exhaust velocity of the combustion products through the nozzle; from there we can actually find out, what is the thrust developed by the, this particular engine and therefore, all the efficiencies and so on.

So, in a non afterburning turbojet, the next component is the nozzle. So, in a nozzle, we know that we may basically be making use of the nozzle efficiency to calculate, how much is the loss of performance occurring in the nozzle as a result of its efficiency, so, **if** if there is no afterburner that is used, this stagnation temperature at the exit of the turbine will be equal to the stagnation temperature at the inlet of the nozzle, because there is no afterburning. And there is no loss in stagnation temperature, because we assume that this process is adiabatic. And if there is no afterburner, we can also assume that there is no pressure loss occurring in the afterburner or in the jet pipe, so $P_0 6$ will be equal to $P_0 5$.

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So, with these assumptions, what we have is that the temperature is the same. If there is an afterburner stream present and still not being used, there may be some loss in stagnation pressure in which case P_{06} will be equal to loss in stagnation pressure, if present in the afterburner $p_{pi} A B$ multiplied by P_{05} .

So, how the nozzle exit kinetic energy is equal to v_e square by 2 where v_e is v_e is the exit stagnation velocity of the nozzle is equal to $h_{07} - h_7$ which is basically equal to efficiency of the nozzle η_n into $h_{07} - h_{7s}$. Now, since h_{07} is equal to h_{06} , the exit velocity v_e is equal to square root of $2 C_p \eta_n T_{06}$ multiplied by $1 - (P_e / P_{06})^{(\gamma-1)/\gamma}$. So, this is basically coming from simplification of this equation that is the velocity square by 2 is efficiency multiplied by $h_{07} - h_{7s}$. So, this again is equal to efficiency into C_p into $T_{07} - T_{7s}$ and so on. So, if you simplify that we can get the exhaust velocity which is equal to square root of $2 C_p g$ into η_n which is the nozzle efficiency into T_{06} into $1 - (P_e / P_{06})^{(\gamma-1)/\gamma}$.

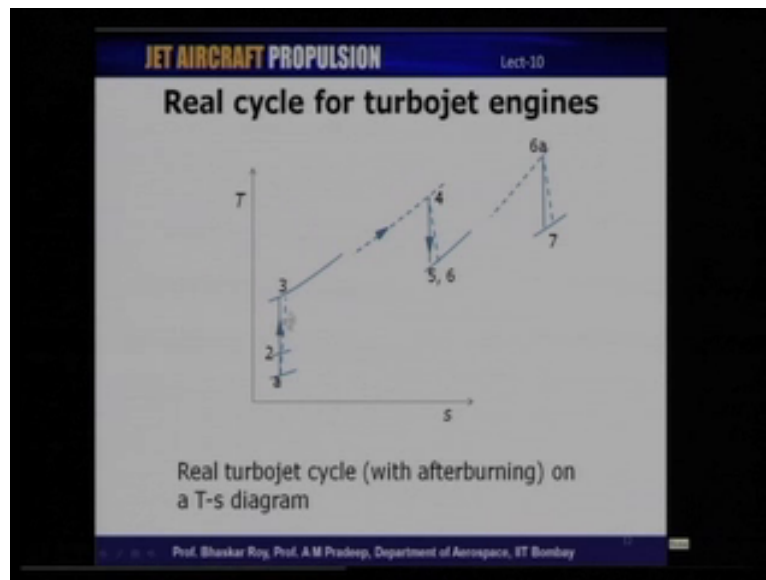
So, we have now calculated all the parameters required for determining the thrust and the other performance parameters like efficiencies and so on. So, once we calculate the exhaust velocity, we know that thrust is now equal to the mass flow rate that is \dot{m} into $1 + f$ into v_e minus v plus the pressure thrust term which **which** I now determine. So, we have already seen the thrust **thrust** and efficiency definitions and the specific fuel consumptions in

the earlier lectures. So, the **the the** entire process of the cycle analysis is basically to determine the exit velocity and of course, all the other pressure and temperature terms and **which** in which case, we can also now find out the fuel consumption and the various efficiencies like the mechanical efficiency, overall efficiency, thermal efficiency and so on. So, all these efficiencies can also be determined, once we are able to calculate the various temperatures and pressures across the different components.

And so, in this cycle analysis process that we have done, what we have basically done is to modify the ideal cycle by making use of these various efficiencies in all these components. And then using those efficiencies, we can estimate and calculate, what are the properties across each components leading towards the nozzle and from the nozzle exhaust velocity, we can now calculate the thrust developed during this jet engine process. So, what we have been discussing now for, as of now is process involving an a simple turbojet without any afterburning.

Now, what happens if an afterburner is used? So, if an afterburner is used, all the processes all the way up to the turbine exit remains unchanged, we have the same processed or same cycle analysis that we have just now discussed, all the way up to turbine exit, turbine exit onwards we have slightly different process involving a heat addition there. So, it is like reheat Brayton cycle, so there is heat addition taking place at the turbine exit. So, the basic cycle itself will get modified and therefore, cycle analysis after the turbine would need **would need** to be modified accordingly.

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So, let us take look at real cycle of a turbojet with afterburning. So, in this cycle that we have, I have shown here is a TS diagram we can easily see that all up to station 5, it is exactly same as that of a simple turbojet. After 5, we have afterburning taking place and therefore, there is reheating of the exhaust products and because of reheating the temperature, cycle temperature is now higher then what it was earlier. And so, since this is a process which is identical to that of a combustion chamber, there is a pressure loss occurring in this process and that is why we have a broken line here of the pressure, it is no longer a constant pressure process. At the end of the afterburning that is the nozzle entry, nozzle expansion process is shown here between 6 a and 7 which is again non isentropic, so the non isentropic process is indicated by this dotted line.

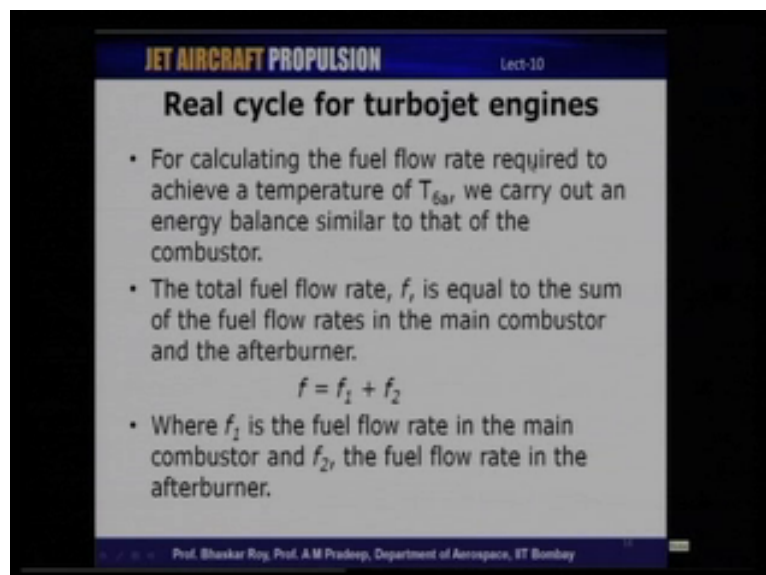
So, in a turbojet cycle with afterburning, the cycle gets modified due to this additional heat which is added after the turbine and this heat addition leads to higher temperature at the nozzle entry and therefore, higher pressure. And so, the nozzle has additional scope for expansion **in** its, because of its higher inlet temperature and pressure and that leads to an additional thrust to which is what is the basic purpose of using an afterburner. So, in this afterburning turbojet cycle, how is the cycle analysis going to be different, as I mentioned it is going to be exactly the same all the way up to the turbine exit and from there, we have another process of heat additions.

So, we need to calculate the fuel that needs to be added at the nozzle at in the afterburner to achieve a certain fixed temperature which again would be known. So, the nozzle entry temperature would now be fixed and that temperature would be known, to achieve that temperature how much fuel is to be added is what needs to be determined from cycle analysis.

And how do you do that, it is basically exactly the same way which we used for calculating fuel to air ratio in the main combustor. So, we now have two different fuel to air ratios, let us denote the first one by f_1 that is the fuel added in the main combustor, the second fuel added in the afterburner is **is** denoted by f_2 . So, the total fuel that is been added in both these process will be sum of the few are added in the main combustor and the fuel added in the afterburner.

So, in an afterburning turbojet, the basic purpose of afterburning I think it is **is** already known that we need an afterburner, because we in **in** some certain situations, you would require an additional thrust. And where does that additional thrust come from? It can come from modifying the basic turbojet cycle to a reheat turbojet cycle which is an afterburning turbojet cycle, you add more fuel just after the turbine exhaust, and then you gain additional thrust.

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Real cycle for turbojet engines

- For calculating the fuel flow rate required to achieve a temperature of T_{6a} , we carry out an energy balance similar to that of the combustor.
- The total fuel flow rate, f , is equal to the sum of the fuel flow rates in the main combustor and the afterburner.
$$f = f_1 + f_2$$
- Where f_1 is the fuel flow rate in the main combustor and f_2 , the fuel flow rate in the afterburner.

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And so, in an afterburning turbojet cycle, we basically need to calculate the fuel to air ratio, we need to achieve a temperature of denoted by T_{6a} , and for that we have to carry

out an energy balance similar to that of the main combustor. So, let us say that the total fuel flow rate is now the new total fuel flow rate is equal to f , this would be comprising of two different fuel flow rates, one is because of the main combustor and the second is the fuel flow rate in the afterburner. So, let us denote f_1 as the fuel flow rate in the main combustor that in the second combustor that is the afterburner will be f_2 .

So, total fuel flow rate f will be equal to f_1 plus f_2 , and how does this alter the performance, it will basically change the fuel efficiency of the turbojet, besides of course it will change the thrust, because the nozzle entry, pressure and temperature are higher than what it was earlier. So, the exhaust velocity will be higher, therefore the thrust developed by the **afterburner will also** afterburning turbojet will be higher than that developed by the simple turbojet engine.

So, the excess fuel that is added in the afterburner will also change the specific fuel consumption of the turbojet. Obviously, because we are now adding additional fuel, even though you are getting higher thrust, it will definitely change the specific fuel consumption of such engines. And something we will discuss a little later or may be in the next lecture is that, afterburning turbojets will generate substantially higher thrust than simple turbojets, but it will also increase the specific fuel consumption of such engines. And therefore, normal turbo, normal will it a similar aircraft wherein the use turbofan engines, we normally do not use afterburners, because firstly, because it requires higher fuel. And secondly, of course, you do not really need to fly at a supersonic speeds all the time.

And that is one of the reasons why the only operational supersonic transport aircraft which was the concord was not an economically viable proposition, because the cost, the associated with this additional fuel was exorbitant. And so, the cost of flying at supersonic speeds was quite high and of course, we will need to radically rethink our strategy and find out how we can probably still use a supersonic similar aircraft, because its saves a lot of time of travel.

Now, let us take a look at the next cycle that we are going to analysis, that is a turbofan and that is the cycle or the engine that is being used most popularly for civil aviation, that is a turbofan engine and the with the basic working of turbofan is something that we have already discussed in some of the earlier lectures. So, you now known how a turbofan engine works and why it is used in the first place and so on. We will now discuss about the cycle analysis involved in a turbofan, now turbofan as you have seen can consist of different configurations,

it could be a twin spool could be three spool geared turbofan and so on, there are different multiple ways in which turbofan can operate.

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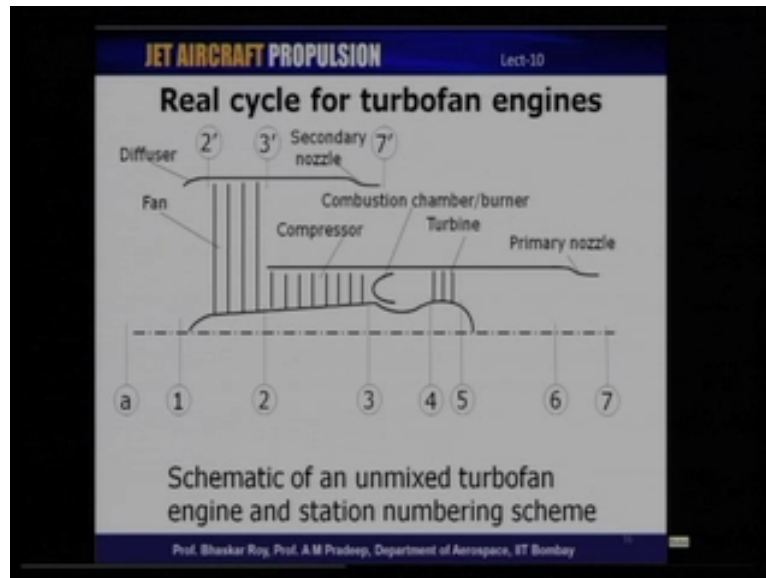
Real cycle for turbofan engines

- A turbofan engine can have different configurations: Twin-spool, three-spool, and geared turbofan. These may be either unmixed or mixed.
- Cycle analysis of a turbofan can hence be slightly different depending upon the configuration of the engine.
- We shall now carry out a real cycle analysis of an unmixed twin-spool turbofan engine.

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And also a turbofan can be operating in mixed mode or an unmixed mode, cycle analysis will now be different depending upon, what is the type of the turbofan that is been used. We will carry out the real cycle analysis of one of these cases and unmixed twin spool turbofan engine; the others are identical to this just that you will need to incorporate, small differences that will be occurring as a result of these changes.

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Now, here is a schematic of an unmixed turbofan engine which is what we will be discussing now, the various components are shown here. The core engine which is shown between stations 2 and 7 is exactly the same as what we had seen for a turbojet engine, the difference is the presence of a fan and a secondary nozzle. So, we will denote these components, let us say the fan entry by station 2 prime, fan exit by station 3 prime which is equal to 2, which is the compressor entry and then the nozzle exit secondary nozzle exit by 7 prime, primary nozzle exit is still 7. So, these are the different stations which are involved in a turbofan. So, we have compression taking place all the way from the free stream in the diffuser, then the fan, and then the compressor.

So, there are three different processes here where in compression takes place and then there is the combustion chamber between stations 3 and 4, 4 to 5 is the turbine expansion and then we have the primary nozzle, there is a secondary cycle here which is basically the fan compression and then subsequent expansion in the nozzle, there is no heat addition taking place in the secondary nozzle.

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Real cycle for turbofan engines

- Intake: Ambient pressure, temperature and Mach number are known, P_a , T_a and M
- Intake exit stagnation temperature and pressure are determined from the isentropic relations:

$$T_{02} = T_a \left(1 + \frac{\gamma-1}{2} M^2 \right)$$

$$\eta_d = \frac{T_{02} - T_a}{T_{02s} - T_a} = \frac{T_{02}/T_a - 1}{T_{02s}/T_a - 1} = \frac{T_{02}/T_a - 1}{(P_{02}/P_a)^{\frac{\gamma-1}{\gamma}} - 1}$$

$$\therefore P_{02} = P_a \left(1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right) \right)^{\frac{\gamma}{\gamma-1}}$$

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So, for such an engine, let us try to carry out the cycle analysis, the real cycle analysis of such engines. So, we have the intake as the first component, the ambient pressure temperature, mach number are known. So, based on this we can calculate the exit conditions, exactly the same way that we did for the turbojet. So, the fan inlet stagnation temperature T_{02} is equal to T_a multiplied by $1 + \frac{\gamma-1}{2} M^2$.

So, once the stagnation temperature is known, we can calculate the pressure based on the efficiency, the diffuser efficiency which is equal to $\frac{T_{02} - T_a}{T_{02s} - T_a}$, and in the denominator we have the pressure ratios. Therefore, the intake exit **well**, the intake exit or the fan inlet stagnation pressure T_{02} is equal to P_a multiplied by $1 + \eta_d \left(\frac{T_{02}}{T_a} - 1 \right)$, the whole thing raise to $\frac{\gamma}{\gamma-1}$.

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Real cycle for turbofan engines

- Fan: Fan pressure ratio is known, $\pi_f = P_{03} / P_{02}$

$$P_{03} = \pi_f P_{02}$$

$$T_{03} = T_{02} (\pi_f)^{(\gamma-1)/\gamma}$$

The fan efficiency is

$$\eta_f = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} = \frac{T_{03s}/T_{02} - 1}{T_{03}/T_{02} - 1} = \frac{\pi_f^{(\gamma-1)/\gamma} - 1}{\pi_f^{(\gamma-1)/\gamma} - 1}$$

Simplifying,

$$T_{03} = T_{02} \left\{ \frac{1}{\eta_f} \left[\pi_f^{(\gamma-1)/\gamma} - 1 \right] + 1 \right\}$$

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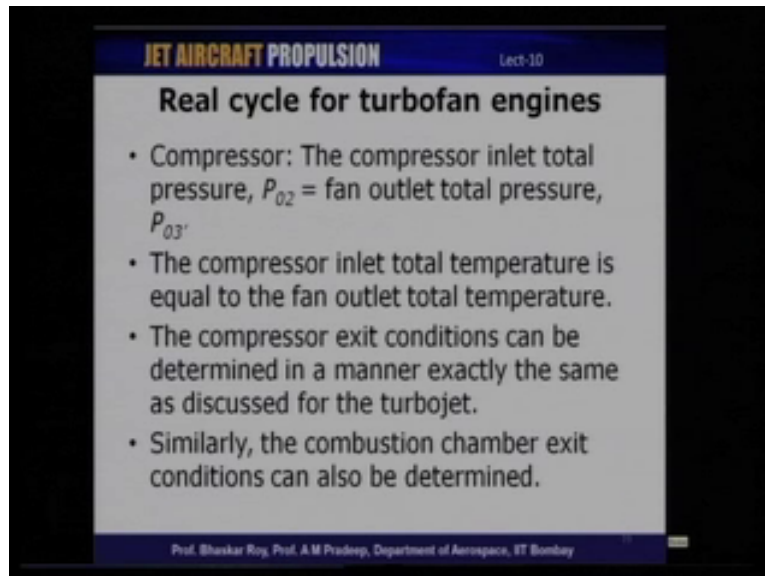
Now, once we calculate the intake exit conditions which is basically the fan entry conditions, the **the** fan pressure ratio like the compressor pressure ratio is yet another design parameter. So, π_f which is the fan pressure ratio is equal to P_{03} prime divided by P_{02} prime. So, P_{03} prime is equal to π_f which is fan pressure ratio multiplied by P_{02} prime and the temperature ratio T_{03} prime, we will now calculate based on the efficiency. So, fan efficiency we know is defined as T_{03} prime s minus T_{02} prime divided by T_{03} prime minus T_{02} prime.

So, if this is the efficiency, we can now relate the efficiency to the pressure ratio which is π_f raise to γ minus 1 by γ minus 1 divided by the temperature ratio. So, if you simplify this, we get T_{03} prime is equal to T_{02} prime into 1 by η_f which is the fan efficiency multiplied by π_f raise to γ minus 1 by γ minus 1, this whole thing plus 1. So, this will help us in calculating the fan exit stagnation temperature which is T_{03} prime which in turn is equal to the compressor inlet stagnation temperature. So, T_{03} prime will be equal to T_{02} which is basically the compressor inlet conditions. Similarly, P_{03} prime which is the fan exit stagnation pressure will be equal to the compressor inlet stagnation pressure.

So, the next component we will take up is compressor, the analysis for compressor and the combustion chamber is exactly the same as that for the turbine. So, I will not repeat it here again, because compressor exit stagnation pressure will be equal to that inlet stagnation

pressure multiplied by the pressure ratio, the compressor pressure ratio. The compressor exit stagnation temperature will **will** **can** be calculated based on the compressor efficiency, the same way as we did for the fan, the same way as we did for the turbojet engine, so it is exactly the same way. Similarly, the combustion chamber calculations are exactly the same as what we did for a turbojet engine.

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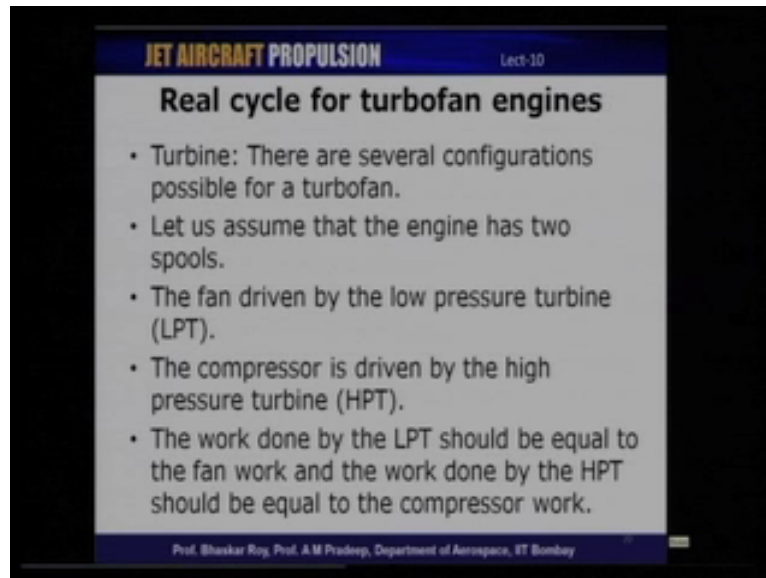
Real cycle for turbofan engines

- Compressor: The compressor inlet total pressure, P_{02} = fan outlet total pressure, P_{03} .
- The compressor inlet total temperature is equal to the fan outlet total temperature.
- The compressor exit conditions can be determined in a manner exactly the same as discussed for the turbojet.
- Similarly, the combustion chamber exit conditions can also be determined.

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So, for the compressor and the turbojet, we will not repeat the calculations, because it is the same. For a compressor, the inlet total pressure P_{02} will be equal to fan outlet total pressure, compressor inlet total temperature is equal to the fan outlet total temperature, and the exit conditions of combustion chamber as well as the compressor can be determined in exactly the same way as we carried out for the turbojet, the pure turbojet engine.

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Real cycle for turbofan engines

- Turbine: There are several configurations possible for a turbofan.
- Let us assume that the engine has two spools.
- The fan driven by the low pressure turbine (LPT).
- The compressor is driven by the high pressure turbine (HPT).
- The work done by the LPT should be equal to the fan work and the work done by the HPT should be equal to the compressor work.

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The difference comes in the turbine; in the case of turbine there are different configurations which are possible like twin spool, three spool, single spool and so on, let us assume that there are this engine has two spools. Now, in a two spool engine, we know that the fan is usually driven by the low pressure turbine and the compressor is usually driven by the high pressure turbine, which means that the work done by the LP turbine will be equal to the work required for the fan and work done by the HP turbine will be the work required by the compressor. So, we will equate them separately and find out the corresponding exit conditions, let us take up the high pressure turbine first, because that is the first component that comes up after the combustion chamber.

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Real cycle for turbofan engines

- High pressure turbine:

$$\eta_m (\dot{m} + \dot{m}_H) c_{pg} (T_{04} - T_{05}') = \dot{m}_H c_{pa} (T_{03} - T_{02})$$

Here, T_{05}' is the temperature at the HPT exit.

$$T_{05}' = c_{pg} T_{04} - c_{pa} (T_{03} - T_{02}) / c_{pg} \eta_m (1 + f)$$

$$\text{Hence, } P_{05}' = P_{04} \left[1 - \frac{1}{\eta_t} (1 - T_{05}' / T_{04}) \right]^{\gamma / (\gamma - 1)}$$

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So, in the case of an HP turbine, the work done by the turbine will be equal to the work required by fan. And so, we have by the compressor and so, we have mechanic efficiency multiplied by the fan the turbine work, the HP turbine work which is equal to mass flow rate \dot{m} dot a which is mass flow rate of air plus \dot{m} dot h which is basically the mass flow rate of fuel multiplied by $C_p g$ into T_{04} minus T_{05}' . This is equal to mass flow rate of air given here by \dot{m} dot h which is mass flow rate of the hot engine that is the core engine multiplied by $C_p a$ which is C_p of the air and the work required by the compressor T_{03} minus T_{02} , T_{05}' is it temperature at the HP turbine exit.

So, here if we simplify this, we can calculate T_{05}' which is the temperature at the HP turbine exit which is $C_p g$ into T_{04} minus $C_p a$ into T_{03} minus T_{02} divided by $C_p g$ into η_m into 1 plus f . Now, in a same manner as we did for a turbojet, we now can calculate the pressure P_{05}' which is equal to P_{04} into 1 minus 1 by η_t into 1 minus T_{05}' divided by T_{04} , whole thing raise to γ by γ minus 1 . So, we have now calculated the temperature and pressure at the HP turbine exit which is also the LP turbine inlet.

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Real cycle for turbofan engines

- Low pressure turbine:

$$\eta_m (\dot{m} + \dot{m}_c) c_{pg} (T_{05}' - T_{05}) = \dot{m}_c c_{pa} (T_{03}' - T_{02}')$$

Here, T_{05}' is the temperature at the HPT exit/LPT inlet.

$$\therefore \eta_m (1 + f) c_{pg} (T_{05}' - T_{05}) = B c_{pa} (T_{03}' - T_{02}'), \text{ where, } B = \frac{\dot{m}_c}{\dot{m}_h}$$

$$T_{05}' = c_{pg} T_{05} - B c_{pa} (T_{03}' - T_{02}') / \eta_m c_{pg} (1 + f)$$

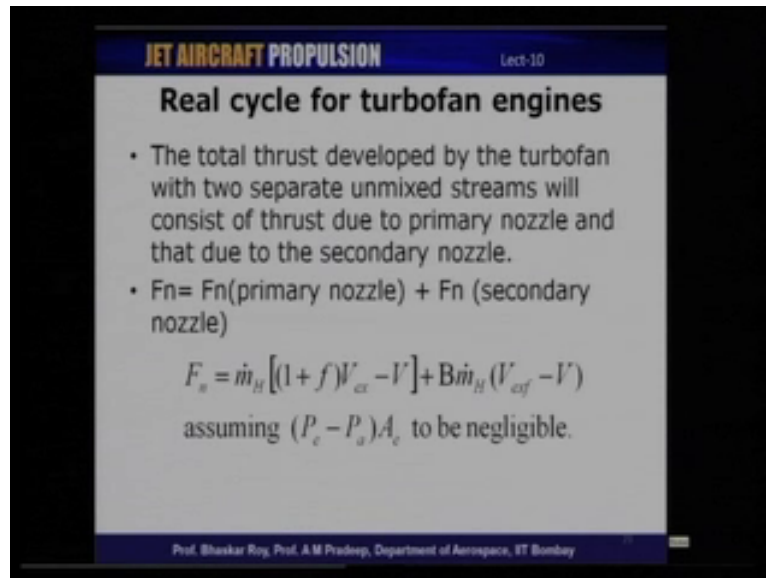
$$\text{And, } P_{05}' = P_{05} \left[1 - \frac{1}{\eta_t} (1 - T_{05}' / T_{05}) \right]^{\gamma/(\gamma-1)}$$

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So, the next component will be the LP turbine. So, at the LP turbine inlet, LP turbine work output will be equal to the fan work input. So, here again we equate the work required by the fan to that of the LP turbine output, mechanical efficiency times mass flow rate into C p into T 0 5 prime minus T 0 5 is equal to mass flow rate through the fan m dot c, because that is the cold mass flow passing through the bypass duct into C p into the temperature difference.

So, if we simplify this, we can calculate the turbine exit stagnation temperature T 0 5 which is equal to what is shown here, C p g into T 0 5 prime minus B which is the bypass ratio which is m dot c by m dot h into C p a into T 0 3 prime minus T 0 2 prime divided by mechanical efficiency into C p g into 1 plus f, similarly find the turbine exit, low pressure turbine exit stagnation pressure as well.

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Real cycle for turbofan engines

- The total thrust developed by the turbofan with two separate unmixed streams will consist of thrust due to primary nozzle and that due to the secondary nozzle.
- $F_n = F_n(\text{primary nozzle}) + F_n(\text{secondary nozzle})$

$$F_n = \dot{m}_H [(1+f)V_{ex} - V] + B\dot{m}_H (V_{exf} - V)$$

assuming $(P_c - P_o)A_c$ to be negligible.

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So, in the case of a turbofan we have, let us say an unmixed turbofan we have thrust developed by two different nozzles, one is the core nozzle that is the hot nozzle which, through which the hot exhaust products are expanded and then we also have the cold nozzle, the bypass nozzle, the secondary nozzle. So, thrust developed will be equal to sum of the thrust developed by the primary nozzle and the thrust developed by the secondary nozzle, this is true for a unmixed turbofan. In a mixed turbofan, there is only one nozzle through which the, or both the streams are mixed and exhausted, so there is only a single thrust developed there.

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Real cycle for turbofan engines

- The total thrust developed by the turbofan with two separate unmixed streams will consist of thrust due to primary nozzle and that due to the secondary nozzle.
- $F_n = F_n(\text{primary nozzle}) + F_n(\text{secondary nozzle})$

$$F_n = \dot{m}_H [(1+f)V_{ex} - V] + B\dot{m}_H (V_{exf} - V)$$

assuming $(P_c - P_o)A_c$ to be negligible.

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So, the total thrust here is equal to F_n is equal to F_n due to the primary nozzle that is thrust due to the primary nozzle plus thrust due to this secondary nozzle. So, total thrust will be equal to \dot{m}_H , which is mass flow rate through the core that into $1 + f$ into V_{ex} which is exhaust velocity minus V flight speed plus the bypass ratio into \dot{m}_H , or \dot{m}_c into the exhaust velocity through the bypass nozzle that is V_{exf} minus V , here we are assuming that the pressure thrust is negligible. If the flow is not fully expanded, then of course, there will be a pressure thrust which also needs to be considered while carrying out the cycle analysis.

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Real cycle for turbofan engines

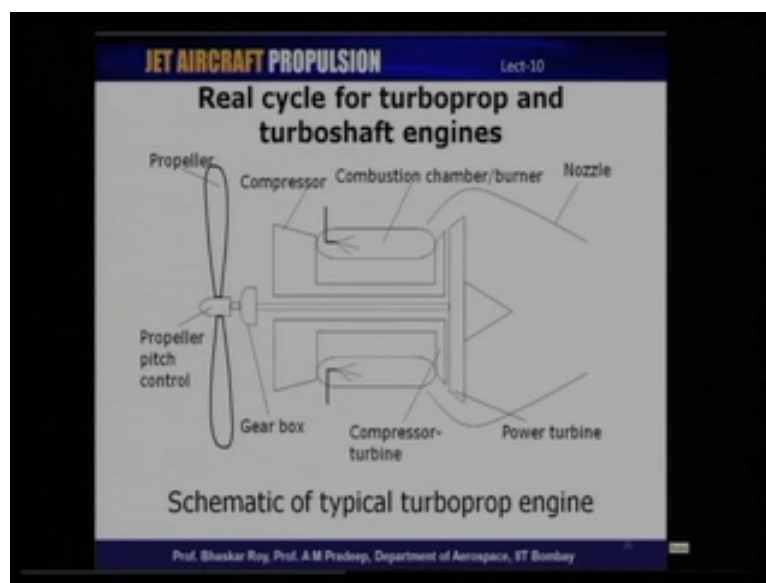
- The cycle analysis procedure will need to be slightly modified depending upon the turbofan engine configuration.
- The differences in the various configuration arise because of the number of spools and turbine-compressor/fan arrangements as well as mixed and unmixed exhausts.
- If the turbofan is of a mixed configuration, then, we will have to calculate the temperature at the nozzle entry from enthalpy balance of the two streams.

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So, in this ideal turbofan, the real turbofan, cycle analysis what we have discussed is for turbofan with two separate nozzles, cycle analysis procedure will be slightly different depending upon the turbofan configuration.

And so, those differences will need to be taken into account while carrying out cycle analysis for these different types of differences of the turbofan. So, if it is mixed configuration, then we will need to basically calculate the temperature at the nozzle entry from enthalpy balance of the two streams. So, we will not go into details of those configurations.

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Let us now quickly go to discussion of cycle analysis for turboprop and turbo shaft, we have already seen this schematic of a turboprop, usually consist of a propeller with a gear box attached to what is known as a power turbine which basically drives the propeller, then we have a compressor, combustion chamber and the compressor turbine which drives the compressor. In many of the turboprops in addition to the propeller thrust, we may also have thrust developed by the nozzle because of expansion through the nozzle.

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Real cycle for turboprop and turboshaft engines

- Turboprops and turboshafts usually have a free-turbine or power turbine to drive the propeller or the main rotor blade (turboshafts).
- Stress limitations require that the large diameter propeller rotate at a much lower rate and hence a speed reducer is required.
- Turboprops may also have a thrust component due to the jet exhaust in addition to the propeller thrust.
- In turboshafts, however, there is no thrust component due to the nozzle.

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And so, turboprops and turbo shafts usually have a free turbine or a power turbine which drives the propeller or a main rotor blade in the case of turbo shaft. And so, there may be a gear box which is because the propellers cannot run at very high speeds. And in turboprops, they may have additional thrust component because of jet exhaust, turbo shaft, of course do not have any additional thrust because of nozzle.

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Real cycle for turboprop and turboshaft engines

Enthalpy-entropy diagram for power turbine-exhaust nozzle analysis

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Now, will just take up the difference in turboprop or turbo shaft as compared to the turbojet, which is basically the power turbine. So, which is why I have shown only the power turbine here which is between stations 5 and 6 and so, expansion here is non ideal, so this dotted line shows the actual expansion. So, enthalpy drop across that is given by delta h, fraction of the total enthalpy drop taking place through the turbine is given by alpha times delta h which is where alpha is the fraction of the total enthalpy drop which occurs in just the power turbine.

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Real cycle for turboprop and turboshaft engines

- Δh is the enthalpy drop in an ideal isentropic power turbine and exhaust nozzle.
- α is the fraction of Δh that would be used by an isentropic turbine.
- The propeller thrust power, $F_{n,p} V_p$ is

$$F_{n,p} V = \eta_p \eta_g \eta_{PT} \alpha \Delta h \dot{m} \quad \text{or, } F_{n,p} = \frac{\eta_p \eta_g \eta_{PT} \alpha \Delta h \dot{m}}{V}$$

η_p = propeller efficiency, η_g = gear box efficiency,
 η_{PT} = power turbine efficiency

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So, alpha is fraction of delta h that would be used by the turbine which is basically isentropic, we will multiply the isentropic work with all the efficiencies which are involved here. So, the propeller thrust power here is given by F subscript n comma p r which is the propeller thrust multiplied with the flight speed. So, this gives us the thrust power, this should be equal to alpha times delta h into mass flow rate multiplied by various efficiencies.

And what are those efficiencies? We have three efficiencies here, one is the propeller efficiency, then the gear box efficiency which is eta g, and the power turbine efficiency which is eta power turbine. So, there are three efficiency terms here which will get multiplied to the total propeller thrust power. So, thrust is equal to, the propeller thrust will be equal to these efficiencies, propeller thrust, the gear box efficiency and the power turbine efficiency multiplied by alpha times delta h m dot divided by the flight speed that is V.

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Real cycle for turboprop and turboshaft engines

- The exhaust nozzle thrust, F_n ,
$$F_n = \dot{m}(V_{ex} - V), \text{ where, } V_{ex} = \sqrt{2(1-\alpha)\eta_n\Delta h}$$
- Thus, the total thrust is given by,
$$F = F_{n,pr} + F_n = \frac{\eta_p\eta_t\eta_{pr}\alpha\Delta h\dot{m}}{V} + \dot{m}(\sqrt{2(1-\alpha)\eta_n\Delta h} - V)$$

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And the exhaust nozzle thrust, because nozzle also may generate some thrust will be equal to \dot{m} into V_{ex} exhaust velocity minus V , where exhaust velocity is basically equal to square root of $1 - \alpha$ into the nozzle efficiency times Δh . And why is it $1 - \alpha$? Because $\alpha \Delta h$ has been expanded in the turbine, the power turbine, therefore $1 - \alpha$ is what is available for expansion in the nozzle. So, $1 - \alpha$ times Δh is the enthalpy which is available for expansion in the nozzle.

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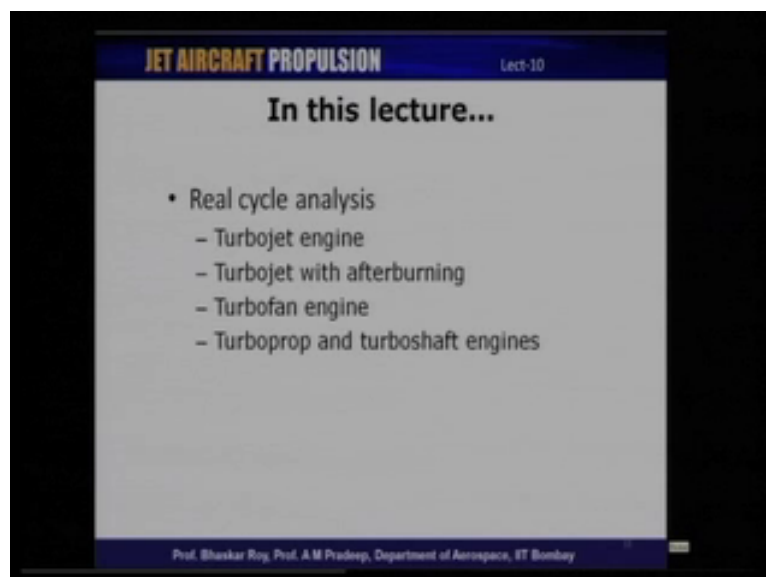
Real cycle for turboprop and turboshaft engines

- The exhaust nozzle thrust, F_n ,
$$F_n = \dot{m}(V_{ex} - V), \text{ where, } V_{ex} = \sqrt{2(1-\alpha)\eta_n\Delta h}$$
- Thus, the total thrust is given by,
$$F = F_{n,pr} + F_n = \frac{\eta_p\eta_t\eta_{pr}\alpha\Delta h\dot{m}}{V} + \dot{m}(\sqrt{2(1-\alpha)\eta_n\Delta h} - V)$$

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And therefore, we get this term that is $1 - \alpha$ into Δh multiplied by the efficiency and 2 will give us the exhaust velocity through the nozzle. Therefore, the total thrust which is available or which is developed by a turboprop, let us say will be equal to the thrust developed by the propeller plus the nozzle thrust. In a turbo shaft engine usually the nozzle thrust will be equal to 0 which is basically equal to, the propeller thrust is equal to this that is efficiency of propeller times gear box times power turbine into α times Δh into \dot{m} by V plus the nozzle thrust that is \dot{m} into square root of 2 into $1 - \alpha$ eta and Δh minus V . So, these are the two thrust terms which are associated with a turboprop or a turbo shaft engine.

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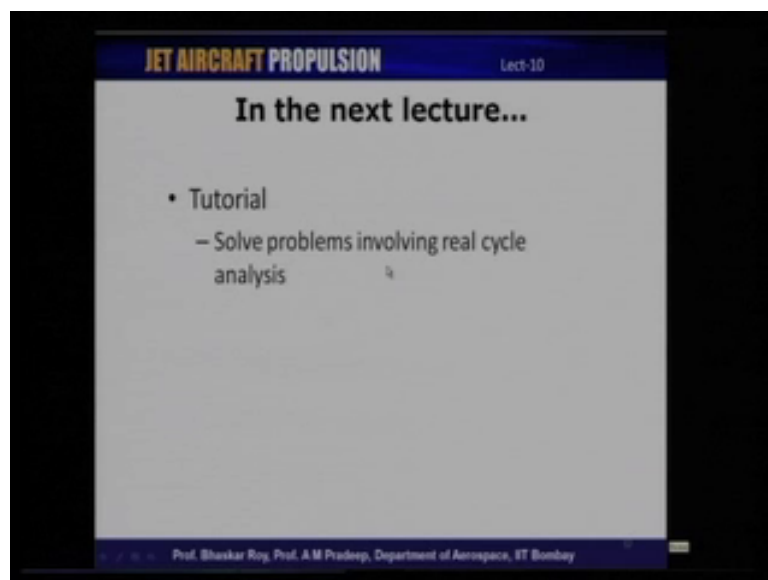
So, today we have been discussing about the cycle analysis associated with different types of engines, the real cycle analysis which wherein we have accounted for various efficiencies like the diffuser efficiency, the compressor efficiency, the combustion efficiency and the pressure drop in the combustion chamber, subsequently the turbine efficiency and then the nozzle efficiency. So, these different efficiency terms put together will help us in getting a realistic estimate of what happens in a jet engine, we have already been discussing about ideal cycle and its analysis. So, putting all these efficiencies in two places, we can get a realistic estimate of how these cycles behave.

So, we have discussed about a simple turbojet, we started our discussion today with a simple turbojet where there was no afterburning, we then took up an afterburning turbojet engine

where there is an additional thrust term, because of additional fuel added there. And then we discussed about one of the forms of a turbofan engine that is an unmixed turbofan, where we have two thrust components, then we have also discussed about turboprop and turbo shaft engines.

So, these are the different cycle analysis we had carried out for these different engines. And in the next lecture we will, what **what** we are going to do is to use our discussion, based on our discussion today, solve a few problems on cycle analysis, on real cycle analysis of some of these engines which we have discussed.

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So, in the next lecture, it will basically be a tutorial, we will solve some problems involving real cycle analysis, we will take up may be a turbojet with afterburning, with and without afterburning and probably a turbofan, may be an unmixed or a mixed turbofan engine. So, we will take up these problems for discussion in our next lecture.