Introduction to Aerospace Propulsion

Prof. Bhaskar Roy

Prof. A. M. Pradeep

Department of Aerospace Engineering

Indian Institute of Technology, Bombay

Module No. # 01

Lecture No. # 34

Tutorial

Hello and welcome to lecture number 34 of this lecture series on Introduction to Aerospace Propulsion. In the last 2 lectures we had a chance to understand and analyze different types of gas turbine engine cycles which are primarily operating on the Brayton cycle.

The different cycles or different types of engines that we have looked at over the past 2 lectures involved the simple turbojet engine which is a simplest, well not really the simplest form, the simplest form of a jet engine is the ramjet engine, but the turbojet engine includes all the different components that has essentially constitute a jet engine like a compressor, a combustion chamber, turbine, nozzle and intake and so on.

So, turbojet engine includes all these different components. What we had discussed in the last lectures were the simple forms of these engines in the form - in the sense - that we had considered the idealized version of these different engines. We had not really looked at what are the different loses that could take place or what are the efficiencies associated with different components and so on. What we had discussed were the idealized versions of these jet engine forms and then subsequently, we also looked at variance of the turbojet engine. One of the immediate variations of the turbojet engine is the turbojet with after burning, which is burning additional fuel after the turbine and so as to achieve extra thrust.

That is primarily used in cases where the turbojet engine is required to give additional thrust like when it has to accelerate to supersonic Mach numbers and crews at supersonic Mach numbers. Then besides the turbojet with after burning, there are different other variants of the cycle. One of the other variations or the most popular variation is the turbofan engine; the turbofan engine constitutes an extra duct over the core engine where in mass flow rate is driven by a fan which has a larger diameter.

Turbofan engines have been used in most of the commercial engines and the advantage of using a turbofan engine is that it gives an additional thrust because of the presence of the fan duct. So, the fan duct and the mass flow rate associated with the fan generate an additional thrust and also it has other benefits like better fuel efficiency, lower noise and so on.

Then, the other form of the jet engine is what is known as a turboprop engine; turboprop engine constitutes a propeller which generates a thrust which is driven by a power turbine and also some amount of thrust is generated by the nozzle. So, turboprop engines may have thrust components, majority of the thrust being generated by the propeller, it may also have some thrust generated by the nozzle.

A slightly different version of that is known as a turbo shaft engine; turbo shaft engines are used in helicopters, where the major thrust is generated by the main rotor blade. Turbo shaft engines do not generate any thrust because of the nozzle, the net power that is generated by the engine goes into driving the main rotor blade.

Then, we also discussed about a very simple form of a jet engine which is known as a ramjet engine. Ramjet engine does not have any of the turbo machines in the sense that does not have compressor, it does not have turbine, but it has an intake which has to be very efficient then there is a combustion chamber and a nozzle.

These three components does not have any rotating components in a ramjet which makes it very simple in terms of construction, but ramjets have an inherent disadvantage in the sense that they cannot be used for generating static thrust. So, ramjets can generate thrust only after it has reached a certain Mach number which means that it has first flown to a certain Mach number after which it can start generating thrust, so ramjets do not generate any static thrust.

What we shall discuss today is we will try to solve some problems which are related to some of these types of engines. Basically, carry out an ideal cycle analysis of some of these types of engines. Ideal cyclic analysis as we have discussed involves that by using some of the known design parameters like compress of pressure ratio, turbine inlet temperature and so on and also the operating conditions like Mach number, temperature and pressure.

To analyze each of the components which constitute the engine like intake, turbine, compressor, combustion chamber, turbine nozzle etcetera and in the end determine what is the thrust fuel consumption and the efficiencies associated with the engine. We will carry out cycle analysis in a systematic way for different types of engine. We will begin with a turbo jet then, we will move on to a turbofan engine and then, we will take up one example of a turbo shaft engine. Then, also I will give you some examples or exercises which you can solve at your leisure time later based on whatever we have discussed.

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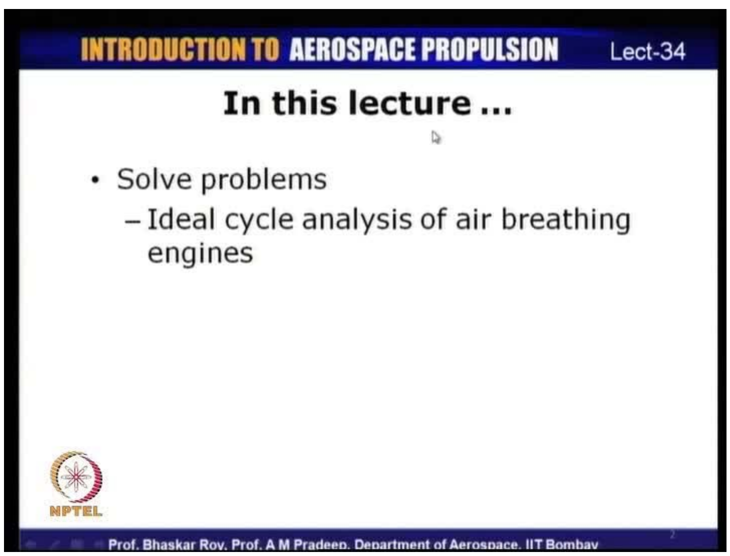

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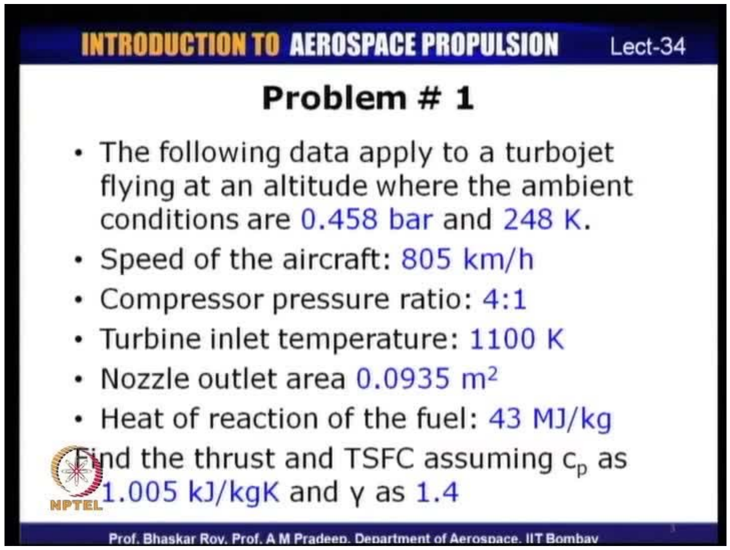

In today's lecture, we shall primarily be discussing about cycle analysis as applied to different types of air breathing engines. So, let us take a look at the first problem, the problem number 1 is that it is basically a turbojet problem, the problem statement is the following. The following data applied to a turbojet flying at an altitude where the ambient conditions are 0. 458 bar and 248 kelvin. Speed of the aircraft is 805 kilometers per hour, the compressor pressure ratio is 4 is to 1, turbine inlet temperature is 1100 kelvin, nozzle outlet area is 0.0935 meter square, heat of reaction on the fuel is or calorific value of the fuel is 43 mega joules per kilogram. We are required to find the thrust the specific fuel consumption assuming the C p as 1.005 kilojoules per kilogram kelvin and gamma as 1.4.

This particular problem applies to a pure turbo jet without any after burning and some of the data that is given to us are the compressor pressure ratio, the turbine inlet temperature; these are the design parameters of the engine. The ambient conditions are given as 0.458 and temperature of 248 and the flight speed is also given to us, so based on this we are required to find the thrust and fuel consumption for this particular turbojet cycle.

Before we start to analyze and try to solve the problem, let us first understand the basic turbojet construction in the form of what are the different components which constitute a turbojet.

We already discussed that in a last couple of lectures; let us take a relook at the different components and what are the station numbers associated with these components. Then, we will also take a very quick look at the ideal cycle of a turbojet before we start solving the problem in terms of each of these components.

So, a turbojet cycle as we have seen or turbojet engine constitutes of certain salient components. First component is the diffuser and following the diffuser we have a compressor - it could be an axial compressor or a centrifugal compressor. Then, after the compressor we have a combustion chamber, where heat is added or the fuel is added there and following the combustion chamber we have the turbine where the combustion products are expanded.

After the turbine we may or may not use an after burner, where additional fuel can be added to generate additional thrust. Then, we have the nozzle which generates the thrust. In a turbojet engine the sole objective of the turbine is to drive the compressor. So, the turbine work will be equal to the compressor work, which is not necessarily true in other types of engines like in a turboprop or a turbo shaft, where the turbine generates only part of the work to drive the compressor, rest of the work is used for either driving the propeller or for driving the main rotor blade, whereas in a turbojet the work done by the turbine will be equal to the work done by the compressor.

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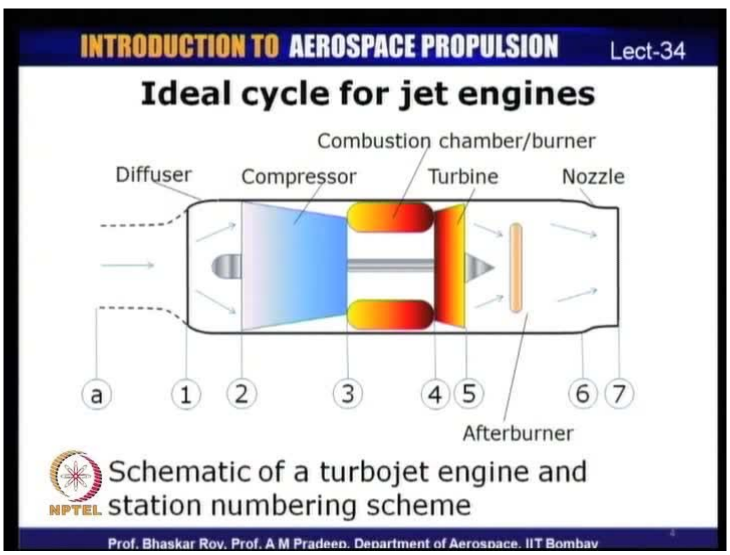

So, if we look at a schematic of the turbojet engine; turbojet engine as I mentioned constitutes of a diffuser, following the diffuser we have a compressor then, the combustion chamber. Compressor delivers air at high pressure and temperature - reasonably high temperature - to the combustion chamber. Here, additional fuel is added, combustion takes place and maximum cycling temperature takes place right here at the turbine inlet.

Air in the turbine, the combustion products in turbine inlet are expanded through the turbine and then it passes through the nozzle which generates the thrust. So, for cycle analysis we have designated different numbers for each of these components. The subscript would denote properties in the free stream that is away from the engine influence.

These are the different other subscript; subscript 2 would be used for the compressor inlet or diffuser exit, 3 for compressor exit or combustion chamber inlet, 4 for combustion chamber outlet or turbine inlet, 5 is for the turbine exit, 6 is for nozzle entry and 7 is for the nozzle exit.

Air as it passes through these different components we will experience changes in its pressure and temperature, those pressures and temperatures are denoted by some of the subscripts. Suppose for example, we write the stagnation temperature as T 03 that denotes the stagnation temperature at the compressor exit or combustion chamber inlet and T 04 it denotes stagnation temperature at the turbine entry or the combustion chamber outlet and so on.

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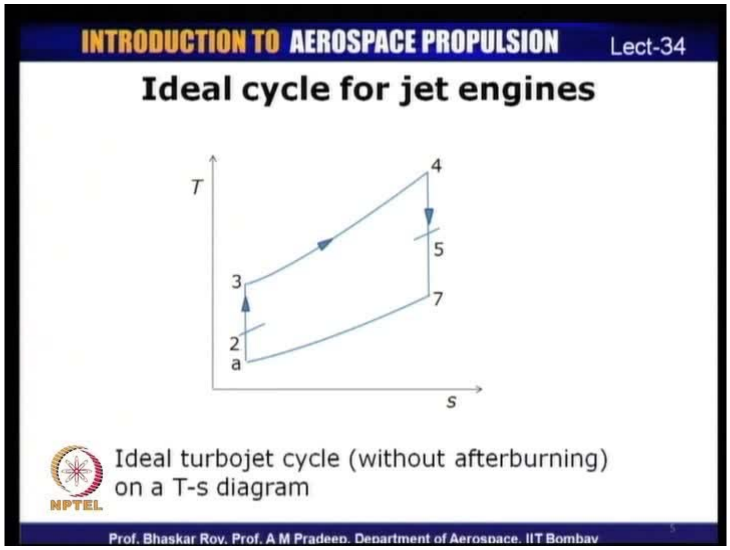

Now, if you look at the cycle diagram that is the Brayton cycle diagram for this particular turbojet engine. The ideal turbojet cycle without any after burning would look like this on a T s diagram, we have temperature on the y axis and entropy on the x. Since it is an ideal cycle, we have all the processes which are idealized, so process begins at state a which is the ambient condition then, the first process is the compression process in the intake, a to 2 is the compression in the intake.

So, 2 denotes the combustion compressor entry, 2 to 3 is the ideal compression or isentropic compression in the compressor, so process all the way from a to 3 isentropic compression then, we have a constant pressure heat addition which is in the combustion chamber. So, process from 3 to 4 is the heat addition or combustion process taking place in the combustion chamber which as in an ideal cycle takes place at constant pressure.

Process 4 to 5 is the turbine which is an ideal cycle and isentropic expansion, 5 to 7 is isentropic expansion in the nozzle. This completes the entire Brayton cycle starting from state a, all the way from state 2, 3 and so on. First process is the isentropic compression in the intake. Then, we have an isentropic compression in the compressor, and then there is a constant pressure heat addition. At the end of heat addition process that is the turbine entry, we first expand the combustion products through the turbine followed by expansion through the nozzle.

So, the net thrust that is generated by the turbojet engine is purely because of the expansion taking place through the nozzle. The compressor is driven by the turbine, turbine does not generate an additional power, it generates just enough power to drive the compressor. The entire power or entire thrust which is generated by the turbojet is because of the nozzle which is not true for some of the other types of engines like turboprops or turbo shafts and so on. There are other components in fact the turbine generates more power than what is required to drive the compressor and that extra power required is used for driving the propeller in a turboprop and so on.

For a cycle analysis, what we shall do is to analyze the turbojet engine in a very systematic fashion looking at each of these components and seeing what are the parameters which are known and how do you analyze the next component based on the parameters which are known to us. Let us start the cycle analysis process by first looking at the parameters which are known to us.

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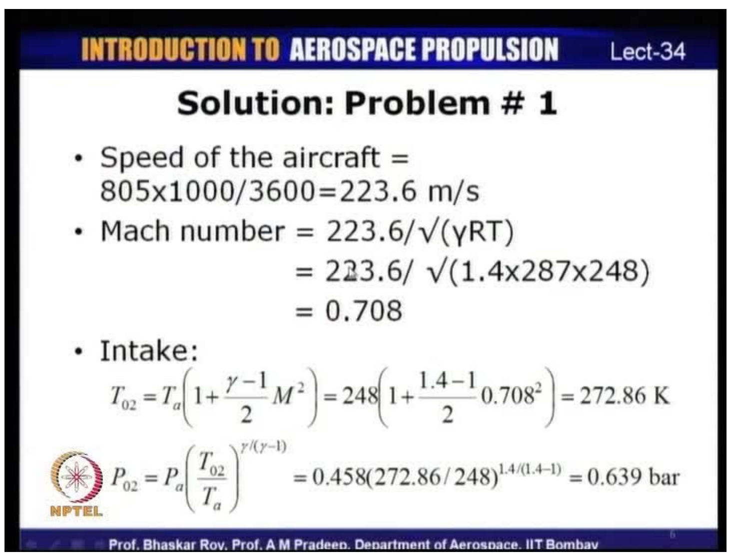

What are known is firstly the speed of the aircraft is specified, it is 805 kilometers per hour, let us convert that to Mach number. So, 805 kilometers per hour can be converted to meters per second that is 805 into 1000 dived by 3600 converts kilometer per hour to meters per second, so this corresponds to 223.6 meters per second.

How do you convert this into Mach number? Mach number, we know is the speed of aircraft divided by speed of sound and speed of sound is defined for an ideal gas as square root of gamma R T, where T is the static temperature. Temperature that is given to us, the ambient temperature is 248 kelvin and square root of gamma R T square root of 1.4 into R, for air is 287 joules per kilogram kelvin and T is the ambient temperature. Therefore, the Mach number of the aircraft is 223.6 divided by square root of gamma R T which is 1.4 into 287 into 248.

So, this Mach number comes out to be 0.708, the Mach number at which this aircraft is operating is 0.708. The first component we will analyze is the intake and for intake exit, we need to calculate the stagnation temperature and stagnation pressure. We know the intake entry temperature and pressure, the ambient conditions are given to us. From isentropic relations, we can calculate the stagnation temperature at the exit of the intake. How do we calculate that? It is basically using the isentropic relations which relate the ratio of stagnation temperature to the stagnation static temperature that is T naught by T is 1 plus gamma minus 1 by 2 m square f.

Similarly, we can use the same expression for pressure as well. Using these isentropic relations we can calculate the intake and exit stagnation pressure and stagnation temperature, which will form the entry conditions for the compressor.

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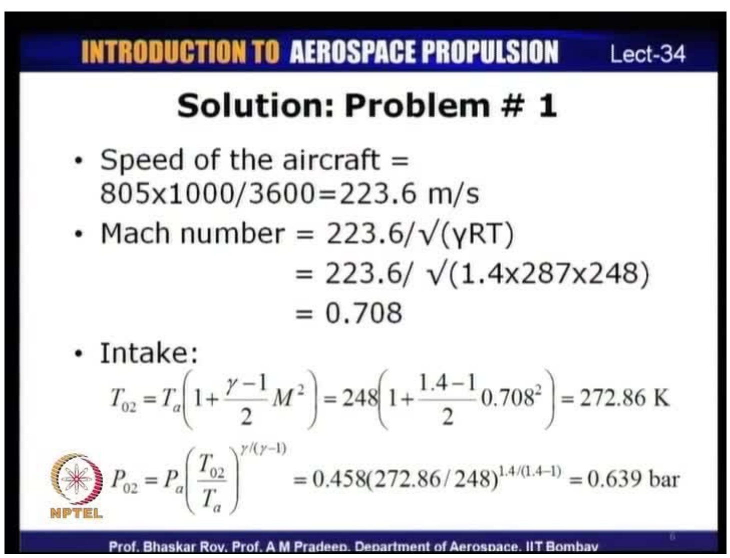

For the intake we have T 02 which is the intake exit temperatures stagnation, this is T a into 1 plus gamma minus 1 by 2 M square. T a is given as 248 kelvin, so this multiplied by 1 plus gamma is 1.4; 1.4 minus 1 by 2 into M square, where M Mach number is 0.708, we have just calculated that. This temperature comes out to be 272.86 kelvin. Similarly, stagnation pressure T 02 can be calculated from the isentropic relations, P 02 is P a into T 02 by t a raise to gamma by gamma minus 1 and P a is given as 0.458 bar, so this multiplied by 272.86 divided by 248 raised to 1.4 by 1.4 minus 1, so this is equal to 0.639 bar.

So, these are parameters at the intake exit and these form the entry conditions for the compressor. In the compressor, we have the compressor pressure ratio which is a designed parameter, we have been given the compressor pressure ratio, so using that we can calculate the compressor exit stagnation pressure. Then, again from the isentropic relations we will be calculating the compressor exit stagnation temperature.

Compressor exit stagnation temperature and pressure will form the entry conditions for the combustion chamber and that the exit of the combustion chamber we know the stagnation temperature which is the turbine inlet temperature. So, from an energy balance we can calculate the fuel to air ratio which is used in the combustion chamber, then by balancing the compressor work and the turbine work and turbine exit conditions in terms of pressure and temperature which forms the nozzle entry conditions because there is no after burning. Then, from the nozzle conditions we can find the nozzle exit velocity and hence the thrust.

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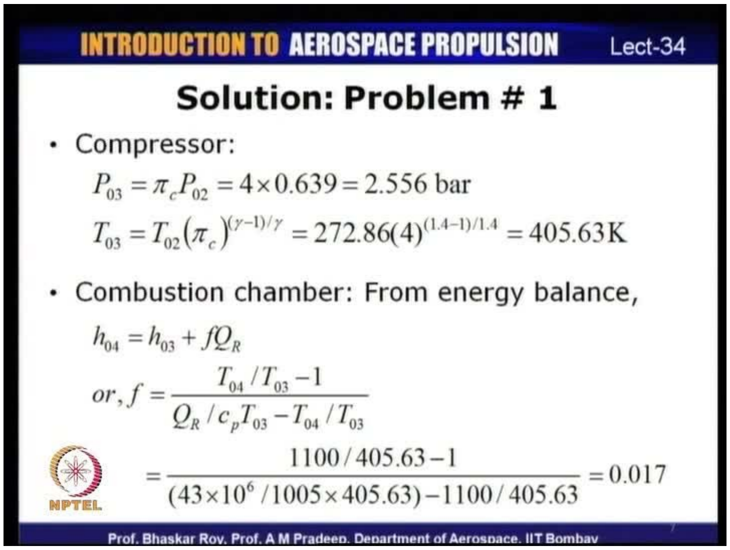

Let us take up the next component which is the compressor, so the compressor exit stagnation pressure is equal to the compressor pressure ratio multiplied by inlet static stagnation pressure. The compressor pressure ratio is given as 4, this multiplied by the compressor inlet stagnation pressure which we have just now calculated which is 0.639 bar, so this is equal to 2.556 bar.

Compressor exit stagnation temperature is T 02 into pi c, which is the compressor pressure ratio raise to gamma minus 1 by gamma, so this is equal to 272.86 into 4 raise to 1.4 minus 1 by 1.4, so this temperature comes out to be 405.63 kelvin. We have calculated both the compressor exit stagnation pressure, as well as the exit stagnation temperature. We also know the combustion chamber exit stagnation temperature which is the turbine inlet temperature, so all we have to do is carry out an energy balance.

Combustion chamber exit enthalpy h 04 is equal to h 03 which is entry enthalpy plus f times QR, where f is the fuel to air ratio, QR is the heat of reaction in of the fuel. Therefore f, if we simplify this from assuming air to be ideal gas we get T p into T 04 is c p into T 03 plus f into QR. If we simplify that we get T 04 by T 03 minus 1 divided by QR by c p T 03 minus T 04 by T 03, all these parameters are known to us, so we have to just substitute for these different values.

We have T 04 which is 1100 divided by T 04 that is 405.63 minus 1 divided by QR which is 43 mega joules per kilogram - so we have converted that to joule - so 43 into 10 raise to 6 divided by c p which is 1005 joules per kilogram kelvin multiplied by 405.63 minus the temperature ratios. If we simplify this we get the fuel to air ratio as 0.017.

Fuel to air ratio as we have defined is ratio of mass flow rate of fuel to mass flow rate of air and we see the number to be 0.017 which means that the amount of fuel that is added as compare to the mass flow rate of air is very small. In fact, in some of the earlier analysis we had assumed that the fuel to air ratio will be very small it can be neglected. In fact, if you look at many of the text books very often you would see that in many numerical problems f is considered to be very small and it is many a times is neglected.

The basic reason is that this ratio f is much less than 1 and if you neglect this and calculate thrust, the error that is introduced because of neglecting fuel to air ratio is usually very small but of course, calculating fuel to air ratio always is a good idea because you will get much more accurate estimate of the thrust that is generated.

At the same time, if you have to calculate specific fuel consumption or thrust specific fuel consumption, you need to know the fuel to air ratio. It is always a good practice to calculate the fuel to air ratio though in many books you might notice that it is not always done.

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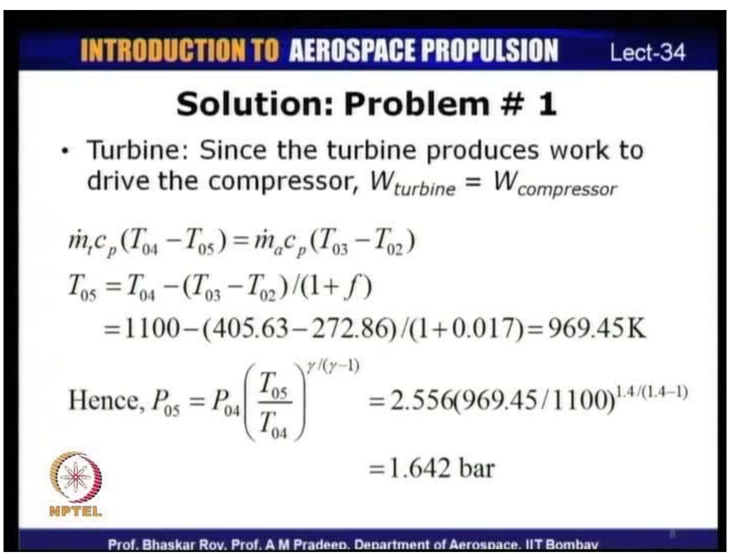

Now that we have calculated the properties at the fuel combustion chamber exit that is we now have the fuel to air ratio, let us move on to the turbine. Now, in the turbine as I have mentioned for a turbojet engine; the turbine generates work only to drive the compressor therefore turbine work is equal to compressor work.

Turbine work is mass flow rate through the turbine into c p into temperature difference that is T 04 minus T 05 this is equal to m dot a which is mass flow rate of air into c p into T 03 minus T 02. If we simplify this we get T 05 is equal to T 04 minus T 03 minus T 02 divided by 1 plus f. All these numbers are known to us T 04 is 1100 kelvin, T 03 is 405.63, T 02 is 272.86 divided by 1 plus 0.017. This temperature once we calculate we get 969.45 kelvin, so the turbine exit stagnation temperature is 969.45 kelvin. Now how do you calculate the turbine exit stagnation pressure?

Since, the expansion process is assumed to be isentropic, we use the isentropic relations again here, P 05 is equal to P 04 multiplied by T 05 by T 04 raise to gamma by gamma minus 1. This will be equal to P 04 which is 2.556 we have calculated that from the combustion chamber inlet, so combustion chamber there is no loss taking for the pressure and P 03 is equal to P 04 which is 2.556 bar. This multiplied by 969.45 which is T 05 divided by 1100 which is T 04 raise to 1.4 divided by 1.4 minus 1.

The turbine exit stagnation pressure is 1.642 bar, so you have seen that in the case of a turbine we need to just equate if it is a turbojet engine we can equate the compressor work to the turbine work and therefore, determine the turbine exit conditions from the equality of the compressor work to the turbine work.

In one of the later examples we will see the third one, we cannot really equate the turbine work to the compressor work because the turbine is generating more work than what is required to drive the compressor and that extra work actually reviews to drive the propeller and so on.

After we have determined the properties for a turbine, the next component is the nozzle. In the case of a nozzle, we need to take a little precaution here in one sense that we need to first determine whether the nozzle is operating under choked condition or is it operating under unchoked condition because depending upon whether the nozzle is choked or not the exit conditions of the nozzle will be different.

That is, if the nozzle is operating under choked condition this means that the nozzle exit Mach number is 1 and if Mach number is 1, then we know that the exit velocity will be equal to the speed of sound and therefore, u e will be equal to square root of gamma r static temperature. Whereas, if it is unchoked then the nozzle exit velocity is what we had derived in our cycle analysis discussion earlier which is square root of 2 into c p into T 06 1 minus p a by P 06 raise to gamma minus 1 by gamma.

So, how do you find out whether the nozzle is choked or not, to determine whether the nozzle is choked or not we need to find the stagnation pressure ratio across the nozzle as well as the critical pressure ratio across the nozzle which is a function of the ratio of specific heats. Then, if the actual pressure ratio is greater than the critical pressure ratio than it means that the nozzle is choked.

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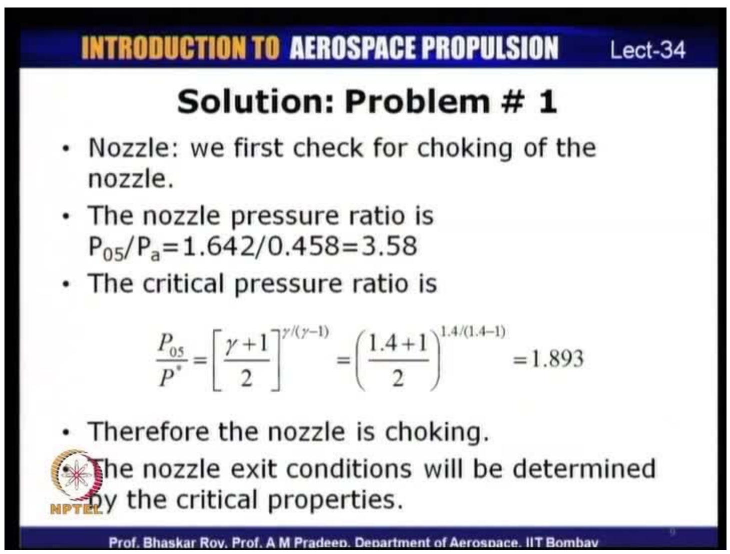

If it less than the critical pressure ratio it means, the nozzle is unchoked. So, depending upon whether it is choked or unchoked the exit conditions can be different. To check for the choking of the nozzle, let us find out the nozzle pressure ratio. P 05 that is turbine exit stagnation pressure is 1.642, this divided by the ambient pressure which is 0.458, so this ratio comes out to be 3.58. What is the critical pressure ratio? If you go back to some of the earlier lectures we had discussed on compressible flows, we had derived this expression for critical pressure ratio.

Critical pressure ratio was derived as P naught by P star is equal to gamma minus; well, gamma plus 1 by 2 raise to gamma minus gamma by gamma minus 1. So, it is purely of a function of the ratio of specific heats. Since, gamma is 1 point 4 here we get critical pressure ratio P 05 by P star where P star is the critical pressure as 1.4 plus 1 divided by 2 raise to 1.4 divided by 1.4 minus 1, this pressure ratio is 1.893.

What do you see here is that the actual pressure ratio is 3.58 whereas, the critical pressure ratio is 1.893. So, the actual nozzle pressure ratio is much greater than the critical pressure ratio and therefore, this means that the nozzle is choking. So, what if the nozzle is choking? This means that if the nozzle is choking then, the exit conditions will be fixed accordingly that is, the exit pressure will be equal to the critical pressure, exit temperature will be equal to the critical temperature and density will be equal to critical density and so on.

From the critical temperature you can actually find out the exit velocity because exit velocity will be equal to square root of gamma R T star. How do you find out the critical pressure, temperature and density? We have to first find these critical parameters because that is the exit condition and then accordingly, we can find the exit velocity and therefore, the thrust and a fuel consumption and so on.

Let us know find the critical pressure, to find the critical pressure we already have the ratios with us and from those ratios, we can easily find out the critical pressure. The critical pressure, temperature and density will need to be determined for calculating the nozzle exit conditions.

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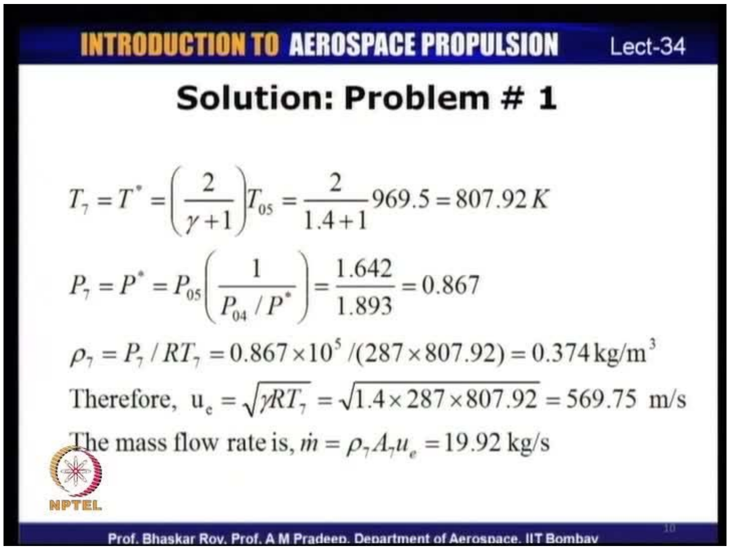

T 7 which is the exit temperature is equal to T star is 2 by gamma plus 1 into T 05 and T 05 is known, it is fixed as 969.5 kelvin, so 2 by 1.4 plus 1 into 969.5, so this comes out to be 807.92 kelvin.

T 7 that is the exit temperature - static temperature - is equal to 807.92 kelvin. Similarly, exit pressure will be equal to P 05 into 1 by P 05 by P star and P 05 by P star is already known that is 1.893 - we had calculated that already - this is 1.893. P 05 is known as 1, is already calculated, as turbine exit stagnation pressure 1.642 bar and so this exit pressure comes out to be 0.867 bar.

The exit pressure of the nozzle is 0.867 and density would be equal to P by RT because air is an ideal gas here therefore, P is 0.867 bar - bar is converted to Pascal's - so 80.867 into 10 raise to 5 divided by r that is, gas constant for air 287 into the temperature that is 807.92, the exit density is 0.374 kilograms per meter cube. Since, Mach number at the exit of the nozzle is 1 the exit velocity is equal to square root of gamma RT 7 which is equal to 1.4 into 287 into 807.92, so this is equal to 569.75 meters per second.

We know the exit velocity, the density and the area; area already been given in the question, we can calculate the mass flow rate. The total mass flow rate is equal to rho 7 a 7 into u e which is 19.92 that is, rho is 0.374, area is given exit velocity is also given, so it is 19.92.

The mass flow rate at the exit of the nozzle is also calculated. Now, we know all the parameters required for calculating the thrust, so thrust is defined as equal to m dot a into 1 plus f u e minus u plus p minus p a into e. So, all these parameters are known to us and we can calculate the thrust generated by the turbojet engine and subsequently also the thrust specific fuel consumption that is TSFC.

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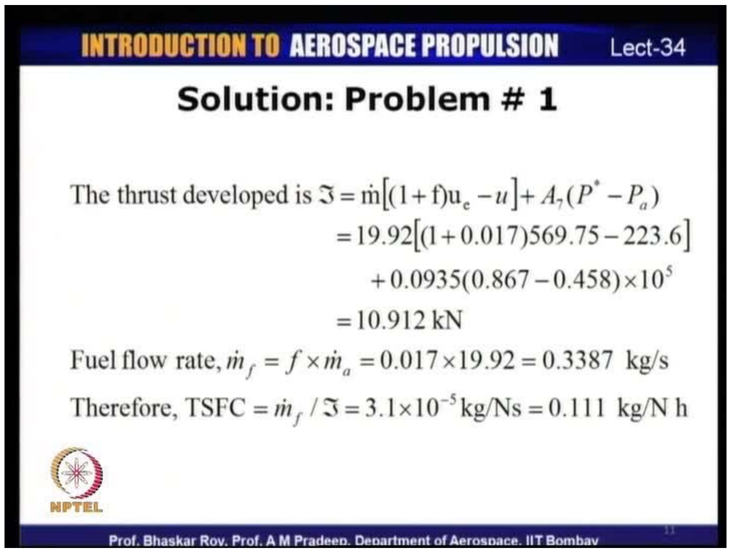

The thrust developed is equal to m dot into 1.1 plus f u e minus u plus a 7 into P star minus P a because P e is equal to P star. Mass flow rate is 19.92, this multiplied by 1 plus f that is 1 plus 0.017 into u e that is 569.75 minus 223.6 plus a 7 that is 0.0935 into P star 0.867 minus 0.458 into 10 raise to 5, so this comes out to be 10.912 kilonewtons. The total thrust developed by the turbojet engine here is 10.912 kilonewtons.

Therefore, fuel flow rate is equal to which is m dot f is equal to f into m dot a that is, 0.017 into 19.92 which is equal to 0.3387 kilograms per second. Therefore, Thrust Specific Fuel Consumption - TSFC - is m dot f divided by thrust and m dot f is 0.3387 divided by thrust which is 10.912. So, if you divide it by the thrust we get 3.1 into 10 raise to minus 5 kilograms per newton second which is in turn equal to 0.111 kilograms per newton hour.

In most of the books and as well as in industry standards the fuel consumption is usually expressed in terms of kilograms per newton hour, because that is kilograms per newton second comes out to be a very small quantity.

So, they convert that to hour by multiplying it by 3600, which is why I have expressed it also in terms of kilograms per newton hour, because that is in terms of some form of an industry standard where they would express the fuel flow rate as in the form of kilograms per newton hour. Once we calculate kilograms per newton second, it just have to be multiplied by 3600 to convert it to kilograms per newton hour.

This completes our cycle analysis for the first problem where we have looked at a turbojet engine and we have systematically carried out cycle analysis by considering each of these components in steps that is starting from intake then proceeding towards compressor, combustion chamber, turbine and then the nozzle.

Only thing is, at the nozzle we have to check for choking that is, if the nozzle is choked then the nozzle exit conditions are fixed by the choking conditions. Therefore, the exit velocity temperature etcetera needs to be calculated based on the choked properties or the critical properties.

If it is un choked of course, nozzle exit velocity is determined from the enthalpy and so, from their enthalpy drop we can calculate the exit velocity and the thrust is basically a function of the mass flow rate 1 plus f, the exit velocity flight speed and also the pressure thrust term that is, if the pressure thrust term is significant that will also contribute towards the thrust in some form or the other.

This is how we calculate or carry out a cycle analysis for a turbojet engine and in the next problem that we shall discuss about cycle analysis to be carried out for a turbofan engine. The analysis is very similar to that of turbojet just that we have an additional set of calculations to be carried out for the bypass duct that is, the fan and the secondary nozzle. The total thrust will be equal to sum of these 2 separate nozzles which are there.

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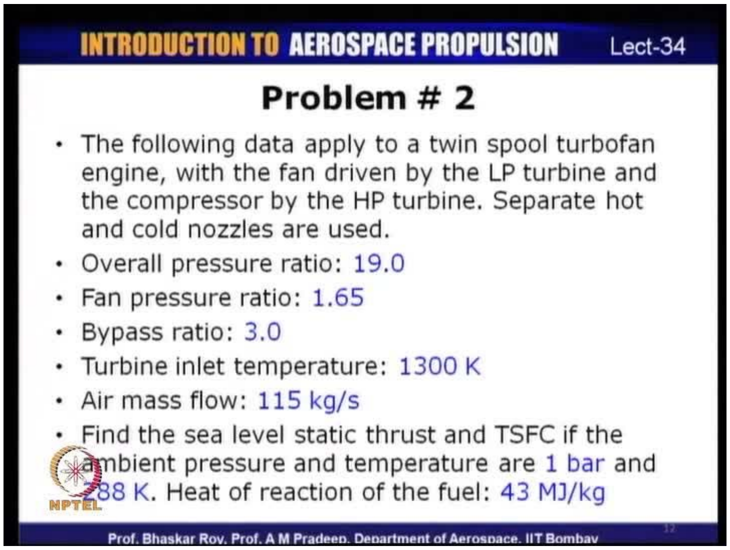

Let us look at the second problem statement, the second problem states that the following data applied to a twin spool turbofan engine with the fan driven by the LP turbine and the compressor driven by the HP turbine. Separate hot and cold nozzles will be used that means that in this turbofan the exhaust streams do not mix before exiting the nozzles, so you have separate hot and cold nozzles.

The overall pressure ratio is given as 19, the fan pressure ratio is 1.65, the bypass ratio is 3 and turbine inlet temperature is 1300 kelvin, air mass flow rate is given as 115 kilograms per second. We need to find the sea level static thrust, the specific fuel consumption, the ambient pressure and temperature are 1 bar and 288 kelvin, heat of reaction of the fuel is 43 mega joules per kilogram.

This is a problem which involves a turbofan; it is basically a twin spool turbofan engine. The fan is here driven by the LP turbine, compressor is driven by the HP turbine and we have separate hot and cold nozzles which are used that is, the two streams do not mix before exiting the nozzle. So, we have thrust components because of the core nozzle as well as the bypass nozzle.

Some of the parameters of the engine which are the overall pressure ratio, fan pressure ratio, bypass ratio, the mass flow rate and turbine inlet temperature these have been specified. Based on this, we need to carry out the cycle analysis basically find the sea level static thrust.

Now, sea level static thrust means that the engine is stationary, it is generating thrust while it is stationary which means, Mach number is 0. So, the ambient conditions are also specified like temperature and pressure which correspond to the sea level conditions.

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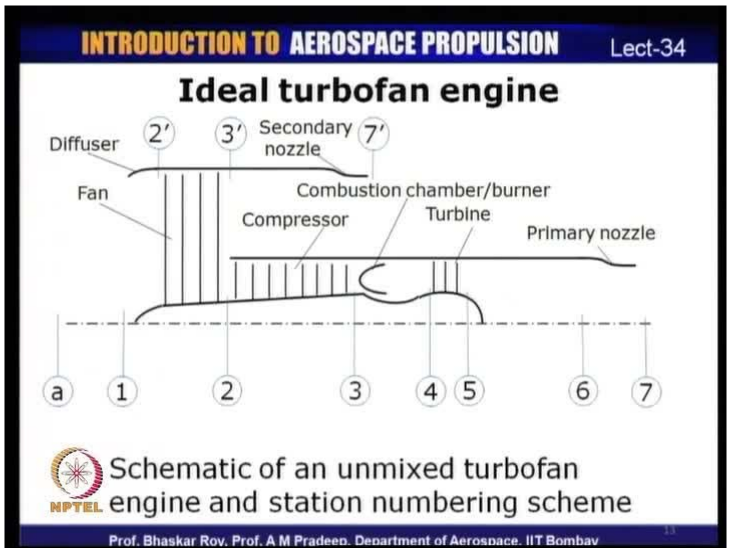

This is the schematic of the turbofan engine which we have seen in the earlier lectures, so turbofan engine as we know consists of these additional components that is a fan and the bypass duct and also the secondary nozzle other components are identical to that of a turbo jet like the compressor, combustion chamber, turbine and then a primary nozzle.

For cycle analysis, we shall be using as additional symbols which denote the bypass duct that is, symbols with the prime symbols that is 2 prime, 3 prime and so on, which correspond to properties in the bypass duct. 2 prime for example, corresponds to the fan inlet, 3 prime corresponds to fan exit and also the compressor pressure inlet and 7 prime is the secondary nozzle exit. So, other symbols are same as what we have used for the turbojet.

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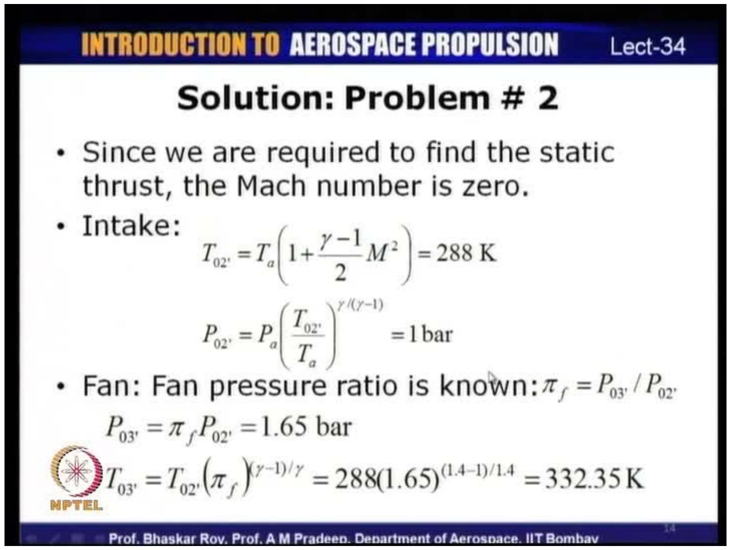

We will begin the cycle analysis by calculating properties across the intake and since the Mach number here is 0, because we require finding the static thrust that means, Mach number is 0, so the intake exit properties will be same as the ambient conditions.

Intake exit stagnation temperature T naught 2 prime is equal to ambient stagnation temperature which is 288 kelvin and P 02 prime is equal to 1 bar. Now, the first component that we will take up for analysis is the fan after the intake and in the case of fan the pressure ratio is fixed it is known to us that is bar pressure ratio is 1.65 bar and therefore, P 03 prime which is the fan exit stagnation pressure is equal to the pressure ratio that is pi f multiplied by P 02 prime which is 1 bar. Therefore, fan exit stagnation pressure is 1.65 bar.

To calculate stagnation temperature at the fan exit, we use the isentropic relation therefore T 03 prime is equal to T 02 prime into pi f raise to gamma minus 1 by gamma, so this is equal to 288 which is the stagnation temperature multiplied by 1.65 raise to 1.4 minus 1 by 1.4, so fan exit stagnation temperature is 332.35 kelvin.

In the case of the compressor in this question we have been given the overall pressure ratio as well as the fan pressure ratio, the compressor pressure ratio will be equal to the overall pressure ratio divided by the fan pressure ratio. So, overall pressure ratio is given as 19 and fan pressure ratio is 1.65. Therefore the compressor pressure ratio will be equal to 19 divided by 1.65 and once we know the compressor pressure ratio we can find the compressor exit conditions that is the exit stagnation pressure as well as stagnation temperature and then, we shall take up the combustion chamber of the turbine.

Turbine will be in two stage, we have the HP turbine first and then the LP turbine. So, HP turbine properties are determined by equating the HP turbine work to the compressor work. LP turbine we equate that to the fan work and from there we can find the exit conditions of the turbine and then, the nozzle - that is the primary as well as the secondary nozzle - and we add up the thrust which is generated by the primary nozzle as well as the secondary nozzle that gives us the total thrust develop by the turbofan.

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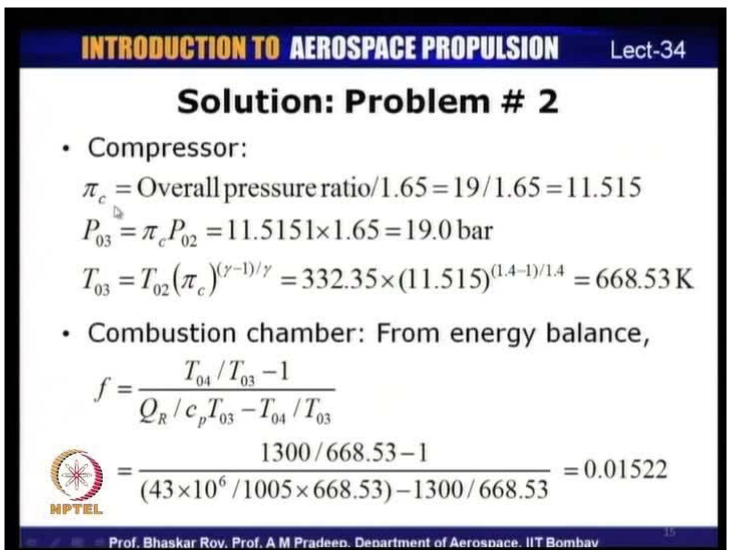

For the compressor we have the pressure ratio which is equal to overall pressure ratio divided by 1.65, so this comes out to be 11.515. P 03 is compressor exit stagnation pressure this is equal to pi c - which is compressor pressure ratio - into P 02 which comes out to be 19 bar, because we are looking at static thrust therefore, P 02 is actually equal to 1.65.

T 03 which is the compressor exit stagnation temperature is equal to T 02 which is inlet stagnation temperature multiplied by pressure ratio raise to gamma minus 1 by gamma, so this is 332.35 which is T 02, fan exit stagnation temperature multiplied by 11.515

raise to 1.4 minus 1 by 1.4, so this is 668.53 kelvin. In the combustion chamber we carry out the energy balance as we have discussed for the turbine.

Turbine inlet temperature is fixed that is 1300 kelvin and the other parameters are already known that is, T 03 and the heat of reaction of the fuel. We substitute all these different values here, we get 1300 divided by 668.53 minus 1 divided by 43 into 10 raise to 6 divided by c p which is 1005 joules per kilogram kelvin into 668.53 minus 1300 by 668.53.

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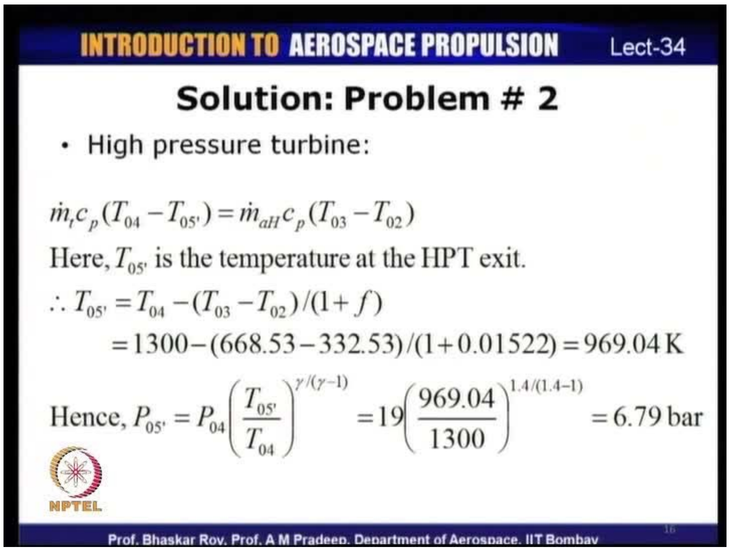

The fuel to air ratio comes out to be 0.01522, so after the combustion chamber we shall first take up the HP turbine and in the case of HP turbine, we equate the HP turbine work to the compressor work because HP turbine is driving the compressor. Therefore, the enthalpy across the HP turbine which is mass flow rate time c p into the temperature difference T 04 minus T 05 dash this is equal to mass flow rate through the compressor into c p into T 03 minus T 02.

The HP turbine exit stagnation temperature can be determined which is T 05 prime this is equal to T 04 minus T 03 minus T 02 divided by 1 plus f, if you substitute for all these values we get the HP turbine exit temperature as 969.04 kelvin.

Similarly, we can calculate the exit pressure that is HP turbine exit stagnation pressure which is P 05 prime this is equal to P 04 into T 05 prime divided by T 04 raise to gamma by gamma minus 1. The combustion chamber inlet pressure is known that is P 03 which is equal to P 04 that is 19 bar multiplied by 969.04 divided by 1300 raise to 1.4 divided by 1.4 minus 1, so this is equal to 6.79 bar. So, HP turbine exit stagnation pressure is 6.79 bar.

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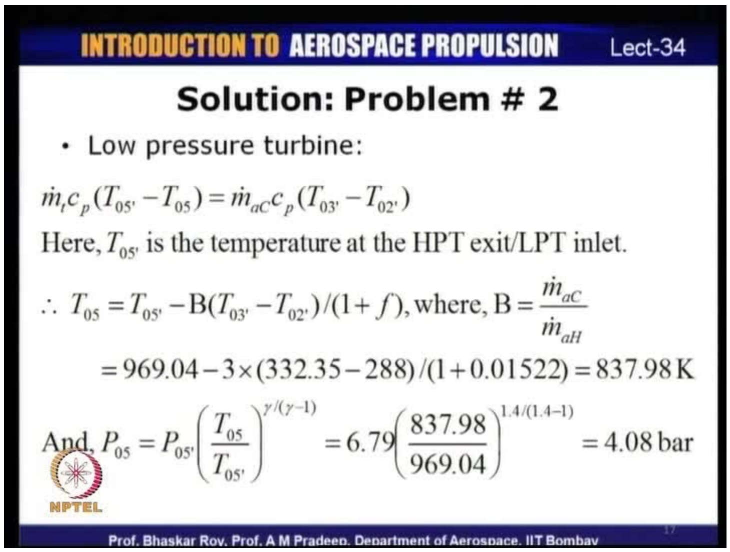

Similarly, we can also calculate the properties at the exit of the LP turbine because we know the LP turbine inlet conditions. So, for the LP turbine which drives the fan enthalpy across LP turbine is equal to enthalpy across the fan, this is mass flow rate through the Lp turbine into c p into T 05 prime minus T 05, this is equal to m dot aC which is mass flow rate through the fan multiplied by c p into T 03 prime minus T 02 prime.

So, from this we can determine the LP turbine exit stagnation temperature, this can be simplified in terms of the bypass ratio. Here, we also have a bypass ratio because that will be equal to fan mass flow divided by the core mass flow. We have 969.04 minus 3 which is the bypass ratio into 332.35 minus 288 divided by 1 plus 0.01522, so this is 837.98 kelvin.

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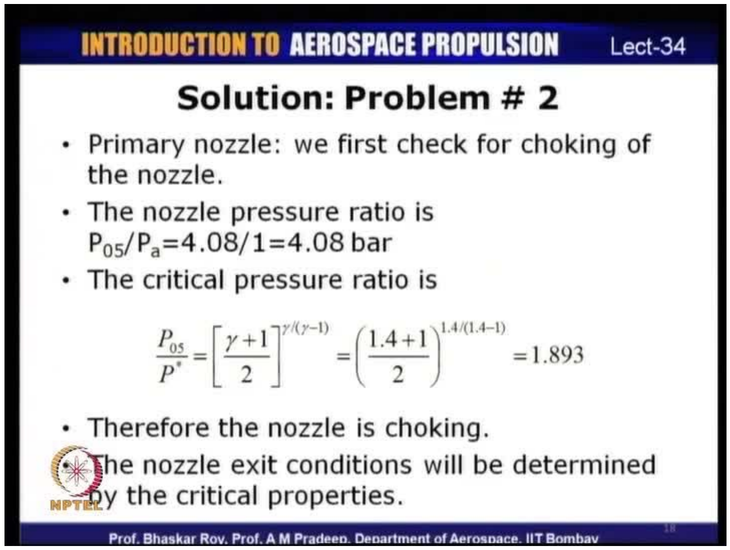

Similarly, the pressure we can calculate as 4.08 bar, the primary nozzle - we again check for choking as we did for the turbojet - we shall check for choking for the primary nozzle. Here, the primary nozzle pressure ratio is 4.08 bar whereas, the critical pressure ratio is 1.893 which means that the primary nozzle is choking and therefore, the primary nozzle exit conditions will be determined by the critical properties.

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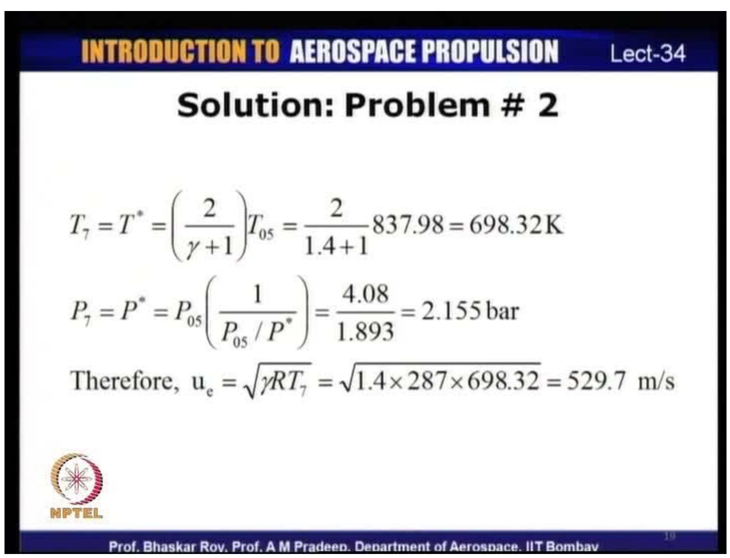

Therefore, T 7 is equal to T star which is 2 by gamma plus 1 into T 05 this is 2 by 1.4 plus 1 into T 05 comes out to be 698.32 kelvin. P 7 is equal to P star this is again from the pressure ratios we get 2.155 bar. Therefore, primary nozzle exit velocity is square root of gamma RT 7; this is 1.4 into 287 into 698.32 that is 529.7 meters per second.

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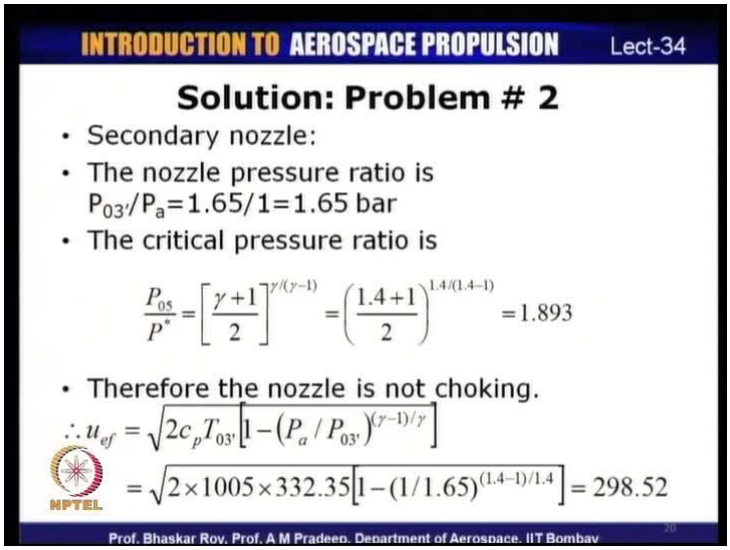

Now, for the secondary nozzle we again check for choking here, the nozzle pressure ratio is less than the critical pressure ratio which means, the secondary pressure ratio is not choking and therefore, the secondary nozzle exit velocity is calculated as this square root of 2 c p T 03 prime into 1 minus P a by P 03 prime raise to gamma minus 1 by gamma, so this comes out to be 298.52 meters per second.

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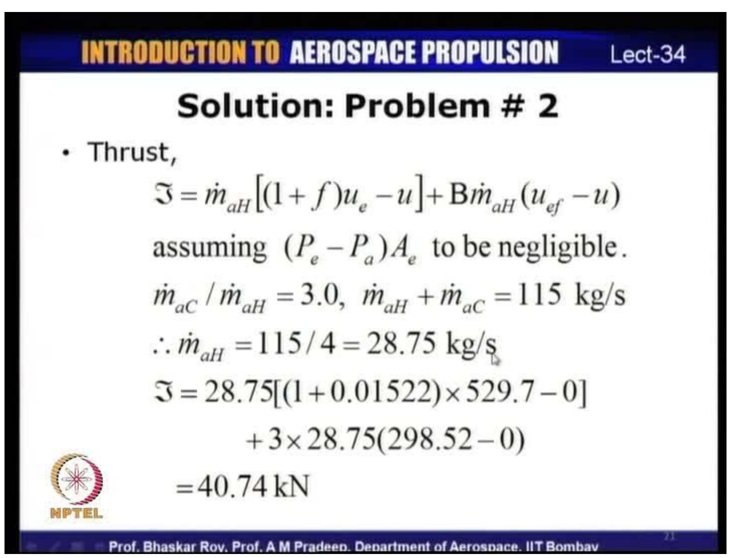

Therefore, the total thrust generated is equal to the sum of the primary nozzle thrust and the secondary nozzle thrust this is m dot aH into 1 plus f into u e minus u plus bypass ratio into m dot aH into u e f minus u assuming that the pressure thrust is negligible. From the mass flow ratio that is m dot aC divided by m dot aH is equal to 3 bypass ratio, sum of these two is given as 115 kilograms per second therefore, the core mass flow is m dot aH should be equal to 28.75.

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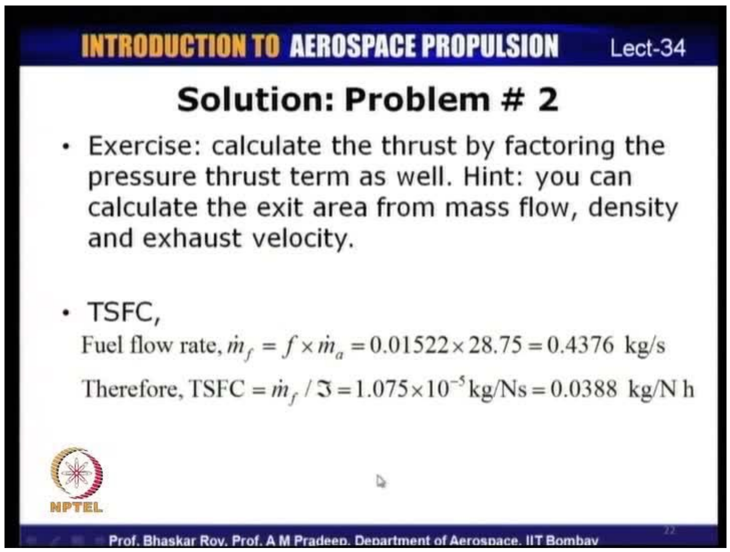

If you substitute it for all these values we get the total thrust which is 40.74 kilo newtons. I leave it as an exercise for you to calculate the pressure thrust term as well, because you now know the exit velocity, the density and mass flow, so you can calculate exit area and therefore, the pressure thrust.

The fuel consumption is equal to fuel to air ratio divided by mass flow rate, this is 0.4376 kilograms per second from there we can find the thrust specific fuel consumption which is 0.0388 kilograms per newton hour. So, this was a problem were we are analyzing a turbo fan engine which is a twin spool turbofan engine with 2 separate streams. Third problem which we shall quickly look at is that of a turbo shaft engine, we need to calculate the specific power that is generated and the efficiency.

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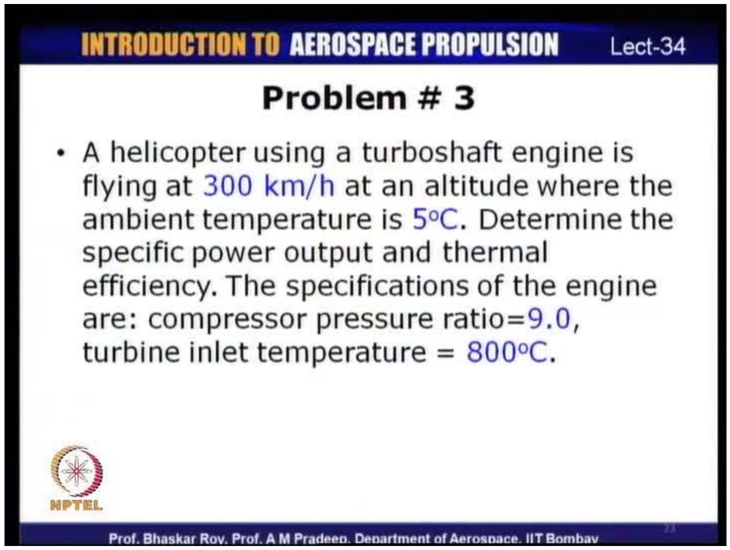

Let us look at the third numerical problem which is a helicopter using a turbo shaft engine is flying at 300 kilometers per hour at an altitude where the ambient temperature is 5 degree Celsius.

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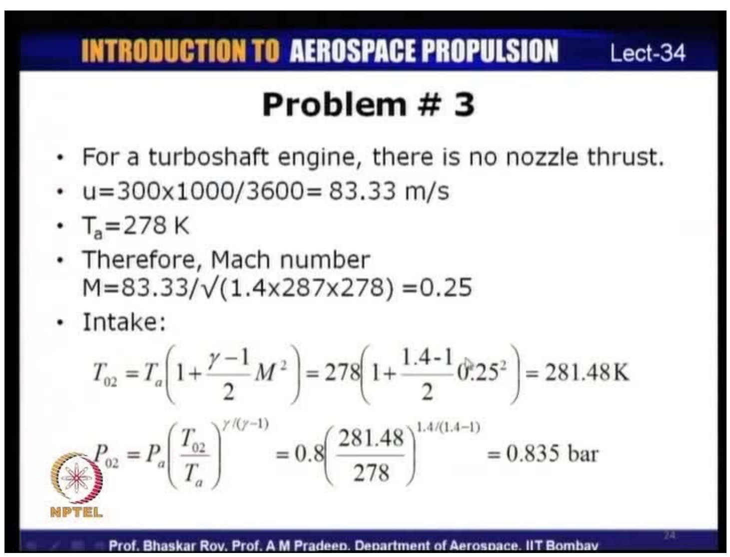

Determine the specific power output and the thermal efficiency. The specifications of the engine are, the compressor pressure ratio is 9 and turbine inlet temperature is 800 degree celsius. In this case, we first calculate flight speed which is 300 kilometers per hour, so we convert that to meters per second that is 83.33 meters per second.

Ambient temperature is 278 therefore, Mach number is 0.25. At the intake exit we have stagnation temperature and pressure which we can calculate as 281.48 and pressure as 0.835 bar.

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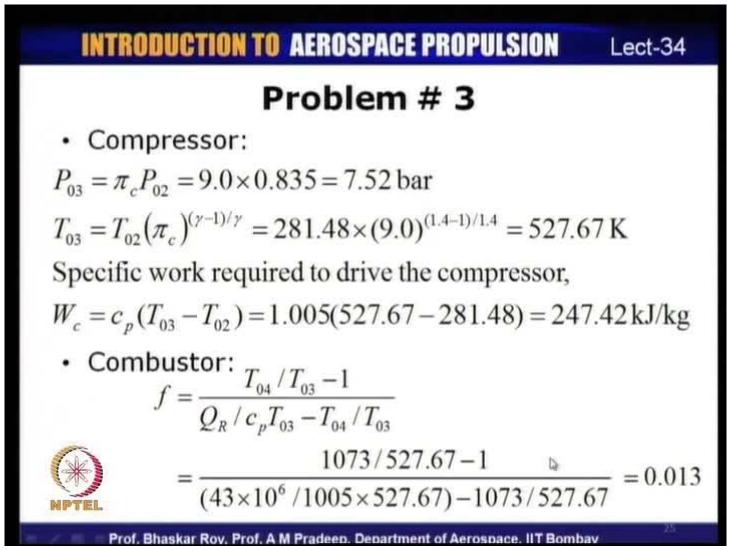

For the compressor similarly, we will calculate exit stagnation pressure which is 7.52, exit stagnation temperature which is 527.67 and therefore, specific work required to drive the compressor is c p into T 03 minus T 02 which is 247.42 kilo joules per kilogram.

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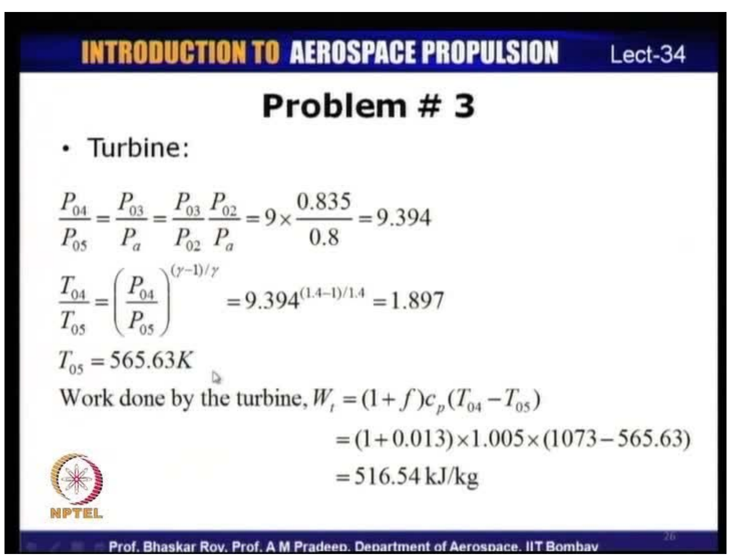

Combustor we get the fuel to air ratio which we can calculate as 0.013. Now, for the turbine we cannot equate turbine work to the compressor work because this is no longer a turbojet engine. So, for calculating turbine exit conditions we have the turbine pressure ratio P 04 by P 05, this should be equal to the cycle pressure ratio which is P 03 by P a which is in turn equal to P 03 by P 02 into P 02 by P a this is equal to 9 into 0.835 divided by 0.8 that is 9.394.

From this, we can calculate the turbine exit stagnation temperature from isentropic relations which comes out to be 565.63 kelvin. Therefore, work done by the turbine is equal to 1 plus f into c p into T 04 minus T 05, this is 516.54 kilo joules per kilogram.

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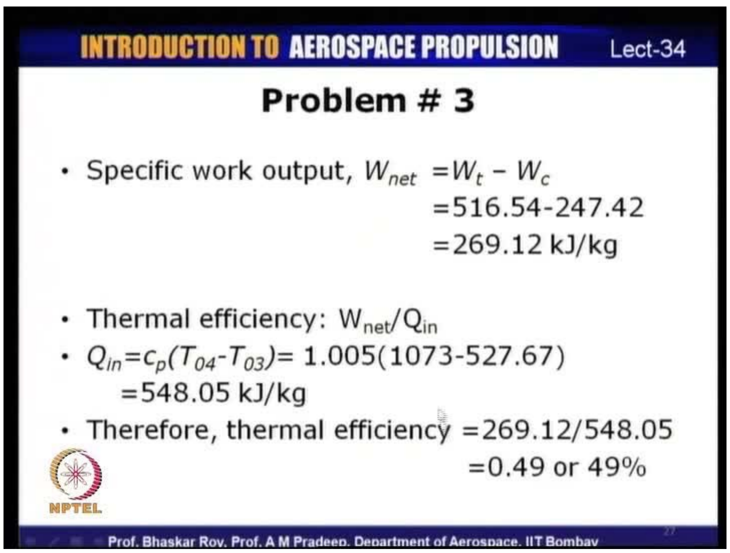

Therefore, specific work output is W net that is W t minus W c comes out to be 269.12 kilojoules per kilogram. Then, how do you calculate thermal efficiency? Thermal efficiency as by definition is network output by heat input. Heat input is C p into T 04 minus T 03 that is across the combustion chamber. Since both these temperatures are known we can calculate heat input as 548.05 kilojoules per kilogram. Therefore, the thermal efficiency is the network output 269.12 divided by heat input 548.05, so this is 49 percent.

So, I have now 3 exercise problems for you which you can solve based on what we have discussed in today’s lecture. The first exercise problem for you is on a turbojet engine.

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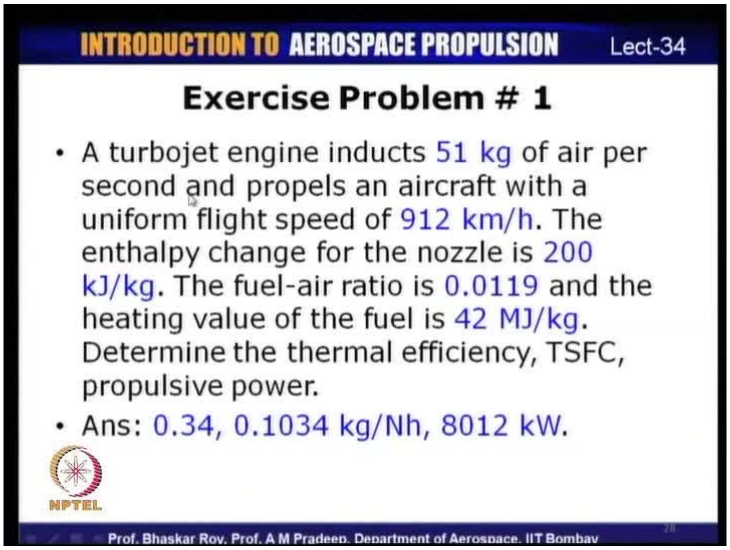

A turbojet engine inducts 51 kilogram of air per second and propels an aircraft with a uniform flight speed of 912 kilometers per hour. The enthalpy change for the nozzle is 200 kilojoules per kilogram. The fuel to air ratio is 0.0119 and the heating value of the fuel is 42 mega joules per kilogram. Determine the thermal efficiency, the fuel consumption and propulsive power. Answer for thermal efficiency is 0.34, the fuel consumption is 0.1034 kilograms per newton hour and the propulsive power is 8012 kilowatts.

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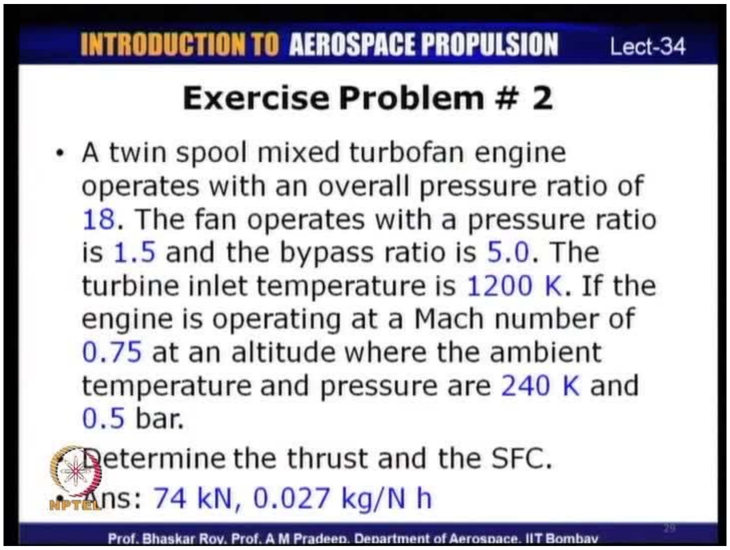

The second exercise problem is of a turbofan, a twin spool mixed turbofan engine operates with an overall pressure ratio of 18. The fan operates with a pressure ratio of 1.5 and the bypass ratio is 5, the turbine inlet temperature is 1200 kelvin. If the engine is operating at a Mach number of 0.75 at an altitude where the ambient temperature and pressure are 240 kelvin and 0.5 bar. Determine the thrust and specific fuel consumption. In this case, the thrust is 74 kilonewtons and fuel consumption is 0.027 kilograms per newton hour.

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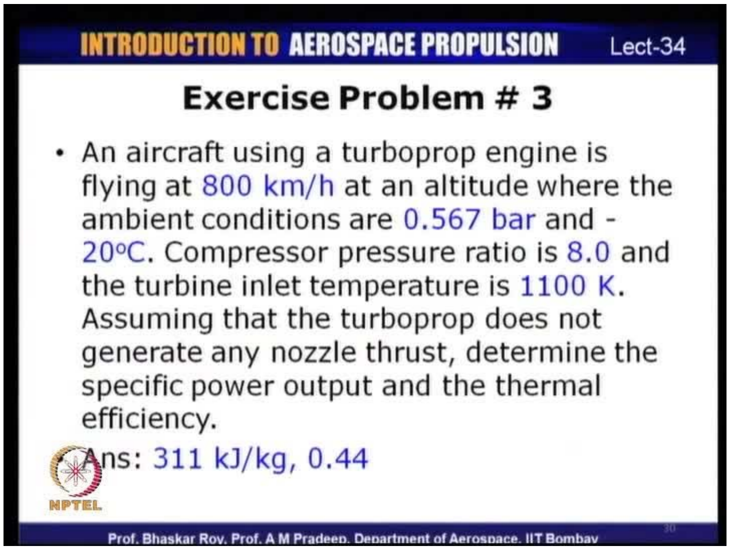

The third exercise problem is of a turboprop engine an aircraft using a turboprop engine is flying at 800 kilometers per hour at an altitude where ambient conditions are 0.567 bar and 20 degree Celsius, compressor pressure ratio is 8, turbine inlet temperature is 1100 kelvin. Assuming that turboprop does not generate any nozzle thrust, determine the specific power output and thermal efficiency. Here, the power output comes out to be 311 kilojoules per kilogram, thermal efficiency is 0.44.

So, these are three exercise problems for you based on cycle analysis exactly the same way we had discussed in today's tutorial session on 3 different cases the turbojet engine, turbofan engine and a turboprop engine. In the second problem of course, it is a mixed turbofan, so there is only one nozzle which generates thrust. You have to calculate nozzle entry properties determined based on an enthalpy balance between the 2 streams.

So, I hope you have been able to at least understand the basic procedure involved in the ideal cycle analysis of jet engines. In today's lecture, we had discussions on ideal cycle analysis of a turbojet a twin spool turbofan engine and a turbo shaft engine.

So, this brings us to the end of today's lecture which was basically a tutorial session on ideal cycle analysis of jet engines.