

## **Introduction to Aerospace Propulsion**

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**Module No. # 01**

**Lecture No. # 30**

### **Propeller aerodynamic theories - II**

We have been doing propeller theories, we did some propeller fundamentals and we try to understand how the propeller actually creates thrust. In the process of creating thrust, the propellers need to be supplied with power, for which you need an engine, which could be a piston engine or a gas turbine engine or any other engine, any other device that produces power. Propellers' main job is to make use of that power and create thrust for flying an aircraft.

Typically, the propellers are made up of three or more number of blades, number of blades could be even two. These blades are made up of airfoil sections, which put together, create the blade shape; these blades when in rotation, in a predetermined manner, helps in producing thrust.

In the last class, we did a theory of propellers, in which we looked at propellers as an actuator disk. That means physically and mathematically the propeller was replaced by an actuator disk. This actuator disk essentially is modeled, to be a thrust creator or energizer, which then produces the thrust; this is what we did in the last class. We have been introduced to the fact that propellers are made up of airfoil sections; we had a look at one or two of these airfoil sections.

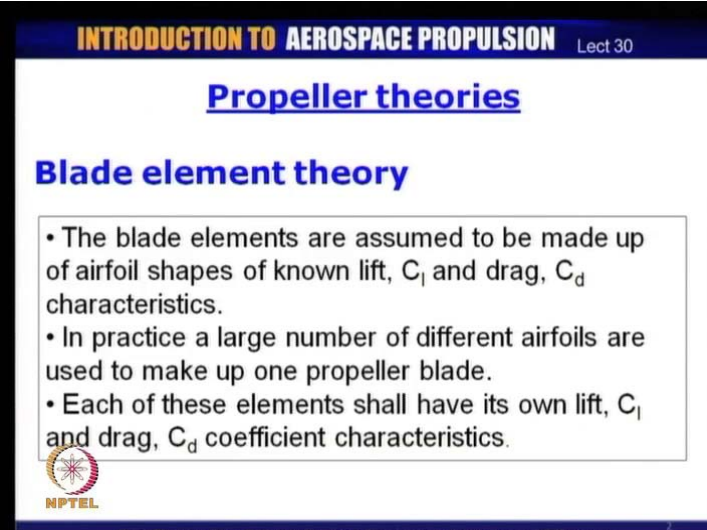
Today, we shall look at a propeller theory, which again makes use of the propeller airfoil sections, the proper propeller blade shapes, how this theory then uses the airfoil sections to actually create thrust, how this thrusts is determined with a help of simple mathematical formulations. As a result of which, the designer can make use of this theory to design a propeller and then create a model for prediction of the propeller

performance. We shall see how the prediction of the propeller performance is actually created, which are called propeller characteristics. These characteristics essentially, as the name suggest, characterizes the propeller, this particular propeller which has been designed.

Every propeller that is designed needs to be immediately accompanied with propeller characteristics, which determine the entire propeller operation and its capability. Only then the aircraft designer can make use of the propeller, because he has to match the aircraft characteristics with the propeller characteristics. On the other hand, matching the propeller with the engine requires that the propeller characteristics are matched with the engine characteristics.

Propeller as we see, is interface again between the engine and the aircraft, its characteristics must match with the aircraft characteristics, its characteristics must match with the engine characteristics, only then we have a propeller that can fly an aircraft with the help of the engine providing the power. Let us take a look at the propeller theory that uses the propeller blade shape, as it is actually is and the airfoil sections which actually provides the conversion of lift and drag, to thrust and matching of the torque that is provided by the engine.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

**Propeller theories**

**Blade element theory**

- The blade elements are assumed to be made up of airfoil shapes of known lift,  $C_l$  and drag,  $C_d$  characteristics.
- In practice a large number of different airfoils are used to make up one propeller blade.
- Each of these elements shall have its own lift,  $C_l$  and drag,  $C_d$  coefficient characteristics.

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The theory that uses the propeller blade shape is simply known as blade element theory. There are various versions of the blade element theory; we will be doing the elementary

version. There are more advanced versions of blade element theory, which are beyond the scope of this course, we shall not be doing them. But, the fundamental blade element theory that we will be doing provide sufficient backdrop, for understanding of how the propeller blades are designed, how the propeller blades finally go on to create thrust, how the airfoil shapes with characteristic values of  $C_l$  and  $C_d$  actually help in creation of thrust. The blade elements, which create the propeller blade shape, are assumed to be made up of airfoil shapes.

Now, these airfoil shapes can actually vary from root to the tip of the blade. So, along the length of the blade, the airfoil shapes can vary quite a lot really. As a result of which, their  $C_l$  and  $C_d$  characteristics would also vary substantially from the root of the blade to the tip of the blade.

Now, this is something which you need to understand very quickly, because when the propeller is in a rotation, as we have seen before, in the simple velocity triangle that you can construct on a blade section, the incident velocity on a blade section would vary from root to tip. One of the reasons is that near the root, your rotational speed is rather low; near the tip, the rotational speed is quite high. As a result of which, the incident velocity at the root is likely to be low, the incident velocity at the tip is likely to be quite high, the difference is quite substantial.

Near the roots, you could have velocities, which we could say are low subsonic velocities. The airfoil sections - therefore you should be using, they are essentially meant for low subsonic applications. On the other hand, near the tips, the incident velocity, which we called  $v_r$ , would be quite high, they could be high subsonic. In the more modern airfoils, they are in fact going almost transonic. Some of these would then correspondingly require airfoils, which are either high subsonic airfoils or in the modern propellers transonic airfoils.

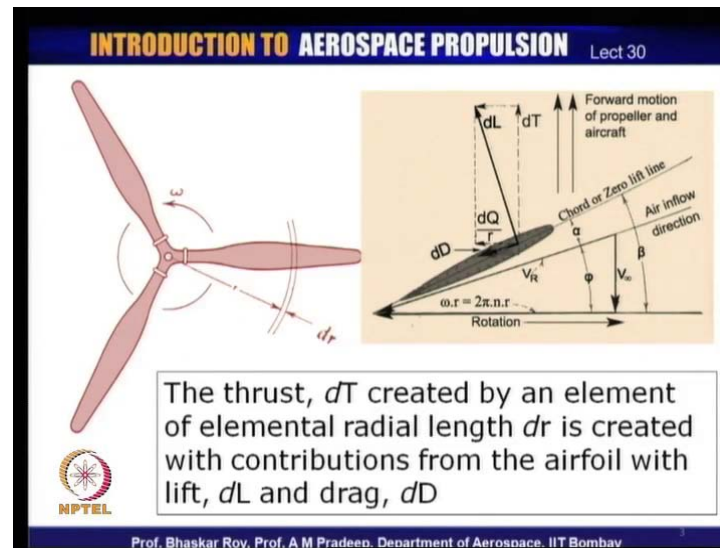
You see from the root to the tip of the blade, the airfoil sections would vary quite a lot from somewhat thicker airfoil sections, which are good for low subsonic applications, to thin airfoils, which are good for high subsonic or transonic applications. So, in the airfoils, you would be using all waves from the root to the tip of a single blade, vary substantially in its shape, in its camber and its basic characteristics, normally denoted in terms of  $C_l$  and  $C_d$ .

The other difference is these airfoils would also have different kind of incident angle range. The  $C_l$   $C_d$  that they use, actually, automatically would have the airfoil characteristics built into it, these airfoil characteristics would show the  $C_l$   $C_d$  applicable over a certain incidence range. Now, this incidence range also varies from low subsonic airfoil to high subsonic or transonic airfoils. Typically, the low subsonic airfoils would have a somewhat higher incidence range, of the order of 15, 16 degrees of incidence range of operation. On the other hand, near the tip, where the incidence range is going to be very low, because there are thin airfoils meant for high subsonic to transonic applications, those incidence ranges are often of the order of 4, 5, 6, 7 degrees. So that is the range that you have of variability of the incidence, which means your incident flow velocity  $v_r$  could change its direction by that much depending on the airfoil that is deployed at that particular section.

So, this is the difference that happens from root to tip of a single blade. This is what the designer has to be very careful about when choosing the airfoil sections, blending them together into one blade shape, because their actual sectional properties vary substantially.

A large number of different airfoils are used to make up one propeller blade, could be something like 10, 15, 20 different airfoil sections, with each having its own  $C_l$   $C_d$  and incidence range characteristics. All of them will have to be blended together into one single blade shape. Each of these elements they have their  $c_l$   $c_d$  characteristics, in addition, they have their - as I just mentioned their incidence range, which is built into the shape of the propeller that is being deployed over here.

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Now, let us look at the various flow geometry and the flow parameters that every blade section can be said to be dealing with. On the left side, we have a three bladed propeller, which is in rotation with an angular velocity  $\omega$ . If we take one blade section here, which has a radial depth of  $dr$ , this is the blade section, which we say is representative of the propeller.

Typically, a propeller is quite often represented by a blade section, which is somewhere near about around 60 to 75 percent of the blade. Most of the subsonic propellers would have their representative blade section at around 75 percent of the blade length; some of the transonic propellers may be even higher than that. So, it is somewhere between 50 to 75 percent that is what can be called a representative blade section. This representative blade section is the first section that is typically designed by the designer. The designer creates its own characteristics, which is the average characteristics of that particular propeller.

Every propeller is first designated with average sectional characteristics of sectional property that is required to create thrust. This average sectional property is built into the reference section of the blade, which is as I mentioned, normally someone near 75 percent of the blade length. This section is the first section that is often designed and then the rest of the sections are designed from there onwards, from root to the tip of the blade.

Let us take a look at this representative section over here, in which, you have a blade, which is expected to rotate with angular velocity  $\omega$ , as a result of which, it acquires a rotational speed of  $\omega r$ , which can be written in terms of  $2\pi n r$  being the rpm. This propeller along with the aircraft, which it is flying, is moving with a forward velocity, with which is of the order of  $V_\infty$ . So, we say that this is the velocity with which the air is coming and meeting the propeller, so that this is the relative forward velocity with which the air is meeting the propeller and this is the rotational velocity of the propeller section.

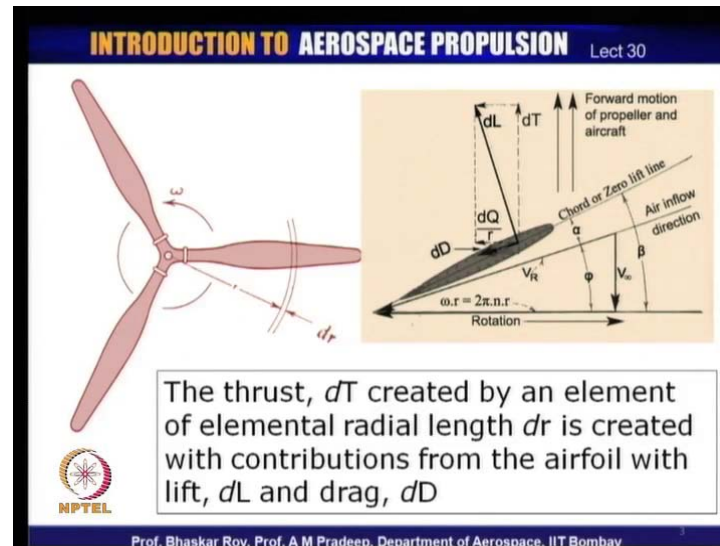
The two of them together make up this resultant velocity  $V_R$ , which is now said to be actually incident on the propeller at this particular section. So, this particular section now has to be aligned as close to this direction of the resultant velocity. Now, this resultant flow is at an angle  $\phi$  with respect to the peripheral direction or tangential direction of the propeller. Then, you need to set the blade, this particular blade section, which here is shown as a blade setting angle of  $\beta$ . If you set it at an angle  $\beta$  that means the chord or the 0 lift line of the particular blade section, is set at this angle  $\beta$  with respect to the rotational direction. Then, what you have is  $\beta - \phi$  is  $\alpha$ , which is then the angle of attack of this particular blade section. Now, this is the angle of attack or sometimes called angle of incidence, which characterizes this particular airfoils  $C_l$   $C_d$  characteristics.

One needs to be very careful, what is the angle at which this blade section is being set with reference to the angle at which the flow is expected to come into the propeller, so the angle of  $V_R$ , the angle at which the blade section is set need to be very close to each other. So that you have an angle of attack  $\alpha$ , which is within the operating range of this particular airfoil as characterized by its  $C_l$   $C_d$  characteristics. You have here the propeller blade section, being typically harnessed on the one hand using the  $C_l$   $C_d$  characteristics. On the other hand, aligning it to the actual propeller operation in terms of rotation and its forward velocity.

Now, let us say that the blade has been aligned, the blade has been set and the flow is coming into the propeller at an angle  $\alpha$  to the blade section, with a velocity  $V_r$ . Then, it creates a small elemental lift of  $dL$ , born out of the lift characteristics  $C_l$ , so this is the elemental lift that is created born out of  $C_l$ . Correspondingly, this section would

experience a small drag of  $dD$ , which is born out of its drag characteristics  $C_d$ . These are then the characteristic values of this particular blade element.

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Now, these are as per the definitions of lift and drag, are parallel and perpendicular to the flow direction, as a result of which, the lift is in this direction, which is perpendicular to the flow direction; drag is in this direction, which is parallel to the flow direction.

Now, if you decompose the elemental lift  $dL$ , the elemental drag  $dD$  into two other direction, which is the direction of motion of the propeller, the aircraft together and perpendicular to that you get two forces, the one which is in the direction of motion of the propeller and the aircraft together, gives you the thrust. So, the component of  $dL$  in the forward direction or what can be called the direction which is parallel to the axis of the propeller, would give you the thrust component, which we call  $dT$ , which is the elemental thrust of this particular element. So, as a result of which, you get a thrust now, born out of the lift component of the propeller.

Of course, here, you would get a very small negative thrust in the actual direction, so the net thrust would be composed out of positive contribution from the lift and a small negative contribution from the drag. On the other hand, if you take perpendicular to the thrust in the peripheral direction, the two components of the lift and drag is actually additive. They together create a force, now this force is what needs to be countered or matched by the torque that is applied by the engine. So, a torque, which is elemental

torque, which is shown here as  $dQ$  divided by  $r$ , gives you the peripheral force that is coming in from the engine on this particular section and that needs to be matched with the peripheral component of  $dL$  and  $dD$ . If they are matched, if the torque supplied by the engine matches with this torque requirement of the propeller, of this particular section, then we have a situation where we get the thrust that we want. So, the lift and the drag are the characteristic of the propeller, the torque is what is supplied by the engine. If all these things are matched together properly, we get the elemental thrust  $dT$ , which is what you would require to fly aircraft, when all the elements are put together of a particular propeller.

The thrust that is created by an element, at let us say radial length  $dr$ , is created with the contributions from the airfoil, which are characterized by  $dL$  and  $dD$  of this particular element and the torque which is supplied by the engine. So, this is how the thrust is elementally created by any particular element of a particular propeller. All the elements put together and then create the total thrust of a blade, which is in rotation.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

Using the blade elemental lift and drag characteristics the working capacity of the blade element may be found as :

**Thrust produced,**

$$dT = dL \cdot \cos \phi - dD \cdot \sin \phi$$

$$= \frac{1}{2} \cdot \rho \cdot V_R^2 \cdot c \cdot dr \cdot (C_l \cos \phi - C_d \sin \phi)$$

**Torque to be supplied ,**

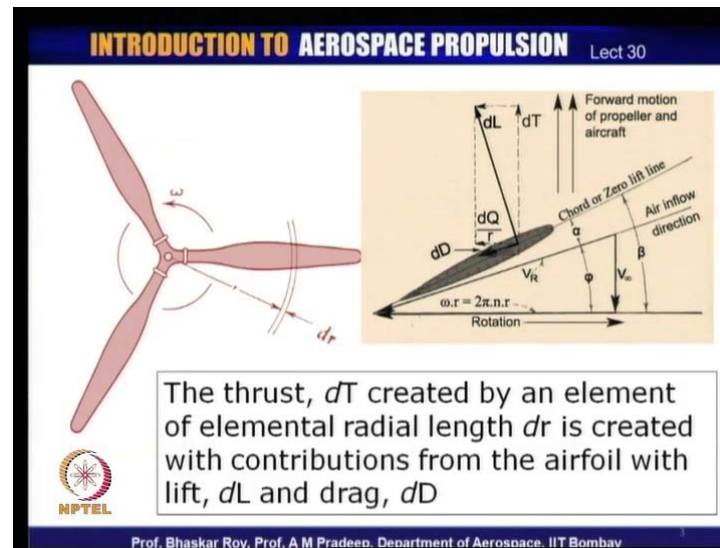
$$dQ = (dL \cdot \sin \phi + dD \cdot \cos \phi) \cdot r$$

$$= \frac{1}{2} \cdot \rho \cdot V_R^2 \cdot c \cdot dr \cdot (C_l \sin \phi + C_d \cos \phi)$$

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If you now look at this elemental lift and drag characteristics, which then create the working capacity of the propeller. We have a blade element, which is in work, which is in rotation - in rotation it is doing work, in the process of doing work, it is actually creating thrust. So, the thrust that is produced by this particular element can be now written down in terms of  $dT$ , which is shown in the diagram. They can be written down in terms of the elemental lift  $dL$ , elemental drag  $dD$ . As I have stated, the drag component actually gives you a slight negative component, as a result of which, you can now write down the entire thrust equation. The first part that is  $\frac{1}{2} \rho V R^2 c$ , actually is the dynamic head which is created.  $C_l$  is the characteristics of the propeller blade section,  $C_d$  also is a characteristic of the propeller blade section, which is under consideration,  $\phi$  is the angle at which the flow is coming into the blade,  $c$  is the chord of the particular blade section. As we have seen in the earlier diagram, the propeller blade section could vary from quite a lot - the actual value of the chord could vary quite a lot from the root to the tip of the blade.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

Using the blade elemental lift and drag characteristics the working capacity of the blade element may be found as :

**Thrust produced,**

$$dT = dL \cdot \cos \phi - dD \cdot \sin \phi$$

$$= \frac{1}{2} \cdot \rho \cdot V_R^2 \cdot c \cdot dr \cdot (C_l \cos \phi - C_d \sin \phi)$$

**Torque to be supplied ,**

$$dQ = (dL \cdot \sin \phi + dD \cdot \cos \phi) \cdot r$$

$$= \frac{1}{2} \cdot \rho \cdot V_R^2 \cdot c \cdot dr \cdot (C_l \cdot \sin \phi + C_d \cdot \cos \phi)$$

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Chord of the blade would vary from one section to another, so in this thrust creating equation, rho is the density of the air which is operation, which let us say is invariant from root to the tip of the blade. On the other hand, as we have seen from this diagram, V R would vary from root to the tip of the blade, c would vary from root to the tip of the blade, dr the elemental length could be same for each and every element that you take. The C l value would vary from root to the tip of the blade, depending on the airfoil section you are using; same with C d, its value would vary from root to the tip of the blade - of any particular blade. Then, the value of phi also would vary from root to the tip of the blade, depending on the rotational speed, which depends on r omega, angular velocity being constant, so phi would vary also from root to the tip of the blade.

In this thrust creating equation, as you can see, except for rho, all the other parameters actually are varying from root to the tip of the blade. Now, supposing, if you want to have the thrust created by each and every section of the same order, you would probably have to manipulate these values in terms of lifting capacity, the drag penalty, the resultant velocity that is coming in and the chord - dimension of the chord, to get the value of dT constant from root to the tip of the blade. Now, quite often that may not always be possible, so quite often the actual elemental thrust produced by each section could indeed vary from root to the tip of the blade.

Now, let us look at the torque that is to be supplied. Now, this is to be matched by the engine supply, so torque is required for the propeller to be operated, this is to be supplied by the engine. So, again using the same diagram, we can write down the torque equation here, in terms of the elemental lift and the elemental drag, in terms of the local flow angle phi. Again, if we write down in terms of the fundamental propeller blade characteristics, the airfoil characteristics, we can write down half rho V R square; c again is a chord, dr is the elemental length of the elemental blade, C l is again characteristics, phi is the flow angle and C d is a characteristic, implies the flow angle.

Here, we can see that the components from C l, or lift and drag are additive, they are on the same side of the axis, as a result of which, they are additive. They add up together to create a force and torque, which needs to be matched by the engine supply. This when multiplied by r gives you the torque. Now, this torque has to be supplied by the engine, without which the propeller would not operate at all. This is how you create thrust, this is how the torque is needed to be supplied by the engine, is matched by the propeller requirement. If this requirement is matched, if we have a proper airfoil deployed there with proper C l C d characteristics, then you have a thrust production that is required for flying an aircraft.

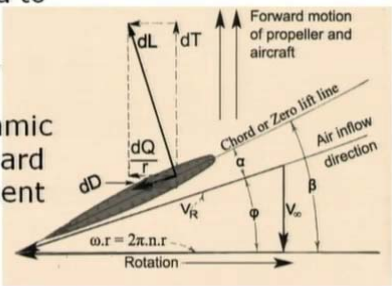
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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

Substituting for Resultant inflow velocity Incident and aligned to the blade element,  

$$V_R = V_\infty / \sin \phi$$
 and for Incoming flow Dynamic head based on forward velocity of the element  

$$q = \frac{1}{2} \rho V_\infty^2$$



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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

The **elemental thrust** is :

$$dT = \frac{q \cdot c \cdot dr}{\sin^2 \phi} (C_l \cos \phi - C_d \sin \phi)$$

and

The **elemental torque** is :

$$dQ = \frac{q \cdot c \cdot r \cdot dr}{\sin^2 \phi} (C_l \sin \phi + C_d \cos \phi)$$

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Now, if you proceed along those lines, if you say from the flow geometry that is we have created, the resultant inflow velocity is a line to the blade element. If we right down  $V_R$  as  $V_\infty$ , which is the forward velocity or the incoming axial velocity by  $\sin \phi$ , which is the angle which it subtends, the incoming flow dynamic head is based on the forward velocity of the particular element. If we make these two substitutions in the equations that we are written down for thrust and torque, from the flow geometry that is available over here, the elemental thrust can now be written down in terms of  $q$ , which is now the dynamic head;  $c$  is the chord,  $dr$  is the elemental length,  $\phi$  is the flow angle again in terms of  $C_l$  and  $C_d$ .

The thrust - elemental thrust can now be written down in a slightly different form, but using the same characteristic values of  $C_l$ ,  $C_d$  and flow angle correspondingly, the elemental torque can be written down again in terms of dynamic head  $q$ , the chord  $c$ ,  $r$  is the length of the element from the axis,  $dr$  is the elemental length of the particular element,  $\phi$  is the flow angle,  $C_l$ ,  $C_d$  are the characteristics of the airfoil being deployed at that particular section. The elemental thrust and the elemental torque can be now written down in terms of the characteristics of the airfoil, the geometry of the airfoil and the flow which is coming in at an angle  $\phi$ . So, these are the things that are built into the equations that create the elemental thrust and the elemental torque.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

Propeller **thrust** and **torque** are now computed by integrating from the root to the tip of the blade and for number of blades,  $B$

$$T = q.B \int_0^R \frac{c.dr}{\sin^2 \phi} (C_1 \cos \phi - C_d \sin \phi)$$
$$Q = q.B \int_0^R \frac{c.r.dr}{\sin^2 \phi} (C_1 \sin \phi + C_d \cos \phi)$$

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If we now try to put all together, as we have seen, the blade is a blended version of all the airfoils put together. When all the airfoils are put together, you get a total propeller thrust and the total propeller torque. So, the elemental thrust and torque we were talking about are now to be integrated from the root to the tip of the blade. Let us say, we have a number of blades, which is  $B$ , which as we have seen could be 2, 3, 4, 5, 6 or 8 that is typically the number of blades normally used these days. If you put them all together, you get the total thrust of the propeller and the total torque of the propeller that needs to be supplied by the engine.

These two are to be varied accurately, estimated only when the total torque is properly supplied, we get the thrust that is required to fly the aircraft. This is the thrust you would require to fly the aircraft in a predetermined manner, to do that you need to be supplied with that kind of a torque from the engine. The propeller power needs to be matched with the engine power, to be supplied by the engine. So these are the parameters that you required finally for the propeller to create thrust, based on the aircraft, on which it is mounted, based on the engine, to which it is attached.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

- Thus, the net thrust and the torque are seen to be directly proportional to the number of blades,  $B$  and the chord,  $c$ .
- *This is not quite true in practice*, as more is the number of blades and wider the blade chord - it shall result in more surface area, more flow blockage and higher consequent aerodynamic losses.
- The optimum number of blades need to be found separately and not from the blade element theory.




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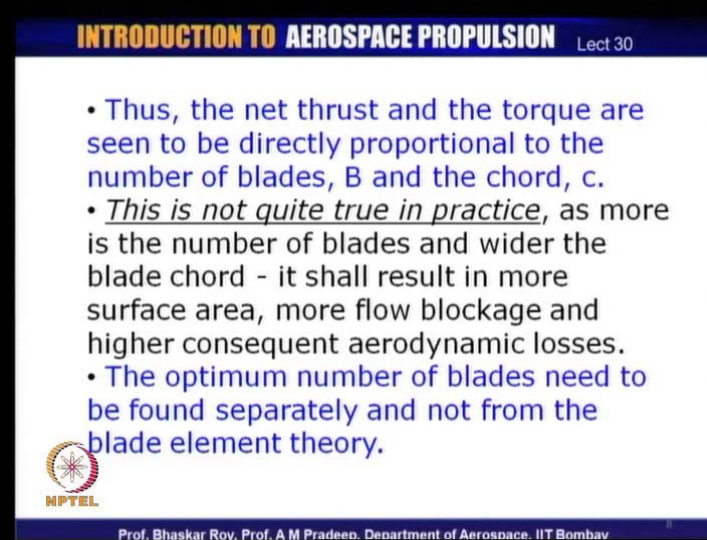
**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

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$$T = q \cdot B \int_0^R \frac{c \cdot dr}{\sin^2 \phi} (C_l \cos \phi - C_d \sin \phi)$$
$$Q = q \cdot B \int_0^R \frac{c \cdot r \cdot dr}{\sin^2 \phi} (C_l \sin \phi + C_d \cos \phi)$$


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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

- Thus, the net thrust and the torque are seen to be directly proportional to the number of blades,  $B$  and the chord,  $c$ .
- *This is not quite true in practice*, as more is the number of blades and wider the blade chord - it shall result in more surface area, more flow blockage and higher consequent aerodynamic losses.
- The optimum number of blades need to be found separately and not from the blade element theory.

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Hence, we see that the net thrust and the torque are seen to be directly proportional to the number of blades  $B$ , the chord  $c$  as given in the final thrust and torque relationship. Now that gives an impression that if you keep on increasing the number of blades, let us look at the equation quickly, you see here those thrust and torque are directly proportional to the number of blades  $B$ , the chord  $c$ ; both of them. Now that gives you an immediate impression that if you simply increase the number of blades, you get more thrust, you would require more torque or if you simply increase the size of the chord, let us say all the way from root to the tip of the blade, you would actually get more thrust and you would need to be supplied with more torque. So, if you are supplied with more torque, you would simply get more thrust. Now that is the impression one would get from the blade element theory, the torque and the thrust equation that we have put together, in practice that is not quite true.

What happens is that if you increase the number of blades or the size of the blades by increasing the chord, it shall result in more surface area of the blades. If you have more blades around, it will create more flow blockage, as a result of which, very high aerodynamic losses, so the efficiency of the propeller blades would start falling.

If we increase the number of blades and the size of the blades or the chord, the efficiency of the propeller would be affected, the propellers would create more and more blockage to the flow. The thrust created is directly proportional to the amount of mass flow that it



actually activates; so, by increasing the surface area, you are decreasing the efficiency of the propeller, by increasing the number of blades, you are increasing the blockage of the propeller. Two of them together actually reduce the aerodynamic efficiency of the propeller.

So, just by increasing number of blades or the size of the blade shapes, you would actually be reaching a situation where you would not get more thrust, there would be what can be called a point of diminishing return, at which you have to stop your increase of number of blades. That is how the number of blades is decided quite often that is how you see today, even today the number of blades is of the order of 3, 4, 6 or 8 and very really more than that. The reason is here if you just simply increase the number of blades, you are not going to get more thrust.

The Optimum number of blades that need to be decided, would need to be found separately, does not directly come from the blade element theory. So, blade element theory is not a correct indicator of the number blades that need to be deployed for reaching a certain value of thrust.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

The blade element efficiency,

$$\eta_{el} = \frac{\text{Thrust power produced}}{\text{Torque power supplied}}$$

In terms of elemental airfoil characteristics  $C_l$  and  $C_d$ , blade efficiency is :

$$\eta_{el} = \frac{v dT}{2\pi r dQ} = \frac{V}{2\pi r} \cdot \frac{C_l \cos \phi - C_d \sin \phi}{C_l \sin \phi + C_d \cos \phi} = \frac{C_l \cos \phi - C_d \sin \phi}{C_l \sin \phi + C_d \cos \phi} \cdot \tan \phi$$

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The next thing we need to talk about is the blade element efficiency. Every element is creating thrust, as we have seen, they use the airfoil characteristics  $C_l$  and  $C_d$ . As a result, fundamentally they are aerodynamic entities. Any aerodynamic entity would have a certain aerodynamic efficiency, so these blade elements would have basic elemental



efficiency, which when blended together would give us the total propeller efficiency. The elemental efficiency typically, as per known efficiency definition, is the thrust power that is produced by the particular element and the torque power that is supplied by the engine.

The elemental airfoil characteristics that we have seen before now can be made use of, if you do that the elemental efficiency comes out in the form of  $C_l$  and  $C_d$ . The elemental efficiency of any particular blade element of a propeller can be written down in terms of  $C_l$ ,  $C_d$  and  $\phi$ , which is the flow angle into that particular element. These three together finally give you the elemental efficiency of the propeller, which stands to a reason really, because three of them together actually decide the aerodynamic working capability of that particular element, which as you know is an airfoil section.

This airfoil section is operative in a particular situation, which is incident flow at an angle of  $\phi$ . This creates the local aerodynamic flow angle, which creates a lift and the drag. They together then finally decide what is the efficiency with which this element is going to perform? So, we can directly calculate the elemental efficiency from the airfoil characteristics and the local flow condition.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

Applying maxima condition it can be shown that maximum efficiency,  $\eta_{el-max}$  occurs at

$$\phi = \frac{\pi}{4} - \frac{C_d}{2.C_l}$$

for a blade element airfoil characterized by its  $C_d$  &  $C_l$

The estimations from blade element theory is within 10% of the actually obtained results.

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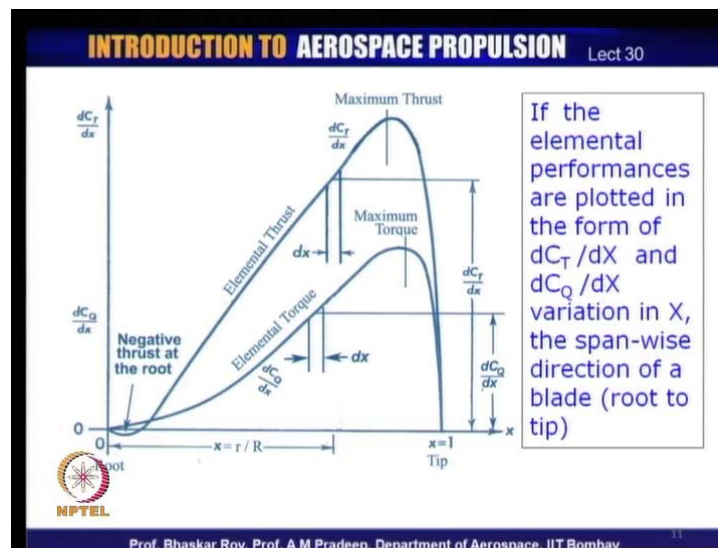
Now, what can be done is, you can find the maximum efficiency of this element from the earlier equation that we have written down. It can be shown by simple algebraic derivation that the elemental maximum efficiency occurs at this value,  $\phi$  by 4 minus  $C$

d by twice  $C_l$ . This is how you can quickly calculate what could possibly be the maximum elemental efficiency.

Now, you know, the elemental airfoil is by characterized by the  $C_l$  and  $C_d$ , hence it stands to reason that they together decide what the maximum efficiency is likely to be. It is generally found that all the things that we have been finding, the thrust, the torque, the efficiency and the maximum elemental efficiency, all this can be found with a reasonable engineering approximation, which normally gives us of the order of 10 percent approximation, which is a fair approximation to begin with, given the simplicity of the theory.

It allows us to design the propeller; it allows us to predict the propeller performance with reasonable approximate accuracy. If we if do that we can create a propeller, which then can be deployed in an aircraft flight. So, these are the some of the simple things that can be derived out of this elemental propeller efficiency, the thrust and the torque that we can calculate.

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If we can now put together, all of it into typical blade, every element is now created with a lifting characteristic, let us say that we get a value of a variation of lift, variation of thrust and torque in the direction from root to tip that is designated as  $x$  from root to the tip of the blade. If we can show that there is a gradient of thrust coefficient and torque

coefficient, which can be found, if they are plotted again from root to tip, we would get a characteristic curve like this for any particular propeller.

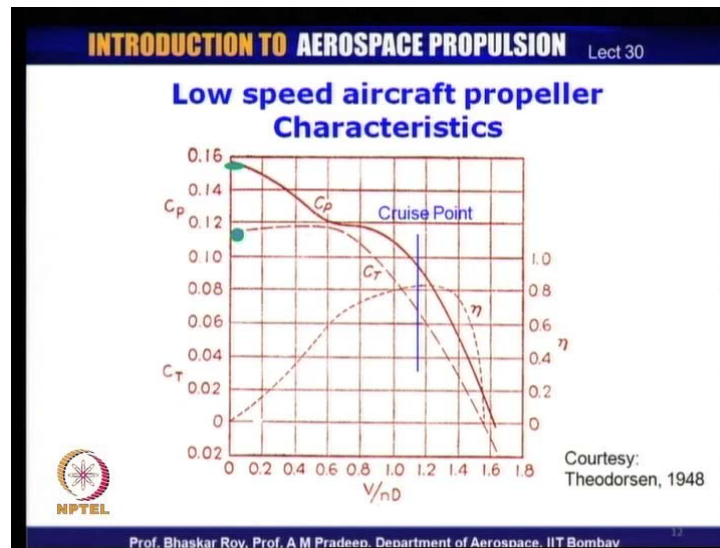
The elemental thrust variation would look like this; you would probably get a maximum thrust somewhere over here. The elemental torque characteristics would look something like this, you would get a maximum torque somewhere over here. What you can see here is that the maximum thrust and the maximum torque may not occur at the same element.

The same element may not be giving you maximum thrust, just because maximum torque is being supplied. The thrust there could be little less than maximum and you could get maximum thrust not necessarily at the maximum torque, may be something little less than maximum torque.

As you can see, the maximum thrust and torque typically occur on the outer half of the propeller blade, the lower half actually is less contributory to the thrust and they also consume less of torque. Towards the root of the propeller, you could see that it could actually be creating negative thrust, because that is the portion which is often not properly aerodynamically shaped. They may not have very good airfoil shapes over there, in fact they may not have airfoil shapes there at all, because that portion is structurally strengthened and hence they may be creating actually negative thrust around the root area.

Most of the thrust need to be created on the outer half of the propeller, however near the tip of the propeller, again as we can see, the torque and the thrust dips very fast. So, very near that tip of the propeller, the tip flow actually highly influences the propeller elemental behavior there. You are unlikely to get much thrust contribution from the tip area, so one needs to be very careful in designing the propeller, in which, the thrust needs to be distributed in a manner. So that together the blended propeller would give the necessary thrust that is required for flying the aircraft. So, this is the variation that you need to build into a propeller shape, to get a propeller that is useful for flying an aircraft.

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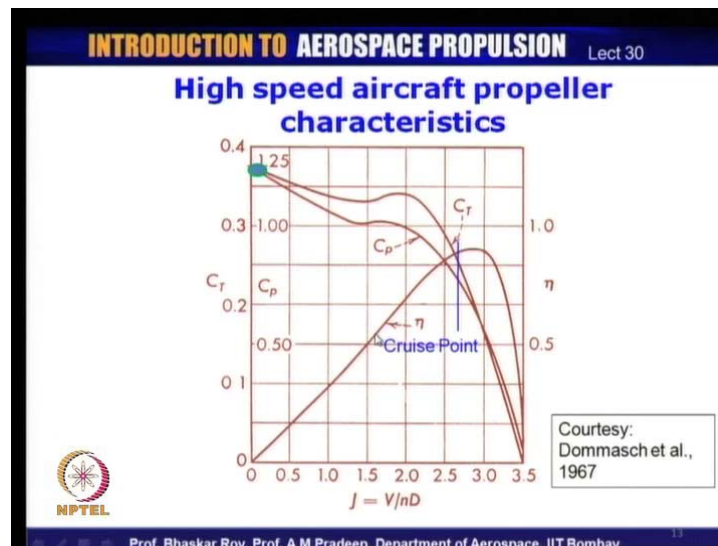


A typical low speed aircraft propeller characteristic is being shown here, which is shown in terms of  $C_T$ ,  $C_P$ . The  $C_T$  as we have seen, is the thrust coefficient,  $C_P$  is the power coefficient. As we can see here, the efficiency of the propeller is being shown over here, which shows that the efficiency of the propeller maximizes somewhere here at one point. This is plotted with reference to advance ratio  $V$  by  $nD$ , this shows that the efficiency of the propeller could be maximum at one point, whereas the thrust and the power could be maximum at some other point of advance ratio.

Now, advance ratio as you know, would vary with the forward speed of the propeller, which is the flying speed of the aircraft and the rotational speed. The ratio of the two gives the advance ratio. Typically, higher the aircraft flying speed, higher would be the advance ratio. Now, this is a low speed aircraft, so advance ratio values we are looking at is somewhat on the lower side. So, typically that is the variation one would probably get from a low speed aircraft propeller. First thing we can see here is that the maximum efficiency operation, at the cruise or the longest deployment of the propeller, which is during cruise, is somewhere near the maximum efficiency. That is not the point where you use maximum thrust or maximum power, you would probably use less than the maximum power, you would probably use less than the maximum thrust coefficient and still creating sufficient thrust to fly the aircraft.

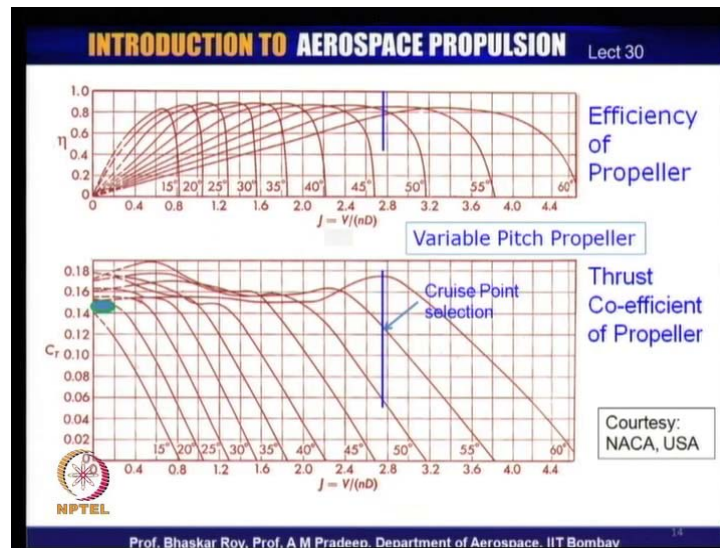
On the other hand, when you are taking off, you would be somewhere near zero of advance ratio. You would be using maximum - near about maximum power to create near about maximum thrust. You would probably need to create a good thrust during the climb operation of the aircraft, so you would need to create good thrust. Then, during cruise, you come down to lower thrust and lower power requirements, where you have very high efficiency of the propeller. So, good propeller efficiency is often obtained near the cruise, whereas as you can see here, during the takeoff, the propeller efficiency is indeed quite low actually, but you operate for a very short period to create high thrust using high power.

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We can have a quick look at the high speed aircraft, where again the cruise is somewhere near the maximum propeller efficiency, to get the best fuel efficiency of the power plant. It does not use the maximum power or maximum thrust coefficient, which are typically high - the highest near the takeoff. That is where you need to create maximum thrust, for the takeoff and the climb operation. This is where you can see the advance ratios are little on the higher side, compared to the earlier one. The advance ratio here shown is of the order of 3.5 maximum, you are probably operating at advance ratio somewhere near 2.6. This is a comparatively high speed aircraft, on which a propeller has been deployed for providing thrust to the aircraft.

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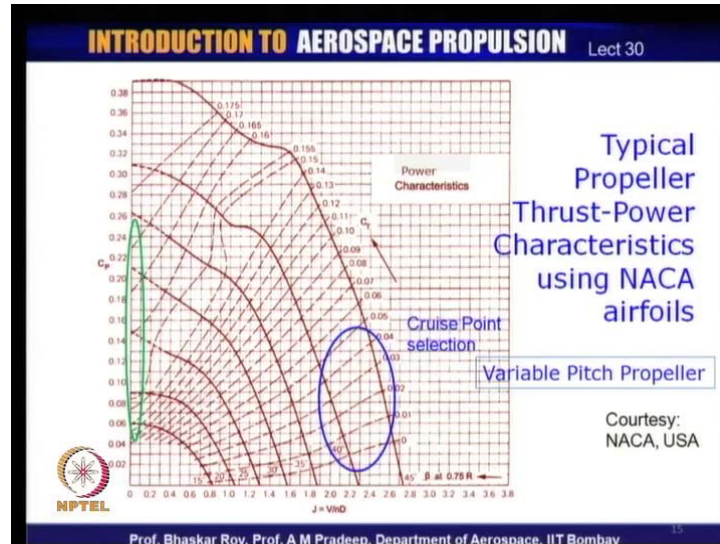
Now, this is what you would probably get of propeller characteristics, which is a variable pitch propeller. Now, you see, we have discussed the fact that propellers can be fix pitch or variable pitch; most of the modern propellers used in most of the aircraft today, are variable pitch propellers.

Now, these variable pitch propellers then would need to have their thrust versus advance ratio, efficiency versus advance ratio characterized with different pitch angles. This is the blade angle that is beta that we have seen before. Each value of beta then would create one characteristic graph like this. So, if you have a variable pitch propeller, one of the job of the propeller designer is to create a variable pitch propeller characteristics, within which then the aircraft would need to be flown, the engine would need to be operated, so that all the time the propeller is matched on one hand with the engine, on the other hand with the aircraft.

Now, what we see here is a probable a cruise point selection, which is where you are likely to operate at a comparatively high advance ratio. You are working at a high pitch angle. Now, during the cruise, your pitch angle is likely to be higher order. During the takeoff, your pitch angle of the propeller would likely to be rather low or as it is often called they are finally said, whereas during cruise, it is going to be a core setting of the propeller. So, during the cruise, the efficiency of the propeller can be used for a high pitch angle, you would get a good efficiency there, during the cruise operation of the

aircraft. So, this is the kind of variable pitch propeller characteristics that you would get of thrust coefficient and efficiency.

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We can get similar characteristics for the power coefficient, this is a thrust power characteristic, so you have the thrust variation here on the y axis and you have the power variation. What you would see here is a thrust power characteristics, the cruise point is likely to be somewhere over here, where you would use modest amount of power and create modest amount of thrust sufficient for the aircraft to fly. On the other hand, near the takeoff, you would need to create more thrust; hence you would probably use more power. As a result of which, you would be operating somewhere here, which is near 0 advance ratio, whereas over here, you are operating at high advance ratio during high forward flight speed of the aircraft.

So, these are the typical characteristics that characterize the propeller. Every propeller once designed and created would need to have a characteristic plots like these, for the engine designer and the aircraft designer to match to. This is absolutely necessary for matching the propeller with the aircraft and the engine.



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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

$C_s$ , the speed power coefficient, defined by,  
$$C_s = (\rho \cdot V^5 / P \cdot n^2)^{1/5}$$

Is often used for design / selection of propeller

If coeff of power,  $C_p$  as a function of  $J$ , is known,  $C_s$  can be obtained from  
$$C_s = J / C_p^{1/5}$$

The usefulness of  $C_s$  is in the process of defining it -- diameter was eliminated. Thus the propeller design or selection related flow parameters may be estimated even before the propeller size is fixed.

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There is a fourth characteristic which is often used. The first three being the thrust coefficient, the power coefficient and the efficiency, a fourth characteristic which is often used for propeller design or selection, is simply called speed power coefficient. This is defined as  $C_s$ , is defined as  $\rho V$  to the power 5 divided by power  $P$  into  $n$  square,  $n$  being the rotational speed, all of it together power of 1 by 5 that is 0.2. This is often used for designer selection of propeller.

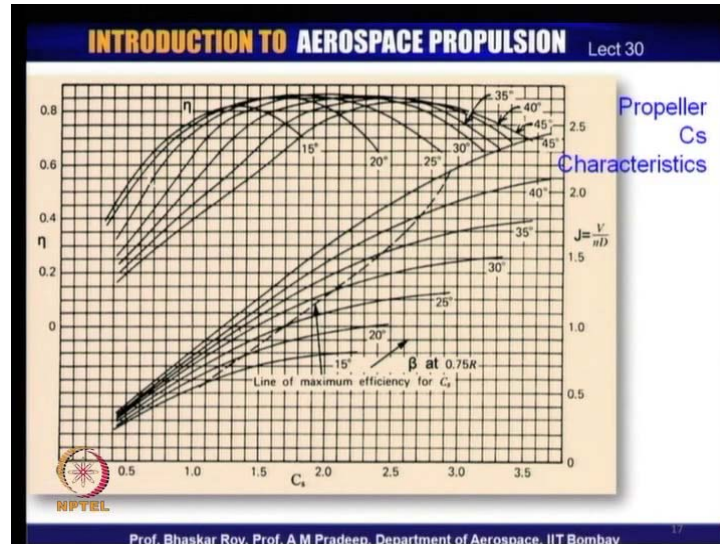
Now, this speed power coefficient can be related to the power coefficient, simply with the advance ratio  $J$ , is related as  $C_s$  equal to  $J$  by  $C_p$  to the power 1 by 5. Now, why this speed power coefficient has been created? This is used only by the designers and the propeller selectors; this is because, in the process of creating this definition, the diameter of the propeller has been eliminated. As a result of which, you have a parameter - normalized parameter, in which the size of the propeller has been taken out of the equation, as a result if which, you can create a propeller characteristics conceptually, without to begin with a priori bothering about the size of the propeller, which can be then factored little later in the design process.

So, this speed power coefficient allows you to create a conceptual propeller without fixing the size right away. The fixing of the size can be slightly deferred to a later date, so that you a have propeller conceptually, already created and its characteristics is



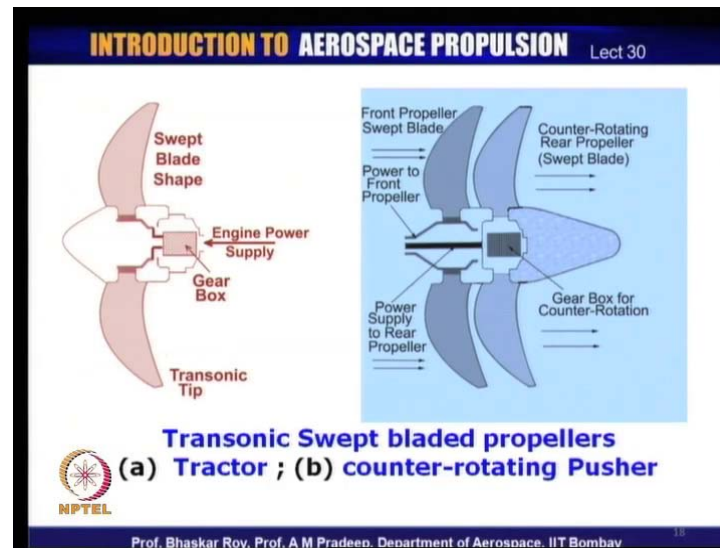
already created. This is the advantage of the speed power coefficient, which is defined in many of the literature.

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This is a typical speed power coefficient characteristic of a propeller, again a variable pitch propeller. This shows that under various pitching angle operation, the speed power characteristics would vary. This is the advance ratio, this is the speed power characteristics on the x axis and it is plotted against advance ratio J, the efficiency eta. These are the efficiency curves; these are the advance ratio versus  $C_s$  curves. These are plotted at various pitch angles, so we are characterizing typically again a variable pitch propeller. So, this is the kind of characteristic plot, which helps a selection or design of a propeller for a particular operation, in which thrust needs to be created and the power needs to be supplied by the engine.

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As a result of this, we get propellers, which can be matched to aircraft and engine. Some of the modern propellers, we can take a quick at - have shapes, which have sweeps. These swept propellers are being used in the modern propellers, which have gone transonic. The airfoil shapes of these are of transonic airfoil shapes, some of the new propellers that are coming up are counter rotating; that means, you have two propellers one behind the other, the second one is rotating in the opposite direction to the front one; so these are called counter rotating propeller.

The propellers is fundamentally of two types; one is known as a tractor type, in which, the propeller is deployed somewhere at the front of the aircraft and it is pulling the aircraft that is why it is called tractor type. Sometimes, they are deployed in the rear of the aircraft, quite often they are referred to as pusher type, as if they are pushing the aircraft from behind. The tractor type is what is a deployed somewhere in the front of the aircraft, the pusher type is deployed somewhere near the rear of the aircraft. So, these are the various kinds of propellers, which are typically used in aircraft.

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**INTRODUCTION TO AEROSPACE PROPULSION** Lect 30

In an aircraft application:


Propeller Power,  $P_{prop} = P_{Engine} \cdot \eta_{shaft} \cdot \eta_{prop}$

Propeller Torque,  $Q_{prop} = Q_{engine}$

Typically,

at Take off,  
 $Q_{prop}$  is low,  $\beta$  is low,  $P_E$  is High, rpm is high

at Cruise ,  
 $Q_{prop}$  is high,  $\beta$  is high,  $P_E$  is low, rpm is low

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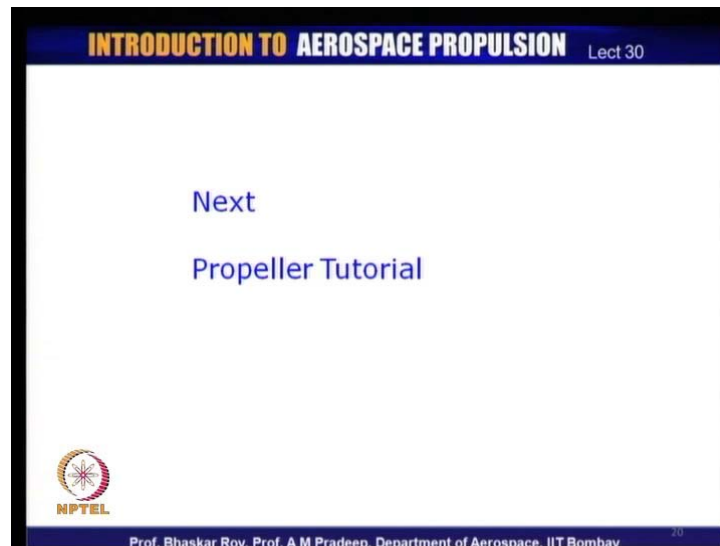
In typical aircraft application, we can say that the propeller power will be equal to the engine power multiplied by the shaft efficiency, multiplied by the propeller efficiency. So, we have to keep an eye on the propeller efficiency, which is a composite of all the elemental efficiencies.

The propeller torque will have to be matched exactly with the engine torque that is being supplied with the engine. Till you do that you are not going to get the proper thrust that is required. Typically, during takeoff, the torque requirement is low, the blade setting angle, or the pitch angle is low, the power required is high and the rpm is high. Because, during takeoff, you require very high thrust, normally you require very high thrust. During cruise, on the other hand, the torque requirement is very high that is because the blade pitch setting angle is very high, as a result of which, the torque requirement is very high. But, on the other hand, the power requirement is rather low; the rpm is also rather low compared to the takeoff rpm.

So, during the cruise, during the takeoff, the propellers operate at quite different operating conditions to create thrust; one for takeoff, which is very high, one for cruise, where the thrust requirement is indeed rather low, but the torque requirement is rather on the higher side. So, these are at least two different operating conditions, which a propeller has to cater, to ensure that the aircraft flies properly. So, these are some of the fundamental issues that comes out of propeller theories, you need to ensure that you have

a propeller which supplies sufficient thrust during takeoff, meeting all these requirements. You have a propeller, which supplies thrust during takeoff, again meeting all the requirements that are shown over here. So, only then you have a propeller, which is worthy of putting on an aircraft for flying the aircraft.

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In the next class, we will be trying to make use of all these propeller theories that we have done, the propeller fundamental definition that we have used, the C P and the C T. When you use all these fundamental theories, it should be possible for us to solve some very simple problems, so that you get a feel of the numbers. I will be bringing along a problem, I will solve a problem for you, may be a variable pitch problem, so that you get an idea of what happens when these theories are used to solve real life problems. So that is what we will be doing in the next class, trying to solve problems, making use of the theories that we have done over the last three lectures.