

Introduction to Aerospace Propulsion

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Module No. # 01

Lecture No. # 29

Propeller aerodynamic theories - I

We are talking about propellers. In the last class, we deal with various fundamental issues related to the propellers, how the propellers look, how they are made, how they actually function and to some extent, how they create thrust.

Propellers as we have seen are made up of a number of blades and these blades have sections, which are of airfoil sections. We stack airfoils to make 1 blade and then we decide to have a number of blades in a propeller; number of blades as we have seen could be 2 or 3 or 4 or even more sometimes 6 or 8 blades. When all these blades put together, rotate, they create thrust. Now, the thrust creation is based on - also we had a quick look - on the fundamental issue that blades are made up of airfoils and these airfoils intrinsically have the capability to create lift and drag. Now, this lift and drag creating capability of the fundamental airfoil sections are harnessed towards creating thrust in a composite manner.

All the airfoils in a composite manner contribute towards finally, creating thrust. Now, this is an aerodynamic principle. It is aerodynamics of the blade shape or the airfoil shape which is finally utilized for creating thrust and this is something which we had a look at in the last class.

Today, we shall have a look at some of the propeller theories, we will start off with one of the theories today and we will do the other theory in the next class. We will try to see how the propeller theories actually try to create various propeller fundamental performance parameters, because these fundamental performance parameters are what the propellers would be characterized with.

Propeller being a thrust making body, being fundamentally a propulsor, its various characteristics or the performance characteristics or the figures of merit need to be clearly defined. When a performance is going on, its actual performance numbers are to be written down or clearly quantified. How you quantify these parameters is what we shall be talking about today through one of the propeller theories, which has been in existence for little more than 100 years.

Now, the propellers we have seen are actually made up of blades which again, are made up of airfoils. The first theory that we will look at today is actually using a propeller, which is not made up of airfoils, which does not have blades and the whole propeller is mathematically modeled into another actuator disk, which **per pets** to actually model of active propeller and gives us a reasonable notion of how this propeller is going to function under various operating conditions.

The second theory that we will do in the next class of course, takes into account the actual propeller blade shapes, so that a more exact quantification of the various performance parameters can be actually done with the help of that theory.

Both the theories have been in existence for quite some time. Most of the propeller designers, propeller performance analysts, they use both the theories, because the first one which we will do today is actually a simpler theory. It gives you a very quick estimation of what the propeller performance could be. It also has a number of other advantages and of course, a few simplifications, limitations which we will talk about today.

(Refer Slide Time: 04:58)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Propeller Theories

- 1) Momentum Theory
- 2) Blade Element Theory

- The first one does not use the blade shape for design or analysis at all.
- The second set of theories use the propeller blade shapes made up of stacked airfoils.
- Both the theories are used, to design and predict the propeller performance, using the fundamental parameters defined earlier.

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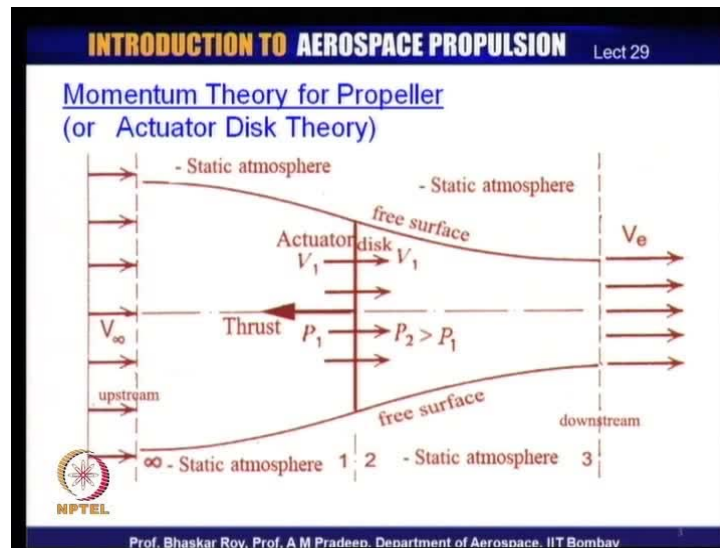
Some of these things together make up for what is called propeller theories and that is what we will be starting today. The two theories that we will be discussing are called momentum theory and the second one the blade element theory.

The momentum theory has been in existence as I mentioned for quite a long time certainly more than 100 years and this is the theory that does not use the blade shape. It replaces the propeller blade with an actuator disk but, it has been used by the designers and the analysts for little more than 100 years and it is still a very handy and useful basic theory for analyzing propeller performance.

The second theory, which actually is a set of theories, we will be doing the simple version of the theory use the propeller blade shapes, which are as we know, made up of airfoils.

Quite often, the analysts use both the theories in conjunction with each other. The first one being the simpler version is used very quickly to estimate the basic propeller functioning. The second one gives a little more detail about the airfoils which are being used. As a result of which both the theories have been in existence and in use by the propeller designers and the propeller performance analysts. So, let us take a look at the momentum theory today in some detail.

(Refer Slide Time: 06:35)



The momentum theory what it does is, it replaces the propeller with an actuator disk in a free stream flow and in a technical power lens it is often referred to as actuator disk. Hence, this theory is often also referred to as actuator disk theory. So, in many of the books or literature, you may find it mentioned as actuator disk theory.

What this theory does is it models the entire propeller flow. First of all, it replaces the actual propeller with this actuator disk, which is working in a free environment. You have static atmosphere all around and then you have this virtual domain which is been created by the flow of air into the propeller, which is actuator disk. This domain is now assumed to have a free surface like this, so it moves into the actuator disk and then goes out of the actuator disk in a shape something like this (Refer Slide Time: 07:41).

It is a kind of circular shape all around the particular domain that we are considering. So, the front domain is the inlet domain and the rear domain is the exit domain from the actuator disk and the flow comes from far upstream distance; let us say the velocity far upstream of the actuator disk is V_∞ .

The exit velocity for downstream of this entire domain is V_e that is the exit velocity. So, it comes in with a velocity V_∞ taken at for upstream of the actuator disk and for downstream of the actuator disk you have exit velocity let us say V_e . This entire domain has this free surface, so it is bounded by this free surface which is actually as you know

air, so the air is flowing inside this free surface domain and the air, which is static atmosphere outside this free surface domain.

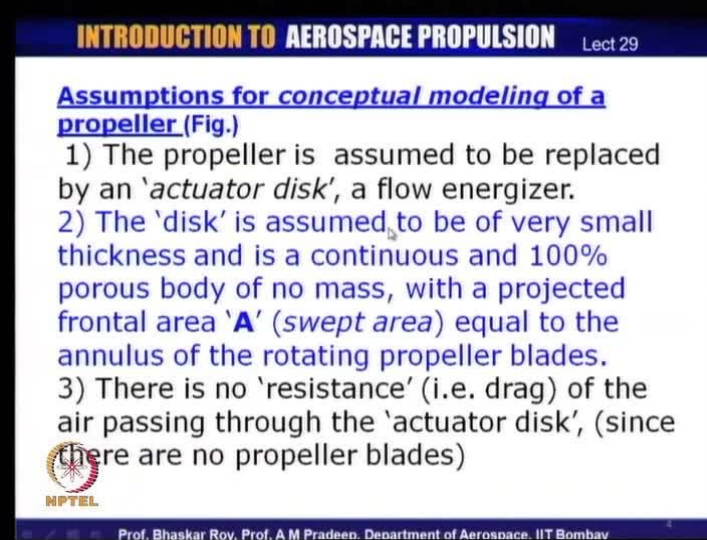
Now, as the flow is kind of drawn into the actuator disk now a propeller is in reality is a suction mechanism. It draws in air from all sides and that is exactly what is actually shown here, in this domain definition; that the flow is being drawn in from here and its area is actually decreasing as it is approaching the actuator disk (Refer Slide Time: 09:21).

Then of course, it gets thrown backwards by the propeller and it goes out finally, large distance downstream with a velocity V_e . The shape of the free surface is what is generally known to happen that the air gets drawn into the actuator disk, the area decreases, it continuous to decrease a little and then again it becomes more or less uniform as it goes to large distance downstream of the actuator disk.

Now, as the flow moves into the actuator disk, it acquires a velocity V_1 . Now, V_1 is the velocity at the actuator disk which you change from V_∞ , which is what it started off with large distance upstream of the actuator disk and then it goes to the actuator disk with a velocity V_1 . So, the velocity through the actuator disk or the axial velocity more specifically actual velocity is V_1 through the actuator disk.

Across the actuator disk there is a change of pressure, it changes from P_1 to P_2 this is a jump change from P_1 to P_2 ; P_2 will have to be more than P_1 and the differential pressure on the two sides of the actuator disk actually creates the thrust.

(Refer Slide Time: 11:11)



INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Assumptions for conceptual modeling of a propeller (Fig.)

- 1) The propeller is assumed to be replaced by an '*actuator disk*', a flow energizer.
- 2) The '*disk*' is assumed to be of very small thickness and is a continuous and 100% porous body of no mass, with a projected frontal area '**A**' (*swept area*) equal to the annulus of the rotating propeller blades.
- 3) There is no '*resistance*' (i.e. drag) of the air passing through the '*actuator disk*', (since there are no propeller blades)

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Now, a little more about this actuator disk, let us see a few things that we can write down quite easily actually the propeller are replaced by an actuator disk. The most important thing is this disk is assumed to be of very small thickness and it is a continuous and 100 percent porous body of no mass. So that is what an assumed actuator disk is that is it is a disk of infinitesimally small thickness but, it is a continuous body and it is 100 percent porous body of no mass what so ever. It has a projected frontal area of A through which all the air passes and this A is also often called the swept area, which is normally equal to the area of the rotating propeller blades. The rotating propeller blade which is being replaced would have the same amount of swept area which this actuator disk would have.

(Refer Slide Time: 12:36)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Assumptions for modeling of a propeller

4) The axial velocity, V_1 through the 'disk' is uniform over the 'actuation' area and is considered to be smooth across the disk i.e. no abrupt changes are 'experienced'.

5) The received energy manifests itself in the working medium (i.e. air) finally in the form of differential pressure ($p_2 - p_1$), a jump change across the actuator disk, uniformly distributed across the disk surface.

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The other physical property of the actuator disk is that it has no resistance. That means, it is not offering any drag at all, so the porosity of the body is absolutely 100 percent and it offers no resistance or drag whatsoever to the air passing through this actuator disk. The axial velocity which it acquires as it approaches or comes near the disk is uniform over the entire actuator disk and is considered to be smooth across the disk. So, it does not experience any abrupt change or jump change or anything as it passes through the actuator disk. So, it simply passes through the actuator disk as if it does not exist and that is where the 100 percent porosity actually is manifested.

However, it is an actuating disk, so it is doing some work and this work or energy manifests itself in the working medium which of course, for our case is always air and it manifests itself in the form of differential pressure that is P_2 minus P_1 and this is a jump change across the actuator disk. In the mathematical model that we are putting forward, we are now saying that the flow as it passes through the actuator disk experiences a differential pressure of P_2 minus P_1 and this is a jump change and this is uniformly distributed across the entire disk surface; actuator disk surface.

(Refer Slide Time: 14:11)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Assumptions for modeling of a propeller

6) The fluid medium, air, is assumed to be a perfect incompressible fluid. Flow is assumed 'irrotational' in front of and behind the disk, but not through it.

and

7) The static pressures far from the disk, i.e. far upstream and far downstream, are both assumed equal to the atmospheric pressure. The corresponding velocities are independent values, to be determined separately.

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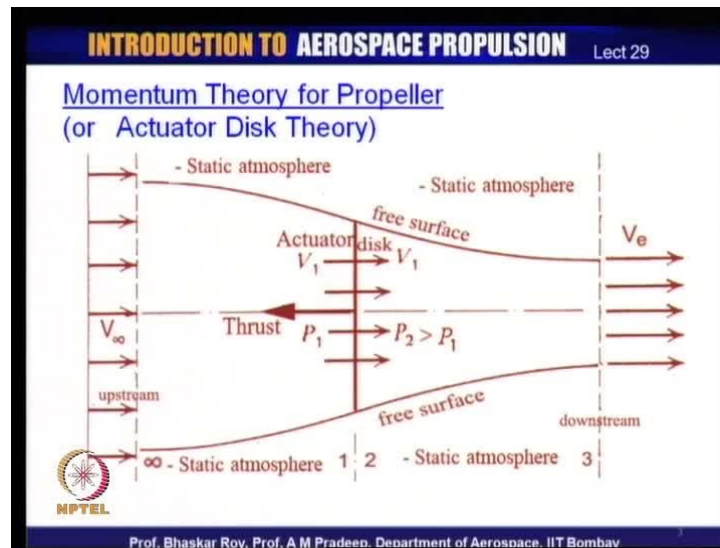
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The actuation that the actuator disk does involves essentially a jump change in the pressure across this disk. It does not involve any change in velocity it involves only change in pressure. The fluid medium which is being used here is assumed to be a perfect incompressible fluid to begin with that is a simple theory that we are doing we will take it as an incompressible fluid.

The flow is assumed irrotational in front of and behind the disk, irrotational in the fluid mechanics sense that you may have done before but, it is not assumed to be irrotational when it is passing through it, which means it may acquire certain amount of rotationality inside the disk. However, in front and behind the disk the flow is assumed to be irrotational.

The static pressures far from the disk that is for upstream and for downstream are both assumed equal to the atmospheric pressure let us say, P_A . On the other hand, the velocities are two independent velocities that is for upstream and for downstream they are different from each other. These two velocities are to be found out or determined separately for upstream velocity may be given for a particular problem and the downstream velocities is to be determined or estimated depending on how the actuator disk is modeled.

(Refer Slide Time: 15:38)



Let us go back to the diagram which we were looking at. We have now defined the entire mathematical or physical model that we have in front of us. We have an actuator disk which essentially replaces a propeller it has infinitesimal thickness, it is 100 percent porous and it has no mass what so ever, the velocity of the flow as it approaches the actuator disk acquires a value V_1 and remains V_1 through the disk and across the disk it remains V_1 .

However, the pressure across the disk jumps from P_1 to P_2 and this jump in pressure manifest itself in the form of creation of thrust and that is what of course, the fluid static tells us. This thrust creation is now entirely due to the change in pressure across the actuator disk. After the flow leaves the actuator disk it continues to flow downstream and acquires uniform flow for downstream with a velocity V_e , so this V_e would have to be - - as per the thrust creation principle that we have studied before: in any form of jet propulsion this V_e will have to be somewhat more than V_a or V_∞ , how much more depends on the propeller you using.

The general concept that we have used is that the change in velocity across a propeller is normally somewhat small compared to any other kind of jet propulsion. So, this velocity would be probably not very high and that would again give us another estimate of the thrust, which we shall see would have to be pretty close or equal to the thrust creation determined by the jump in pressure across the actuator disk.

This is the physical model basically, physics and mathematics model that has been created to replace an actual propeller and then allow us to make a very quick estimation of the propeller performance and acquire its performance parameters. We shall see later on that it has number of advantages which of course, can be made use of by the propeller designers or propeller analysts for very quick estimation of propellers performance. So, let us go ahead look at and develop this theory in its full form.

(Refer Slide Time: 18:24)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

The mass flow through the disk from continuity, is $\dot{m} = \rho \cdot A \cdot V$

The thrust produced by the disk from Newton's II and III laws (change in momentum in air) resulting in reaction force, **thrust**.

$$T = \dot{m} \cdot \Delta V = \rho \cdot A \cdot V \cdot (V_e - V_\infty)$$

From **simple fluid statics**, thrust is produced by the differential static pressure on either side of the disk, multiplied by its surface area (**swept area**)

$$T = A (P_2 - P_1)$$

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The mass flow through the disk from general continuity that we normally use can be written down as \dot{m} that would be equal to $\rho A V$ and ρ being the density of air, A is the area of the actuation and V of course, is the velocity through which the flow is flowing at any given point in that domain.

Now, the thrust produced by the disk from Newton's principles or laws that we have done before, because of the change in momentum in air would be available from the momentum change and that reaction which normally gives us the thrust would be in terms of $\rho A V$, which is the mass of air; A and V can be taken anywhere in that domain and the change across the entire domain from V_∞ to V_e and that gives us the thrust. Now, this is the momentum method of the estimating thrust.

On the other hand, the physics model that we have put forward now from the fluid statics the thrust can be estimated by simply taking the pressure differential across the two sides of the disk. When that is multiplied by the surface area activation area of the actuator

disk, which we also call the swept area we get another value of thrust which is $A(P_2 - P_1)$. The thrust can be indeed actually calculated by two different methods: one is the momentum method and other is the pressure differential method using the pressure differential across the disk.

(Refer Slide Time: 20:22)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

Applying Bernoulli's equation on either side of the disk, but not through it, gives
[note : Bernoulli's theory is not valid if any energy is added within the flow domain.]

$$P_\infty + \frac{1}{2} \rho V_\infty^2 = P_1 + \frac{1}{2} \rho V_1^2 \quad \text{--upstream}$$

$$P_2 + \frac{1}{2} \rho V_2^2 = P_\infty + \frac{1}{2} \rho V_e^2 \quad \text{--downstream}$$

Using, $V_1 = V_2 = \text{constant through the disk,}$

$$P_2 - P_1 = \frac{1}{2} \rho \cdot (V_e^2 - V_\infty^2)$$

From above equations $V_1 = \frac{1}{2} \cdot (V_e + V_\infty)$

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Now, what we can do is we can use the energy principles. We have assumed that the flow across the domain is essentially incompressible in nature. This incompressibility allows us to use the Bernoulli's equation; Bernoulli's theorem which all of you know very well. Here, we have to be a little careful the Bernoulli's equation needs to be used on either side of the disk separately but not through it, because the actuator disk is a working medium and it is energizing the flow. As a result of which Bernoulli's equation is not valid through the disk.

Typically, this is one thing you need to remember that you cannot use Bernoulli's equation and Bernoulli's theory if any energy is added within the flow domain. We have two domains: one upstream domain and one downstream domain. In the upstream domain you can use the Bernoulli's equation to equate the energy of the flow, which is coming in at the entry that is $P_\infty + \frac{1}{2} \rho V_\infty^2$ and that has to be equal to $P_1 + \frac{1}{2} \rho V_1^2$ that is in front of the disk. From upstream to in front of the disk, we can have equality of energy upstream of the actuator disk.

Now, downstream of the actuator disk as we have seen it acquires a pressure P_2 and then it has the kinetic energy $\frac{1}{2} \rho V_2^2$, V_2 we have seen actually equal to V_1 and for downstream we have $P_\infty + \frac{1}{2} \rho V_e^2$ and that is a downstream Bernoulli's energy equation, downstream of the actuator disk.

Since, we have already said that velocity through the actuator disk actually remains constant V_1 is equal to V_2 . As result of which we can now write down that the pressure differential $P_2 - P_1$ through the actuator disk is actually $\frac{1}{2} \rho V_e^2 - V_\infty^2$. As a result of this from the above equations, we can now say that V_1 which is the flow at the actuator disk is half of $V_e + V_\infty$, which is just the mean of the upstream and the downstream velocities. If you have the upstream velocity V_∞ and if you can estimate the downstream velocity V_e then you can find simply by averaging the two the velocity at the actuator disk.

Now, this is a very important and useful concept really coming from very simple formulae or simple mathematical simplifications or equations. This gives us that if the upstream velocity is measured let us say, a propeller is being tested in a propeller wind tunnel. The easiest thing to do is measure the velocity for upstream of the propeller in the wind tunnel and then measure the velocity for downstream of the propeller in that huge big wind tunnel. Average of these two actually gives us the velocity through the propeller. Now, this is a very important concept which comes from this momentum theory and allows even the experimentalists to find out what the velocity of the air is at the propeller which is of course, as you know extremely difficult to measure.

Measurement for upstream and downstream of the propeller very quickly gives us a quantified value of the velocity at the propeller. As I said, this is a very important concept used by the analysts and by the designers or even experimentalists can make use of this to find out the velocity at the propeller.

(Refer Slide Time: 24:49)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

- This simple analysis shows that the *air flow velocity through the actuator disk is the mean of the velocities far upstream and far downstream of the propeller.*
- This simple conclusion drawn out of the simplified flow model permits design, analysis, and even experimental verification of the propeller performance rather quickly. Thus, thrust $T = \frac{1}{2} \cdot \rho \cdot (V_e^2 - V_\infty^2) \cdot A$

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Now, this shows that the air velocity through the actuator disk is the mean of the velocities far upstream and far downstream of the propeller. This allows us to draw a simplified model and this tells us that the thrust now can be written down in terms of half rho V square minus V infinity square into A.

Now, this is a very simple way of putting the thrust, if the density of the air is known, if the far upstream velocity and far downstream velocities are known and the activation area or the propeller swept area is known, we get a very quick estimation of the thrust it can create. Needless to say this is a somewhat idealistic and theoretical estimation of the thrust but, nevertheless it gives us a very quick estimation of the thrust this propeller can possibly create and given the working condition as defined in these parameters, one can say that would be about the maximum thrust that this propeller would be able to create.

This is a very quick method by which the maximum possible thrust of a propeller can be estimated with the help of the most important and the fundamental parameters that are activating the propeller. The momentum theory does allow us to make very quick estimation of the thrust if some of the fundamental operating parameters are available to either by experimental verification or by analysis, which could involve CFD analysis or for design purposes, which the designer would then make use of those parameters for designing a propeller.

(Refer Slide Time: 26:49)


INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

The velocity at the disk comes out to be the free stream axial velocity, V_∞ plus induced (axial) velocity (v), whereas, the far downstream velocity is equal to the free stream velocity plus two times the induced velocity, v .

$$V_1 = V_\infty + v; \text{ and } V_e = V_\infty + 2.v$$

Therefore, $T = \rho A (V_\infty + v) 2.v = 2. \dot{m}.v$

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Now, as we see, the velocity at the disk comes out to be the free stream axial velocity plus induced velocity. Now, this induced velocity is what is the average of the two and as a result of which we can now write down that the thrust of this actuator disk, which is modeling a propeller is in terms of $\rho A (V_\infty + v) 2v$ and that would be equal to twice the mass flow into v . So that is a very simple neat way of writing what the thrust of a propeller would be which is being modeled by this actuator disk.

As you can see, it is born out of number of simplified assumptions which we made right in the beginning. It gives us as I said a very simple handy method by which the thrust of a propeller can be very quickly estimated.

(Refer Slide Time: 27:58)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29


Momentum Theory for Propeller

From the equation the *induced velocity*, v , can be found as,

$$v = \frac{[-V_\infty + \sqrt{V_\infty^2 - (2T / \rho A)}]}{2}$$

For a static thrust, where the propeller is not in forward motion (at take off), $V_\infty = \text{zero}$,

$$v = \sqrt{\frac{T}{2 \rho A}}$$

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(Refer Slide Time: 28:16)


INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

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Therefore, $T = \rho A (V_\infty + v) 2.v = 2. \dot{m}.v$

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(Refer Slide Time: 28:20)

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Momentum Theory for Propeller

From the equation the *induced velocity*, v , can be found as,

$$v = \frac{[-V_\infty + \sqrt{\{V_\infty^2 - (2T / \rho.A)\}}]}{2}$$

For a static thrust, where the propeller is not in forward motion (at take off), $V_\infty = \text{zero}$,

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Now, we need to know what the induced velocity is and this is of course, the velocity that has been created through the actuator disk. This can now be written down in terms of this relationship from the earlier relationships that we had developed and if you do a simple mathematical analysis you would find that the induced velocity comes out to be minus V_∞ plus root over V_∞^2 minus $2T$ divided by ρA (Refer Slide Time: 28:10). Now, this whole thing divided by 2. Now, this is of course, the induced velocity that one can see in front of an actuator disk.

Now, using this induced velocity if one can estimate this induced velocity with the help of those parameters if they are available. The static thrust where the propeller is not in forward motion; for example, during takeoff, when the aircraft is not moving, the propeller is not moving and the V_∞ large distance is 0; that means the flow is being sucked into the propeller from static atmosphere with no movement of the propeller at all. The V_∞ could be simply written down in terms of root over T by twice ρA .

Now, this is a very simple way of trying to figure out, what the induced velocity would be during takeoff and then allows us to quickly estimate what this thrust would be during takeoff, when the aircraft and the propeller are actually static and they are not moving. This is again a very simple handy way of quickly figuring out what the static thrust of a propeller would be during the takeoff of an aircraft.

Now, some of these things are born out of the number of simplifications that we did in the initial stages of the setting down this momentum theory. So, you need to remember those things the flow is incompressible and all the physical definitions of the actuator disk that has been put forward in this theory.

(Refer Slide Time: 30:24)


INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

So, power input needed for static thrust production (at Take off),

Power at T.O. $P_m = T^{3/2} \sqrt{2\rho A}$

Where A is the swept area of the propeller
And, ρ is the air density

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The next thing we can do is the power input. Now, you remember the power input is what would have to be supplied by the engine, so the engine needs to be matched with the propeller. This power input at takeoff can be found out using the thrust at takeoff and A of course, is the swept area of the propeller and ρ is the air density at the takeoff condition.

This power input is what the engine has to supply. In various discussions, in the some of the earlier lectures, we have seen that in most cases for aircraft operation the input power during the takeoff is one of the highest powers; probably the highest power required during any phase of the aircraft flight; which means the power required by the propeller would be probably about the highest that is required by the propeller and that has to be supplied by the engine during the takeoff. Equally, the thrust created by the propeller during the takeoff is likely to be also one of the highest thrust required during the aircraft flight. The highest power and highest thrust creation of the propeller quite often is asked for during the takeoff.

The power required has to be supplied by the engine and only then the thrust that is needed to fly the aircraft or for the takeoff would be possible through the activation of this propeller. So, this power supply estimation is extremely important because that allows the propeller to match with the engine. We shall see later on through these lectures that propeller would have another requirement and that is torque, which also needs to be supplied by the engine in a very accurately matched fashion otherwise, the propeller operation would be quite difficult.

(Refer Slide Time: 32:28)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

The ideal efficiency can be calculated by using classical definition of efficiency ,

$$\eta_p = P_{out} / P_{in}.$$

Power output needs to be equal to thrust generated by the disk multiplied by velocity of the actuator disk through the air medium (i.e. flight velocity of the aircraft). The power input is the thrust generated by the disk multiplied by the airflow velocity through the disk at the disk plane,

$$P_{out} = T V_{\infty} \quad \text{and} \quad P_{in} = T V_1$$

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The efficiency with which the whole thing can be designated normally is the efficiency definition is in terms of the power output by power input. In our case, the power output of a propeller would have to be the thrust work done by the propeller. Now, thrust is the work or the final output that the propeller creates. The input of course, is the power input that is coming from the engine. In our case, we can say in this momentum theory that power input is the thrust generated by the disk, when the thrust is multiplied by the air velocity at the disk plane whereas, thrust is the power which is created by multiplying the air velocity with the flight velocity of the aircraft.

So, if you write them down we will find that power output is thrust into the V_{∞} velocity that is, with which the whole aircraft is flying which we have taken as a far upstream velocity and power input is the activation power which is the thrust created by

the actuator disk into the velocity through the actuator disk. So that is the actuation power required for actuating the propeller.

This calculation allows us to find out, what the values of power input would be and what the value of power output would be, from which then we can find out what the propeller efficiency would be using this actuator disk model.

(Refer Slide Time: 34:17)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

$$\begin{aligned}\text{Therefore, } \eta_i &= P_{out} / P_{in} \\ &= T \cdot V_{\infty} / T \cdot V_1 \\ &= V_{\infty} / [1/2(V_{\infty} + V_e)] \\ &= 2V_{\infty} / (V_e + V_{\infty})\end{aligned}$$

Therefore, $\eta_i = 1 / [1 + (v / V_{\infty})]$

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The efficiency which we mentioned can now be written down in terms of power output and power input. We can see that they can now be written down in terms of the velocity fields that we have been defining and it comes out to be in terms of V infinity and V e. The final efficiency parameter can be written down as 1 divided by 1 plus small v which is the induced velocity by V infinity.

Now, as you can see here, the efficiency parameter is shown here with a subscript i and this is because the efficiency of propellers computed through the momentum theory or actuator disk theory is often called an induced efficiency. This induced efficiency is what is normally obtained through the momentum theory of the propeller. Remember that we had made a number of assumptions in setting together this momentum theory.

All those assumptions need to be understood to appreciate the fact that the induced efficiency that we are getting now is based on the model that has been created in the actuator disk theory. Hence, the induced efficiency is likely to be slightly different from

the actual efficiency of the propeller. So, we shall be able to see what this difference is going to be.

(Refer Slide Time: 35:16)

INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

- The efficiency estimated using momentum theory is referred to as "*induced efficiency*".
- The induced efficiency is zero for zero forward velocity and approaches 1.0 as induced velocity, v_i , tends towards zero.
- The induced efficiency reaches a maxima but does not show any fall with increasing J

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There is efficiency estimated then using the momentum theory is simply called induced efficiency in most of the literature. The induced efficiency as calculable through this theory is zero for zero forward velocity and approaches 1.0 as the induced velocity, v_i tends to zero. The induced efficiency reaches maxima but, does not show any fall with increasing J which is the advanced ratio which we introduced in the last lecture.

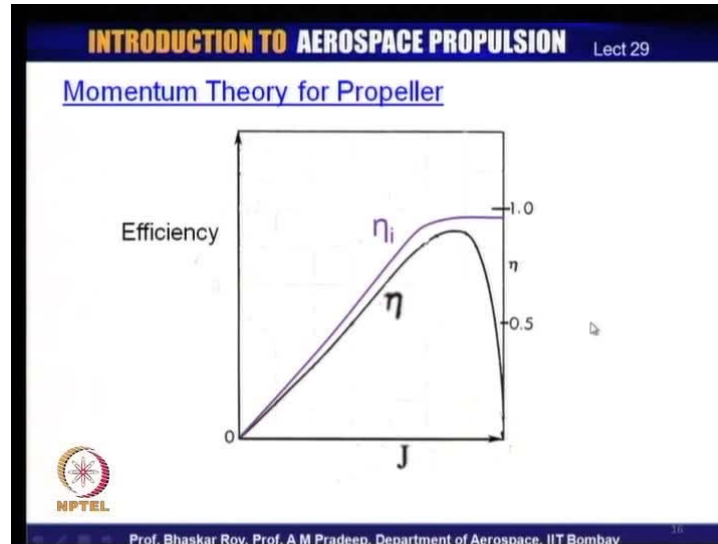
Now, this is a bit of a dichotomy which appears over here that induced efficiency would tend to be shown as 0 by this theory for 0 forward velocity, which is how you takeoff. At the top of the takeoff run the aircraft, the propeller has no forward velocity but, theoretically the induced efficiency there would be shown as **zero**.

So that is of a singularity which this particular theory shows us and then it approaches 1 as the induced velocity v_i tends to zero, which means you can have a 100 percent efficiency if the induced velocity or the additional velocity through the actuator disk is zero.

Now, as you know, if the induced velocity is zero your thrust is going to be zero. Your efficiency would be 1 when the thrust is zero and when the thrust is maximum the efficiency is going to be zero and this is the anomaly of this particular theory born out of

various models that we have put forward. As a result of that we have to take these two points as singularities and not to be actually accounted for with the help of this theory.

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The third point, which I mentioned is that the induced efficiency reaches a maxima and does not show a fall with increasing J . Now, this is what is shown in here in the figure (Refer Slide Time: 38:10). The actual efficiency of a propeller would go up and then somewhere along the way the propeller efficiency would start falling with the advanced ratio J , J is designated this as advanced ratio. So, if you plot efficiency verses advanced ratio the efficiency keeps on increasing depending on the propeller design and at a particular advance ratio it reaches its maximum.

Now, this is a typical propeller characteristic (Refer Slide Time: 38:38). It depends on the propeller design it depends on what advance ratio the propeller is designed for and typically most propellers are designed for actual operation within certain advance ratio. Somewhere over there at one of the higher advance ratios the propeller efficiency is suppose to be maximum and somewhere out there we may have a cruise point where the aircraft is likely to cruise.

On the other hand, the efficiency estimation through the momentum theory actually tells us that the efficiency would go along this line; the violet line and then it would reach maxima over here and it will not fall any more. First thing you see is the induced efficiency would be always a little higher than actual efficiency of the propeller, whether

it is estimated through any other means or experimentally acquired. That extra efficiency comes out of the simplified model that we have used in the momentum theory.

Then in the momentum theory it shows maxima more or less at the same place where the actual propeller would show maxima. After that the induced efficiency obtained from momentum theory does not show any fall any more, it remains constant thereafter.

Now, this is a big difference and you need to understand why this difference actually happens. This is due to the fact that we assumed that we have an actuator disk, which does not have any mass, it is 100 percent porous but, probably the most important thing is the actuator disk is not made up of blades and it has no surface area over which it can offer any resistance. So, the actuator disk does not offer any resistance at all, it has no drag and since it has no drag what so ever, it does not show any fall as an airfoil section would show in performance. Once it reaches a maximum performance efficiency of the performance it remains there; there is no reason by this theory for it to fall thereafter.

So, this is another; one can say a simplification of this momentum theory that it gives us very quickly a handy method of estimating the thrust and the efficiency but, both are likely to be slightly idealistic and somewhat on the higher side compared to what one would probably get actually in actual estimation.

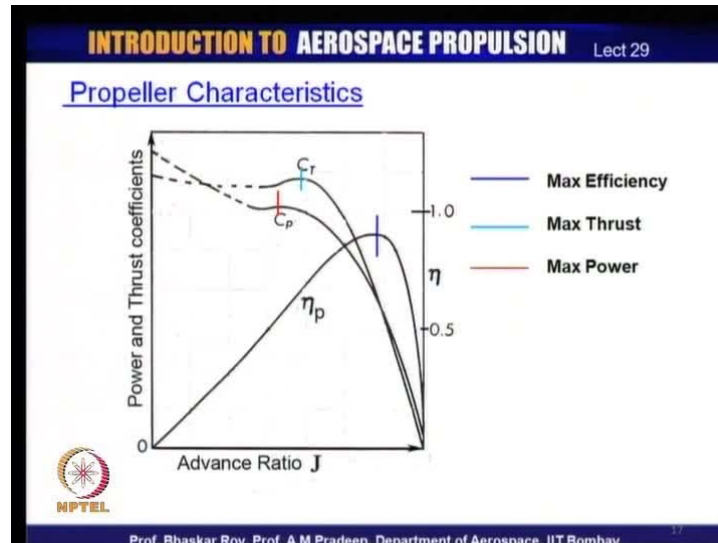
Now, we can look at the propeller characteristics. Now, propeller characteristics as we have seen has to be shown in terms of thrust, in terms of power, in terms of efficiency and all the three put together and their variation with advanced ratio what gives us a propeller characteristics. The reason it is called characteristics is because this particular characteristic plot actually characterizes this particular propeller.

Every propeller has its own characteristics almost like a human being. So, every propeller is characterized by such a characteristic plot and shows its own efficiency thrust and power variation. In our case, we shall see that they are shown in terms of thrust coefficient and power coefficient, which we have defined in the last class.

The definition of thrust and power coefficient that has been put forward in the last class is now brought into the fore and is used for characterizing the propellers. Every propeller would have its own characteristic plot and one of the things that a propeller designer need to do is quickly find out what the characteristics of this propeller is going to be,

before it is matched to an engine or before it is matched to an aircraft. The aircraft designer as well as, the engine designer or the engine selector requires this characteristic match to be available to select an engine or to put it on an aircraft.

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Now, let us look at this characteristic what it tells us. You see, as I said they are shown in terms of the advance ratio as the advance ratio increases. Typically, the advance ratio will increase as the forward velocity of the propeller increases that means as the aircraft starts moving and flying, its forward velocity increases and the advance ratio typically would increase.

Depending on the aircraft the maximum advance ratio can be set, a very high speed aircraft would have very high advance ratio range, over which this characteristic map would have to be created. Comparatively, a low speed aircraft would have a lower advance ratio range over which this characteristic map would have to be created, so it also depends on the aircraft on which this propeller is getting mounted on. Because the advance ratio range would have to be decided on the basis of the aircraft flight velocity range starting with takeoff.

Now on the y-axis, you can see the power and thrust coefficients are shown here as C_T and C_P . As you can see here, two things that are very important; one is the efficiency of the propeller maximizes at a certain velocity or certain flight velocity or advanced ratio

whereas, the thrust coefficient and the power coefficient actually maximize during flight at different values of advance ratios.

At lower advance ratios quite often the performance of the propeller is likely to be somewhat uncertain and that is what is shown here in dotted lines. However, you would probably likely to get maximum performance during at the takeoff point. During the takeoff run of an aircraft, maximum thrust and maximum power would be created and then of course, it would go through the climb procedure and then you have the flight during which the propeller characteristics are more clearly available. During this phase often the propeller characteristics may be slightly uncertain using this theory; the momentum theory that we are talking about. You may need slightly more advanced theories to calculate what is happening during the climbing operation of an aircraft.

Now, on a straighten level flight again, if we come back, you will see that the power characteristic goes like this, the thrust or the thrust coefficient characteristic goes like this, the maxima of the two are different and maxima of these two are different from the maxima efficiency point. Now, the maximum efficiency occurs at a higher advance ratio, a maximum thrust occurs over here and as a result of which the aircraft designer or the aircraft operator would have to decide where the propeller would be mostly operative. Quite often, the propeller operation is expected to be at a point where the propeller efficiency is good and the thrust is also good and power is somewhat on the lower side (Refer Slide Time: 46:11).

So, you do not use maximum power all the time, because if we use maximum power you are likely to use more fuel; you do not want more fuel use all the time. So, you would like to have lower fuel consumption with reasonable amount of thrust that is required for straightened level flight. At a good efficiency which will ensure that you have a good propulsive efficiency of the aircraft propeller combined and which will show up finally in good fuel efficiency.

The choice of where the propeller should be operated during the cruise is to be decided based on this propeller characteristics and that is the reason why this propeller characteristics is so important that we need to have these characteristics, before the propeller is mounted on the aircraft.

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INTRODUCTION TO AEROSPACE PROPULSION Lect 29

Momentum Theory for Propeller

- Induced efficiency cannot be realized as the energy lost in the rotational motion acquired by the flow in passing through the propeller.
- Losses due to non uniform thrust loading over the blade length.
- Blade interference losses due the interaction of flows over the neighbouring blades.
- Propeller profile drag losses, incurred over all the blade surfaces , *and*
- Changes in flow properties due to effect of compressibility, which are not accounted for.

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If we look at some of the issues that we have discussed that the induced efficiency which we talked about reaches a maximum. It cannot be realized as the energy lost in the rotational motion, which is possible through the actuator disk in passing through the propeller or the actuator disk, because the certain amount of energy which is going in the rotational motion of the air is not available for thrust creation. Thrust is created only with the help of actual velocity change, so the actual velocity change is what gives thrust. The rotational velocity change or rotational velocity which may appear does not contribute to the thrust creation.

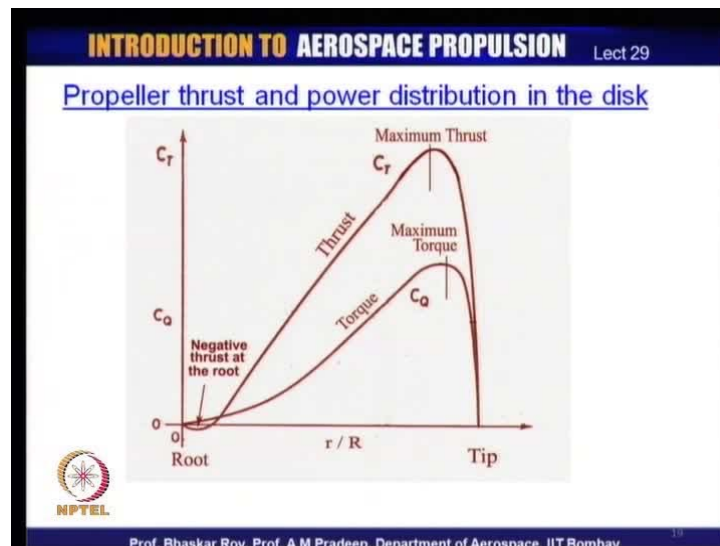
The second point is the losses due to the non-uniform thrust loading over the blade length. What happens is quite often over the length of the blade which is approximated here with actuator disk. The loading is not uniform, which we have assumed here as a uniform loading over the entire actuator disk, in an actual blade that may not actually happen.

The second or the third point is the blade interference losses due to the interaction of the flows over the neighboring blades. In the actuator disk, we have assumed it is a porous body; it is a continuous body with no mass with 100 percent porosity, in an actual propeller you actually have 3 or 4 blades, rest of the area is open. These blades when they are rotating at some speed they actually have certain amount of aerodynamic interference between each other.

For example, wake of one of the blades could interfere with the wake of the other blade or the flow from the one of the blades could flow into the other blade. This aerodynamic interference is totally neglected or assumed to be not present in this particular momentum theory, so that is another reason why there is a difference between what we calculate through momentum theory and what would actually happen in an actual propeller operation.

The next point is the propeller profile drag losses, incurred over all the blade surfaces; now, we do not have blades over here. There are no blade surfaces, so the airfoil is absent and hence there is no surface whatsoever and the airfoils which encounter drag in the process of creating lift is completely absent. The propeller does not experience any loss whatsoever as it is modeled in actuator disk theory. The changes in properties due to compressibility, we have assumed that the flow is incompressible, so in this simplified theory we are not taking into account compressibility at all.

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These are the differences between the momentum theory that we have put together and the actual operation of the propeller that actually happens during the operation of the propeller. If we quickly look at the propeller thrust and power distribution in the disk across the length of the disk, we shall see that this is what actually propeller does at the thrust creation varies from root to tip, as I was just mentioning little while earlier that it is maximum thrust is somewhere near the outer half of the propeller. The maximum

torque is also created by the outer half but, in our actuator disk theory we have assumed that it is uniform over the entire actuator disk or entire **emulous** or entire area of the actuator disk.

This difference between an actual propeller and actuator disk is what manifests itself in the differential between the estimated values of thrust and efficiency and the actual values; one we find by experimental or other more accurate methods.

This is what the propeller theory is; the first propeller theory is that we have put together a very simple theory. A very simple theory that quickly tells us what the thrust is likely to be, even tells us what the thrust is likely to be during the takeoff operation of an aircraft. It gives us a very good estimation of the efficiency over the advanced ratio variation with slight increase in the values of efficiency once we know what these differences are, why these differences are based on this simple theory. We do have a very quick estimation of how the propeller is going to function, because as we have seen, we need to have a good reliable propeller characteristic for it to be matched with an aircraft, for it to be matched with an engine.

So, this is the first cut estimation of the propeller performance. In the next theory, the blade element theory, which we will do next we shall have a closely look at how the propeller functions when the entire propeller blade shape made up of airfoils is brought into the reckoning in the theory and that is what we will be doing in the next class.