Introduction to Aerospace Propulsion

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Module No. # 01
Lecture No. # 22 - B

Flows with friction and heat transfer, normal and oblique shocks

Hello and welcome to lecture number 22, part B. In the last lecture, we were discussing about aspects of compressible flow and what are the different terminologies that one needs to understand in order to analyze and make use of compressible flows.

There are several aspects of compressible flows, which are slightly different from what we have been analyzing so far. In our analysis which we carried during the initial part of our course, there was an inherit assumption that the density does not change or the changes with the flow does not really have much kinetic energy and so, changes associated with that kinetic energy was always neglected.

But if you look at flows which involve higher speeds, then kinetic energy term can no longer be neglected and also, it is possible that density changes cannot really be neglected.

How do we take these into account? These were through stagnation properties; we have already defined and derived equations for stagnation properties like stagnation enthalpy, stagnation pressure, temperature and so on. Also, we have seen that in the absence of any heat or work interactions, stagnation enthalpy does not change which means that for an ideal gas, the stagnation temperature does not change across an area of constant of flow through a duct.

This is in the absence of any heat or work interactions, but it is possible that the static pressure or in fact, this stagnation pressure can change, even if the stagnation enthalpy does not change.

Stagnation pressure may change because of frictional effects. Frictional effects can occur even in the absence of any heat or work interactions. So, in a duct flow, if there are no heat or work interactions and in spite of that if there are frictional losses then it is possible that stagnation pressure might change, but stagnation temperature, enthalpy do not change.

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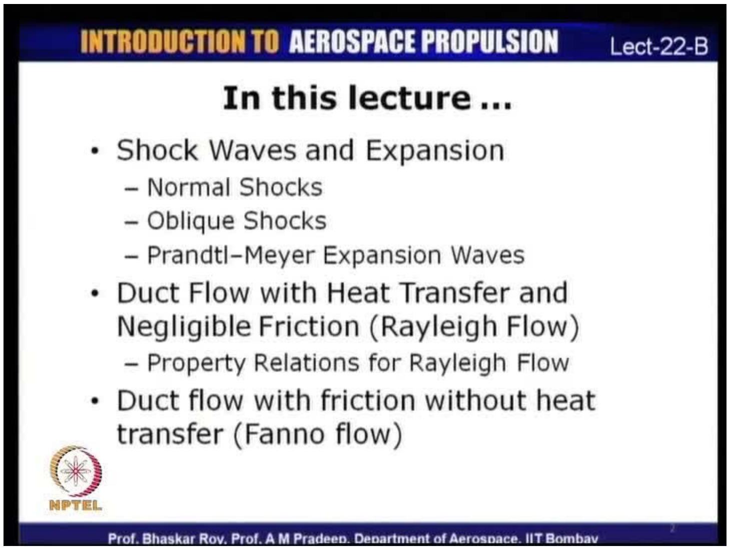

These were some of the aspects we had discussed in the last lecture. What we are going to discuss today is a continuation of some of the aspects, we discussed in the last lecture. Let us take a look at what we are going to discuss in today's lecture.

We shall be talking about what are meant by shock waves and expansion. We shall then continue to discuss about different types of shock waves: normal shocks, oblique shocks and Prandtl-Meyer expansion waves. This is what we will begin our lecture with. We will then continue to discuss about duct flow with heat transfer and negligible friction; these flows are usually classified as Rayleigh flows and we will also look at some aspects of property relations for Rayleigh flows.

Towards the end of the lecture, we will be discussing about duct flow with friction, but without heat transfer; these flows are known as Fanno flows. So, these are some of the topics that we are going to discuss in today's lecture. What we will begin our lecture is on discussion on shock waves. If you recall, during the later part of the previous lecture, I was discussing about flow through a converging, diverging nozzle.

We saw that as we change the back pressure, at a certain back pressure, there is sonic flow at the throat, then the flow becomes supersonic and then abruptly, it becomes subsonic. I mentioned in the passing, during last lecture that this is because of the presence of a shock wave in the diverging section of the nozzle.

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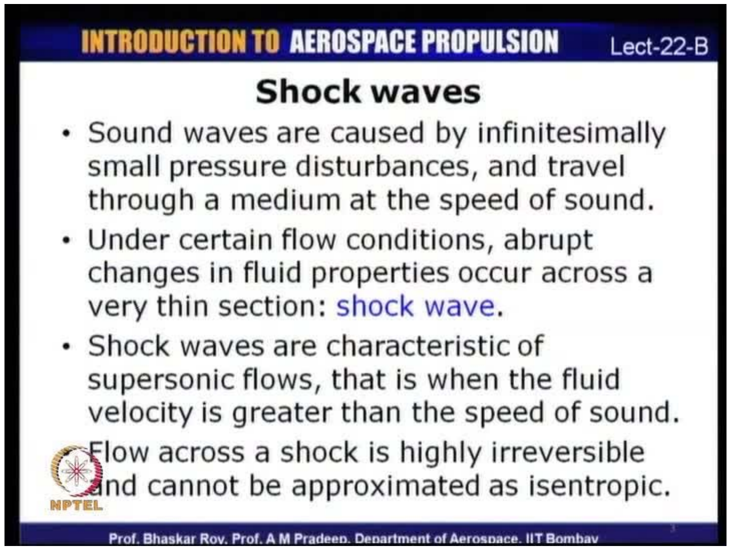

Let us look at what we mean by shock waves. Shock waves are basically certain aspects of a flow in a supersonic flow, where there could be abrupt changes in fluid properties. Now, we have already defined, what is meant by speed of sound. Sound waves are caused by infinitesimally small pressure disturbances and they travel through a medium at the speed of sound.

But under certain flow conditions, there could be abrupt changes in fluid properties which can occur through a very thin section and that is known as a shock wave. So, shock wave is a very thin section in a fluid flow, across which there could be a sudden change, an abrupt change in fluid properties like pressure, temperature, density and so on and also Mach number.

Shock waves are characteristic of supersonic flows. It means that shock waves cannot exist in subsonic flows. Shock waves occur only in supersonic flows and the reason for this is that in a supersonic flow, the speed at which the fluid moves or the vehicle moves is greater than the speed of sound and we have seen that sound waves are essentially pressure waves.

That means that as a vehicle or fluid moves at a speed, which is greater than the speed of sound, then the information travel does not occur upstream. This means that if there is a vehicle which is moving at a supersonic speed then the presence of this vehicle is not known to fluid particles which are ahead of the vehicle. Therefore, what happens is that the fluid particles strike the vehicle and since they have to take the shape of the vehicle, they have to flow over the vehicle. This has to occur through the presence of an abrupt change in fluid properties like velocity, temperature and so on.

That occurs through the presence of shock waves. So, shock waves are very thin sections in a supersonic flow, across which there are abrupt changes in all fluid properties. There are some fluid properties which do not change; we will discuss that. Since, there are abrupt changes taking place through the shock wave, flow through a shock wave is highly irreversible.

Therefore, flow through a shock wave is not to be considered as isentropic. So, flow across a shock wave or in the section of the shock wave is non-isentropic. So, you cannot consider that shock wave flow through shock waves as isentropic.

That is one of the aspects of a shock wave and so, what we will do is we will first analyze what is meant by a normal shock wave? To do that, I will take you back to the pressure variation across a convergent divergent nozzle, where we had discussed that at certain back pressure, there is a shock wave. Let us take a relook at what is happening across a convergent divergent nozzle.

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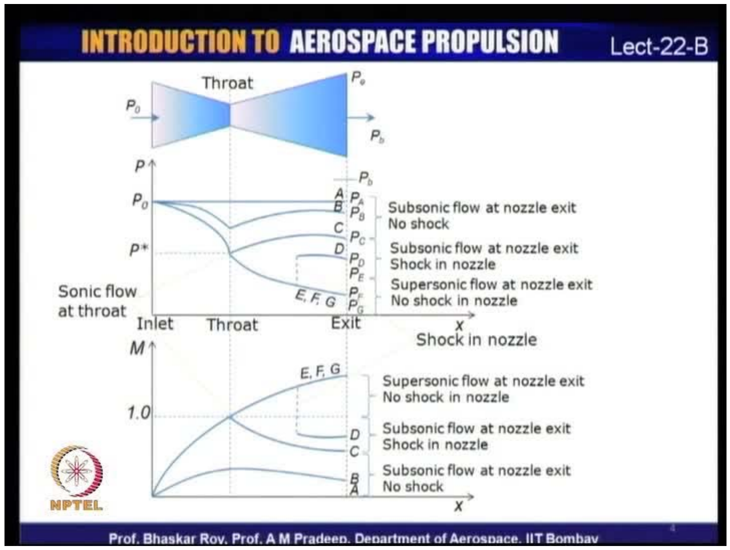

This is what we had discussed in the last lecture - that this is a convergent, divergent nozzle and as you change the back pressure, that is P b, then the fluid properties across the nozzle changes in particular manner, for different values of P b.

As you reduce P b, at a certain pressure, there is no change in the pressure across the nozzle. As you reduce back pressure further, then the flow accelerates in the convergent section of the nozzle, it reaches the minimum pressure at the throat and then, it again increases because it is a divergent section; divergent section in a subsonic flow acts as a diffuser.

So, the pressure rises again and it exits at a certain pressure. Now, if you reduce the back pressure further, it reaches the minimum pressure at the throat and that is the pressure which is the critical pressure basically and at that pressure, the flow becomes sonic that is, you get a Mach number is equal to 1.

Subsequently to that, it again rises. If the back pressure is further reduced to, let us say a value P d then the flow from the throat accelerates and it becomes supersonic, but again at a certain point abruptly it become subsonic.

So, this abrupt change in the pressure and also the Mach number which you can see here - Mach number was 1 at the throat and then it becomes supersonic and after this point it again becomes subsonic, suddenly.

This is because of the presence of a shock. I had mentioned that there is a shock in the nozzle. Usual form of a shock which can occur in such flow is a normal shock. So, this is because of the presence of a normal shock that is, across a normal shock, there is an abrupt change in the properties, which is what has happened here - that across the shock, the fluid properties have changed abruptly.

So, this is because of the presence of a normal shock. What do we mean by a normal shock? A normal shock essentially is a shock were in the shock wave and the flow and directions meet at 90 degrees; that is, the flow directions is normal to the shock wave itself and that is why they are called normal shock waves.

So, we have seen in the case of the convergent divergent nozzle, if there is a normal shock wave, the flow becomes subsonic across the nozzle. So, a normal shock wave can occur in supersonic flows. How much the Mach number downstream of the normal shock waves depends upon the upstream Mach number?

Basically, it just depends upon the upstream Mach number. The property of a normal shock wave is that downstream of the normal shock wave, the fluid becomes subsonic. That is, downstream, which is what we saw in the supersonic nozzle case that after the normal shock wave, the flow become subsonic and the nozzle now becomes or behaves like a diffuser.

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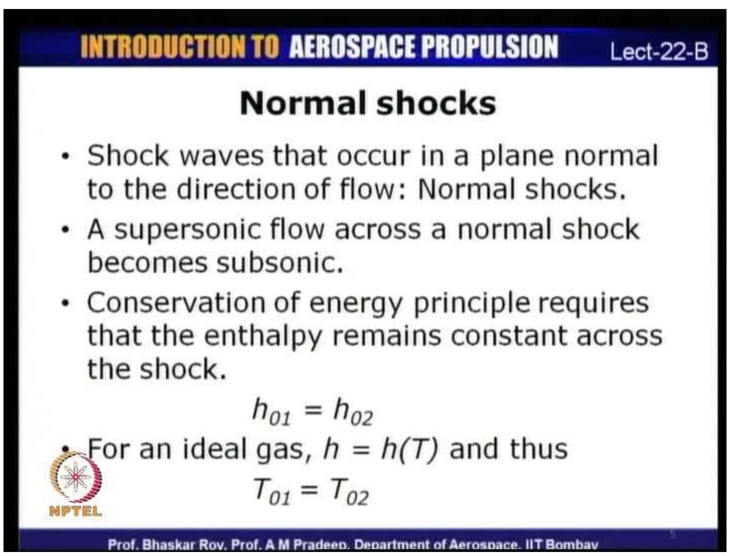

So, downstream of the normal shock, the flow becomes subsonic. Let us look at what are the features of a normal shock wave. Shock waves that occur in a plane which is normal to the direction of the flow are called normal shocks.

A supersonic flow across a normal shock wave essentially becomes subsonic. Since in this case or In the case, where there is no heat or work interactions, we have seen that conservation of energy principle states or requires that the enthalpy remains a constant - which means that the stagnation enthalpy at station 1 just before the shock and at station 2 just after this shock becomes the same or is the same. So, h 01 is equal to h 02. If you consider the gas to be an ideal gas with constant specific heats, then it follows from the energy equation that the stagnation temperature before the shock is equal to the stagnation temperature after the shock.

This is a property of a normal shock that because you do not have any heat or work interactions, the stagnation temperature does not or cannot change across the normal shock, but there are other parameters which change substantially across the shock. Mach number was 1 parameter. I mentioned that Mach number downstream of the shock becomes subsonic; so, that is one parameter which changes. Since Mach number is changing, it follows that the velocity also will change.

What are the other properties or parameters which will change across the shock, which we shall analyze shortly and will take a look at what properties across the shock can change. How these can change? How can you correlate the downstream of the shock with the properties upstream of the shock? Let us take a look at how we can correlate them.

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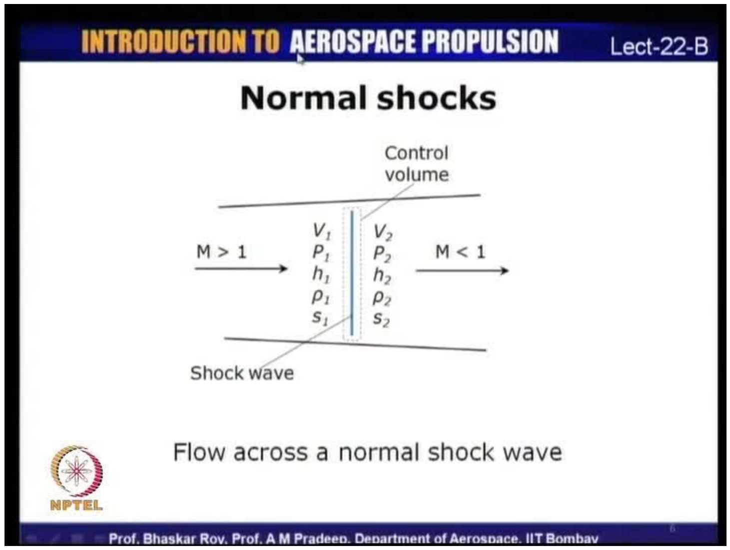

Here we have flow through a duct. It is a generic duct; it could be converging, diverging whatever or a constant area and what is indicated by this blue line here is the shock wave; it is a normal shock. We have taken a very thin control volume which surrounds the shock wave. Upstream of the shock wave, we have a supersonic Mach number and we have properties of the fluid which is velocity of V 1, pressure P 1, static enthalpy h 1, density rho 1 and entropy S 1; downstream of the shock where the Mach number is subsonic, we now have velocity V 2, pressure P 2, enthalpy h 2, density rho 2 and the entropy S 2. Given these properties, we will now try to correlate properties upstream and downstream of the shock.

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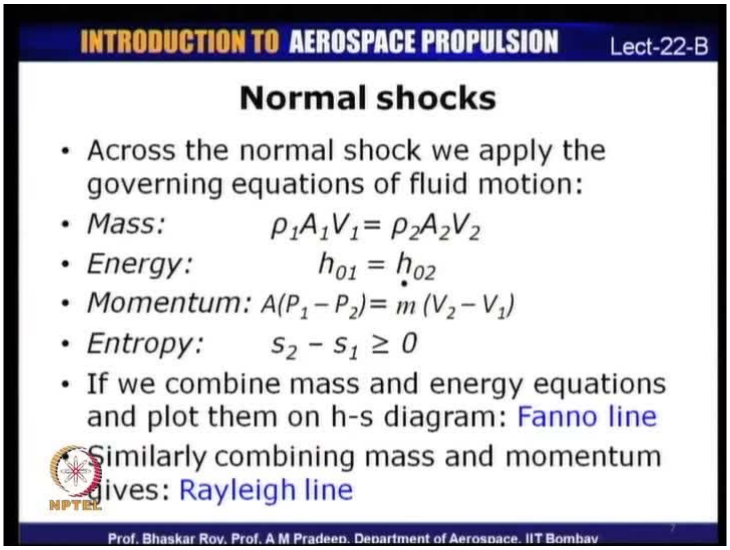

Basically, what we will try to do is to apply the governing equations of fluid motion. We have primarily 4, in fact 5, governing equations: equation for mass, energy, momentum, entropy and the equation of state. If you take up four of these governing equations, let us look at the mass equation or conservation of mass which states that mass flow rate before the shock and after the shock should be the same.

So, rho 1 A 1 V 1 should be equal to rho 2 A 2 V 2. Conservation of energy states that stagnation enthalpy does not change. So, h 01 is equal to h 02. Conservation of momentum states that area into P 1 minus P 2 is equal to m dot into V 2 minus V 1.

Increase of entropy principle states that S 2 minus S 1 is greater than or equal to 0; this is basically a follow up of the third law of thermodynamics. So, what happens is that if you were to combine two of these equations, let us say we combine the mass and energy equation.

Then we plot the combined equations on an h-s diagram that is, enthalpy-entropy diagram, and then the resultant curve that we get which is basically a combination of the mass and energy equation is basically known as the Fanno line. Such flows are basically known as Fanno flows; we analyze Fanno flows later on in the lecture.

Similarly, if we combined mass and momentum equation, we get another equation which when plotted on h-s diagram, we get a line which is known as the Rayleigh line; such flows are known as the Rayleigh flows. So, if you combine the mass and energy equation we get the Fanno line, the mass and momentum equation combines, we get the Rayleigh line and it follows that these two curves when plotted on the same h-s diagram will intersect at two different points and the solution of these two points refers to the flow through the shock wave.

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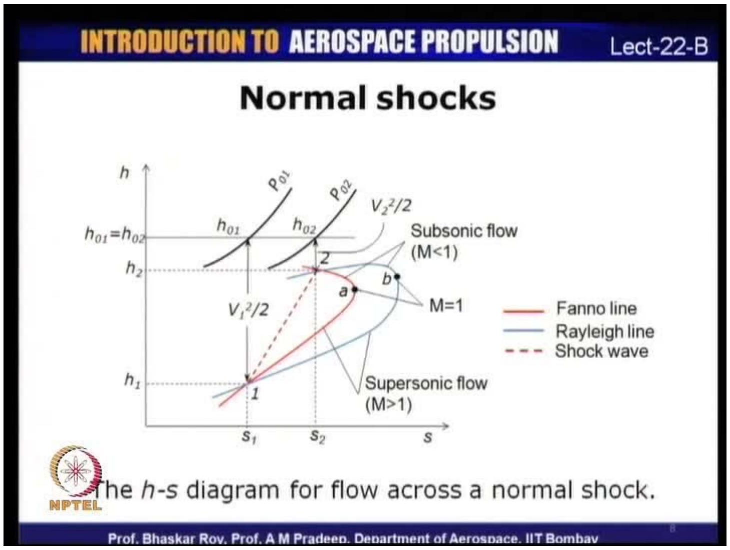

Let me illustrate that through an h-s diagram. On this h-s diagram, you can see that I have plotted the Fanno line equation as well as the Rayleigh line equation. The Fanno line equation is shown by the red line and the Rayleigh line equation is shown by the blue line.

These two curves meet at two points: point 1 and point 2. If you join these two lines which is shown by this dotted line that indicates the flow across the shock wave. Let me explain this h-s diagram in little more detail. Let us take a closer look at what is happening across these two different curves.

We have seen that stagnation enthalpy does not change across a shock wave and therefore, it should mean that at point 1 and point 2, the stagnation enthalpy should be the same, which is what is shown here - h 01 is equal to h 02 and that is, joining these two points which are intersection of the Fanno and the Rayleigh lines.

It means that the static enthalpies obviously can be different. So, static enthalpy at station 1, h 1 plus V 1 squared by 2 is basically equal to h 01; similarly, at station 2, h 02 is equal to h 2 plus V 2 squared by 2 and which means that since across a shock wave, velocities will be different across a normal shock, the Mach number becomes subsonic and velocities also change, it follows that V 2 will be less than V 1.

Therefore, we have h 2 and h 1 which are not equal and h 2 being greater than h 1. What about entropy? Entropy across the shock increases. I mentioned that shock wave is an irreversible process; it cannot be considered to be an isentropic process. Therefore, S 2 is greater than S 1.

So, we have an increase in entropy here. Stagnation pressure: since stagnation enthalpies are same, but static enthalpies are different and there is a loss of stagnation pressure across a nozzle or across a shock wave, we now have P 01 not equal to P 02; P 02 is in fact less than P 01, which is why these two lines are shown as separate lines. The constant pressure lines P 01, P 02 are different. You can also see that I have indicated two different points here; these are the points at which, on the Fanno as well as Rayleigh line which we will analyze in detail little later.

These are the points at which the curve changes its direction and those are the points which correspond to sonic flow; that is, Mach number is equal to 1 occurs at point a and point b on the Fanno and Rayleigh lines respectively. Below these lines, we have a supersonic flow and above the lines, we have a subsonic flow.

Basically, from the h-s diagram, what we can understand is that the shock wave is something which you can derive from solving the Fanno line and the Rayleigh line equations and the point at which these two curves intersect basically refers to the flow through the shock wave and we have also seen that stagnation enthalpy cannot change because of conservation of energy principle; the static enthalpies can be different.

Static enthalpy in fact, downstream of the shock is higher than the static enthalpy upstream. Pressure, drops across the shock wave and similarly, let us also look at what happens to static pressure and static temperature. We have already seen stagnation temperature does not change and stagnation pressure drops across a shock wave. What about static pressure and static temperature? If you were to analyze that, we have to relate the properties upstream and downstream of the shock wave and let us take a look at how we can relate these two upstream and downstream properties.

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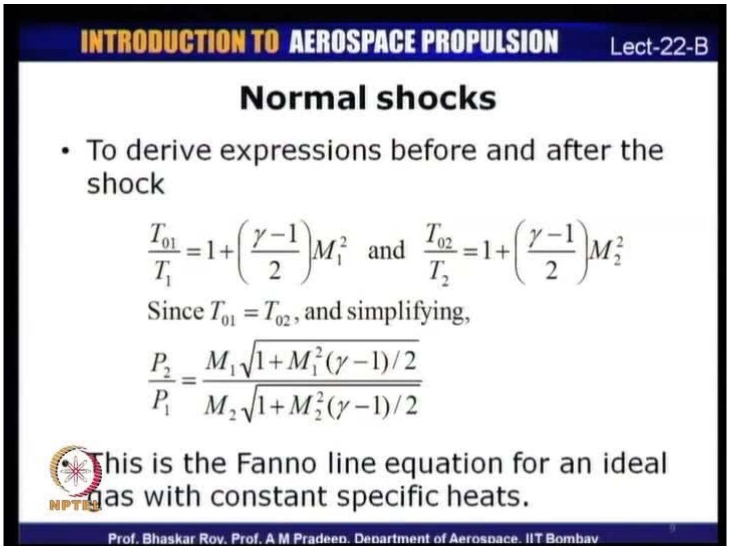

If you have to derive expressions before and after the shock wave, the flow is isentropic before the shock wave and so, we have T 01 by T 1 is equal to 1 plus gamma minus 1 by 2, M 1 squared, where M 1 is the upstream Mach number, T 01 is stagnation temperature upstream and T 1 is static temperature upstream.

Similarly, T 02 by T 2 is equal to 1 plus gamma minus 1 by 2, M 2 squared, where M 2 is the downstream Mach number, T 02 and T 2 are the temperatures downstream of the shock wave.

These are followed from the isentropic expressions. Now, we know that the stagnation temperatures are equal; so, T 01 is equal to T 02. If we do that, we get an expression in terms of temperature ratios and this we can again further simplify in terms of isentropic relations because P 2 by P 1 can be related to T 2 by T 1. So, we have P 2 by P 1 is equal to M 1 into square root of 1 plus M 1 squared gamma minus 1 by 2 divided by M 2 into square root of 1 plus M 2 squared gamma minus 1 by 2.

This is primarily the Fanno line equation for an ideal gas with constant specific heat. This is basically looking at the mass and energy equations and solving them, we primarily get the Fanno line equation, where we can relate the upstream and downstream pressures in terms of the corresponding Mach numbers.

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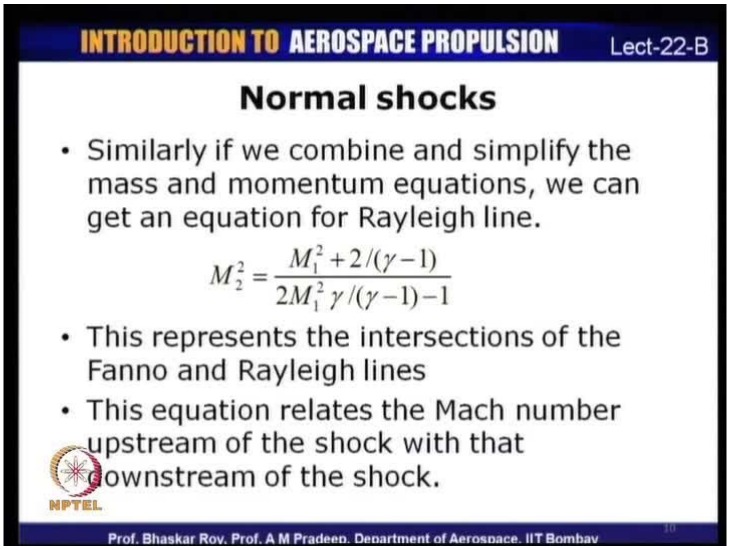

Similarly, we can combine and simplify the mass and momentum equations and what we get is an equation for Rayleigh line. If we were to combine the mass momentum equations, we basically get the Rayleigh line equation and we correspondingly can relate some of the properties that are upstream and downstream.

If we look at these two equations and simplify them, what we can do is that we can relate the downstream Mach number, which is M 2 with the upstream Mach number.

What we get is M 2 squared which is downstream Mach number is equal to M 1 squared plus 2 by gamma minus 1 divided by 2 into M 1 squared gamma divided by gamma minus 1, minus 1.

So, the upstream Mach number and downstream Mach number can be related through this simple equation and what we can see is that, they primarily depend upon the ratio of specific heats which for an ideal gas is 1.4 typically.

So, we can relate the upstream and downstream Mach numbers through a very simple equation. Similarly, the other parameters like the temperature, pressure and so on can actually be related and we can either calculate these. Since, we can see that they depend specifically on certain properties, for an ideal gas where gamma is equal to 1.4, we can actually write down tables for calculating the downstream properties given the upstream properties.

So, these properties are actually available in tabulated form and these are known as the shock tables. If you refer to any book on thermodynamics or on compressible flows, towards the end in the appendix, you will definitely find these properties which are listed in the form of tables and these are known as the shock tables and usually referred to as either the normal shock tables.

In fact, what we will see little later is that, when we talk about oblique shocks that oblique shock properties can also be derived from the normal shock tables assuming or simplifying the velocity vectors which are there on an oblique shock and we can calculate the properties across an oblique shock from normal shock tables. That is something, we will discuss little later. Basically, what we can do is that we can relate the properties that are downstream of a normal shock with that of the upstream properties using simple relations, which we have just seen and which primarily depend upon the ratio of specific heats and a few other properties.

For an ideal gas, it is possible that we can get tabulated forms of these normal shock properties and they are basically related to equations of the Fanno line as well as the Rayleigh line.

So, if you look at the previous equation, I was talking about, where we relate the Mach number which is downstream of the shock with the upstream Mach number, it basically represents the intersections of the Fanno and the Rayleigh lines. If you recall, during the discussion on the Fanno and Rayleigh line, I mentioned that there are two points where they intersect and which is basically the flow through the shock and so, this equation basically represents those two intersection points.

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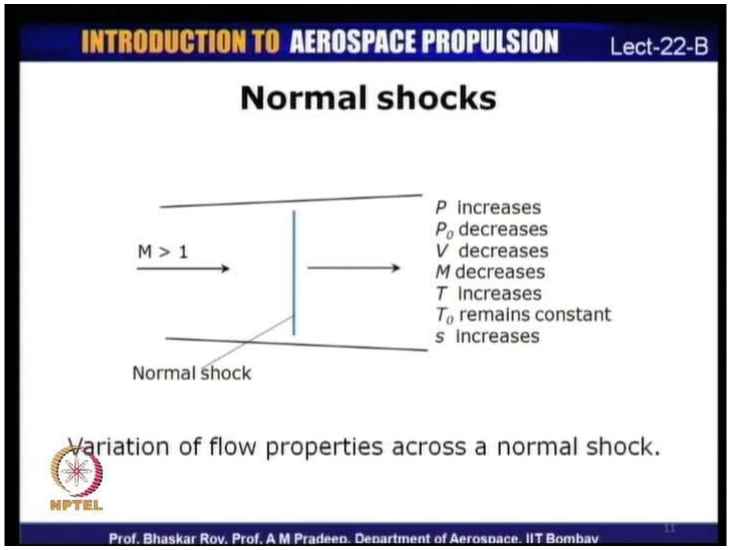

To summarize: across a normal shock what happens is the upstream Mach number is supersonic, downstream Mach number becomes subsonic and across a normal shock these are the variations of different properties.

There is an increase in the static pressure and correspondingly, there is a decrease in this stagnation pressure. So, static pressure across a shock increases; stagnation pressure across a shock decreases, velocity decreases, Mach number also decreases across a normal shock and the static temperature increases across a normal shock whereas, the only property which remains a constant across a normal shock is the stagnation temperature.

So, stagnation temperature across a normal shock remains a constant. It cannot change because there is no heat or work interaction taking place and entropy across a normal shock increases because the process is highly irreversible, entropy increases.

So, these are the different variations of the properties across a normal shock and some of these properties will also be or these variations will also be valid for an oblique shock which is what we will discuss next - that there are flow situations when the shock need not necessarily be normal to the flow. Under these circumstances, the shock wave can be inclined at a certain angle to the flow and such shock waves are known as oblique shocks.

There are several flow situations where we encounter oblique shocks. So, flow downstream of the oblique shocks may be subsonic or it may be sonic or it may remain to be supersonic depending upon the Mach number and the turning angle and so on. Let us look at what we mean by oblique shocks.

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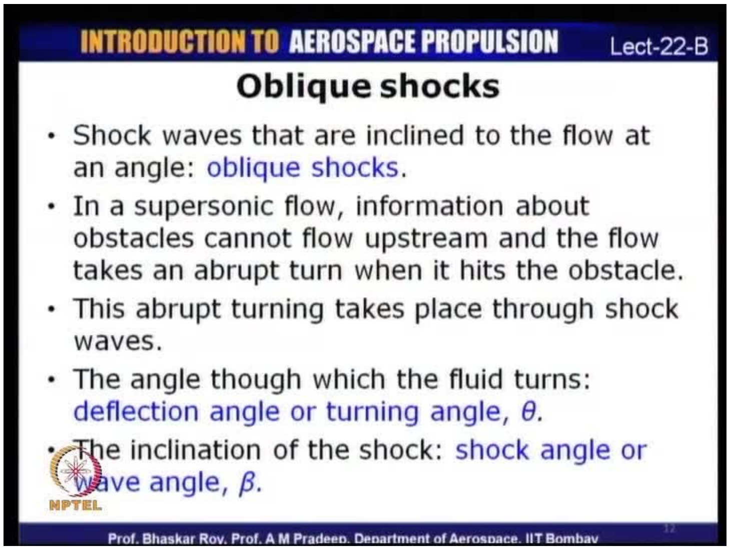

Shock waves that are inclined to the flow at an angle are basically known as oblique shocks. Why do we have an oblique shock in the first place? We have already seen why normal shocks occur; the same reason applies for an oblique shock as well and in a supersonic flow, the presence of obstacles cannot be felt by the flow which is upstream.

Therefore, the flow has to take an abrupt turn, when it hits an obstacle. This abrupt turning basically takes place through the presence of shock waves and in the case of obstacles which are like a wedge or a cone in a supersonic flow, the turning of the flow takes place through the presence of oblique shocks.

The angle through which the fluid turns is known as the deflection angle or the turning angle usually denoted by theta and the inclination of the shock is basically known as the shock angle or the wave angle.

That is, when a supersonic flow hits an obstacle, the angle through which it turns is basically known as the deflection angle of the shock or the turning angle denoted by theta and the angle of the shock wave is basically known as the wave angle or the shock angle.

We will now look at the various terminologies associated with a normal shock like deflection angle, shock angle etcetera. How do you find out these angles given certain geometry in a supersonic flow?

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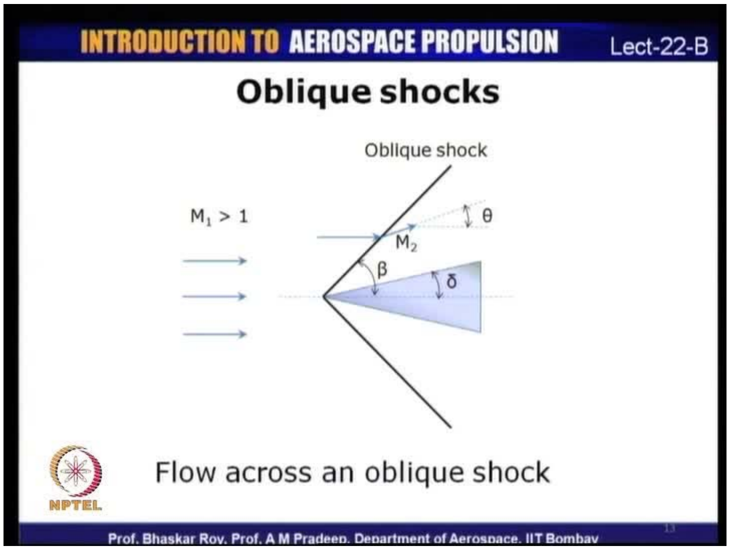

Let us consider a simple example here. What we have here is a two dimensional wedge; wedge has a half angle of delta. We have a supersonic flow Mach number greater than M 1, which is approaching the wedge and as it hits the wedge, because the presence of the wedge is not known to the fluid because information is travelling at a speed greater than the speed of sound.

So, the presence of this in terms of pressure waves cannot travel upstream and what happens is that, the fluid knows that there is an obstacle only after it hits it. So, the fluid has to take an abrupt turn and this abrupt turning of the fluid occurs through the presence of these oblique shocks. The black lines which are shown here are the oblique shocks.

The presence of this oblique shock causes the downstream flow to be deflected by a certain angle which is known as the deflection angle or the turning angle, which is theta and is basically equal to the angle half angle of the wedge itself. So, the flow downstream of the oblique shock takes a direction which is parallel to the wedge itself.

So M 2 will have a direction, which is parallel to this wedge - the side of the wedge and the angle at which the oblique shock is inclined is known as the wave angle or the shock angle and that is denoted by beta.

So, beta is the angle of inclination of the oblique shock. Downstream Mach number M 2 will be a value which is different from M 1; it will be certainly less than M 1, but it need not necessarily be a subsonic Mach number unlike a normal shock, where the downstream Mach number is always less than 1.

In an oblique shock, the downstream Mach number may continue to remain supersonic, but less than the upstream Mach number or it could become sonic or it could become subsonic and that depends upon the upstream Mach number and these deflection angles.

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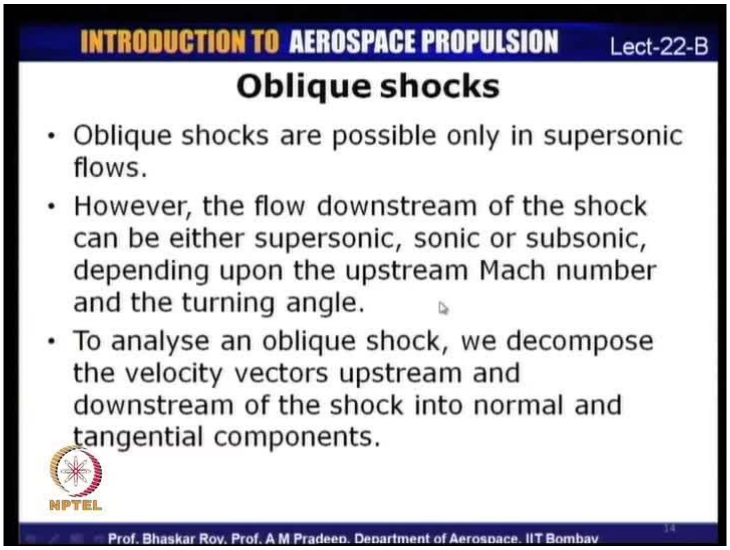

Like we have discussed already for a normal shock, oblique shocks are also possible only in supersonic flows and so, flow downstream of the shock could either be subsonic or it could remain supersonic or it could be sonic and this depends upon the upstream Mach number and the turning angle.

How do we analyze an oblique shock? To analyze an oblique shock, what we do is that we decompose the velocity vectors upstream and downstream of the shock into normal and tangential components. We have seen that the flow approaches an oblique shock at a certain angle or the oblique shock is at a certain angle to the flow, both upstream as well as downstream.

What we do is that, we decompose both the upstream and downstream velocity vectors into their normal and tangential components and then from the normal components, we can use the shock tables - normal shock tables, we have already discussed and calculate the properties downstream of the shock from the normal shock tables and then using algebraic manipulation, we can find out the properties downstream of the oblique shock.

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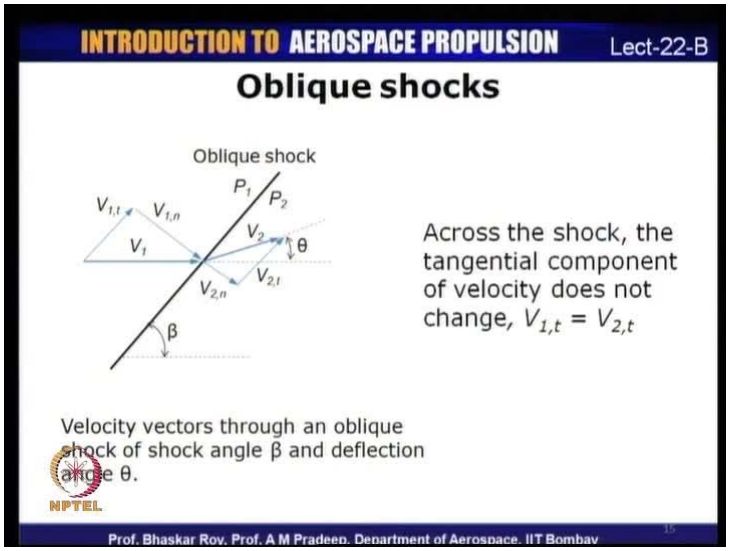

Let us take a look at how we could do this. If you look at this illustration here, we have an oblique shock and there is an upstream velocity which is V 1, velocity downstream of the oblique shock is V 2. What we do is we decompose this velocity vector in terms of its normal component and tangential component, both for upstream as well as the downstream cases.

So, V 1 n is the velocity vector that is normal to the shock upstream, V 2 n is the velocity vector downstream of the shock and normal to the shock. V 1 t is the tangential component, V 2 t is the tangential component of velocity downstream.

It can be shown that for an oblique shock, the tangential component does not change across the shock and so V 1 t will be equal to V 2 t, but V 1 n and V 2 n obviously, cannot be the same and so basically, we can now relate the velocity vectors which are upstream and downstream of the shock using the shock angle beta and the deflection angle theta. It basically depends upon three parameters the deflection angle theta, the shock angle beta and the Mach number upstream Mach number M and all these three parameters are closely interlinked. We will see how they are interlinked in the form of a chart, which relates Mach number, deflection angle theta and the shock angle beta.

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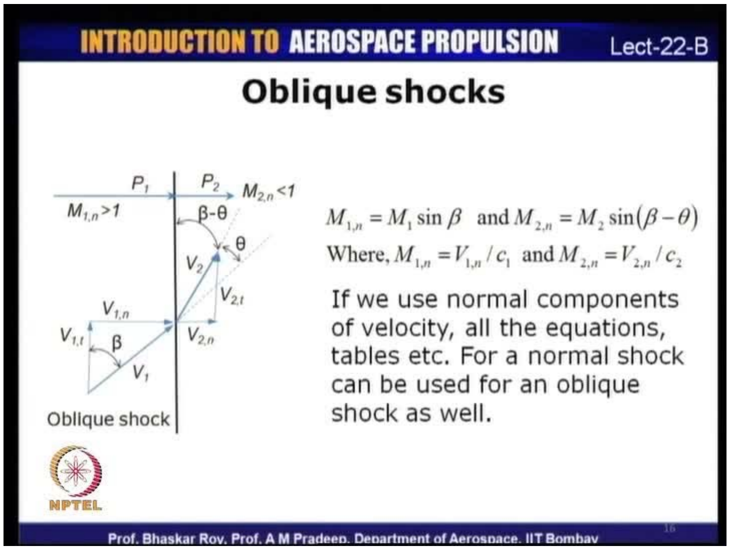

The same set of vectors or the same diagram, if it is tilted and made normal. In the previous case, the oblique shock was inclined at a certain angle, now if we tilt it and make this normal what we get is the following.

So, what we have done is the previous diagram tilted and made normal, so that the shock we have now, the oblique shock is now oriented like this. What we see here is that, the flow or the normal component of this velocity which is approaching the shock which is V 1 n hits the shock at 90 degrees and it leaves the shock also at 90 degrees. This is very similar to what we have discussed for a normal shock.

So, from normal shock tables, we should be able to find out the properties downstream of the shock because now, we have one component of the flow which is normal to the shock itself. In normal shock sense, we have the corresponding Mach numbers M 1 n which is greater than 1 and we have already discussed that flow downstream of the normal shock has to be subsonic. So, in terms of the normal components, V 2 n when converted to Mach number M 2 n will be less than 1, but it is not necessary that M 2 will be less than

1. The normal component upstream and downstream of an oblique shock will be subsonic, but not necessarily the absolute Mach number.

(Refer Slide Time: 34:45) If you look at the flow angles, we have the shock angles, which is beta which is the angle at which the oblique shock is inclined and this is the deflection angle theta and this difference is beta minus theta. From these angles, we can relate that M 1 n which is the normal component of the upstream Mach number is equal to M 1 sin beta - that is, from this velocity triangle, if you see, M 1 n is this component; it is basically equal to M 1 sin beta.

Similarly, M 2 n is for the downstream of the shock; M 2 n is equal to M 2 sin beta minus theta - that is this angle beta minus theta. So, M 2 n is equal to M 2 sin beta minus theta which is primarily this angle here. M 1 n is equal to V 1 n by c 1 and M 2 n is V 2 n by c 2, c 1 is the upstream speed of sound and c 2 is the downstream speed of sound, which will be equal to square root of gamma r t 1 for c 1 and for c 2 it will be square root of gamma r t 2.

So, what follows is that, if we decompose the velocity vectors into normal and tangential component, since the tangential component remains unchanged, we can use the normal shock tables which we discussed for solving an oblique shock; in the sense that we can find the downstream Mach number, the normal component of the downstream mach number from the normal shock tables and once we know the beta and theta angles, we can now calculate the absolute Mach number which is downstream of an oblique shock, which means all the shock tables and equations which are applicable for a normal shock can be extended for the normal components of the velocity vectors that are upstream and downstream of the shocks. Using those relations, we can relate the properties that are upstream and downstream of the shock. That is how you could solve an oblique shock problem very similar to that of a normal shock problem and determine properties which are upstream and downstream of the shock. So, it is possible for us to relate the deflection angle theta to the shock angle beta with the Mach number. If we correlate all the three, we can plot the values of theta, beta and M for different values of or for a range of beta, theta as well as Mach numbers.

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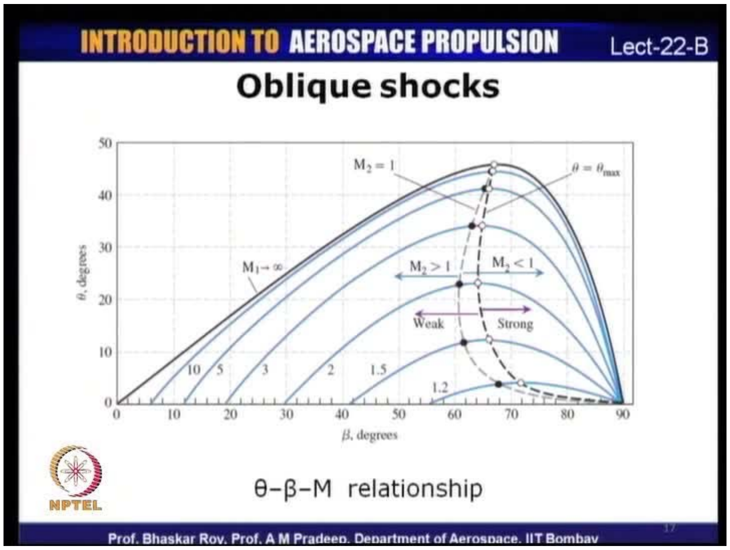

If we do that, we have a chart or a graph or a plot for theta, beta, M, where we have the deflection angle theta on the y axis, the shock angle beta on the x axis and Mach numbers - different contours of the constant Mach numbers.

So, we have Mach numbers starting from very low Mach numbers - sonic Mach numbers all the way up to Mach number tending towards infinity upstream Mach numbers tending towards infinity. You can immediately see that for a particular deflection angle theta and a particular Mach number, it is possible that you have two different wave angles. Let us say for example, we take a deflection angle theta of 10 degrees and an upstream Mach number of Mach number of 2, then it is possible for us to get two different deflection angles - one is around 40 degrees here and another angle, which is on the higher side; it is more than double of that angle.

(Refer Slide Time: 38:42) So, what I have indicated here is that depending upon which angle - the wave angle is the shock wave could either be a weak shock or a strong shock. Correspondingly, it is also possible that the Mach number can either be supersonic or it could become subsonic or in the limiting case it could also be sonic. You can see that at a certain point where Mach number is equal to 1, if you were to join all those points, we get a constant sonic Mach number line which is indicated by this dotted line and the different points at which the Mach number becomes 1 is indicated here. To the left of this line, we have supersonic Mach number and to the right of this line, we have subsonic Mach number.

Similarly, you could also have a maximum theta for a particular Mach number below which, the shock becomes a weak shock or after which, the shock is a strong shock. It basically means that there are certain values of theta and beta as well as upstream Mach number for which the downstream Mach number continues to remain supersonic though it would be less than the upstream Mach number, it would continue to remain supersonic. There are also cases where it could either be sonic or it could become subsonic depending upon the solution of the oblique shock equation.

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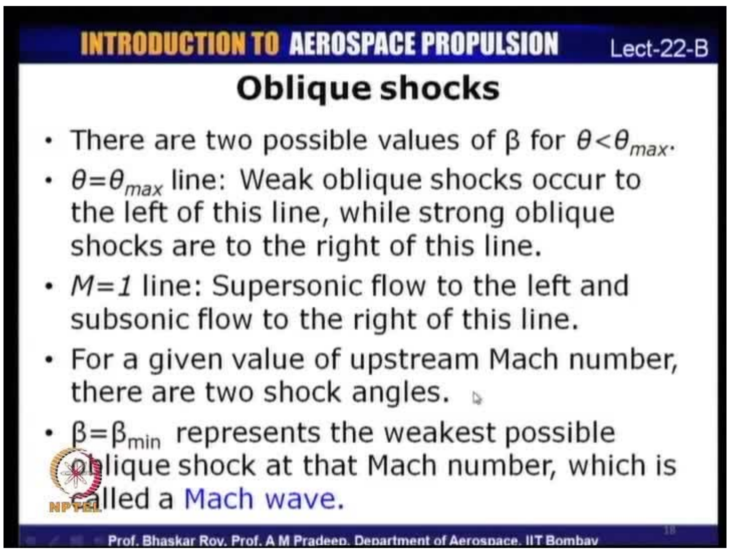

For different values of Mach number, what we can understand from this chart of theta, beta, M is that there are basically two possible values of beta, for any value of theta which is less than theta max. So, if we look at the theta is equal to theta max line, these are lines on the left of which we have weak oblique shocks and on the right of this line, we have strong oblique shocks.

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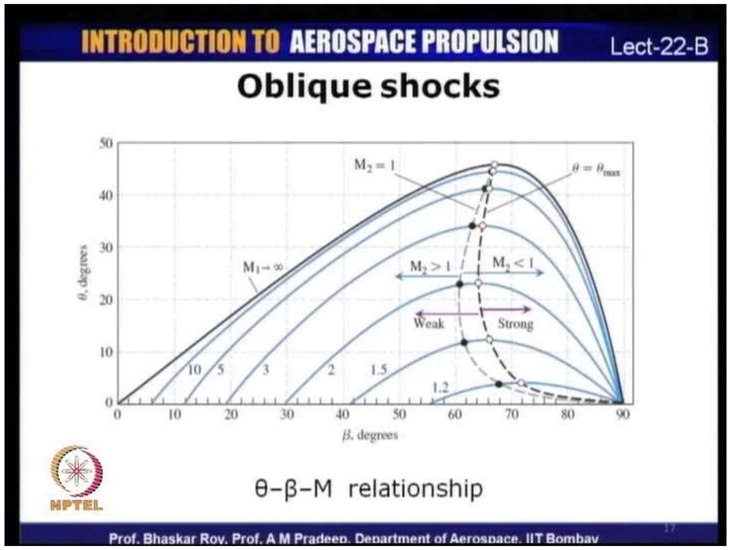

Let us take a look at that once again. This is the line which joins all the theta max lines. That is, if we need to have an attached oblique shock for a given Mach number, let us consider Mach 2, which is what we were discussing. If you look an angle which is greater than this point - that is, around 22 degrees, any angle greater than that for this Mach number, would lead to a shock which is not attached to the surface. So, you would have a case where the shock is detached from the surface. For an attached shock, this is the maximum theta which is permissible for this particular Mach number. It also follows that on the left of this line, we have weak oblique shocks and on the right of this line, we have strong oblique shocks.

We could also join all the Mach number equal to 1 points, on all these Mach number lines and on the left of this line, we have supersonic flow and on the right of this line, we have subsonic flow. For a given upstream value or upstream Mach number, there are basically two shock angles and so, on the left of the constant sonic Mach number line, we have supersonic flow and on the right hand side, we have subsonic flow; which means that, if we have a deflection angle of let us say, 10 degrees and the Mach number is let us say, 2 then if you have a weak oblique shock, then it means that the Mach number continues to remain supersonic because the solution comes on to the left of this line. If it is a strong solution that you have, the Mach number can become subsonic downstream of the oblique shock. In most of the cases, we tend to see the weak oblique shock case that is, the Mach number continuous to remain supersonic, but it is also possible that we can get a subsonic flow downstream of the oblique shock.

For any given value of Mach number and deflection angle, beta is equal to beta min or minimum beta represents the weakest possible oblique shock at that Mach number which is basically known as a Mach wave. (Refer Slide Time: 43:04) That is, for any particular Mach number let us say; Mach 2, the minimum beta that is possible is about 30 degrees here. The Mach number or the shock waves that occur at this particular instance are known as Mach wave. That is, they are very weak shock waves that are present and they are basically known as Mach waves.

So far, we have been discussing about shock waves where in we have an increase in the static pressure and static temperature and so on, downstream of the shock. There are also flow situations which we will discuss now, where in if let us say, the wedge which I had shown for discussing the oblique shock is inclined at a certain angle to the flow, what happens to the flow which is upstream and downstream of the wedge. That is, on certain corners of the wedge we would have shock waves present because the flow is taking a compression corner and on the other side of the wedge we may have what are known as expansion waves or expansion fan, which are present through which the flow will accelerate.

So, in a supersonic flow, which is expanding, we might encounter very weak waves or sonic waves which are basically known as the expansion waves or expansion fan as we denote it.

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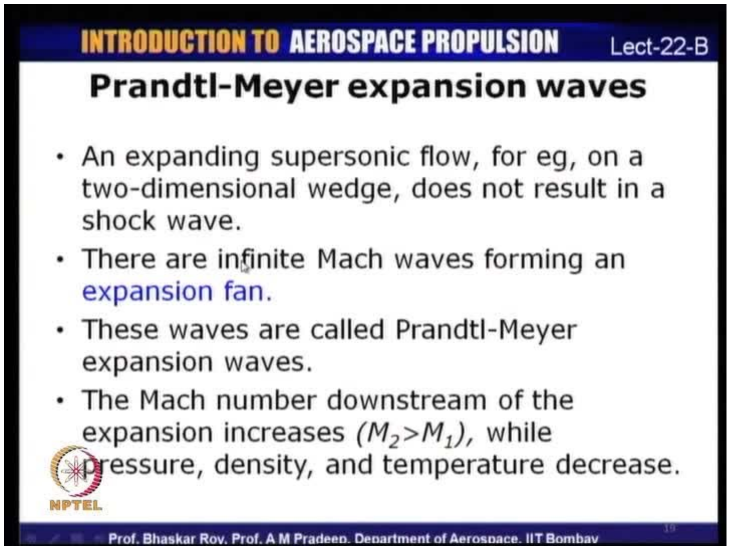

If we look at, for example, a two-dimensional wedge which we had taken for the oblique shock case and if it is inclined at a certain angle and the flow is likely to expand on one of the corners of the wedge, then we see an infinite number of Mach waves which will originate from a particular point on the wedge and that is basically known as a Mach wave. These Mach waves are also often referred to as the Prandtl-Meyer expansion waves and we will take a look at what we mean by Prandtl-Meyer expansion waves. Prandtl-Meyer expansion waves basically can denote an infinite number of Mach waves which form, when we have a wedge which is at a certain angle of attack or under any other expansion corners and the Mach number downstream of the expansion fan increases unlike a shock wave, where the Mach number decreases and also pressure temperature and density decrease, which is exact opposite of what happens across a shock wave.

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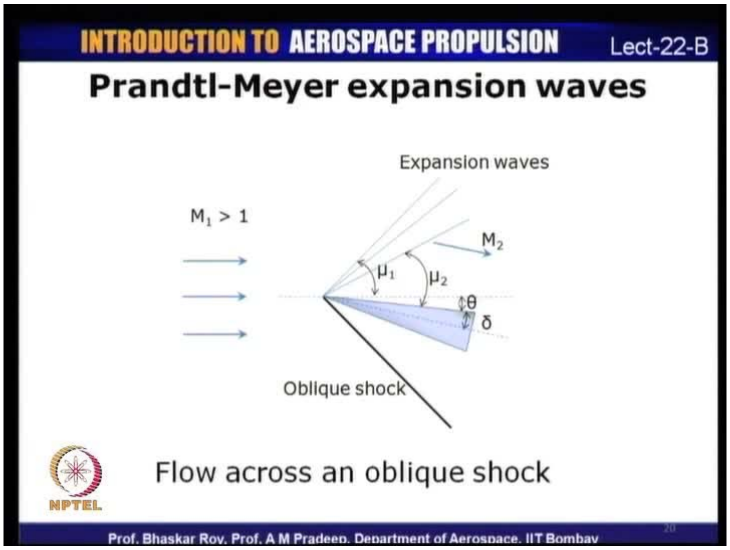

This was the wedge I had shown for an oblique shock. Now, if the wedge was not aligned to the flow and it is at a certain angle, then the flow encounters a compression corner on one surface and on the other surface, it encounters an expansion corner. On the compression corner, we continue to have an oblique shock whereas, on the expansion corner because the flow has to now expand, it is a supersonic flow and there is an increase in area, as you can see here and so, it has to be in expansion flow. This occurs through the presence of these expansion waves and the inclination of these expansion waves are usually denoted by the symbol mu and there could be infinite number of these expansion waves. I have shown only a few of them.

So, downstream of these expansion waves, the Mach number increases. That is, M 2 would be greater than M 1. So, there is an increase in Mach number across an expansion wave whereas, there is a decrease in the Mach number across an oblique shock.

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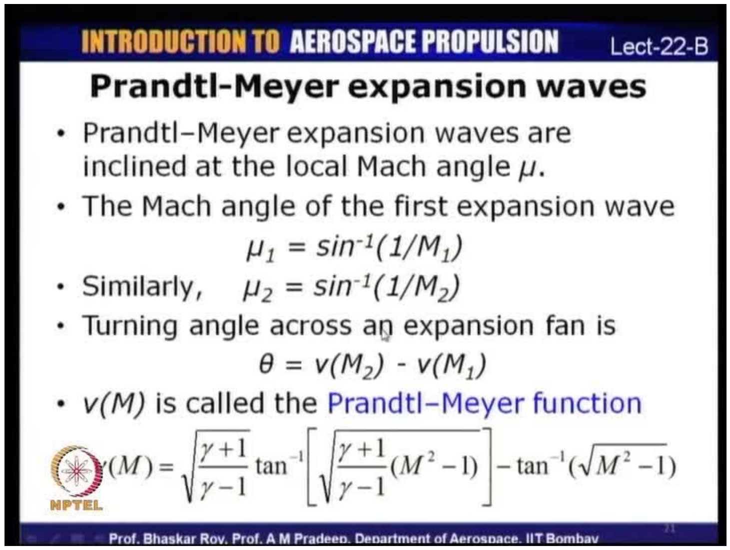

Prandtl-Meyer function or Prandtl-Meyer expansion waves are inclined at a local Mach number or local Mach angle, which is basically denoted by mu. So, mu is the Mach angle. The first expansion wave mu 1 can be shown to be equal to sin inverse of 1 by M 1. Similarly, mu 2 is equal to sin inverse 1 by M 2, where mu 2 is the expansion. It is basically the angle for the last expansion wave. So, the turning angle across the expansion fan theta is equal to nu of M 2 minus nu of M 1, where nu of M is known as the Prandtl-Meyer function. That is, here we denote a function nu, which is a function of the Mach number and it is also related to the ratio of specific heats. Turning angle across an expansion fan can be related to the Prandtl-Meyer function at Mach 2 and at Mach 1, where the Prandtl-Meyer function can be related to the Mach number in the form of this expression, which is basically equal to square root of gamma plus 1 by gamma minus 1 into tan inverse square root of gamma plus 1 by gamma minus 1 multiplied by m squared minus 1, minus tan inverse square root of M squared minus 1.

So, this basically denotes the Prandtl-Meyer function. From the Prandtl-Meyer function for upstream and downstream Mach numbers, you can calculate the Prandtl-Meyer function and the difference between these two functions basically denotes the turning angle for such a case.

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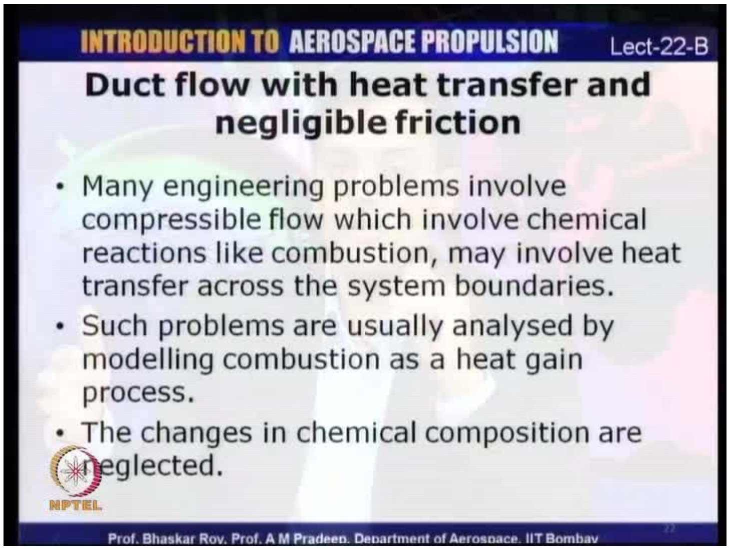

What we will discuss next are slightly different from what we have discussed. It is related in some sense because you are going to talk about Rayleigh and Fanno functions and Fanno processes. We will first take up a duct flow with heat transfer with negligible friction. So, there is a duct flow case, where we consider a duct of constant area and there is heat transfer into or from the system, but there is negligible friction and this is encountered in several engineering problems. For example, in a combustion chamber, we have heat transfer into the combustion chamber, but if we assume friction to be negligible then we can approximate this particular process in a simple way. That is, we basically model combustion as a heat gain process and we neglect chemical composition across the duct.

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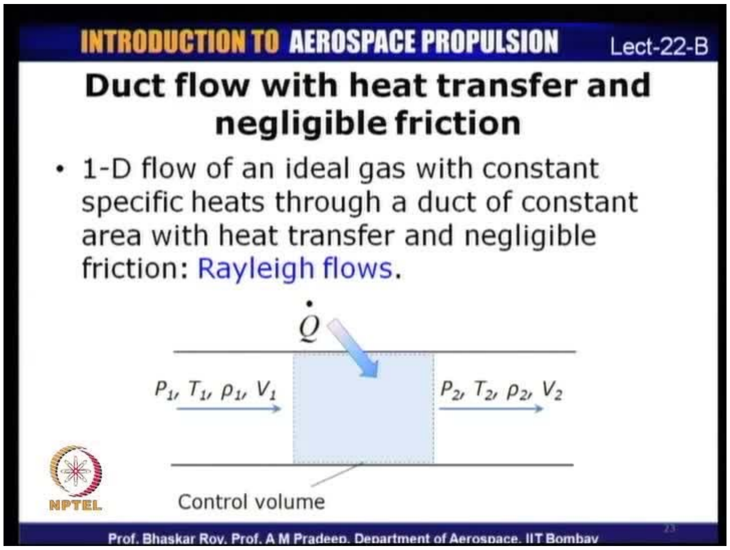

This is what we had done for analyzing heat transfer across a combustion chamber. The 1-dimensional analysis or flow through of an ideal gas with constant specific heat through a duct of constant area with heat transfer and negligible friction are known as Rayleigh flow. Rayleigh flow is basically heat transfer into a duct of constant area of an ideal gas with negligible friction. So, we are going to assume that there is no friction occurring here, there is only heat transfer which causes change in properties across the control volume.

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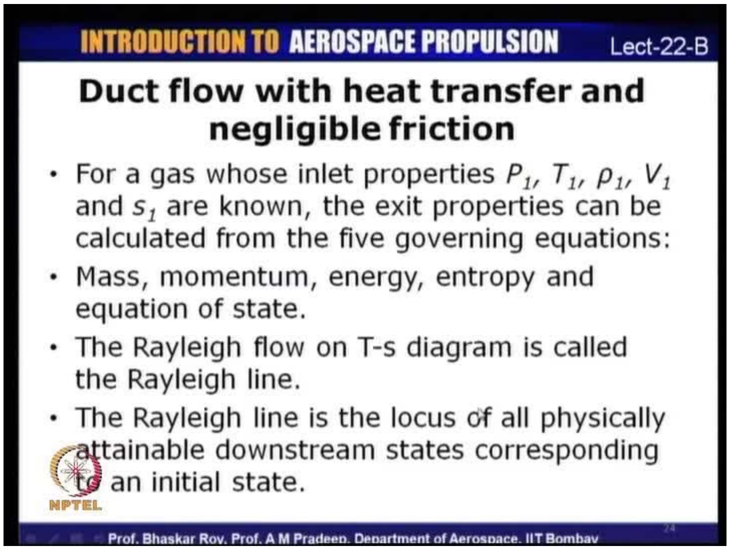

If you have a gas which has a set of inlet properties P 1, T 1, density rho 1, V 1 and entropy S 1 which are known, the exit properties can be calculated from the five governing equations of mass, momentum, energy, entropy and equation of state. If we were to represent the Rayleigh flow on a T-s diagram, that is known as a Rayleigh line. So, Rayleigh line represents the locus of all physically attainable downstream states corresponding to an initial state. If you define a particular initial state with pressure, temperature, density, velocity and entropy, Rayleigh line represents all the properties downstream which are physically attainable, and which primarily come from solution of all the five governing equations.

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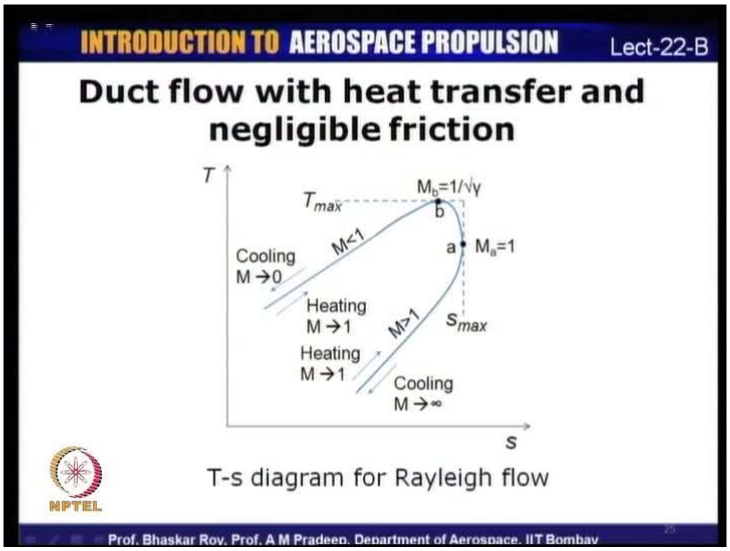

If we plot the Rayleigh process on a T-s diagram - temperature entropy diagram then we have a very interesting phenomena that is taking place here. That is, we have this blue line that is shown that is, the Rayleigh line. We can see that as we continue to add heat in a supersonic flow which is greater than 1 then as we continue to heat, it approaches Mach number equal to 1. Similarly, in a subsonic flow, if we continue to add heat, its Mach number increases and approaches Mach number equal to 1 again and the reverse happens for cooling as well. There are two distinct points I have shown here. One is the point of maximum entropy which in the case of supersonic flow, occurs when it reaches its limiting Mach number - that is, Mach number equal to 1 and that is also happening for a subsonic flow, where its Mach number increases and finally, reaches a Mach number equal to 1 at point a, which is the point of maximum entropy.

In the case of subsonic flow, we also have a point of maximum temperature T max which means that as you continue to add heat in a subsonic flow, it attains a maximum temperature which is given by the Rayleigh line up to Mach T max, beyond which if you continue to add heat, the temperature actually reduces. That means that between points a and b in a subsonic flow, if you add heat, it could actually lead to a drop in temperature.

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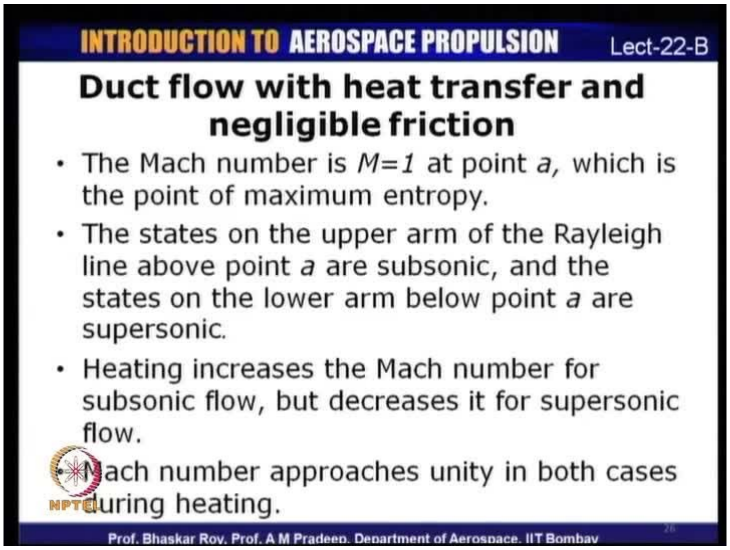

If we were to summarize this Rayleigh line equation, the Mach number at point a is corresponding to sonic Mach number which is Mach 1 - point of maximum entropy. On the upper arm of the Rayleigh line that is, above point a, the flow is subsonic and the states on the lower side of point a are supersonic. Heating increases a Mach number for a subsonic flow, but it decreases for supersonic flow and in both the cases, Mach number approaches unity during heating. That is, in both subsonic as well as supersonic flow, the Mach number approaches unity during heating.

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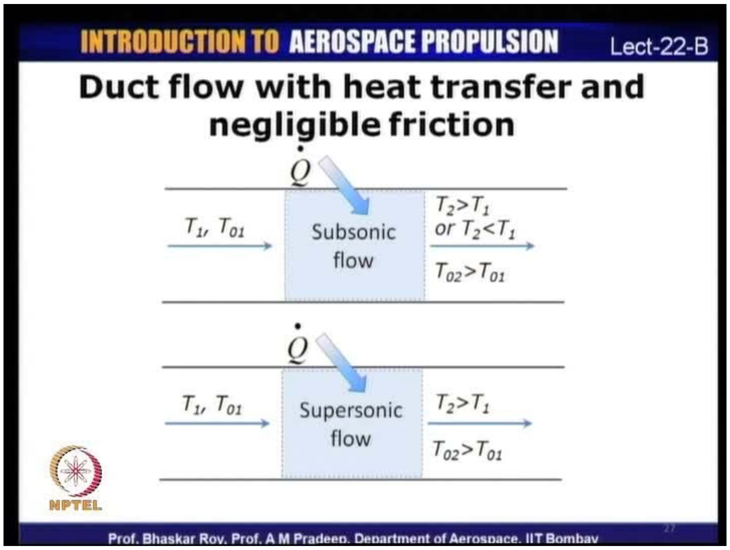

What happens in a Rayleigh line process is that if the Mach number is subsonic and if it is subsonic flow then for a given or predefined temperature upstream, downstream stagnation temperature increases because you are adding heat. So, stagnation temperature has to increase, static temperature may increase or decrease, depending upon where you are on the Rayleigh line; this is on a subsonic flow.

In a supersonic flow, case stagnation temperature increases, static temperature also increases because the limiting case for that is Mach number equal to 1. There is no change of curve there. Whereas in a subsonic flow, there is a T max after which, the temperature reduces for a certain period.

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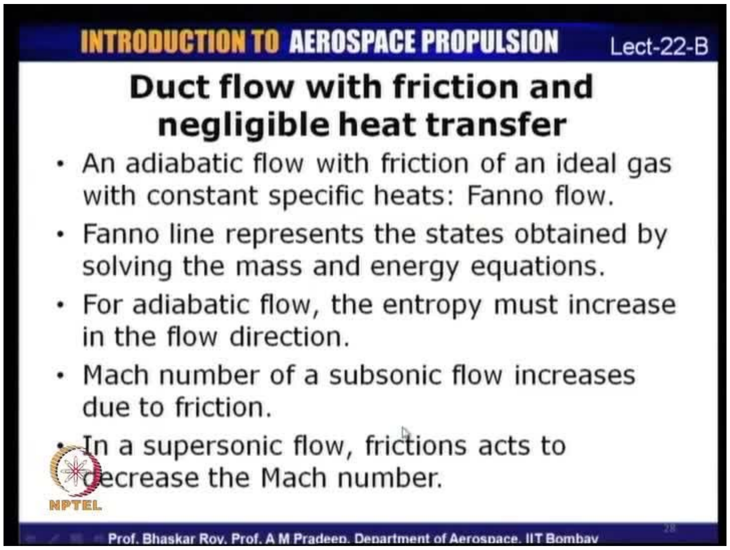

Now, we shall consider another set of flow which is duct flow with friction, but negligible heat transfer. The Rayleigh equation or Rayleigh line represented duct flow with heat transfer with negligible friction. Now, an adiabatic flow with friction of an ideal gas with constant specific heat is known as Fanno flow. Similarly, Fanno line represents the states obtained by solving the mass and energy equations; we have seen this earlier, when we are talking about normal shocks. For an adiabatic flow, the entropy must increase in the flow direction because there is friction and so, in the case of subsonic flow, the Mach number increases due to friction; in supersonic flow, friction acts to decrease the Mach number. in the case of supersonic flow

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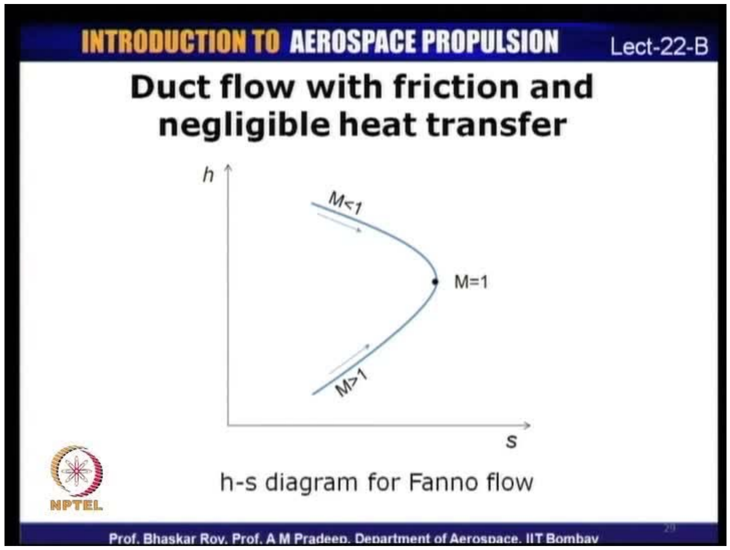

On a h-s diagram, we can represent a Fanno line similar to that of a Rayleigh line. That is, in a supersonic flow because of friction, entropy is increasing. There is no heat transfer and due to friction, the Mach number increases and in the limiting case, it reaches Mach number equal to 1.

Beyond this, the Mach number cannot reduce and this state is known as the choking which we have seen for nozzle flows as well. In the case of subsonic flows with friction or due to friction, the Mach number would increase and in the limiting case that is at choking point, the Mach number reaches 1 beyond which, if you try to pass more flow, it would actually lead to decrease in mass flow.

So, a Fanno line basically states that in supersonic flow due to friction it basically acts to reduce the Mach number to the limiting Mach number of Mach number 1. In subsonic flow, it leads to increase in Mach number up to a Mach number of a sonic Mach number - that is, unity

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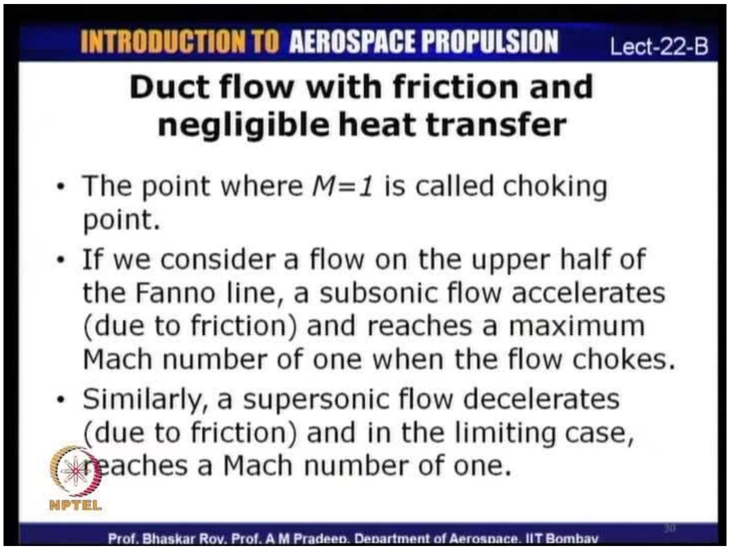

The point where Mach number is 1 is known as choking and so, it is possible that in a subsonic flow, you can accelerate the flow; it basically happens because of friction. In the case of supersonic flow, the flow decelerates and in the limiting case, it reaches a Mach number of 1.

So, let me summarize what we had discussed in today's lecture. We had discussion on few aspects of compressible flows which primarily happened in supersonic flows, the presence of normal shocks, oblique shocks and Prandtl-Meyer expansion waves and subsequently, we discussed about two different duct flow cases: one was duct flow with heat transfer and negligible friction known as the Rayleigh flow and the second case was a duct flow with friction and negligible heat transfer; that was known as the Fanno flow.

In both these cases, we have discussed about how the properties of the fluid vary, how Mach number changes and what is the limiting case for each of these duct flow problems. So, these were some of the aspects we had discussed during this lecture on compressible flows. This was primarily an extension of what we had discussed in the last lecture to begin with on compressible flows. We had more discussion on shock waves and different types of shock waves in today's lecture.