Introduction to Aerospace Propulsion

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Module No. # 01

Lecture No. # 22 - A

One-dimensional compressible flows, isentropic flows

Hello and welcome to lecture number 22 of this lecture series on introduction to aerospace propulsion. Over the last several lectures, we have got introduced to several aspects of thermodynamics, thermodynamic principles, and also laws of thermodynamics, and how to use them in applications like in thermodynamic cycle analysis and so on. What we are going to discuss today is a little bit different from what we have been discussing so far - in the sense that, this particular topic that we are going to take up for discussion today and also in the next lecture combines some of the thermodynamics principles with fluid mechanics in some sense. What we are going to discuss about is on compressible flows and what are the different properties of compressible flows. We shall look at compressible flow through some simple cross-sectional geometries like convergent nozzles and converging diverging nozzles, and so on.

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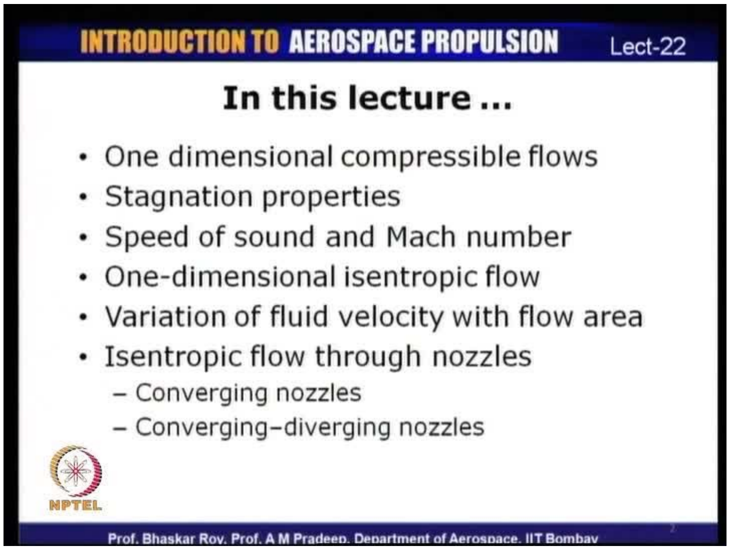

This is a slightly different topic from what we have discussed so far. This is going to also use many of the thermodynamic principles that we have been discussing in the last few lectures. What we shall discuss in today's lecture are the following: we will begin our talk today with discussion on what do you mean by one-dimensional compressible flows.

We shall talk about stagnation properties and then we shall talk about speed of sound and Mach number; how do you define Mach number and then we shall look at one-dimensional isentropic flow and variation of fluid velocity with flow area. We will derive an equation which defines or governs the variation of the fluid velocity with area. We will also be discussing towards the end of the lecture on isentropic flow through nozzles. We will discuss two types of nozzles: converging nozzles and converging diverging nozzles. These are some of the topics that we shall be discussing in today's lecture. You might have got a feeling right now that this is going to be a discussion primarily on compressible flows and also on their significance in terms of analysis of compressible flows.

During our thermodynamic analysis that we have discussed and also we have solved several problems using the thermodynamic principles, we have always made an inherent assumption that at a given state of a system, the density is a constant. So, there is an inherent assumption that the flow is incompressible in the sense that we do not really consider variations of density which is not necessarily true in many of the engineering applications. In most of the day-to-day applications that we are familiar with, the velocities of the fluid are very low and therefore the inaccuracy that we achieve because of assuming incompressible flow is not really high. Whereas, if fluid velocities are very high, then it is not really possible for us to make this assumption that the flow is incompressible. The flow no longer remains incompressible. Therefore, we should be taking into account the compressibility effects and that is one of the aspects or that is one of the reasons why we are taking up this topic for discussion today. There are a lot of engineering applications where the fluid velocities can be significantly high. Therefore, we cannot really assume that the flow is incompressible, changes in density are incompressible or changes in kinetic energy are incompressible for that matter.

What we shall discuss to begin with is that the significance of the compressible flows. Why do we need to discuss them? Basically, there are certain applications where it is important for us to understand or analyze the system in the form of considering the variations in density and also taking into account the variations or effect of kinetic energy; though potential energy may still be negligible, kinetic energy cannot be neglected.

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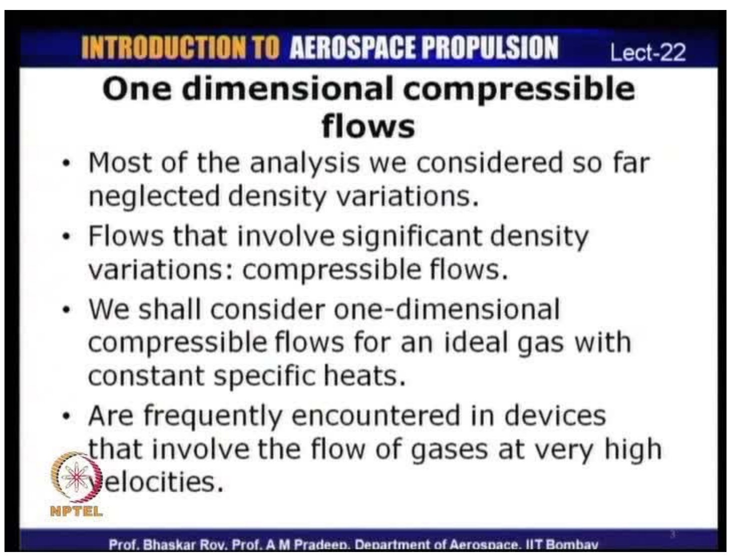

So, the flows that basically involve significant density variations are known as compressible flows. Though, most of the analysis we have considered so far neglected density variations, we shall also take a look at some applications where this effect can no longer will be neglected. We shall be making an assumption here that even when we consider the flow to be compressible, we shall be analyzing it in a one dimensional sense for an ideal gas with constant specific heat. We are going to assume that the specific heat is a constant and the gas is ideal. Therefore, we can assume the ideal gas behavior and so on and. Where do we see such applications? In devices that involve flow of gases at very high velocities like in nozzles and so on. It is important that we have an understanding of how to analyze such systems which involve very high velocities. Therefore, changes in kinetic energies can no longer be neglected.

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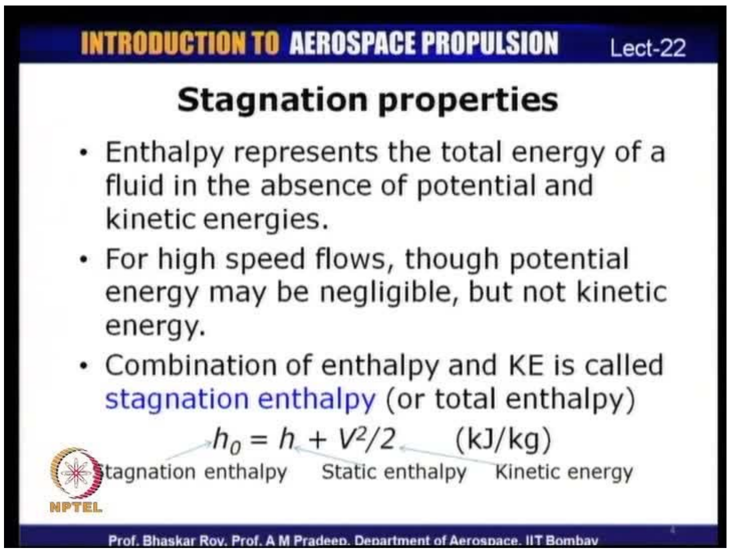

In order that we analyze a system where kinetic energy cannot be neglected, we need to define what is known as stagnation property of a system or different types of stagnation properties. One of the stagnation properties to begin with, we shall understand is the stagnation enthalpy. We have already defined what is enthalpy. We know now that enthalpy represents the total energy of a fluid in the absence of potential and kinetic energy. This comes from the first law of thermodynamics. When we take up this property of enthalpy at high speed flows, potential energy being negligible may still be valid, but not kinetic energy. So, we have to take into account, the changes in kinetic energy.

What we do is, we combine enthalpy with kinetic energy; that is we add up enthalpy and part of the kinetic energy and we define a new property known as the stagnation enthalpy. Why it is known as stagnation enthalpy will be clear when we discuss further. This, so called static enthalpy, which I will now be referring to as static enthalpy and the kinetic energy term put together is known as the stagnation enthalpy. If you look at the definition of stagnation enthalpy which I had defined, stagnation enthalpy which is usually denoted by h subscript 0, 0 is applicable for most of these stagnation properties which we are going to define like temperature pressure and so on.

Stagnation enthalpy, h naught is equal to h which is now the static enthalpy plus V square by 2, which is the kinetic energy. You might notice that these are all per unit mass and therefore, its specific stagnation enthalpy is the sum of the specific static enthalpy plus V square by 2. So, the first term on the right hand side is the static enthalpy which does not have a subscript 0 and the second term is the kinetic energy term.

On the left hand side, we have the stagnation enthalpy. This is how you would define a stagnation enthalpy which is primarily the sum of enthalpy as we had defined earlier, which we are now calling as the static enthalpy. Whenever we take up different properties at high speed cases - that is incompressible flows, we also define these properties as stagnation as well as static parameters. So, static parameter will become stagnation parameter if the velocities are 0 - that is if kinetic energy is 0, then, static and the stagnation properties are the same.

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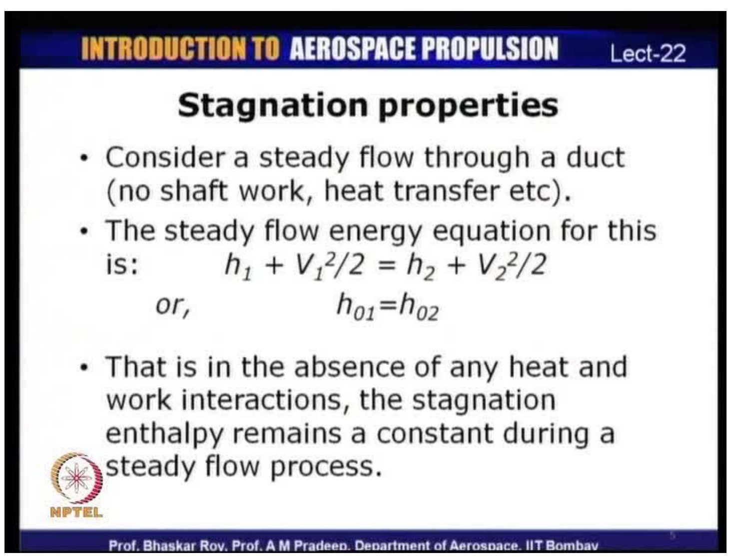

We will now use this principle to define other properties in terms of stagnation parameters like stagnation pressure and stagnation temperature, stagnation density and so on. If you consider a steady flow through a duct, let us say, through some diffuser or a nozzle where there is no shaft work; there is no heat transfer etcetera. Steady flow energy equation for this is something you have already defined - that is h 1 plus V 1 square by 2 is equal to h 2 plus V 2 square by 2. The left hand side as we have not defined is the stagnation enthalpy at state 1, which is at 01. On the right hand side, we have stagnation enthalpy at state 2 - that is h 02. This means that in the absence of any heat and work interactions, if there are no heat transfers or there are no work interactions, the stagnation enthalpy remains a constant during a steady flow process.

This is a very significant property that we need to keep in mind that if there are no heat interactions or there are no work interactions in a steady flow process, then this stagnation enthalpy of such a process remains the same. It does not change. If there are no heat transfers into the system or from the system, stagnation enthalpy does not change. That is something we have to keep in mind; because many of the systems that we are going to consider will have this property to be used - that there is no heat and work interactions taking place like in nozzles and diffusers. Therefore, stagnation enthalpy has to be a constant; but this is not applicable for systems like turbines or compressors, because there is a work interaction taking place in the case of compressor - work done on the system in the case of turbine, work done by the system. So, stagnation enthalpy obviously will not be a constant in such cases; it is only applicable for those cases where there are no work or heat interactions.

We have now seen that if you look at this duct example that I was mentioning, where we had - h 1 plus V 1 square by 2 is equal to h 2 plus V 2 square by 2. Let us say that at state 2, the velocity is now equal to 0. That is, by some means, we bring the velocity at state 2 to be 0. Now, what we have is h 1 plus V 1 square by 2 is equal to h 2 which is also equal to h 02; because as we have discussed, if velocity is 0, static and stagnation parameters are the same; which means that - at state 2, the static enthalpy and stagnation enthalpy are the same. So, what does it mean is that stagnation enthalpy also represents the case where the fluid velocity is isentropically or adiabatically brought to rest.

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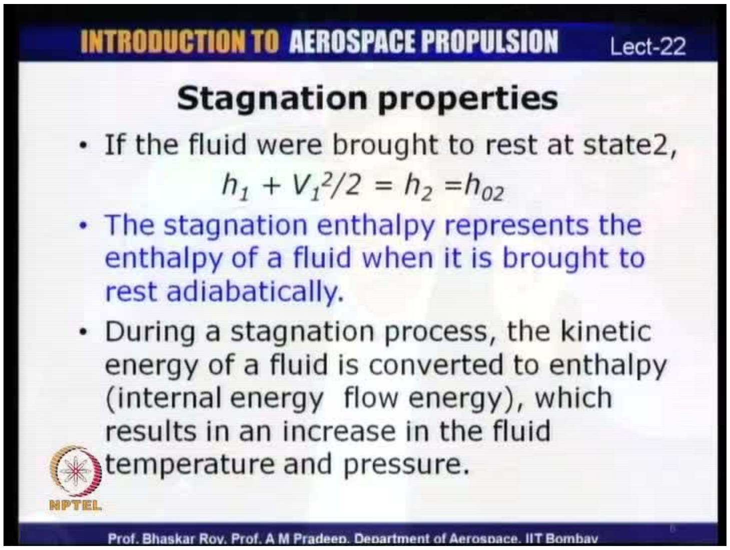

So, at state 2, if you were to bring the fluid to rest, we have - h 1 plus V 1 square by 2 is equal to h 2 which is also equal to h 02, which means that stagnation enthalpy represents enthalpy of a fluid when it is brought to rest adiabatically. That is because in this case, there was no heat interaction or work interaction across the system boundaries. If you bring the fluid to rest adiabatically and in which case stagnation enthalpy basically represents the enthalpy of a fluid when it is brought to rest adiabatically. What happens during this stagnation process? This is why, it is basically called stagnation enthalpy. The term stagnation comes because we are assuming that the fluid is adiabatically coming to rest. That is how, the term stagnation enthalpy arise.

During a stagnation process, kinetic energy of a fluid is converted to enthalpy - that is internal energy and flow energy, which results in increase in the fluid temperature and pressure. If you were to bring a fluid to rest adiabatically, as part of conservation of energy principle, that energy has to get transformed into some other form. What happens is, the fluid energy increases in the form of flow energy and so on so that ultimately leads to an increase in temperature and pressure of the fluid.

Based on this, we shall now define what are known as stagnation pressure and stagnation temperature. On the left hand side, if you recall we had - 1 plus V 1 square by 2 which was equal to h 2 and in turn equal to h 02. In general, we can write h naught, stagnation enthalpy is equal to h plus V square by 2, which means that for an ideal gas, which is where we began our discussion that we are going to assume that the gas is going to be ideal with constant specific heats.

So, with constant specific heats for an ideal gas, enthalpy is simply equal to c p times T. Therefore, we have c p T naught is equal to c p T plus V square by 2 or T naught is equal to T plus V square by 2c p. Left hand side, we have a temperature with a subscript 0 that is known as the stagnation temperature. Stagnation temperature is equal to the sum of the static temperature plus - one term which we are going to define as the dynamic temperature - because that changes with fluid velocity and that is why it is called dynamic temperature.

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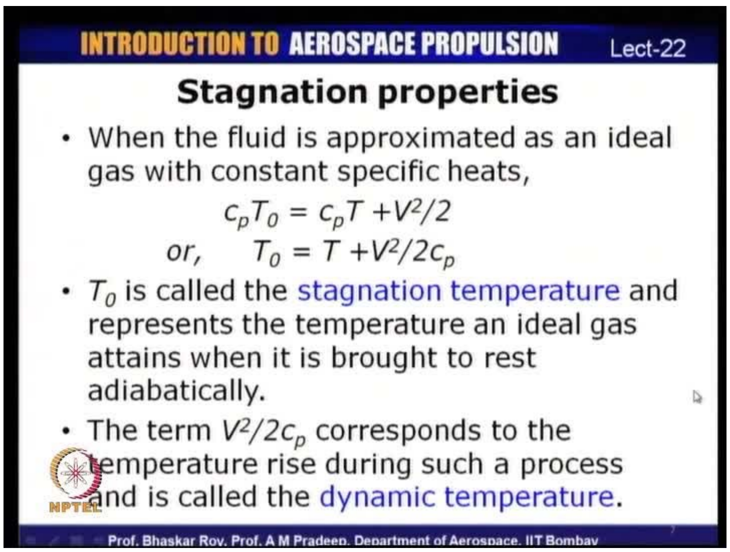

If you write the equation for enthalpy in the form of product of specific heat at constant pressure to the temperature, then, we have - c p T naught is equal to c p T plus V square by 2 or T naught is equal to T plus V square by 2c p. Here, T naught is defined as the stagnation temperature and this represents the temperature an ideal gas will attain, if it is brought to rest adiabatically. That is similar to what we did for enthalpy. If you bring a gas to rest adiabatically, the temperature that the gas attains at the end of this process - that is when it comes to rest adiabatically, is known as the stagnation temperature. The second term, that is - V square by 2c p corresponds to the temperature during such a process and because it can change with velocity, it is called dynamic temperature.

You can immediately see that for non-zero velocities, stagnation temperature will always be greater than the static temperature which is something we have defined earlier as well. That is, as you bring a fluid to rest, the energy gets transformed into internal energy and so on, which ultimately leads to an increase in temperature and pressure, which we have now defined as stagnation temperature and pressure. This has to be higher than the static temperature; because, it has an additional energy term which is the dynamic temperature term. Similarly, we can also define pressure - that is stagnation pressure which is the sum of the static pressure plus the dynamic pressure. Stagnation pressure is equal to P naught is equal to P plus half rho V square. I guess you might have learned this in fluid mechanics and this was basically as part of the Bernoulli equation. We can immediately see that there is a direct correlation between what you get in thermodynamics with what you feel or what you get in fluid mechanics.

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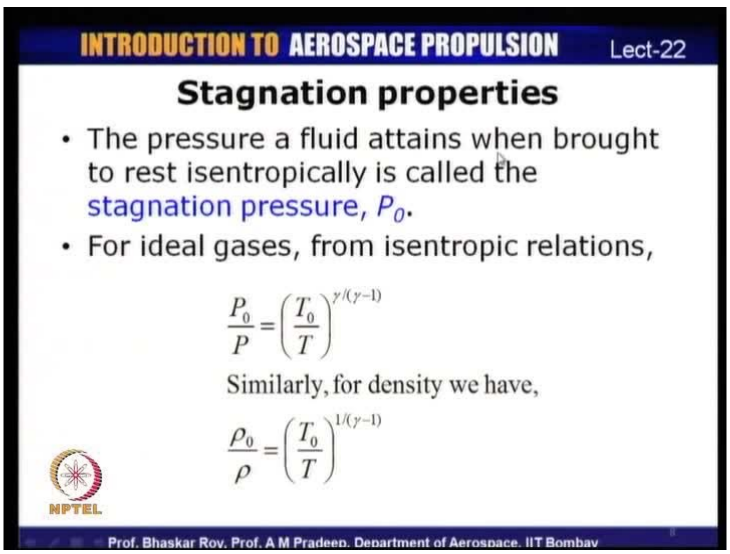

Stagnation pressure is equal to static pressure plus this dynamic pressure. What we are trying to say here is that - there are parameters which we need to take into account when the fluid velocities cannot be neglected. So, kinetic energy needs to be accounted for in calculating parameters. Similarly, the way we have defined for temperature, we can also define stagnation pressure. The pressure that a fluid will attain when it is brought to rest isentropically is known as the stagnation pressure. What we are saying here is that - all these stagnation parameters whether it is temperature or pressure or enthalpy will have a value greater than that of the corresponding static parameters for all non-zero velocities.

From the ideal gas, we have already assumed that all these cases that we are discussing about will be for ideal gas. So, from the isentropic relations that we had discussed, we can relate the stagnation pressure to the stagnation temperature in terms of the pressure and temperature ratios. If you were to relate the pressure ratios to the temperature ratios, we have P naught by P is equal to T naught by T raise to gamma by gamma minus 1, where gamma is the ratio of specific heat which is equal to c p by c v.

Similarly, we can define it for density which is rho naught by rho is equal to T naught by T raise to 1 by gamma minus 1. Again, gamma is equal to the ratio of specific heats. So, these are isentropic relations which we will be using very frequently in our thermodynamic analysis of different cycles. Some of them which we have already discussed was not in the form of the ratio of stagnation and temperatures and pressures; but in the form of static pressures and temperature ratios. In some of the later analysis, we will be doing for aircraft engines where the kinetic energy terms cannot be neglected. These equations will be used very frequently.

Now, what we shall do now is to see what happens if there is a change in - let us say, the stagnation pressure. Stagnation pressure, we have already seen that stagnation enthalpy does not change as long as there are no heat transfer or work transfer across the system boundaries which means that since stagnation temperature is directly related to stagnation enthalpy, this also is applicable to the stagnation temperature. Therefore, stagnation temperature also does not change as long as there are no heat and work interactions across the system boundaries. But it is applicable for pressure, because pressure is a parameter for ideal gases. It is not directly related to the enthalpy; it is the temperature which happens to be related to enthalpy. So, what about pressure?

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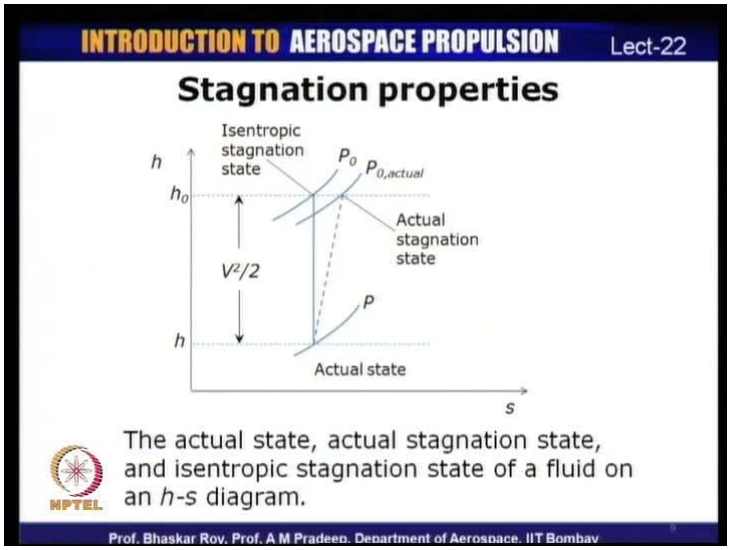

To understand that, what we will do is to look at a process where we could have a loss in total pressure and we will see what correspondingly happens to the enthalpy. What we have done here is that on an enthalpy entropy scale, we have h and s scale here. This is let us say, a compression process, where there is an increase in pressure or even if there is no compression, let us look at fluid which is just a having a certain velocity which means that it will have stagnation parameters which will have to be accounted for. This is basically, the sum of the static parameters plus the dynamic term. So, the actual state of the fluid is represented here at state 1, which is shown there. This is the initial state where it has a certain enthalpy, h correspondingly there is an entropy as well. This is on a constant pressure line. So, this pressure line that is seen here is P; it is a constant pressure line. What are the corresponding parameters for this particular fluid, if you were to look at the stagnation parameters?

We know that stagnation enthalpy - h naught is equal to h plus V square by 2. We have h naught which is equal to h plus -this dynamic term, that is -V square by 2. This is the isentropic stagnation state.; that is, if the process were to be isentropic, then, we get this straight line. Because entropy is a constant; it is a straight line. It is an isentropic stagnation state; where the enthalpy at the end of this process is equal to h naught and the corresponding stagnation pressure is equal to P 0 or P naught. If the process is not isentropic, that is - if you look at a process which has frictional losses and some such irreversibilities, it cannot be any more isentropic.

So, if you look at an actual stagnation state, there has to be an increase in entropy. There will be a certain positive slope for that particular process. There are no heat or work interactions, which means that the enthalpy should not change; because there are no heat or work interactions which are taking place. Therefore, enthalpy cannot change in such a process. Because of irreversibilities, it is non-isentropic and there will also be some pressure loss because of irreversibilities.

If there is a pressure loss, then, we have a certain slope for the process. We have on the same enthalpy line; because, enthalpy does not change, stagnation enthalpy line extended, the process ultimately meets the stagnation enthalpy line and the corresponding pressure is the actual total pressure. So, P naught actual need not be equal to P naught isentropic, because there could be a pressure loss - total pressure loss - taking place because of -let us say- friction. Friction can cause decrease in velocities which means that at the end of the process, you have a lower velocity and correspondingly - a lower total pressure. Therefore, P naught actual can be less than the P naught ideal or P naught isentropic. So, it is not necessary that the total pressure remains the same at the end of such a process. What should remain the same is the enthalpy.Enthalpy does not change because there are no heat or work interactions taking place.

This is a very important aspect that we need to keep in mind. In a process, where there are no heat or work interactions, the stagnation enthalpy cannot change. Therefore, stagnation temperature also does not change. What is possible is that there could be a difference between the stagnation pressure, ideal or isentropic to the stagnation pressure actual - which could be because of frictional losses, which could lead to non-isentropic processes. There could be P naught actual which is less than P naught ideal. In the ideal case, if we assume all irreversibilities to be 0, then P naught actual will be equal to P naught ideal, because there are no more pressure losses.

So, this is a very important aspect that you definitely need to keep in mind. We will keep using this aspect in many of the analysis that we are going to do in some of the later lectures when we analyze ideal cycles and real cycles of gas turbine engines. What we have discussed now are on stagnation parameter, stagnation enthalpy, stagnation pressure, stagnation temperature, density and so on. The bottom line is that - in the absence of heat and work interactions, stagnation enthalpy and stagnation temperature cannot change, but what is possible is that you may have a change in stagnation pressure due to some irreversibility and non-isentropicity of the process.

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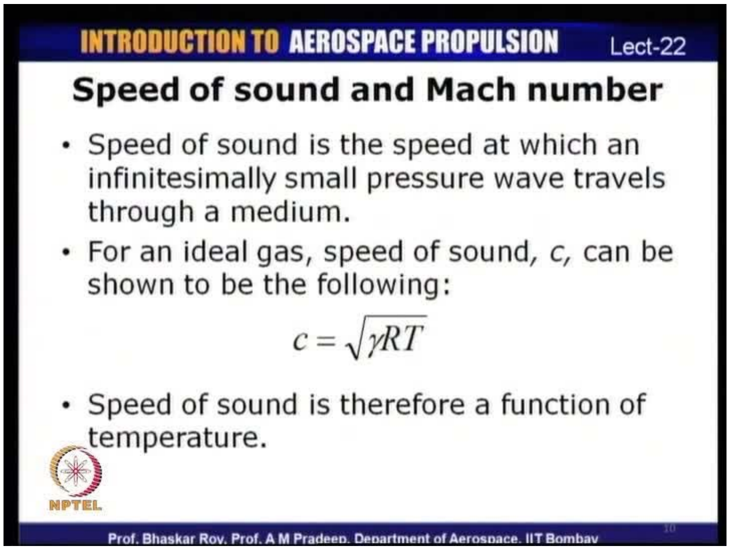

What we shall discuss now is a different aspect related to compressible flows. We will be defining what is known as the Mach number. Before we define that, we need to define what is meant by speed of sound. Speed of sound is the speed at which an infinitesimally small pressure wave travels through a medium. Sound is a pressure wave, a small pressure wave. Speed at which the infinitesimally small pressure wave travels through a medium is basically the speed of sound.

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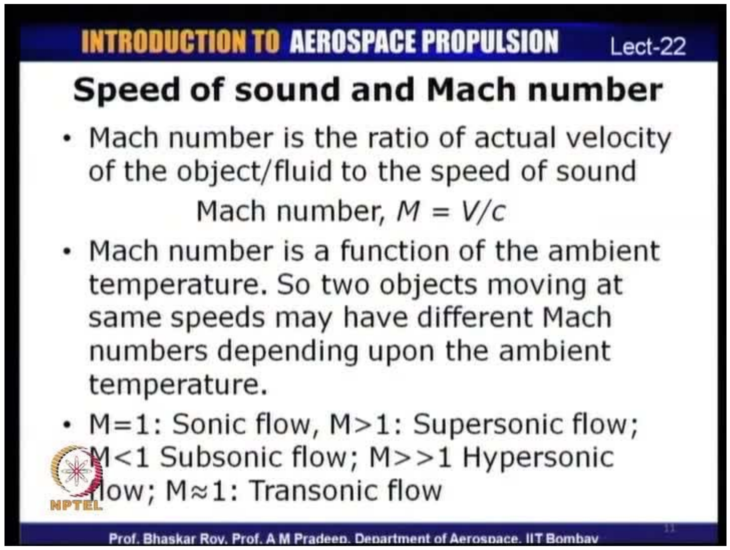

If you assume an ideal gas, speed of sound which is usually denoted by symbol c, c can be shown to be equal to square root of gamma RT - where gamma is the ratio specific heats. R is the gas constant for the medium and T is the temperature, static temperature. So, c is a direct function of temperature. Speed of sound is a direct function of temperature and it also depends upon the ratio of specific heats and the gas constant. For a particular medium, both of these are constant as long as we assume that ratio of specific heat does not change with temperature. We can see that speed of sound is a direct function of the static temperature. Based on this, we are going to define a non-dimensional parameter, which is - taking the ratio of the velocity of fluid or the object to the speed of sound and that is known as the Mach number. So, Mach number is the ratio of the actual velocity of an object or fluid to the speed of sound. In some cases, the fluid may be moving and the object is stationary which is what we would do in internal testing. For example, the fluid is at a certain speed; the object is stationary, whereas, on the other hand, an actual aircraft moves at a certain speed in a medium where the air is relatively at zero velocity.

So, ratio of that speed to the speed of sound is known as the Mach number. Mach number is defined as V by c, where V is the velocity of the object or fluid and c is the speed of sound which is equal square root of gamma RT. So, Mach number is a function of the ambient temperature as we have seen, which means that it is possible that an object which is moving at the same velocity in two different mediums of two different temperatures will have different Mach numbers. Even though their velocities are same, they are in different mediums which have different temperatures, which means that the speed of sound will be different for different mediums depending upon their temperature. Therefore, it is perfectly possible that Mach number of two objects which are moving with the same velocity but in different mediums which have different temperatures, the Mach number can certainly be different. It is not necessary that if the velocity is same, Mach number has to be the same.

Depending upon Mach number, if we have a case where Mach number is equal to 1, then such flows are known as sonic flows. If Mach number is greater than 1, the flow is known as a supersonic flow. If the Mach number is less than 1, it is called a subsonic flow. If Mach number is greater than 5, we refer to such flows as hyper sonic flows. Mach number approximately equal to 1 or around 1, then, we call such flows as transonic flows.

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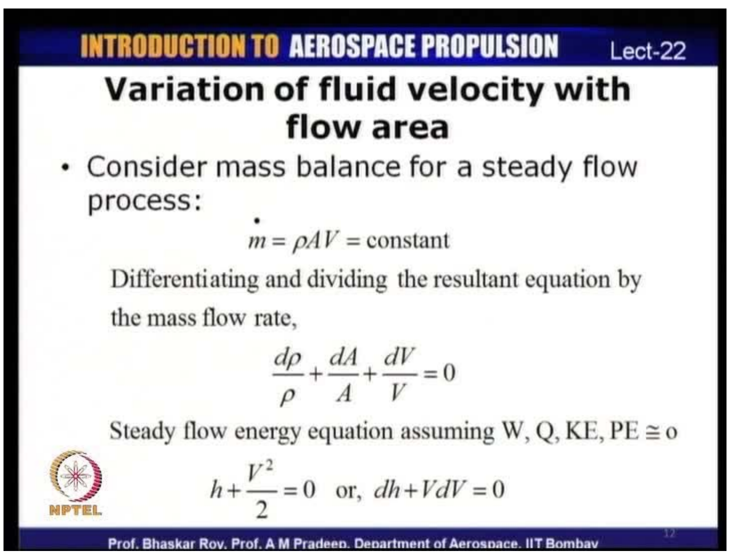

These are different terms we use for flows depending upon their Mach numbers. Mach number less than 1 is subsonic; greater than 1 is supersonic and greater than 5 is usually referred to as hypersonic and so on. So, what we will do now is to look at the variation of fluid velocity with the area and will derive an expression which relates the area ratios and fluid velocity with the Mach number, which means that for different Mach numbers, we can see how area and velocities are related.

For deriving an expression, what we will do is consider a mass balance for a steady flow process. We know that mass flow rate for such processes are equal to the product of density, area and velocity. So, rho AV which will be a constant for a steady flow process. If you differentiate this equation and divide this by the resultant mass flow rate, we can rewrite the above equation as - d rho by rho plus dA by A plus dV by V is equal to 0.

Now, from our steady flow energy equation which we had derived in earlier lectures, if you assume work done, heat transfer, kinetic and potential energy to be more or less 0, then, from the steady flow energy equation, we get - h plus V square by 2 is equal to 0 or dh plus VdV is equal to 0. That is, if you differentiate this equation equation, we get dh plus VdV is equal to 0.

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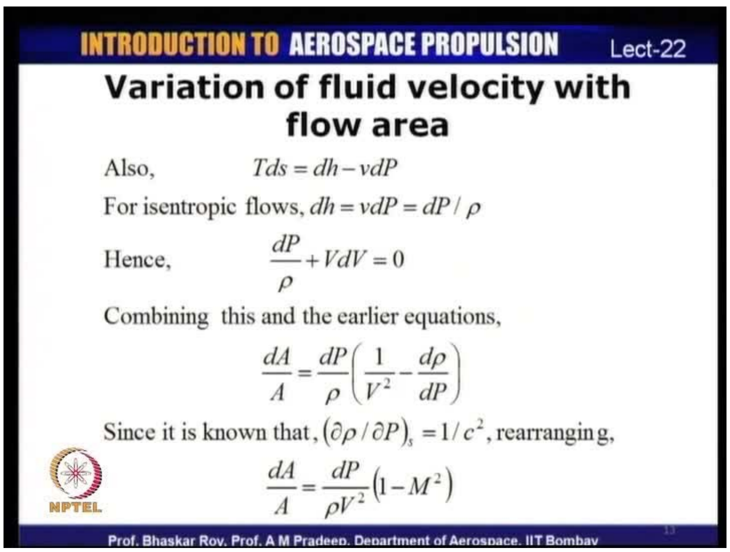

This is coming from the steady flow energy equation, where we assume work done, heat transfer and potential energy to be 0. From the Tds equation, that was the second Gibb’s equation - Tds equal to dh minus vdP. Now, for isentropic flows, Tds will be equal to 0. Therefore, dh is equal to vdP - which is dP by rho, because specific volume is the inverse of density and therefore, dh is equal dP by rho.

Therefore, our previous equation which was - dh plus VdV is equal to 0 becomes dP by rho plus VdV is equal to 0. If we combine this equation with the mass balance equation, we get - dA by A is equal to dP by rho multiplied by 1 by V square minus d rho by dp. It is also known that the ratio, d rho by dP for constant entropy is equal to 1 by c square. If you were to apply this principle, we get dA by a is equal dP by rho V square into 1 minus M square.

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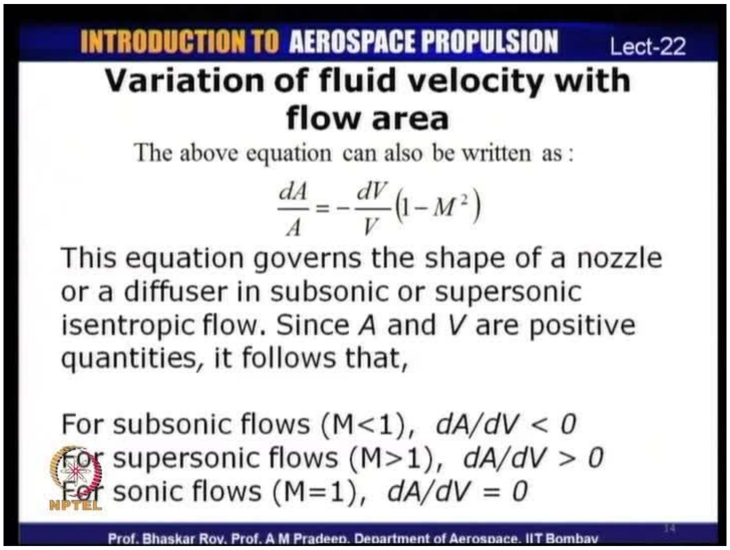

So, M square is coming because we will get a ratio of V square by c square which is equal to Mach number square. From the mass balance equation, we get - dA by A plus dP by rho V square into 1 minus M square. This again, we can rearrange based on our earlier equation. If you rearrange that equation, from dP by rho is equal to minus VdV, we get dA by A is equal to minus dV by V into 1 minus M square. This equation has a lot of significance in the sense that this equation governs the shape of a nozzle or a diffuser in subsonic or supersonic isentropic flows.

In this equation, area and velocity are positive quantities. If that is so, depending upon the Mach number whether it is greater than 1 or less than 1, area velocity changes can be inferred. That is, for subsonic flows where Mach number is less than 1, we have dA by dV less than 0. For supersonic flows, where Mach number is greater than 1, the rate of change of area with velocity is greater than 0 and for sonic flows where Mach number is equal to 1, then we have dA by dV is equal to 0. We will try to understand - what is the implication of the rate of change of area with velocity depending upon the Mach number. As Mach number changes, there are changes in velocities with reference to changes in areas.

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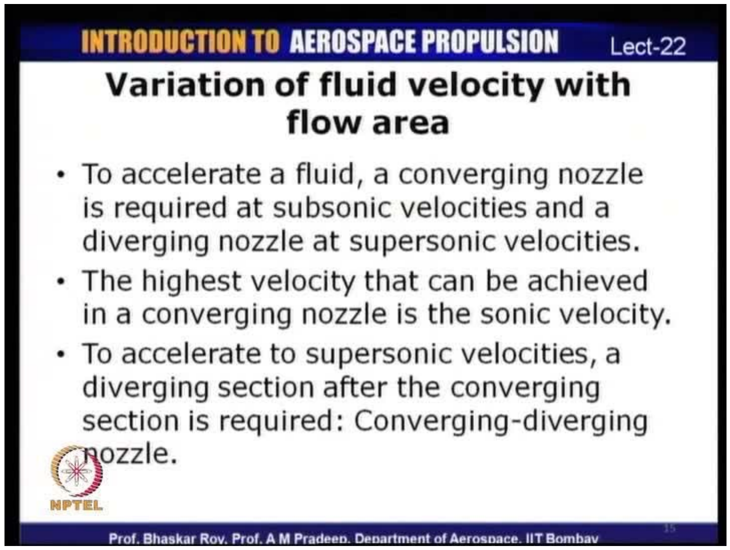

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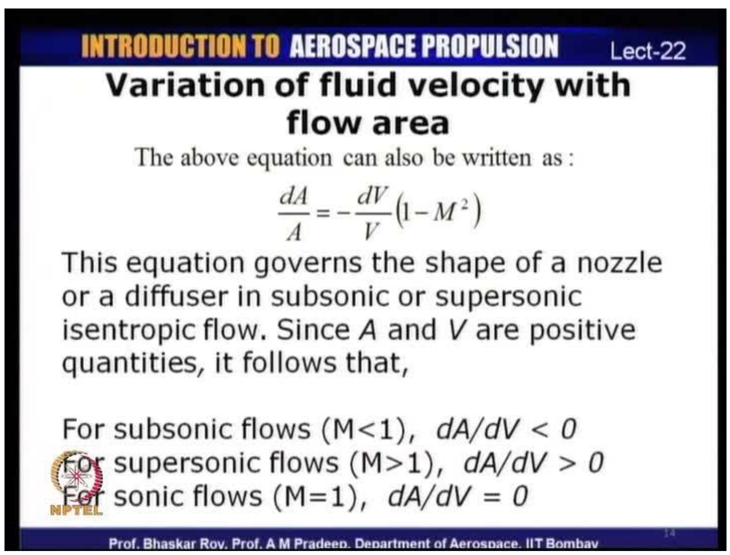

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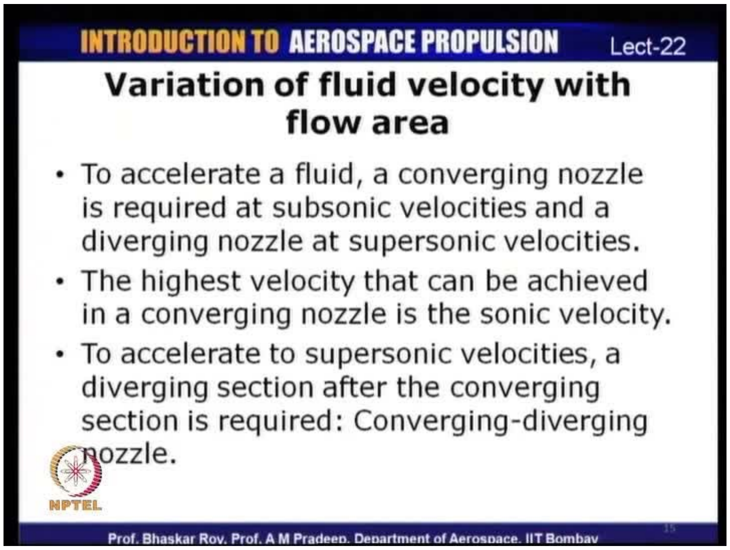

Let us look at what happens as you change the Mach number and what happens to velocity as you change areas. From these equations, it follows that to accelerate a fluid in subsonic flows, you need a converging area - there has to be a decrease in area; because for Mach numbers less than 1, dA by dV is less than 0 - which means that if you have to accelerate a flow, we have to have a corresponding decrease in area in subsonic flows; whereas, in supersonic flows, you will need an increase in area to accelerate fluid. A diverging nozzle is required at supersonic velocities. What we will also see a little later is that the highest velocity that you can achieve in a converging nozzle is the sonic velocity. The maximum velocity that you would be able to achieve in a converging passage in subsonic flows will be that you would get only a sonic velocity at the end of the converging nozzle.

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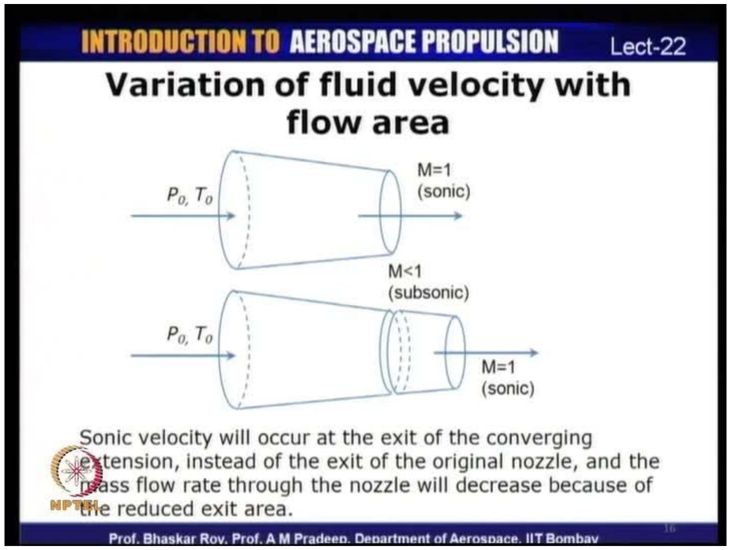

To accelerate a fluid to supersonic velocities, you will need a diverging section or diverging area or increase in area after the flow reaches a sonic velocity at the minimum area, which is known as the throat. After the throughout, there needs to be a diverging section to accelerate a fluid to supersonic velocities. What I was trying to say is that in subsonic flows, if you look at Mach numbers less than 1, as you start accelerating the fluid, the maximum velocity or Mach number that you can get at the end of the acceleration is Mach number equal to 1. This is known as sonic velocity.

So, a flow which has certain total pressure and temperature at the inlet of the nozzle -a nozzle section is shown here - will accelerate. The maximum that it can accelerate is Mach number is equal to 1. What if you reduce the area below this - that is - at the end of this nozzle? Let us attach another nozzle hoping that will accelerate it to supersonic speeds. That is not going to happen; what will happen is that - the section where you get sonic velocity will shift and the section which had sonic velocity earlier will now have a subsonic Mach number.

So, the sonic velocity will occur only at the exit of this converging section instead of the exit of the original nozzle. The mass flow rate will now reduce because you are trying to force a certain amount of mass flow through a lower passage area. So, you would get a decrease in mass flow.

We will see later that as Mach number reaches 1, the mass flow is at its maximum and that is known as the choking of a nozzle. That is, if you were to force fluid through a nozzle till a point that at the exit of the nozzle, Mach number is equal to 1. Then, the mass flow rate that can be passed through such a nozzle has reached its maximum level - that is known as choking of the flow. This means that if you add another convergent section to that, you are not going to increase the Mach number any more; it will in fact lead in to reduction of the mass flow rate because choking area is different now. So, it is not possible for us to achieve supersonic Mach numbers in a converging section.

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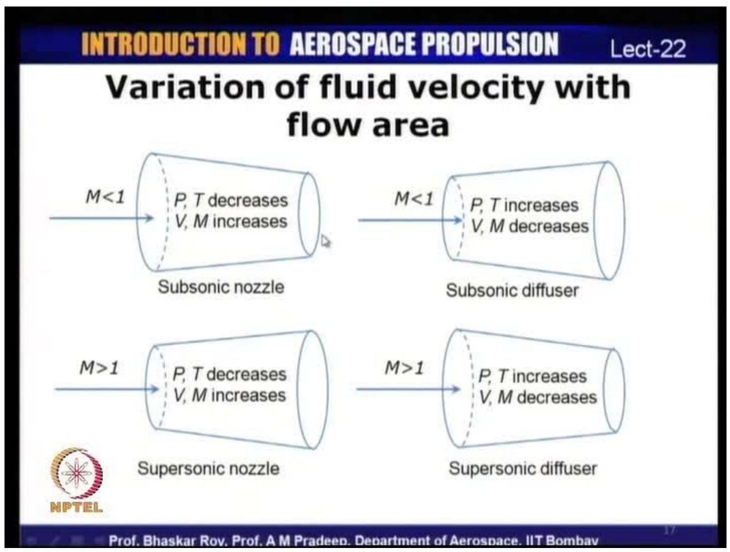

In order to get a supersonic Mach number, we need to have a divergent section at the exit of the convergent section. Such nozzles are known as converging diverging nozzles. We will do some analysis of the variation in pressure across a converging diverging nozzle with change in back pressure. We will do that little later. Now, in summary - from the area, velocity Mach number relation what we have are the following: if you look at the first case here, we have a nozzle which has an inlet Mach number which is subsonic. Along the length of the nozzle, the static pressure and temperature will decrease - the velocity at Mach number increases. This is basically a subsonic nozzle. So, area reduces Mach number increases in subsonic flow. Reverse of that happens -if the area is increasing, Mach number is less than 1 at the inlet; the static pressure and temperature increases along the length of the nozzle, velocity and Mach number decreases. This is basically a subsonic diffuser.

If you look at the supersonic version of this, it is the exact opposite. If Mach number at the inlet is greater than 1, if you have to have an increase in Mach number along the length of the nozzle, area has to increase. So, pressure and temperature decreases; velocity and Mach number increases and this is known as a supersonic nozzle. If you look at a supersonic diffuser, it has decreasing area, so, pressure and temperature will increase and Mach number decreases. So, a supersonic diffuser at least theoretically is a subsonic nozzle. Supersonic diffuser will acts as a nozzle in subsonic flow and a subsonic diffuser will act as a supersonic nozzle at Mach numbers greater than 1 -which is what should happen theoretically.

What we have discussed now is an outcome of the area, velocity, Mach number relation which we had derived. So, we know how the area has to change, given a certain Mach number, so that you get an increase or decrease in velocity accordingly. Now, what we will do is relate the stagnation properties. -that is stagnation pressure, temperature and density to the corresponding static parameters like static temperature, pressure and density through the Mach number.

We have already seen the isentropic relations which relate ratio of temperatures and pressures through the ratio of specific heats - that is T naught by T is equal to P naught buy P raise to gamma minus 1 by gamma. Now, we will relate the stagnation parameters with their corresponding static parameters through the Mach numbers.

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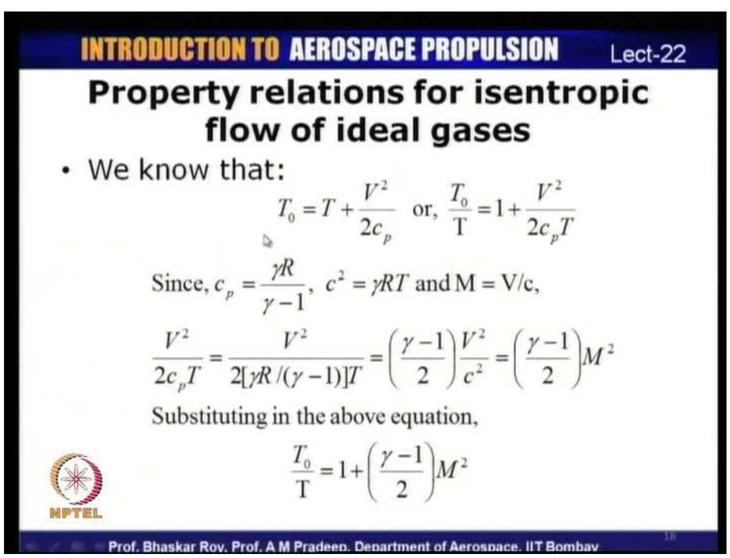

To do that, we have already seen this equation, which was relating stagnation temperature to static temperature and velocity ratios. So, T naught is equal to T plus V square by 2c p. Therefore, T naught by T is equal to 1 V square by 2c p T. Since, we know that c p is gamma R by gamma minus 1 and c square is gamma RT and also Mach number is V by c; if we make these substitutions in the dynamic temperature term, we get V square by 2c p T is equal to V square by 2 into gamma R by gamma minus 1 into T - which basically is gamma minus 1 by 2 into V square by c square which is - gamma minus 1 by 2 M square. So, if you substitute this in the first equation, we get T naught by T is 1 plus gamma minus 1 by 2 M square.

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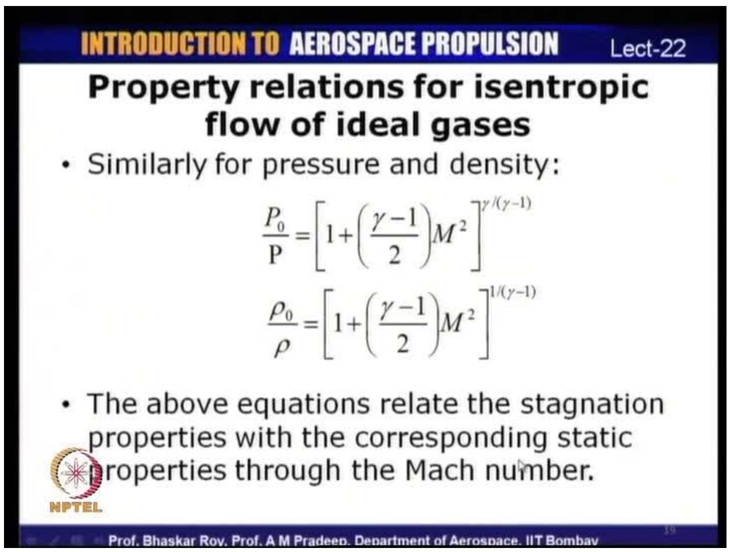

This relates stagnation temperature to static temperature through the Mach number. This can also be extended to the corresponding pressure and density. From the isentropic relations, we get P naught by P is 1 plus gamma minus 2 M square raise to gamma P gamma minus 1 and stagnation density ratio rho naught by rho is equal to 1 plus gamma minus 1 by 2 M square raise to 1 by gamma minus 1. These are property relations which relate the stagnation parameters to the static parameters through Mach number.

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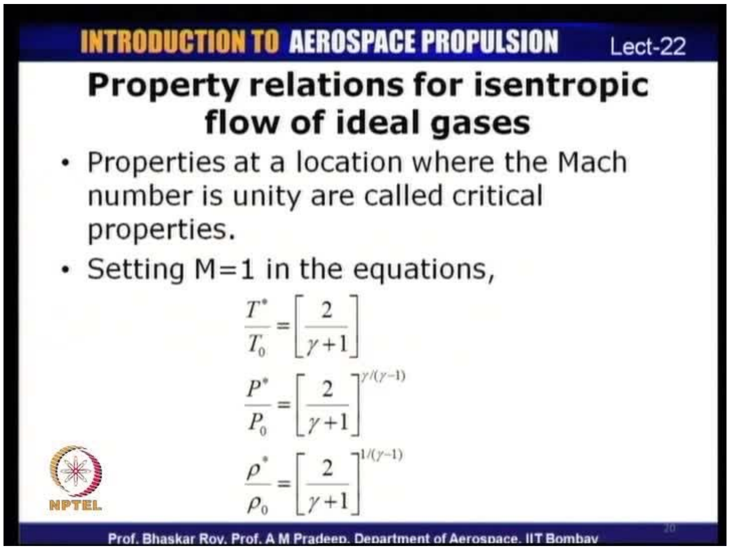

Now, Mach number is equal to 1, we have seen at the end of the nozzle, Mach number is equal to 1. If we equate M equal to 1 in those equations, then, the properties that we get are known as critical properties and they are denoted by a superscript star. If we equate M equal to 1 in them, we get T star by T naught is equal to 2 by gamma plus 1. Similarly, P star by P naught is equal to 2 by gamma plus 1 raise to gamma by gamma minus 1- rho star by rho naught is equal to 2 by gamma plus 1 raise to 1 by gamma minus 1. These are the equations which relate the critical properties to the corresponding stagnation properties and you can see that it depends only on the ratio of specific heats.

This was about property relations for ideal gases; inherently assuming that there is an isentropic flow. Now, what we will do next is to analyze isentropic flow through nozzles. We will take two different types of nozzles: a converging nozzle and a converging diverging nozzle. A converging nozzle in a subsonic flow will have a decreasing area, as we have seen. So, let us look at what happens as you keep decreasing the area. If you keep the area fixed, as you decrease the exit pressure - that is known as the back pressure, how does it affect the flow parameters?

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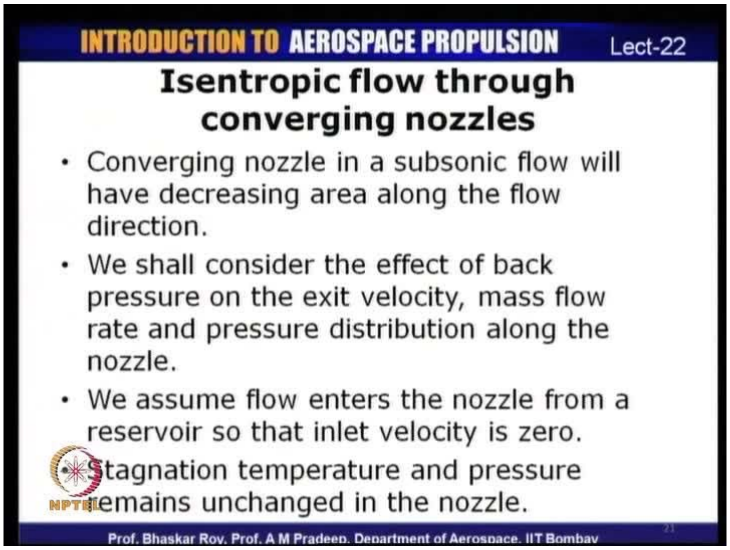

A converging isentropic flow through a converging nozzle will involve a decreasing area along the flow direction. What we shall do is to consider the effect of back pressure on the mass flow rate and pressure distribution along the nozzle. We will assume that the flow enters the nozzle from a reservoir where the velocities can be assumed to be 0. So, the stagnation temperature and pressure will remain unchanged through the nozzle. We will not assume any losses taking place in the nozzle; so that pressure is constant. Since there is no heat or work interactions stagnation, temperature also remains a constant.

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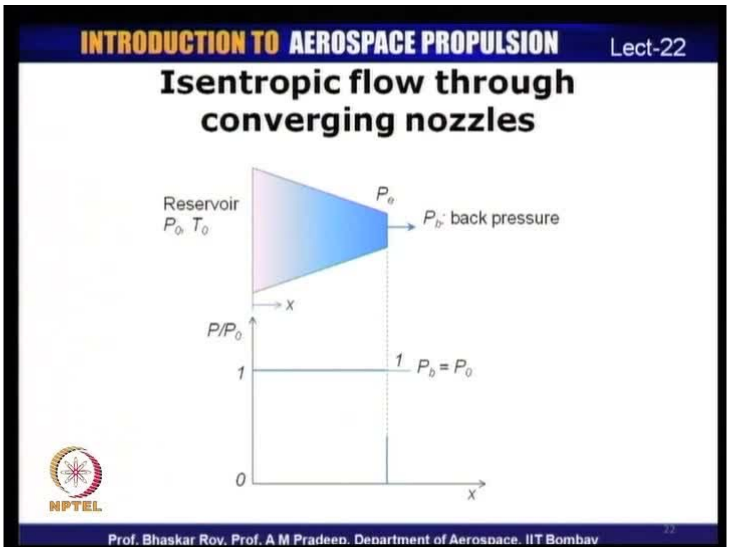

So if that is what the case, this is the nozzle that we are talking about, which has flow entering through a reservoir which is at pressure P naught, temperature T naught; exit pressure is P e and back pressure is the term which we can change. Depending on the back pressure, the pressure across the nozzle also will change. In the first scenario, we have back pressure equal to the reservoir pressure. There is no flow taking place. So, the variation of pressure ratio is a constant. The pressure would vary in the format shown here.

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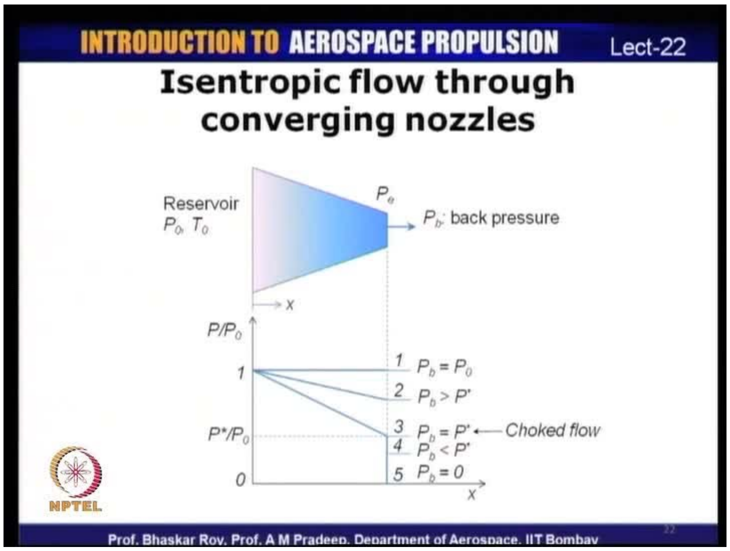

Now, as we reduce the back pressure, but back pressure is still greater than the critical pressure. Now, there is a decrease in the pressure ratio, P by P naught. This is how the variation would be. So, from state 1 to state 2, we have a case where back pressure is still greater than the critical pressure. If you reduce it further at state 3, where back pressure is equal to the critical pressure, the pressure ratios continue to reduce and reaches state 3, let us say. If you were to decrease back pressure below this critical value, what basically happens is that there is no change happening; it will continue to have the pressure ratio which will continue to drop till the extent that if you reduce the back pressure even further, the Mach number at the exit of the nozzle does not change. It will remain the same. So, the point - when we have the ratio or back pressure is equal to the critical pressure is known as the choked condition or the chocked flow. The pressure ratio at that point is P star by P naught. We have seen this depends only on ratio of the specific heats.

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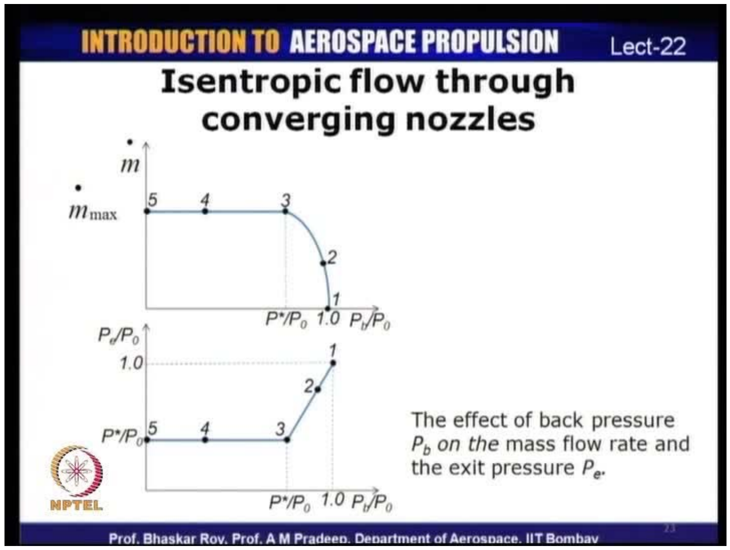

So, for any other pressures, if you continue to drop the back pressure lower than that, we will continuously see a decrease in the pressure ratios till the point, if you continue to reduce back pressure to 0, the Mach number at the exit is still going to be Mach number is equal to 1, because it is a choked flow. There is no more change happening there.

If you look at Mach number and pressure ratio plots, we started off at Mach number state 1. If you look at the mass flow rate, it was equal to 0, because the pressure ratio, the back pressure was equal to stagnation pressure at the reservoir, there was no mass flow. Mass flow was equal to 0. As you start reducing the back pressure, there is an increase in Mach number till a point when it reaches state 3, which was the critical ratio that is, P star, when Mach number reaches maximum mass flow - that is its maximum value. If you reduce back pressure even below that, Mach number does not change. We get the same mass flow rate and which is the Mach number is equal to 1 at the throat; mass flow remains the same throughout for state 4 and 5 as well from state 2 3 4 and 5, the mass flow rate is the same. If you look at the exit to stagnation pressure ratio after state 3, it remains the same, because of the chocked condition and it does not change after the flow reaches the critical state.

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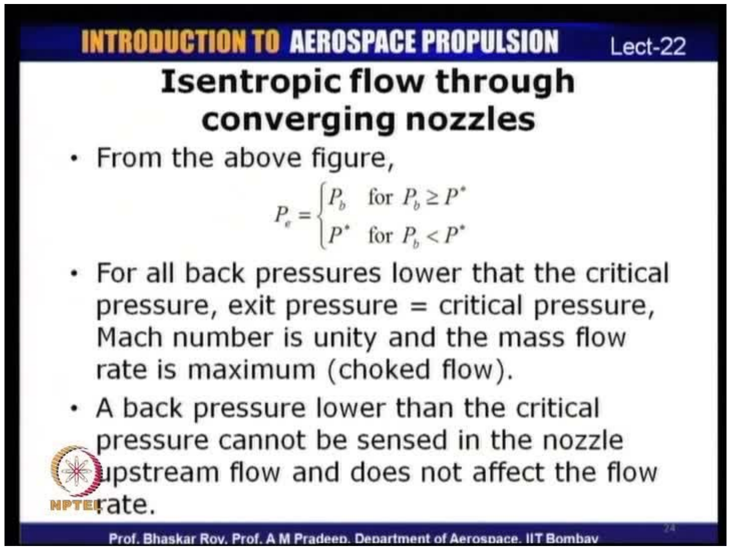

This is how the variation of a flow would be through converging, diverging, nozzle. From the above, what we basically see is that when exit pressure is equal to back pressure. Exit pressure P e will be equal to back pressure, for back pressure greater than or equal to the critical pressure. Exit pressure will be equal to critical pressure for P b less than P star.

So, for all back pressures lower than the critical pressure, the exit pressure will be equal to critical pressure. The Mach number is unity and mass flow rate is maximum, which is basically the chocked flow. Back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and it does not affect the mass flow rate.

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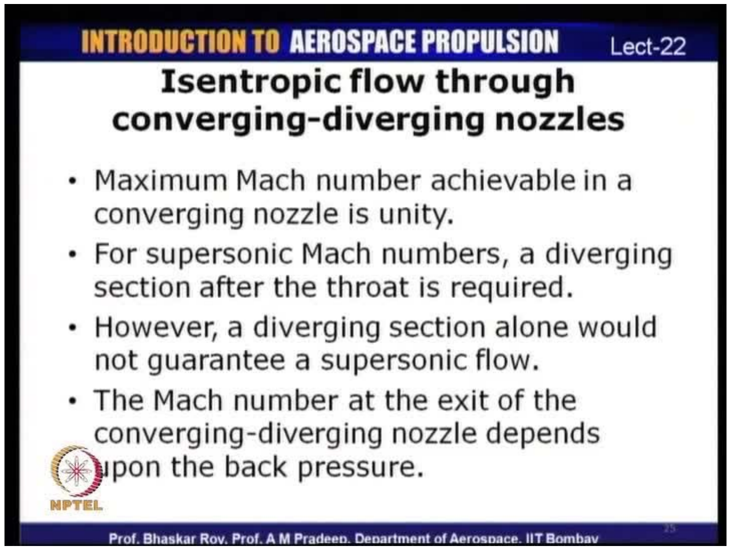

So, back pressure is basically relating the back pressure and exit pressure to the critical pressure as you change the back pressure values. Let us now look at the same scenario for a converging diverging nozzle. We have seen that the maximum Mach number that you can achieve in a converging nozzle is unity; for achieving supersonic Mach number you need a diverging section after the throat.

So, a diverging section alone will not guarantee a supersonic flow. It will happen only as you change the back pressure accordingly. So, for back pressures which are different from what it should be you may not really achieve a supersonic flow.

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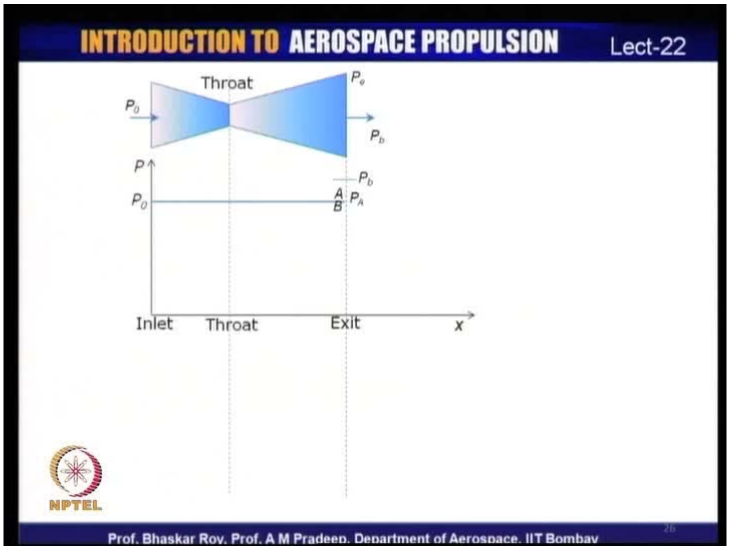

Let us analyze a supersonic flow as we change the back pressure. This is a converging diverging nozzle. There is a converging section, a throat where the area minimum and the diverging section.

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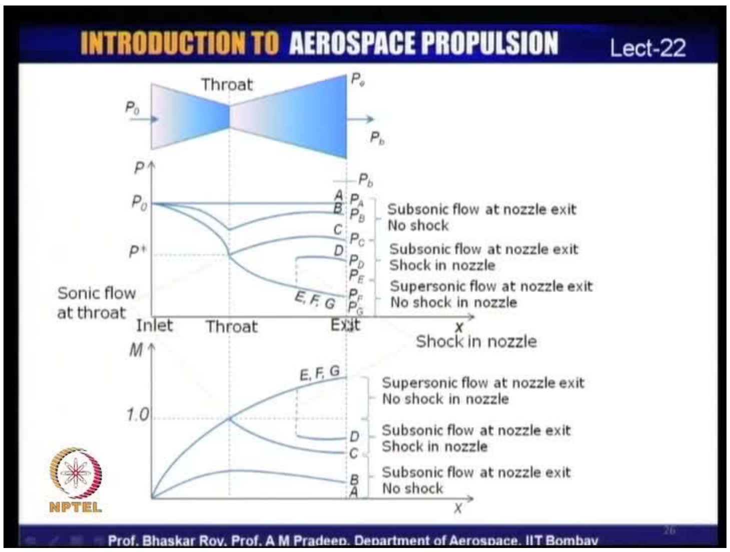

P naught is the pressure in the reservoir and P e is the exit pressure P b is the back pressure. Let us say, P b is currently higher than the stagnation pressure P naught. As you reduced it to P A, there is a flow which takes place. As you increase the P b, what basically happens or as you reduce back pressure, how it affects the flow? As you reduce back pressure which is still greater than the critical pressure, then, we get initially an increase, the static pressure reduces, and velocity will increase and then in the diverging section static pressure will increase.

As you reach the critical pressure which is P star which happens when P is equal to P C or back pressure is equal to P C, the flow reaches its minimum static pressure here which is equal to P star. You get Mach number is equal to 1. But it does not achieve a supersonic flow subsequently, it becomes subsonic and the static pressure continues to rise.

So, if you reduce the back pressure even further what happens is - after throat, the static pressure continues to drop up a point where there is a sudden increase in static pressure which is basically due to the occurrence of a shock. So, in the diverging section of the nozzle there is a shock; after which the flow become subsonic. We will discuss more about shocks in the next lecture. At the end exit of the shock, the flow becomes subsonic and static pressure again rises like in a subsonic flow. You have still not got a supersonic flow at the exit of the nozzle; it is still subsonic flow, because of the presence of a shock.

If you reduce the flow back pressure further, we get a supersonic flow, because the shock which was there in the divergence section will continuously move outward as you reduce back pressure. Eventually the shock will be pushed out of the diffuser and the flow will become supersonic at the exit of the nozzle.

If you look at the Mach number plot, for state A - there was no Mach number because there was no flow; for state B - the Mach number increases and then it decreases in the divergence section; for state C - it reaches Mach number equal to 1 because that is the critical condition and then it does not become supersonic. It again decelerates; then becomes subsonic.

So, we have subsonic flow all the way here. At state D, the flow is supersonic after the throat, it continues so till this point after which there is a shock and because of the presence of a shock, the flow becomes subsonic and we still have a subsonic flow at the nozzle exit. If the back pressure is lower than what is happening at state D, then the shock continuously moves towards the exit and for all other states, which is E and F and G, we will have continuously have a supersonic flow all the way to the exit of the nozzle.

We have now achieved a supersonic flow all the way up to the exit of the nozzle. This was possible only because the back pressure was adjusted to values which were lower than what we have seen here or lower than pressure at D that is P D and P E. For pressures lower than that, we get a supersonic flow cautiously all the way up to the exit of the nozzle. In the Mach number plot, we can see that it goes up to Mach 1 at the throat after which if the back pressure is not low enough, it can again become subsonic or if it is low up to a point, you may get a normal shock in the divergent section which means that you get a supersonic flow up to the shock and downstream of the shock it again becomes subsonic. If the back pressures are lower enough, then, we get supersonic flow all the way up to the exit of the nozzle.

So, this is the variation of a supersonic flow or this is how you would achieve a supersonic flow by changing the exit back pressure. It is not just enough that you put a divergent section at the end from the throat and still get a supersonic flow; that will happen only if the exit back pressure is low enough.

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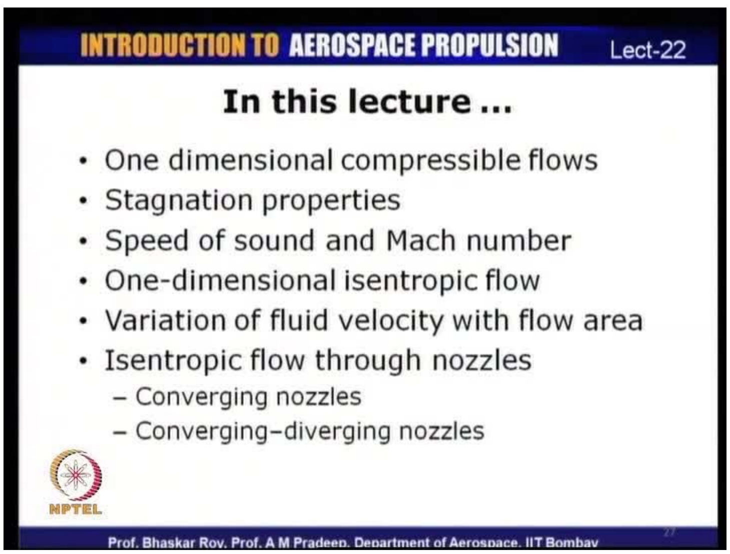

Let me windup today's lecture where we have discussed at least had a very quick introduction to some of these topics which are related to compressible flows. We had some discussions on one dimensional compressible flows, on stagnation properties, on the speed of sound and the Mach number. We have seen variation; we have derived an equation which relates the fluid velocity and Mach number with the flow area. We have also seen isentropic flow through nozzles. Two types of nozzles: converging nozzles and converging diverging nozzles.

Our interaction for this was a very brief one. There are separate courses which are offered on compressible flows which are known as gas dynamics and so on. This is just to give you an idea of what is the importance or significance of compressible flows and why you need to understand stagnation properties and take due care in calculating properties which involve significant kinetic energies.

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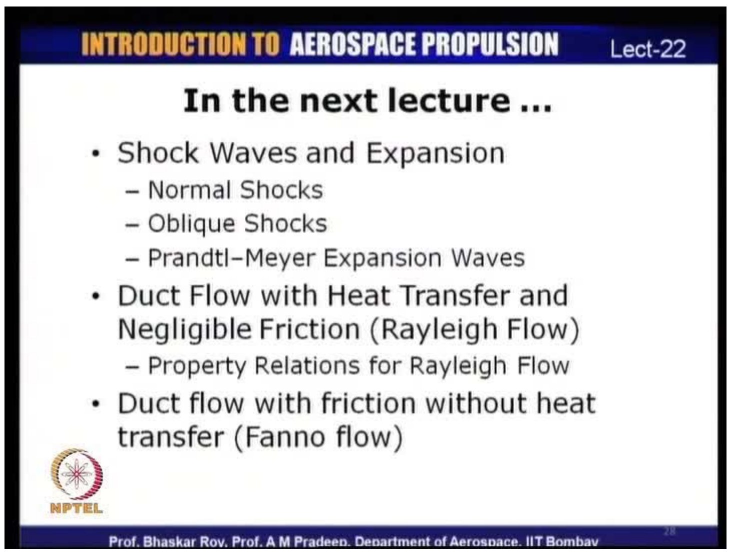

In the next lecture, we will continue our discussion on compressible flows. We will discuss little bit more on shock waves and expansion. We will discuss about normal shocks oblique shocks and Prandtl-Meyer expansion waves. We will be discussing about two duct flow cases; one is duct flow with heat transfer and negligible friction which is basically known as the Rayleigh flow. We will also discuss duct flow with friction but without heat transfer known as the Fanno flow. These are some of the topics which are in continuation with our discussion on compressible flow. We will take up a discussion during our next lecture.